How much color do we really need?
Two-loop subleading-color effects in photon and jet physics

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60th Moriond
27th March 2026

How much color do we really need?
Two-loop subleading-color effects in photon and jet physics,
Czakon, Poncelet [2512.17591]

follow up to the full set of NNLO QCD for massless 2 → 3 process @ LHC:
Chawdhry, Czakon, Mitov, Poncelet [1911.00479][2103.04319]
Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia [2304.06682]
Czakon, Mitov, Poncelet [2106.05331]
+ Alvarez, Cantero, Llorente [2301.01086]

THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

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NARODOWE CENTRUM NAUKI

Leading color picture of ski slope

[La Thuile 2nd April 2025]

Full color picture of ski slope

[La Thuile 4th April 2025]

Precision predictions to keep up with data accuracy

Fixed-order perturbation theory

→ **Core element of event simulation**
→ **Describes high Q regime**

Resummation

Parton-showers

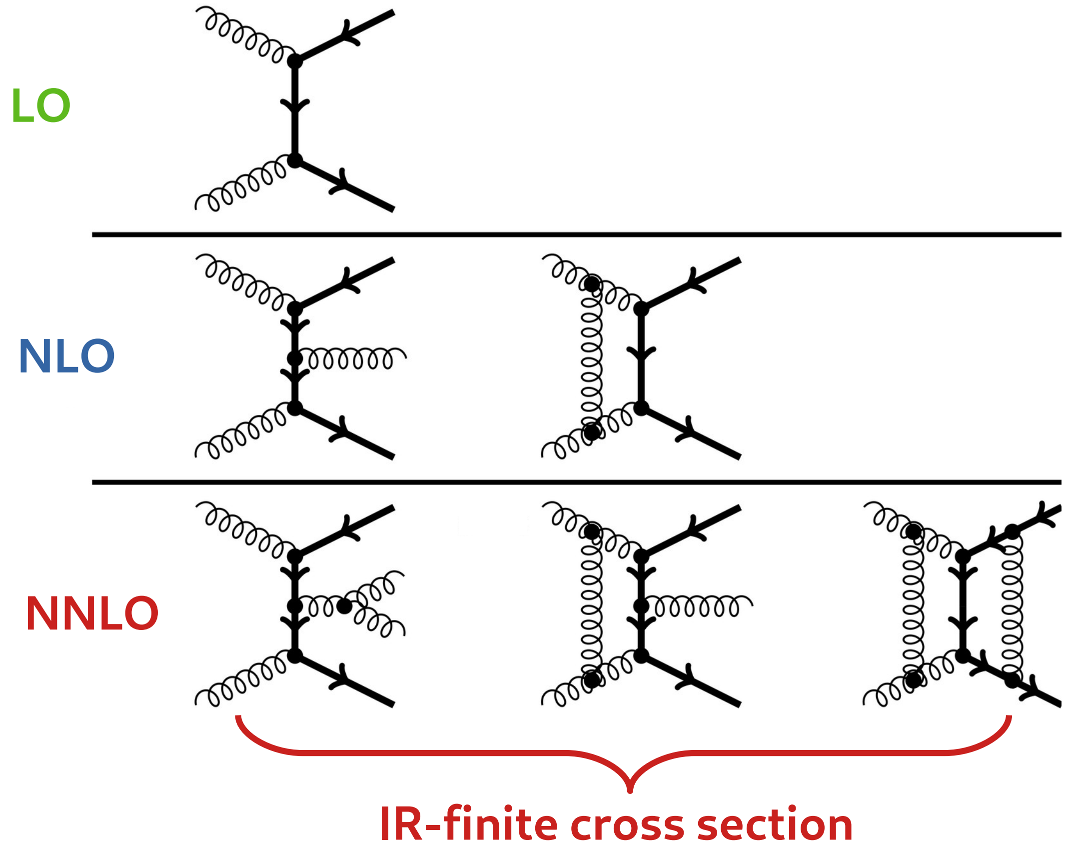
Precision theory predictions

Parametric input:
PDFs, couplings (α_s), ...

Soft physics:
MPI, color reconnection, ...

Fragmentation/hadronisation

NNLO QCD cross sections: real radiation



Infrared finite differential cross sections at NNLO QCD:
→ highly non-trivial IR structure
→ plethora of subtraction schemes

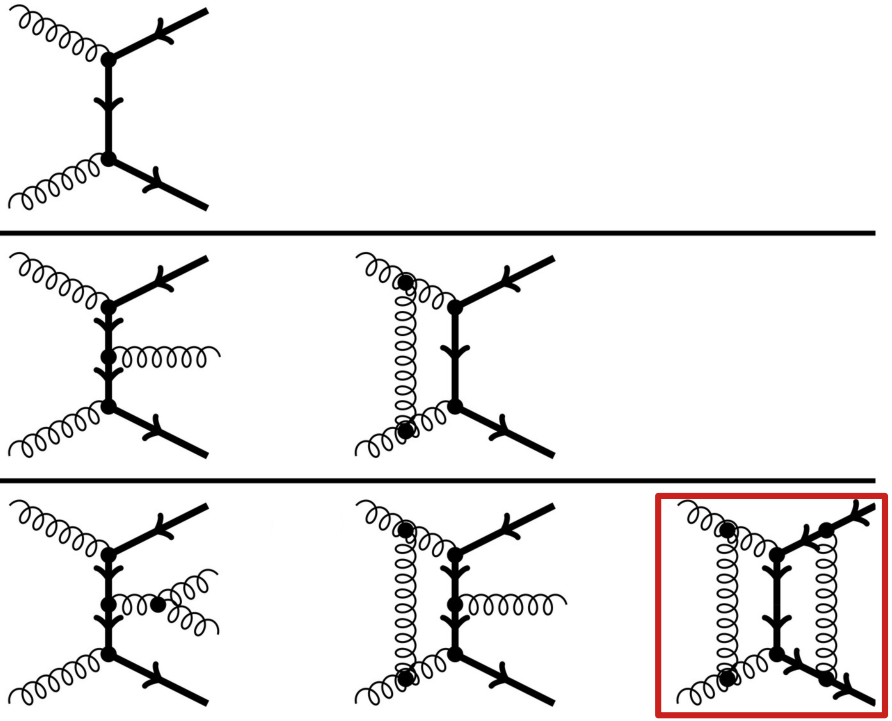
qT-slicing [Catani'07]
N-jettiness slicing [Gaunt'15/Boughezal'15]
Antenna [Gehrmann'05-'08]
Colorful [DelDuca'05-'15]
Projetction [Cacciari'15]
Geometric [Herzog'18]
Unsubtraction [Aguilera-Verdugo'19]
Nested collinear [Caola'17]
Local Analytic [Magnea'18]
Sector-improved residue subtraction [Czakon'10-'14,'19]

NNLO QCD challenges: two-loop amplitudes

LO

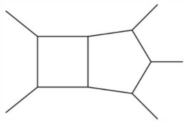
NLO

NNLO

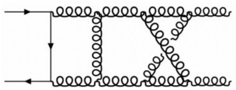


How to compute
multi-scale two-loop QCD amplitudes?
→ **fast growing complexity:**
rational coef. and **special functions**
→ deeper understanding of the analytical properties
→ refinement of computational tools

Two-loop 5-point
[Abreu, Agarwal, Badger, Buccioni, Chawdhry, Chicherin,Czakon, de Laurenties, Febres-Cordero, Gambuti, Gehrmann, Henn, Lo Presti, Manteuffel, Ma, Mitov, Page, Peraro, Poncelet, Schabiner Sotnikov, Tancredi, Zhang,...]

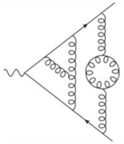


Three-loop 4-point



[Bargiela, Dobadilla, Canko, Caola, Jakubcik, Gambuti, Gehrmann, Henn, Lim, Mella, Mistleberger, Wasser, Manteuffel, Syrrakow, Smirnov, Trancredi, ...]

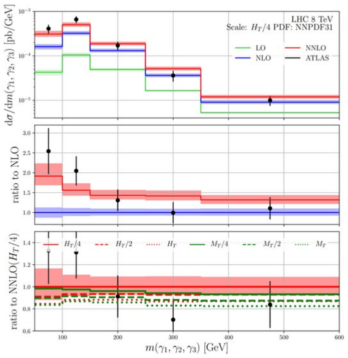
Four-loop 3-point



[Henn, Lee, Manteuffel, Schabinger, Smirnov, Steinhauser,...]

NNLO QCD cross sections for massless 2 → 3 processes

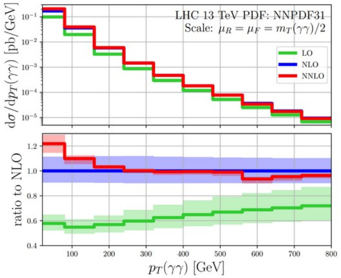
$pp \to \gamma\gamma\gamma$



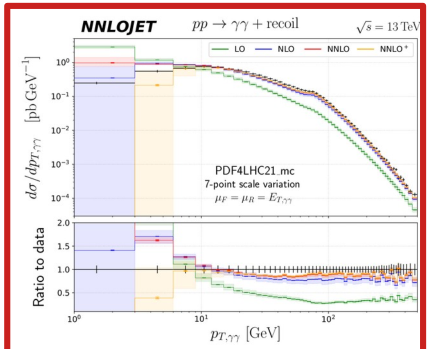
Chawdhry, Czakon, Mitov, Poncelet [1911.00479]

Kallweit, Sotnikov, Wiesemann [2010.04681]

$pp \to \gamma\gamma j$

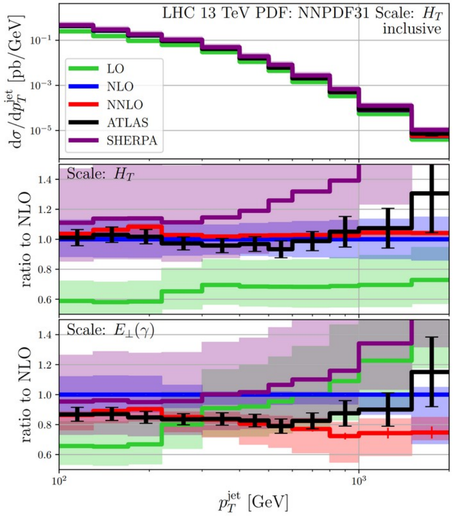


Chawdhry, Czakon, Mitov, Poncelet [2103.04319]



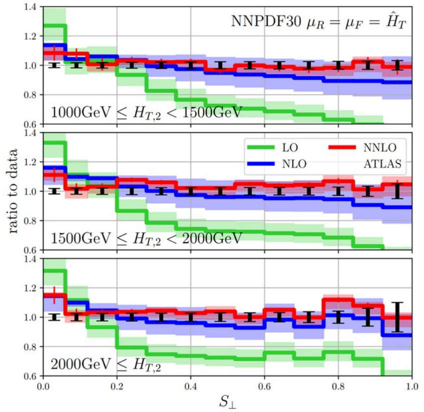
Buccioni, Chen, Feng, Gehrmann, Huss, Marcoli [2501.14021]

$pp \to \gamma jj$



Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia [2304.06682]

$pp \to jjj$



Czakon, Mitov, Poncelet [2106.05331]
+ Alvarez, Cantero, Llorente [2301.01086]

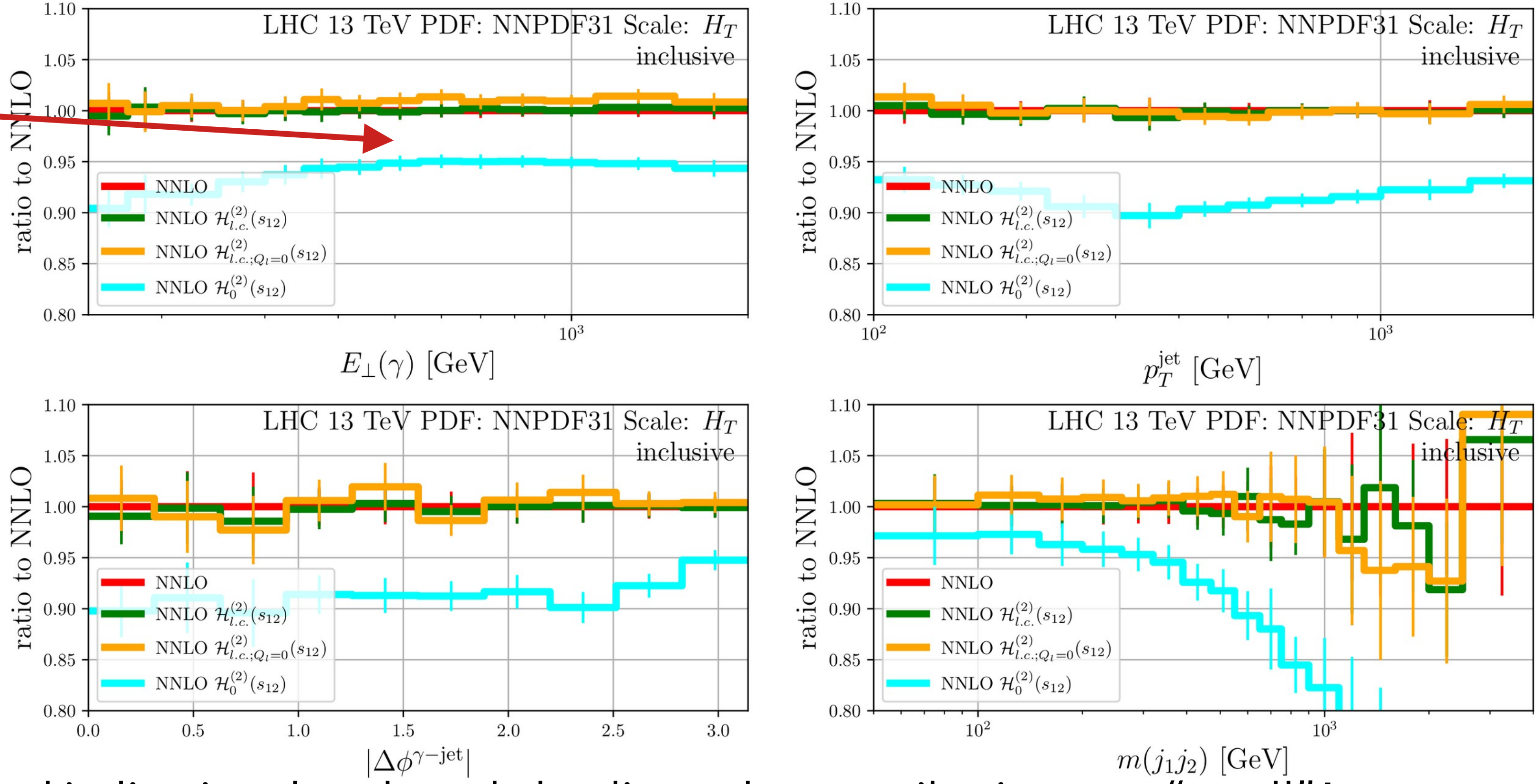
Before [2512.17591] the only cross sections with full color two-loop MEs

Lessons from photon plus dijet

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia [2304.06682]

Two-loop contribution
~ 5-10%
wrt. full NNLO
(scheme dep.)

“Leading colour”
Approximation
”Error” = O(~1%)
wrt full NNLO

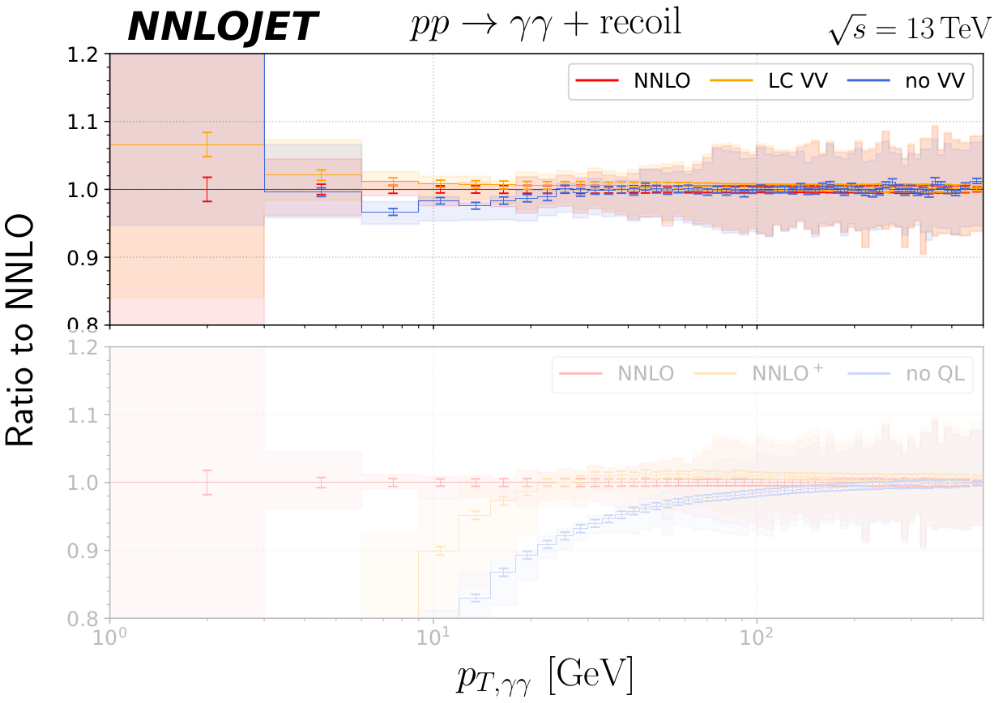


Good indication that the sub-leading color contributions are “small”*

***compared to NNLO QCD cross section**

Subleading color effects in di-photon + jet

Buccioni, Chen, Feng, Gehrmann, Huss, Marcoli [2501.14021]



Observation
two-loop / sub-leading color contribution can be important in some regions of phase space

→ full color crucial for applications like qT subtraction

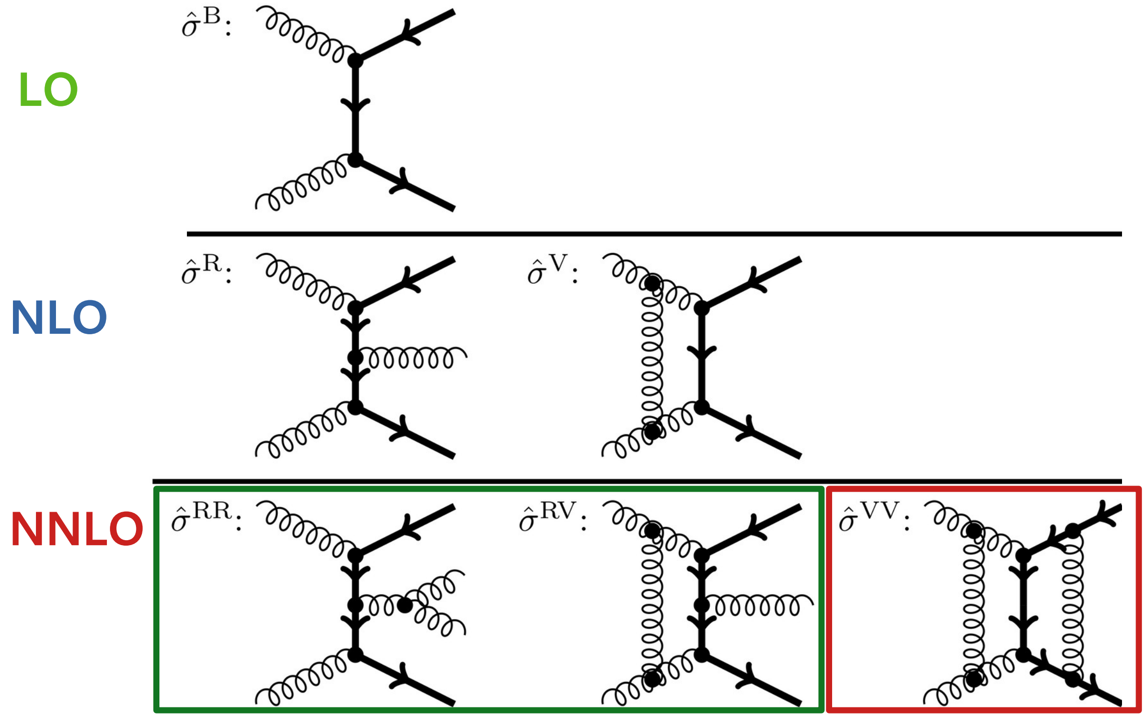
Sub-leading colour in 2 → 3 processes

Since early 2023 all massless five point amplitudes are known with full color/charge dependence (for photon amplitudes “leading color” ~ “planar”)

	Leading colour amplitudes	Full colour amplitudes
$pp \to \gamma\gamma\gamma$	Chawdhry, Czakon, Mitov Poncelet [1911.00479] [2012.13553] Abreu, Page, Pascual, Sotnikov [2010.15834]	Abreu, De Laurentis,Ita, Klinkert, Page, Sotnikov [2305.17056]
$pp \to \gamma\gamma j$	Agarwal, Buccioni, von Manteuffel,Tancredi [2102.01820] Chawdhry, Czakon, Mitov Poncelet [2103.04319]	Agarwal, Buccioni, von Manteuffel,Tancredi [2105.04585]
$pp \to \gamma jj$		Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia [2304.06682]
$pp \to jjj$	Abreu, Dormans, Cordero, Ita,Page, Sotnikov [1904.00945] [2102.13609]	Agarwal, Buccioni, Devoto, Gambuti, von Manteuffel, Tancredi [2311.09870] De Laurentis, Ita, Klinkert, Sotnikov [2311.10086] [2311.18752]

all based in pentagon functions: Chicherin, Sotnikov [2009.07803]
(except [1904.00945, 1911.00479] which use: Gehrmann, Henn, Presti [1807.09812])

Two-loop approximations in NNLO QCD cross sections



Leading color approximation for two-loop finite remainder

Complete colour dependence

Two-loop approximations in NNLO QCD cross sections

In the sector-improved residue subtraction scheme: $\hat{\sigma}^{\mathrm{VV}} = \hat{\sigma}^{\mathrm{VV}}_{\mathrm{F}} + \hat{\sigma}^{\mathrm{VV}}_{\mathrm{FR}} + \hat{\sigma}^{\mathrm{VV}}_{\mathrm{DU}}$

$$\hat{\sigma}^{\mathrm{VV}}_{\mathrm{FR}} = \frac{1}{2\hat{s}}\frac{1}{N}\int \mathrm{d}\boldsymbol{\Phi}_n\, 2\,\mathrm{Re}\langle \mathcal{M}^{(0)}_n | (\mathbf{Z}^{(1)\,\dagger} + \mathbf{Z}^{(1)}) | \mathcal{F}^{(1)}_n \rangle\, \mathrm{F}_n,$$

$$\hat{\sigma}^{\mathrm{VV}}_{\mathrm{DU}} = \frac{1}{2\hat{s}}\frac{1}{N}\int \mathrm{d}\boldsymbol{\Phi}_n \left[2\,\mathrm{Re}\langle \mathcal{M}^{(0)}_n | \mathbf{Z}^{(2)} | \mathcal{M}^{(0)}_n \rangle + \langle \mathcal{M}^{(0)}_n | \mathbf{Z}^{(1)\,\dagger}\mathbf{Z}^{(1)} | \mathcal{M}^{(0)}_n \rangle \right] \mathrm{F}_n.$$

- **only** ε-poles
- depend on lower loop ME

→ taken at **full colour** to cancel full color real poles correctly

Two-loop finite remainder (this contains the actual two-loop information):

$$\hat{\sigma}^{\mathrm{VV}}_{\mathrm{F}} = \frac{1}{2\hat{s}}\frac{1}{N}\int \mathrm{d}\boldsymbol{\Phi}_n \left[2\,\mathrm{Re}\langle \mathcal{M}^{(0)}_n | \mathcal{F}^{(2)}_n \rangle + \langle \mathcal{F}^{(1)}_n | \mathcal{F}^{(1)}_n \rangle \right] \mathrm{F}_n$$

Basic strategy: apply LC approximation **only here**

Two-loop approximations in NNLO QCD cross sections

Pros

- keep full color in real
- easy to implement
- sub-leading color terms additive
  → easy to add later

Cons

- IR scheme dependent
- non perfect cancellation of logs against reals

Leading color approximation of two-loop finite remainders

1. Scale dependence:

Rescaling captures some of the effect

$$F^{(2)}_{\text{l.c. resc.}}(\mu^2) = \frac{F^{(0)}}{F^{(0)}_{\text{l.c.}}} F^{(2)}_{\text{l.c.}}(Q^2) + \underbrace{\sum_{i=0}^{4} c_i \log^i(\mu^2/Q^2)}$$

Only depend on lower order MEs. **Choice:** kept in full color

2. Approximate only "two-loop" or "two-loop plus one-loop squared"?

$$\frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)} | \mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu = \hat{E}_{\mathrm{CMS}}) \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle} \qquad \frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)} | \mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu = \hat{E}_{\mathrm{CMS}}) \rangle + \langle \mathcal{F}^{(\overline{\mathrm{MS}},1)}(\mu = \hat{E}_{\mathrm{CMS}}) | \mathcal{F}^{(\overline{\mathrm{MS}},1)}(\mu = \hat{E}_{\mathrm{CMS}}) \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle}$$

3. defining the leading color for two-loop MEs only is **scheme dependent**
→ access this by comparing Catani and MSbar scheme

Definition of finite remainder: Catani vs MSbar

$$|\mathcal{F}^{(\mathrm{S})}(\{p_i\},\mu)\rangle \equiv \lim_{\epsilon\to 0}\left[\boldsymbol{Z}^{(\mathrm{S})}(\epsilon,\{p_i\},\mu)\right]^{-1}|\mathcal{M}(\epsilon,\{p_i\})\rangle$$

Catani [hep-ph/9802439]
+ Becher Neubert [0901.0722] [0903.1126]
+ Aybat, Dixon, Sterman [hep-ph/0607309]

Catani's scheme:

$$\left[\boldsymbol{Z}^{(\mathrm{Catani})}\right]^{-1} \equiv \boxed{1 - \frac{\alpha_s}{2\pi}\boldsymbol{I}^{(1)} - \left(\frac{\alpha_s}{2\pi}\right)^2 \boldsymbol{I}^{(2)} +} \ldots$$

$$\boldsymbol{I}^{(1)}(\epsilon) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\sum_i \left(\frac{1}{\epsilon^2} - \frac{\gamma_0^i}{2\epsilon}\frac{1}{\boldsymbol{T}_i^2}\right)\sum_{j\neq i}\frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\left(\frac{\mu^2}{-s_{ij}}\right)^{\epsilon}$$

$$\boldsymbol{I}^{(2)} = \frac{e^{-\epsilon\gamma_E}\,\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\gamma_1^{\mathrm{cusp}}}{8} + \frac{\beta_0}{2\epsilon}\right)\boldsymbol{I}^{(1)}(2\epsilon) - \frac{1}{2}\boldsymbol{I}^{(1)}(\epsilon)\left(\boldsymbol{I}^{(1)}(\epsilon) + \frac{\beta_0}{\epsilon}\right) + \boldsymbol{H}^{(2)}$$

MSbar scheme:

$$\boldsymbol{Z}^{(\overline{\mathrm{MS}})} \equiv 1 + \frac{\alpha_s}{4\pi}\boldsymbol{Z}_1 + \left(\frac{\alpha_s}{4\pi}\right)^2\boldsymbol{Z}_2 + \ldots$$

$$\boxed{\boldsymbol{Z}_1 = 2\boldsymbol{I}^{(1)}\Big|_{\substack{\text{principal}\\ \text{part}}}, \qquad \boldsymbol{Z}_2 = \left[4\boldsymbol{I}^{(2)} + 2\boldsymbol{I}^{(1)}\boldsymbol{Z}_1\right]_{\substack{\text{principal}\\ \text{part}}}}$$

only poles, no finite parts

Scale dependence of finite remainder

MSbar scheme

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\ln \boldsymbol{Z}^{(\overline{\mathrm{MS}})} = -\gamma^{\mathrm{cusp}}\sum_{i\neq j}\frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\ln\left(\frac{\mu^2}{-s_{ij}}\right) - \sum_i \gamma^i + \mathcal{O}(\alpha_s^3)$$

finite remainder:

$$|\mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu)\rangle = |\mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu_0)\rangle + \sum_{i=1}^{4}\sum_{i/2\leqslant j\leqslant 2} d_{ij}\,\ln^i\left(\frac{\mu}{\mu_0}\right)|\mathcal{F}^{(\overline{\mathrm{MS}},2-j)}(\mu_0)\rangle$$

$$\mu_0 = \hat{E}_{\mathrm{cms}}$$

Catani's scheme

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\boldsymbol{Z}^{(\mathrm{Catani})} = \mathcal{O}(\alpha_s^3)$$

(If Q in $\mathbf{H}^{(2)}$ independent of μ)

finite remainder:

$$|\mathcal{F}^{(\mathrm{Catani},2)}(\mu)\rangle = |\mathcal{F}^{(\mathrm{Catani},2)}(\mu_0)\rangle + \sum_{i=1}^{2}\sum_{i\leqslant j\leqslant 2} c_{ij}\,\ln^i\left(\frac{\mu}{\mu_0}\right)|\mathcal{F}^{(\mathrm{Catani},2-j)}(\mu_0)\rangle$$

→ different scale dependence

→ translation between these requires only lower loop finite remainders

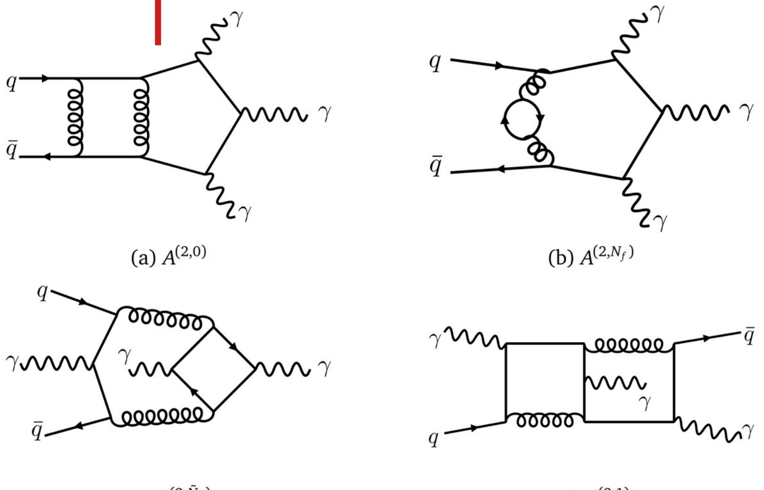
→ use this to compare “LC MSbar” and “LC Cat” (+FC translation)

Phenomenology: three photons

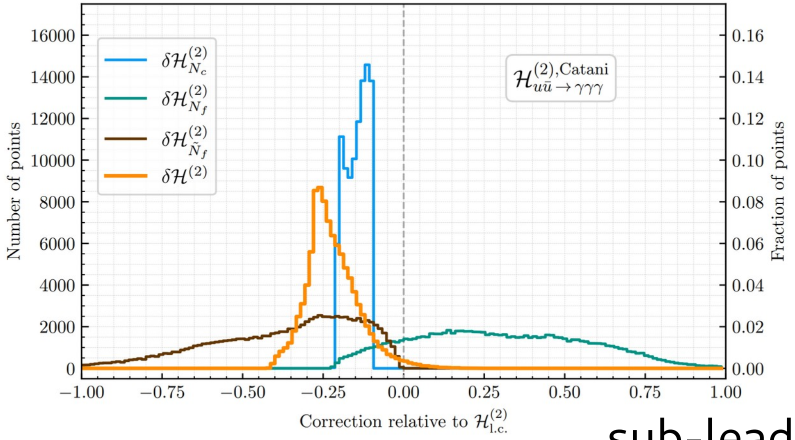
$$\frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)}|\mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu=\hat{E}_{\mathrm{CMS}})\rangle}{\langle \mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle}$$

$$= N_c^2\left[c_{1,0}+\frac{1}{N_c^2}c_{1,1}+\frac{1}{N_c^4}c_{1,2}+\frac{n_f}{N_c}\left(c_{2,0}+\frac{1}{N_c^2}c_{2,1}\right)\right]+\sum_{q'}\frac{Q_{q'}^2}{Q_q^2}N_c\left(c_{3,0}+\frac{1}{N_c^2}c_{3,1}\right)$$ ← full color expansion

$$\approx N_c^2\,c_{1,0}\,,$$ ← leading color approx.



(a) $A^{(2,0)}$ (b) $A^{(2,N_f)}$ (c) $A^{(2,\tilde{N}_f)}$ (d) $A^{(2,1)}$



Abreu, De Laurentis,Ita, Klinkert, Page, Sotnikov [2305.17056]

~35-40%

sub-leading color effect on ME level
→ implications x-sec level expected to be small (two-loop ~1%)

Phenomenology: three photons

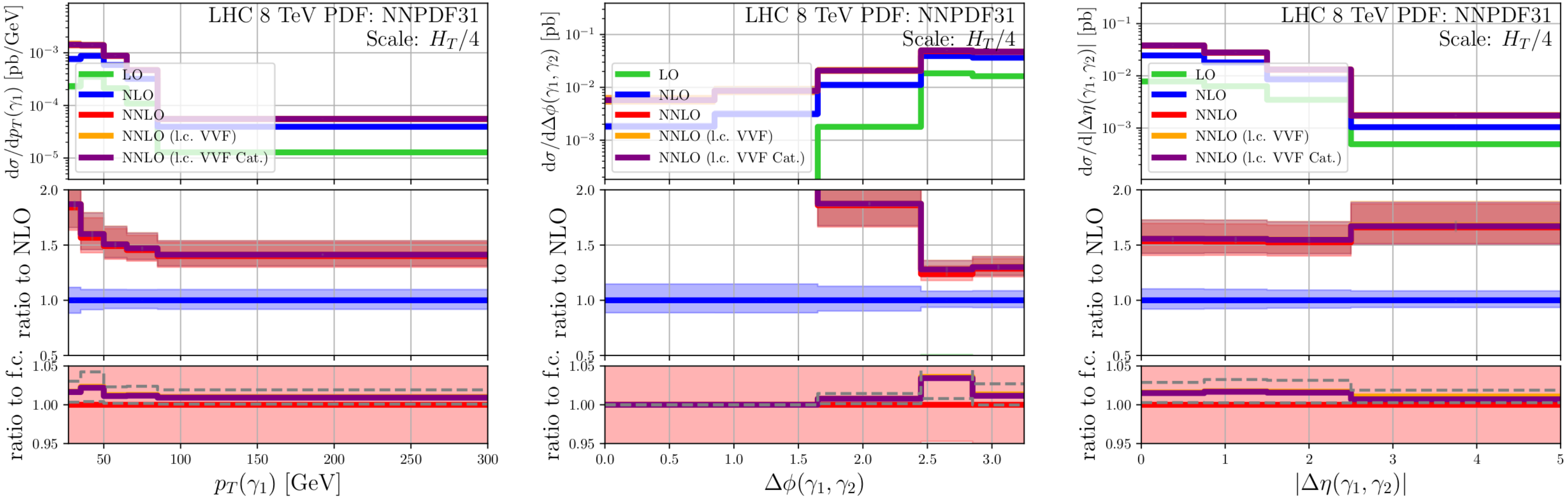
$$\frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)}|\mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu=\hat{E}_{\mathrm{CMS}})\rangle}{\langle \mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle}$$

$$= N_c^2\left[c_{1,0}+\frac{1}{N_c^2}c_{1,1}+\frac{1}{N_c^4}c_{1,2}+\frac{n_f}{N_c}\left(c_{2,0}+\frac{1}{N_c^2}c_{2,1}\right)\right]+\sum_{q'}\frac{Q_{q'}^2}{Q_q^2}N_c\left(c_{3,0}+\frac{1}{N_c^2}c_{3,1}\right)$$

← full color expansion

$$\approx N_c^2\, c_{1,0}\,,$$

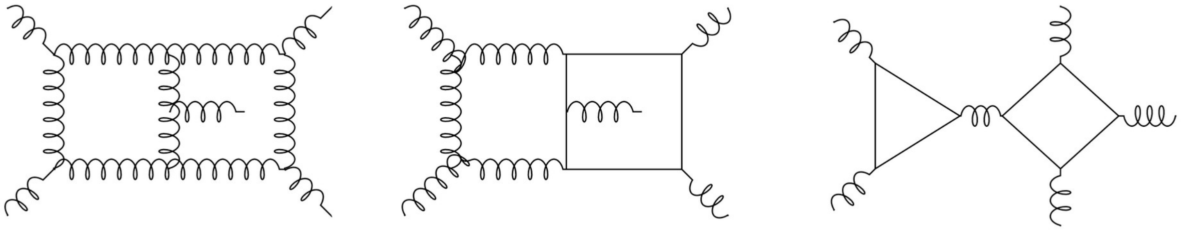
← leading color approximation



Phenomenology: three jets

Color structure of five-parton amplitudes (5 gluon channel):

Agarwal, Buccioni, Devoto, Gambuti, von Manteuffel, Tancredi [2311.09870]
De Laurentis, Ita, Klinkert, Sotnikov [2311.10086] [2311.18752]



$$\mathcal{A}_{\vec{a}} = \sum_{\sigma\in\mathcal{S}_5/\mathcal{Z}_5} \sigma\Big(\mathrm{tr}(1,2,3,4,5)\ A_1(1,2,3,4,5)\Big) + \sum_{\sigma\in\frac{\mathcal{S}_5}{\mathcal{Z}_2\times\mathcal{Z}_3}} \sigma\Big(\mathrm{tr}(1,2)\,\mathrm{tr}(3,4,5)\ A_2(1,2;3,4,5)\Big),$$

$$\begin{aligned} A_1^{(2)} &= N_c^2\ A^{(2),(2,0)} + A^{(2),(0,0)} \\ &+ N_f N_c\ A^{(2),(1,1)} + \frac{N_f}{N_c}\ A^{(2),(-1,1)} \\ &+ N_f^2\ A^{(2),(0,2)}, \\ A_2^{(2)} &= N_c\ A^{(2),(1,0)} \\ &+ N_f\ A^{(2),(0,1)} + \frac{N_f^2}{N_c}\ A^{(2),(-1,2)}. \end{aligned}$$

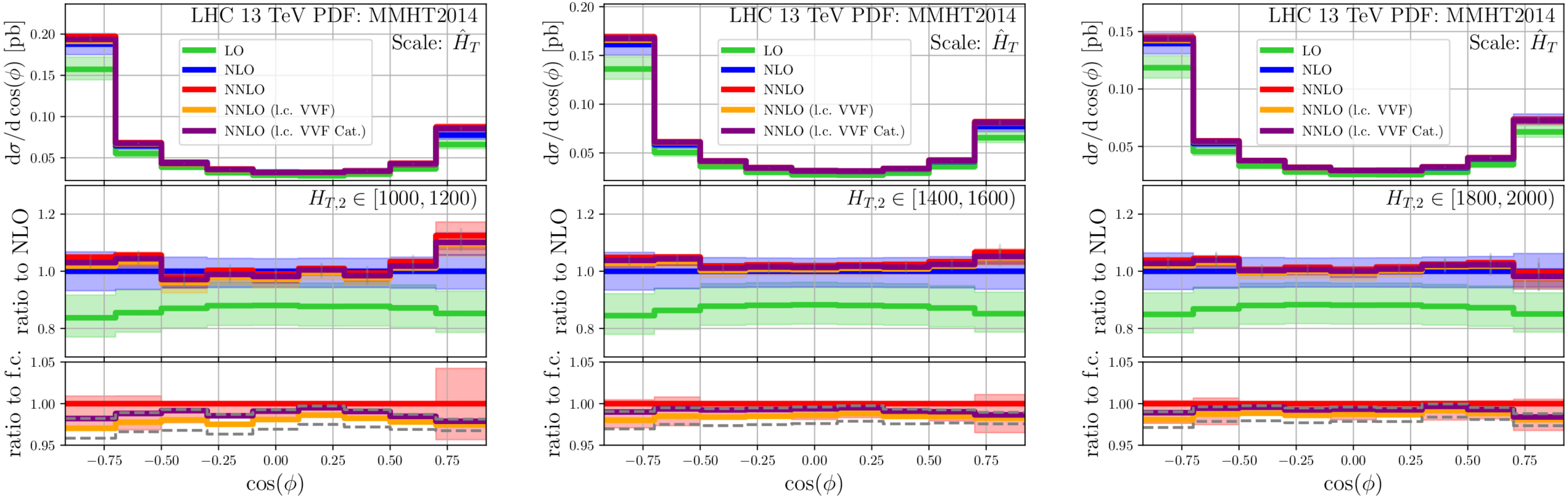
Leading color of squared ME:

$$\frac{2\,\mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu=\hat{E}_{\mathrm{CMS}})\rangle + \langle\mathcal{F}^{(\overline{\mathrm{MS}},1)}(\mu=\hat{E}_{\mathrm{CMS}})|\mathcal{F}^{(\overline{\mathrm{MS}},1)}(\mu=\hat{E}_{\mathrm{CMS}})\rangle}{\langle\mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle} \approx N_c^2 c_{2,0} + N_c n_f c_{1,1} + n_f^2 c_{0,2} + \mathcal{O}(N_c^0)$$

Phenomenology: three jets

$$\frac{2\,\mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu=\hat{E}_{\mathrm{CMS}})\rangle + \langle\mathcal{F}^{(\overline{\mathrm{MS}},1)}(\mu=\hat{E}_{\mathrm{CMS}})|\mathcal{F}^{(\overline{\mathrm{MS}},1)}(\mu=\hat{E}_{\mathrm{CMS}})\rangle}{\langle\mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle} \approx N_c^2 c_{2,0} + N_c n_f c_{1,1} + n_f^2 c_{0,2}$$

HT2 1 TeV to 2 TeV

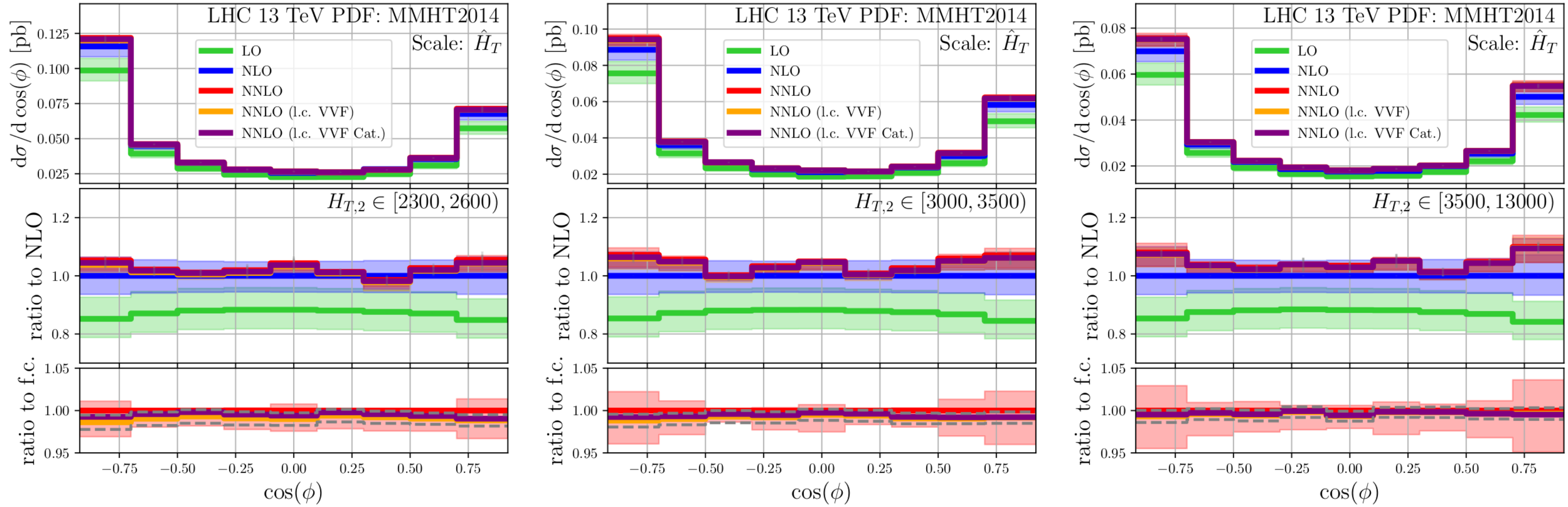


TEEC: $$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} \equiv \frac{1}{\sigma_2}\sum_{\text{jets } i\neq j}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}$$

Phenomenology: three jets

$$\frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)} | \mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu = \hat{E}_{\mathrm{CMS}}) \rangle + \langle \mathcal{F}^{(\overline{\mathrm{MS}},1)}(\mu = \hat{E}_{\mathrm{CMS}}) | \mathcal{F}^{(\overline{\mathrm{MS}},1)}(\mu = \hat{E}_{\mathrm{CMS}}) \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle} \approx N_c^2 c_{2,0} + N_c n_f c_{1,1} + n_f^2 c_{0,2}$$

HT2 > 2 TeV



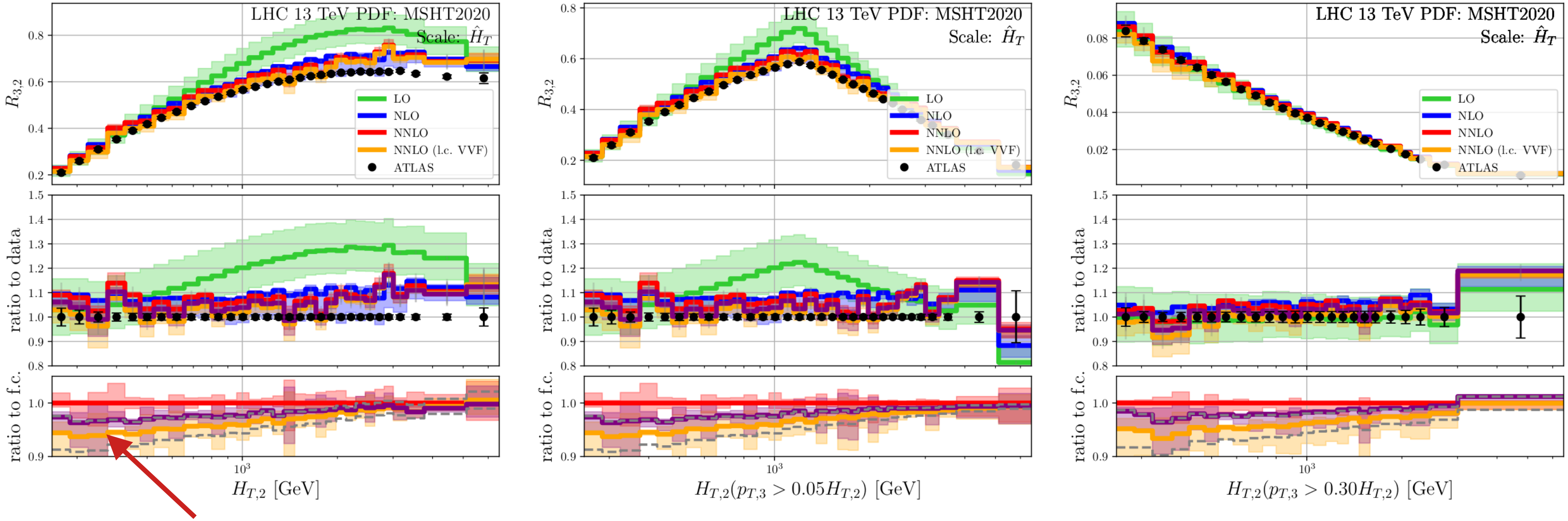
Differences at lower HT2, vanishing at high energy

Phenomenology: three jets

$$\frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)} | \mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu = \hat{E}_{\mathrm{CMS}})\rangle + \langle \mathcal{F}^{(\overline{\mathrm{MS}},1)}(\mu = \hat{E}_{\mathrm{CMS}}) | \mathcal{F}^{(\overline{\mathrm{MS}},1)}(\mu = \hat{E}_{\mathrm{CMS}})\rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)}\rangle} \approx N_c^2 c_{2,0} + N_c n_f c_{1,1} + n_f^2 c_{0,2}$$

R32 ratios:

Data from ALTAS [2405.20206]



More substantial effect O(5%) → leads to a reduction of scale dependence.

Parametrization of sub-leading color terms

For example $pp \to \gamma\gamma j$

$$
\frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)} | \mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu = \hat{E}_{\mathrm{CMS}}) \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle}
$$

$$
= \boxed{\begin{aligned} & N_c^2 \left[c_{1,0} + \frac{1}{N_c^2} c_{1,1} + \frac{1}{N_c^4} c_{1,2} + \frac{n_f}{N_c} \left(c_{2,0} + \frac{1}{N_c^2} c_{2,1} \right) \right] \\ & + \sum_{q'} \frac{Q_{q'}}{Q_q} N_c \left(c_{3,0} + \frac{1}{N_c^2} c_{3,1} \right) + \sum_{q'} \frac{Q_{q'}^2}{Q_q^2} N_c \left(c_{4,0} + \frac{1}{N_c^2} c_{4,1} + \frac{n_f}{N_c} c_{5,0} \right) \Big) \Big] \end{aligned}}
$$

full color structure

$$
\approx \boxed{N_c^2 \left[c_{1,0} + \frac{n_f}{N_c} c_{2,0} \right] + \sum_{q'} \frac{Q_{q'}^2}{Q_q^2} n_f \, c_{5,0}} .
$$

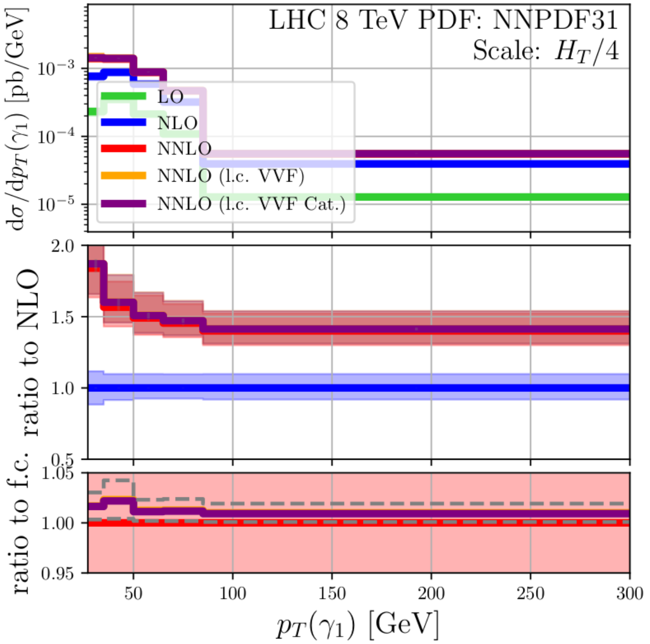
Leading-color / planar approximation

Bonus: a priori estimates of the leading missing term

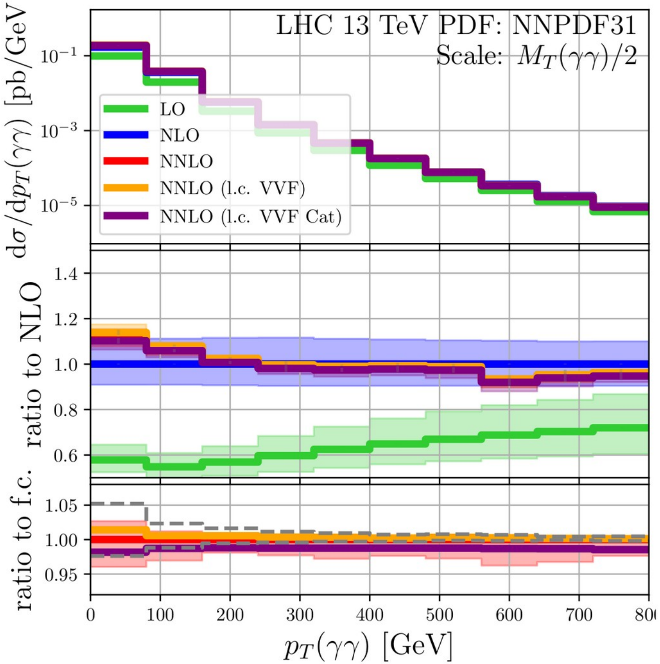
$$
\left. \frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)} | \mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu = \hat{E}_{\mathrm{CMS}}) \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle} \right|_{\mathrm{l.c.}}^{\theta} = N_c^2 \left[c_{1,0} + \frac{n_f}{N_c} c_{2,0} \right] + \sum_{q'} \frac{Q_{q'}^2}{Q_q^2} n_f \, c_{5,0} + \sum_{q'} \frac{Q_{q'}^2}{Q_q^2} N_c \theta \, c_{1,0} \qquad \theta \in [-1, 1]
$$

Sub-leading color estimates vs. reality

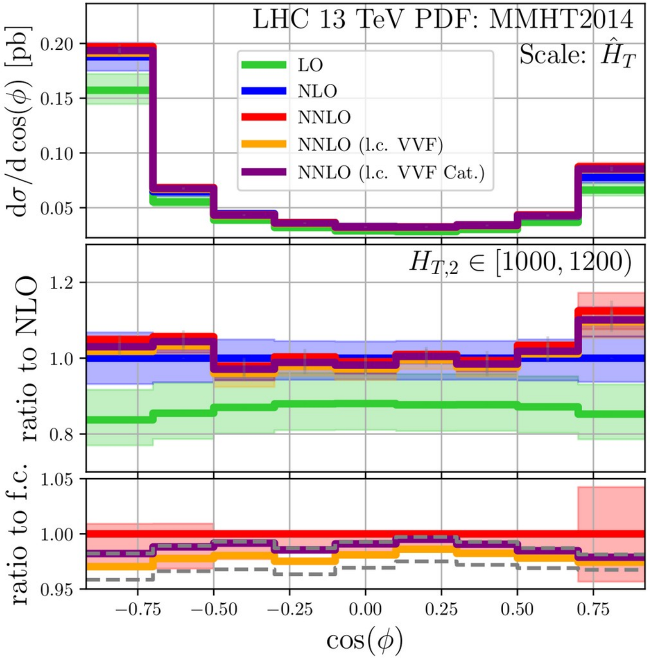
$pp \to \gamma\gamma\gamma$



$pp \to \gamma\gamma j$



$pp \to jjj$



Grey dashed lines: envelopes of variation

A priori estimates → considering the simplicity of the Ansatz satisfying results.

Summary

Sub-leading colour effects in two-loop amplitudes:

→ in generally small with respect to complete NNLO QCD cross section
because two-loop contributions are small
O(100%) **with respect to two-loop** contribution in the case of 2/3 photons
O(10%) in parton-only case (as expected)

→ however specific phase space regions show larger sensitivity
(low pT jets, threshold for example…)

→ a priori estimates based on rescaled leading color coefficients do a good job

Conclusion

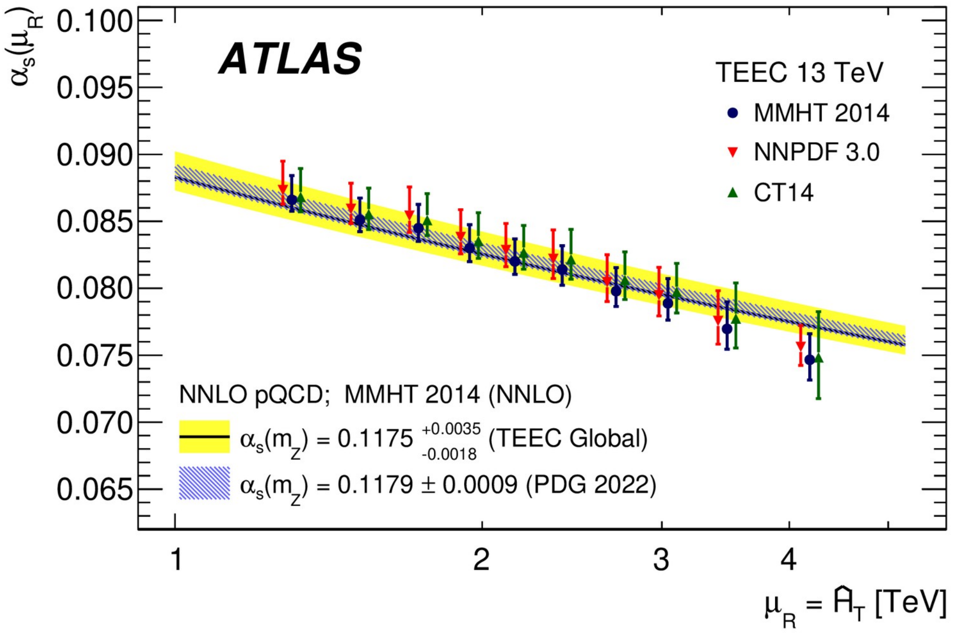
How much color do we really need?

→ LC does work well when two-loop contributions are small and help to estimate the expected size of the full color result

→ full color two-loop amplitudes are crucial when large cancellations between RR+RV and VV occur

Backup

TEEC: where does the slope come from?



- Residual PDF effects → very high Q^2 ?
  → need re-run with modern PDFs
  (a bit expensive, needs improved performance)
- EW corrections? → cancelling in ratio
  Full NLO corrections to 3-jet production and R32 at the LHC Reyer, Schönherr, Schumann, 1902.01763
- Maybe effect from LC approximation in two-loop ME? → apparently not
- Resummation/PS effect? → wip
- **MHOU?**
  wip → **start with inclusive jets are simpler!**

Phenomenology: di-photon+jet

$$\frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)}|\mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu=\hat{E}_{\mathrm{CMS}})\rangle}{\langle \mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle}$$

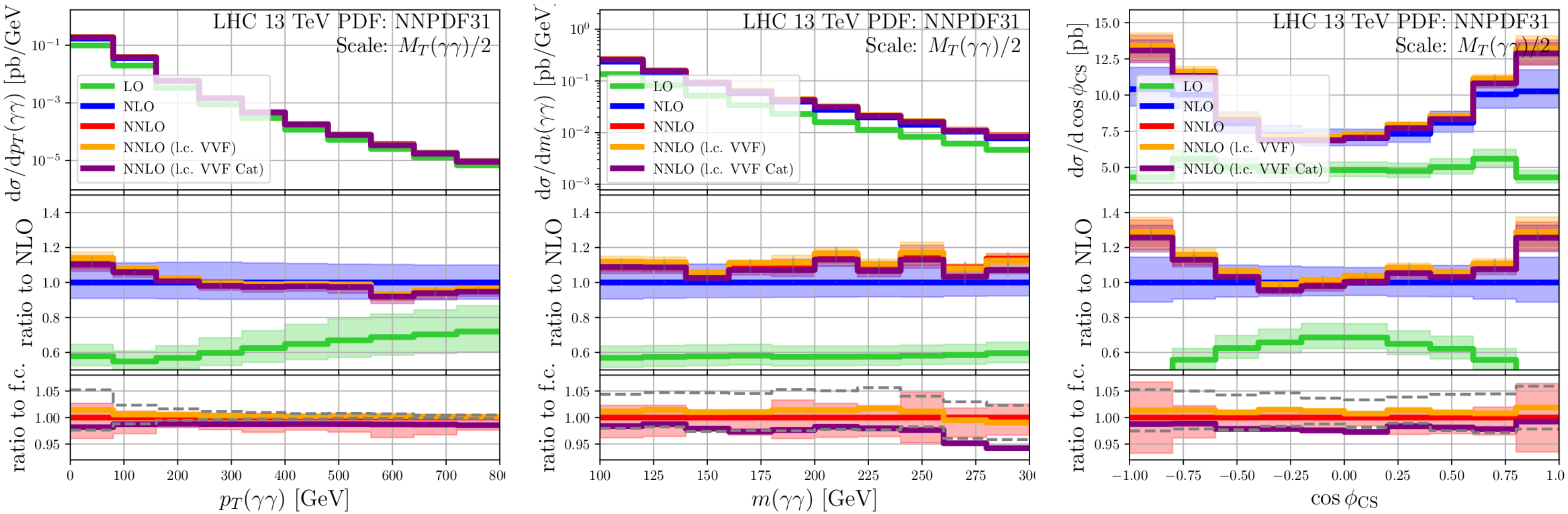
$$= N_c^2\Big[c_{1,0}+\frac{1}{N_c^2}c_{1,1}+\frac{1}{N_c^4}c_{1,2}+\frac{n_f}{N_c}\Big(c_{2,0}+\frac{1}{N_c^2}c_{2,1}\Big)\Big]$$

← **full color expansion**

$$+\sum_{q'}\frac{Q_{q'}}{Q_q}N_c\Big(c_{3,0}+\frac{1}{N_c^2}c_{3,1}\Big)+\sum_{q'}\frac{Q_{q'}^2}{Q_q^2}N_c\Big(c_{4,0}+\frac{1}{N_c^2}c_{4,1}+\frac{n_f}{N_c}c_{5,0}\Big)\Big)\Big]$$

$$\approx N_c^2\Big[c_{1,0}+\frac{n_f}{N_c}c_{2,0}\Big]+\sum_{q'}\frac{Q_{q'}^2}{Q_q^2}n_f\,c_{5,0}\,.$$

← **leading color approximation**



H2 Catani

$$\boldsymbol{H}^{(2)} = \frac{e^{\epsilon\gamma_E}}{4\epsilon\Gamma(1-\epsilon)}\left(\frac{\mu^2}{Q^2}\right)^{2\epsilon}\Bigg[\frac{1}{4}\sum_i\left(\gamma_1^i - \frac{1}{4}\gamma_1^{\text{cusp}}\gamma_0^i + \frac{\pi^2}{16}\beta_0\,\gamma_0^{\text{cusp}}\,C_i\right)$$
$$+\frac{if^{abc}}{6}\sum_{(i,j,k)}\boldsymbol{T}_i^a\boldsymbol{T}_j^b\boldsymbol{T}_k^c\ln\frac{-s_{ij}}{-s_{jk}}\ln\frac{-s_{jk}}{-s_{ki}}\ln\frac{-s_{ki}}{-s_{ij}}$$
$$-\frac{if^{abc}}{32}\gamma_0^{\text{cusp}}\sum_{(i,j,k)}\boldsymbol{T}_i^a\boldsymbol{T}_j^b\boldsymbol{T}_k^c\left(\frac{\gamma_0^i}{C_i}-\frac{\gamma_0^j}{C_j}\right)\ln\frac{-s_{ij}}{-s_{jk}}\ln\frac{-s_{ki}}{-s_{ij}}\Bigg]$$

Different choices for Q^2 in literature:
- $Q^2 = \mu^2$
- $Q^2 = -s$

→ implications for scale dependence of **I** operators