Uncertainties from missing higher orders for precision measurements

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Funded by the European Union

European Research Council
Established by the European Commission

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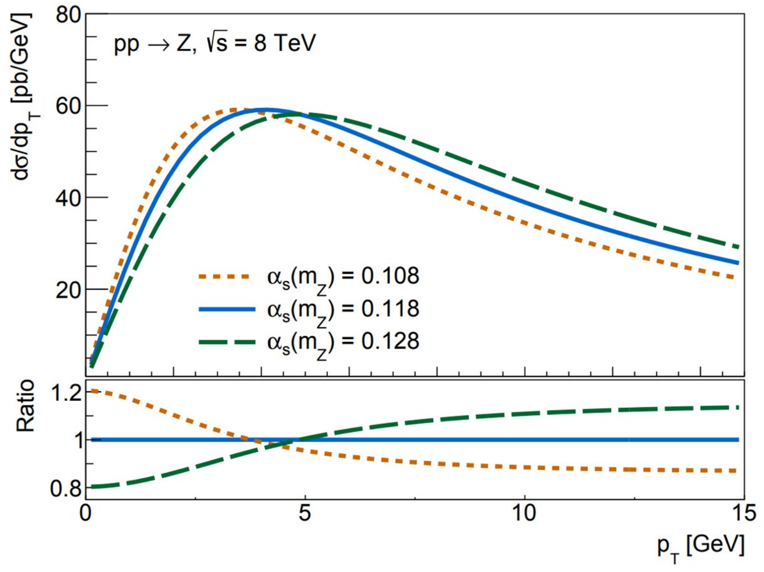
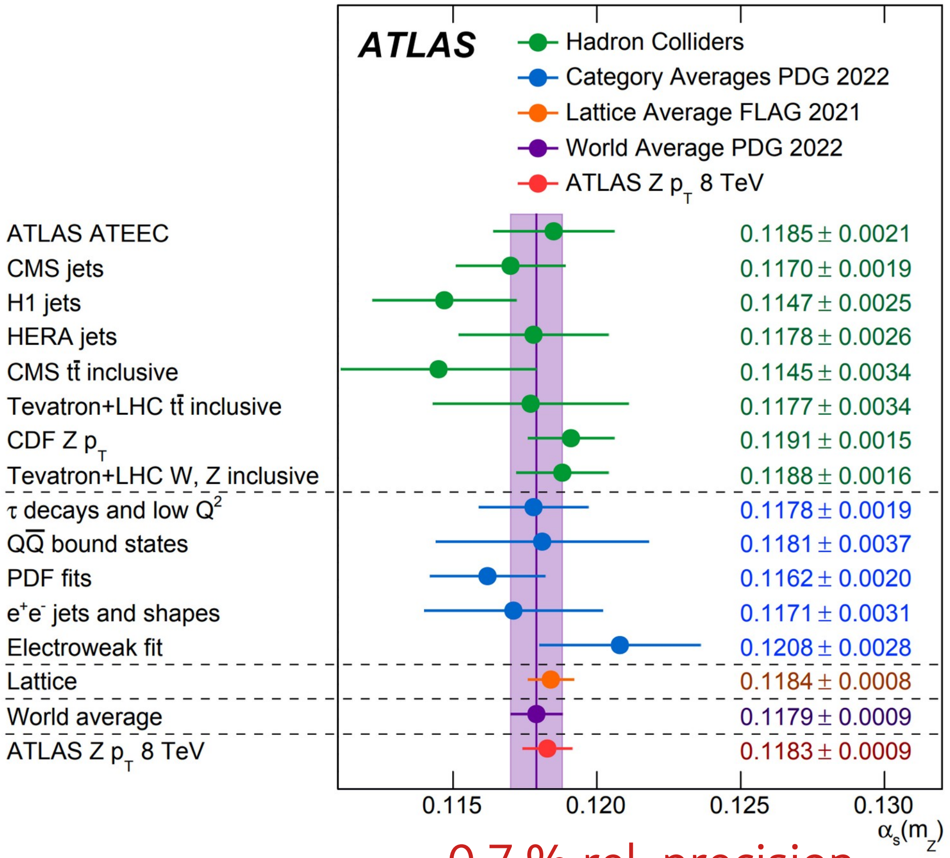
Precision example: strong coupling from pT(Z)

[ATLAS 2309.12986]

Sensitivity of Z-boson's recoil to the strong coupling constant:



→ at low pT resummation regime!
→ theory uncertainty?

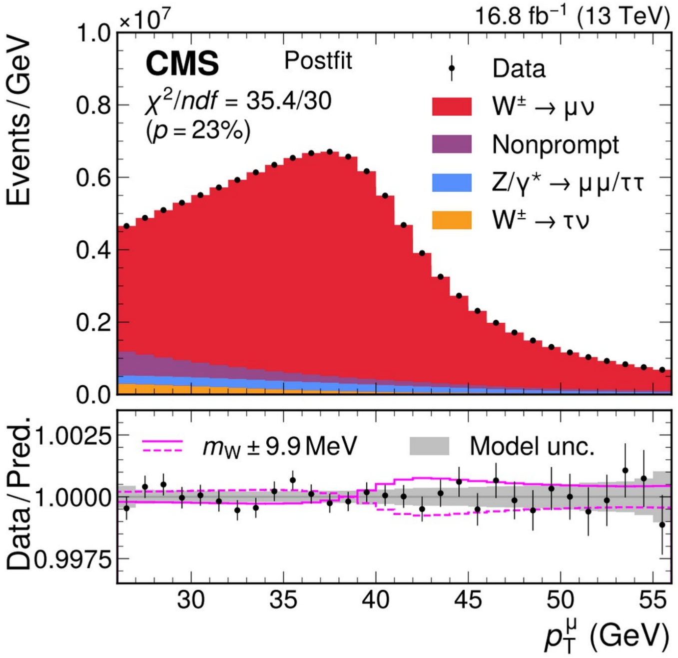


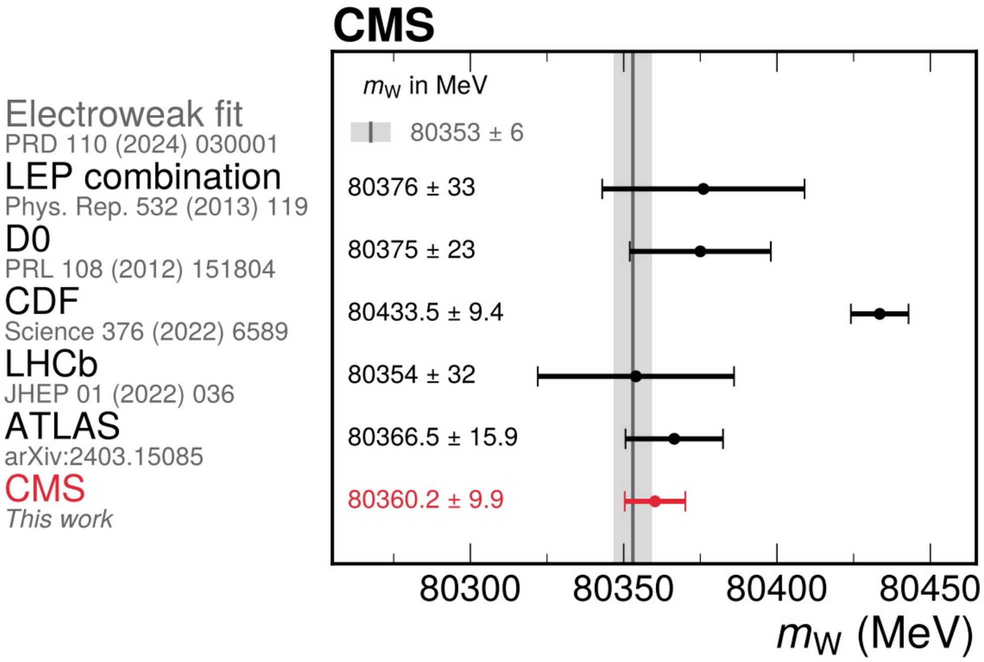
~0.7 % rel. precision

Precision example: W-mass measurement by CMS

[CMS 2412.13872]

Mass dependence of pT(l):





Jacobian peak position ~m(W)/2 → shape sensitive to corrections → theory uncertainty?

Uncertainties in precision phenomenology

Experiments are getting more precise → theory uncertainties matter!

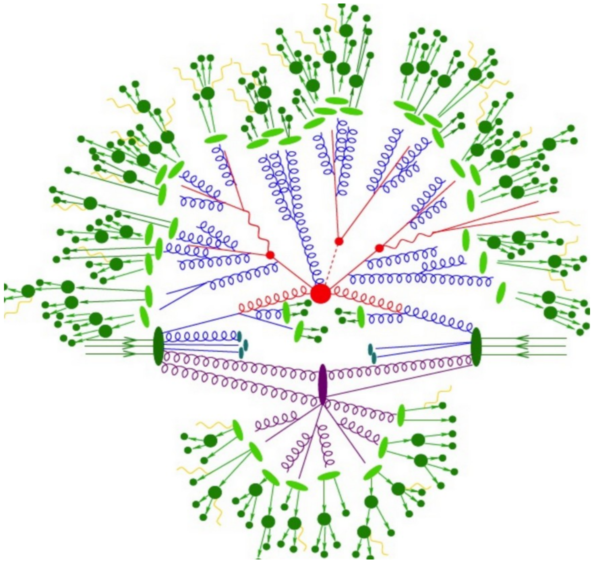
Sources of theory uncertainties:

- parametric (values of coupling parameters etc.)
→ variation of parameters within their uncertainties

- parton distribution functions (PDFs)
→ different error propagation methods (fit parameter, replicas,...)

- non-perturbative parameters in Monte Carlo simulations.
→ needs data constraints by definition. Problematic if dominant effect...

- missing higher orders in fixed-order and resummed predictions (MHOU)
→ tricky because we are trying to estimate the unknown….



[Credit: SHERPA]

Missing higher orders

Notation from: [Tackmann 2411.18606]
Beyond Scale Variations: Perturbative Theory Uncertainties from Nuisance Parameters

Generic perturbative expansion:

$$f(\alpha) = f_0 + f_1\alpha + f_2\alpha^2 + f_3\alpha^3 + \ldots$$

f_i : the coefficient of the series, potentially unknown

We can compute the truncated series: $\hat{f}_i$: the true value, i.e. a value we actually computed

$$f^{\mathrm{LO}}(\alpha) = \hat{f}_0 \qquad f^{\mathrm{NLO}}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha \qquad f^{\mathrm{NNLO}}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha + \hat{f}_2\alpha^2$$

The **missing terms are the source of uncertainty**.
(assume convergence → the first missing is the dominant one)

$$f^{\mathrm{LO+1}}(\alpha) = \hat{f}_0 + f_1\alpha \quad f^{\mathrm{NLO+1}}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha + f_2\alpha^2 \quad f^{\mathrm{NNLO+1}}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha + \hat{f}_2\alpha^2 + f_3\alpha^3$$

Challenge: how to estimate f_1, f_2, f_3, … without computing them?

Theory uncertainties from scale variations

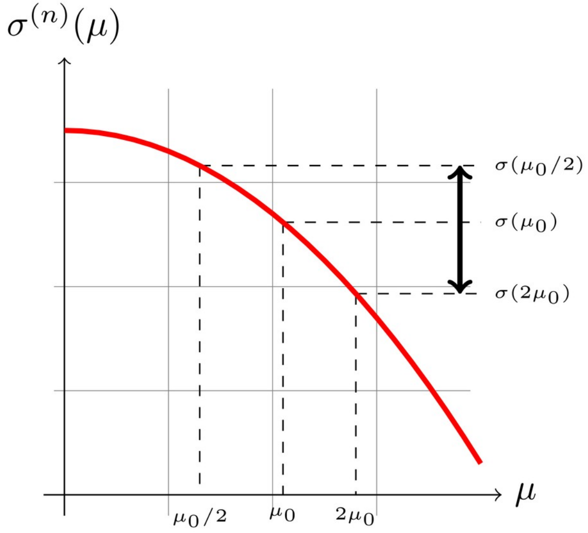
Lets focus on QCD as an example: $\alpha = \alpha_s(\mu_0)$

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(0)} + \alpha_s \mathrm{d}\sigma^{(1)} + \alpha_s^2 \mathrm{d}\sigma^{(2)} + \ldots \xrightarrow{\text{RGE}} \mu \frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\mu} = \mathcal{O}\left(\alpha_s^{(n_0+n+1)}\right)$$

Of same order as the next dominant term → exploiting this to estimate size of $\mathrm{d}\sigma^{(n+1)}$

Scale variation prescription (ad-hoc and heuristic choice)

- choose 'sensible' μ_0
  - → principle of fasted apparent convergence: $\sigma^{(n)}(\mu_{\mathrm{FAC}}) = 0$
  - → principle of minimal sensitivity $\mu \frac{\mathrm{d}\sigma(\mu)}{\mathrm{d}\mu}\Big|_{\mu=\mu_{\mathrm{PMC}}} = 0$
  - → ...
- vary with a factor (typically 2)
- take envelope as uncertainty



Scale variation approach

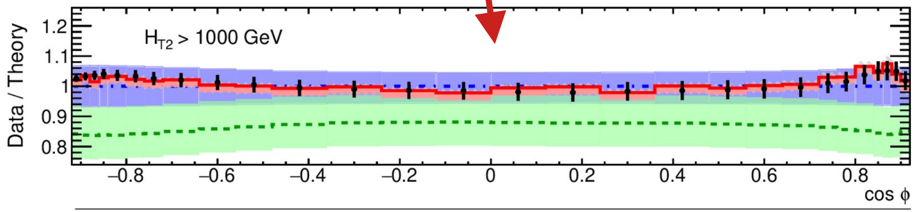
Estimates from scale variations for the unknown f_i:

$$f_2 \equiv \Delta f^{\mathrm{NLO}} = f^{\mathrm{NLO}} - \tilde{f}^{\mathrm{NLO}}(\tilde{\alpha}) = -\alpha^2 b_0 \hat{f}_1 + \mathcal{O}(\alpha^3)$$

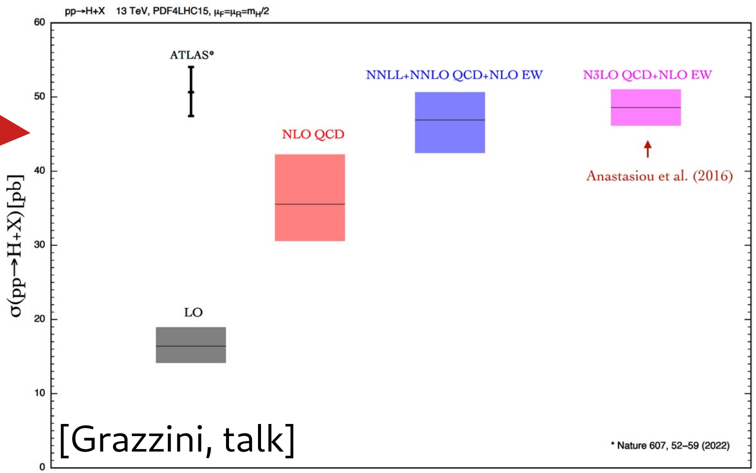
$$f_3 \equiv \Delta f^{\mathrm{NNLO}} = f^{\mathrm{NNLO}} - \tilde{f}^{\mathrm{NNLO}}(\tilde{\alpha}) = \alpha^3(2b_0(\hat{f}_2 - b_0\hat{f}_1) + b_1\hat{f}_1) + \mathcal{O}(\alpha^4)$$

$$b_0 = \frac{\beta_0}{2\pi}L \quad b_1 = \frac{\beta_0^2}{4\pi^2}L^2 + \frac{\beta_1}{8\pi^2}L \quad b_2 = \frac{\beta_0^3}{8\pi^3}L^3 + \frac{5\beta_0\beta_1}{32\pi^2}L^2 + \frac{\beta_2}{32\pi^3}L \quad L = \ln\frac{\mu_0}{\mu}$$

Sometimes it does works … sometimes not



[ATLAS 2301.09351]



[Grazzini, talk]

Short comings of scale variations

- not always reliable … however in most cases issues are understood/expected:
  new channels, phase space constraints, etc. → often we can design workarounds
- however, some issues are more fundamental:
  → how to choose the central scale? → not a physical parameter, no 'true' value
  (Principle of fasted apparent convergence, principle of minimal sensitivity,…)
  → how to propagate the estimated uncertainty, no statistical interpretation!
  → what about correlations? Based on 'fixed form' of the lower orders and RGE.
- Alternative approaches:

"Bayesian"

[Cacciari,Houdeau 1105.5152]
[Bonvini 2006.16293]
[Duhr, Huss, Mazeliauskas, Szafron 2106.04585]

"Theory Nuisance Parameter"

[Tackmann 2411.18606]
→ W mass extraction: [CMS 2412.13872]
[Cal,Lim, Scott,Tackmann Waalewijn 2408.13301]
[Lim, Poncelet, 2412.14910]
[Cridge,Marinelli,Tackmann,2506.13874]

Introducing theory nuisance parameters (TNPs)

[Tackman 2411.18606]

Generic perturbative expansion:

$$f(\alpha) = f_0 + f_1\alpha + f_2\alpha^2 + f_3\alpha^3 + \ldots$$

$$f^{\mathrm{LO}+1}(\alpha) = \hat{f}_0 + f_1(\theta)\alpha$$

$$f^{\mathrm{NLO}+1}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha + f_2(\theta)\alpha^2$$

$$f^{\mathrm{NNLO}+1}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha + \hat{f}_2\alpha^2 + f_3(\theta)\alpha^3$$

Not knowing these constitute the (leading) source of MHOU

Introduce a parametrisation of unknown coefficients in terms of

"Theory nuisance parameters" θ

Key features

1) The parametrization such that there is a true value: $f_i(\hat{\theta}) = \hat{f}_i$

2) Distributions of θ “known” (for example from already existing computations)

→ “Expert knowledge” to construct such a parametrisation

Finding a parametrization

How to find a parametrization for $f_i(\theta)$? What if it depends on an additional parameter x?

- it is a constant → do I understand its normalization?
- it is function of x, but we know the overall functional dependence, e.g.

$$f_i(x,\theta) = \sum_k^{m_i} c_i^k(\theta) \ln^k(x)$$

- it is a function of x, but don't really know the functional dependence

$$f_i(x,\theta) = \sum_k^{\infty} c_i^k(\theta) \phi^k(x)$$

← Some basis of functions: Legendre, Chebyshev,...

If this series converges: $f_i(x,\theta) \approx \sum_k^{m} c_i^k(\theta) \phi^k(x)$

Finding a parametrization

How to find a parametrization for $f_i(\theta)$? What if it depends on an additional parameter x?

- Multi-dimensional dependences: $$f_i(x,y,\theta) = \sum_{k,l} c_i^{kl}(\theta) f^{kl}(x,y) \quad \ldots$$
- Often low-dimensional observables
  → y integrated out: $$f_i(x,\theta) = \int \mathrm{d}y f_i(x,y,\theta)$$

  if it factorizes $$f_i(x,y,\theta) = h_i(x,\theta) g_i(y,\theta)$$

  for example: $$\int \mathrm{d}y f_i(x,y,\theta) = h_i(x,\theta) G_i(\theta) \approx h_i(x,\theta)\tilde{G}_i\theta$$

To summarize: even if you don't fully know the form of f_i you can systematically build it up
→ what are the dominant effects?
→ what dependencies/correlations are actually important?

Example: TNPs in resummed cross sections

[Tackman 2411.18606]

Transverse momentum resummation at leading power ($\mu = Q$):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = [\underbrace{H \times B_a \times B_b \times S}](\alpha_s, \ln\left(\frac{p_T}{Q}\right)) + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right)$$

+ non pert.
+ quark mass
+ QED/EW

$$X(\alpha_s, L) = X(\alpha_s) \exp \int_0^L \mathrm{d}L' \{\Gamma(\alpha_s(L'))L' + \gamma_X(\alpha_s(L'))\}$$

Perturbative expansion of resummation ingredients:
→ functional dependence of H and S on ln(pT/Q) determined through RGE
→ only needs boundary conditions/anomalous dimensions:

$$X(\alpha_s) = X_0 + \alpha_s X_1 + \alpha_s^2 X_2 + \cdots \boxed{+ \alpha_s^n X_n(\theta)}$$

$$\Gamma(\alpha_s) = \alpha_s[\Gamma_0 + \alpha_s \Gamma_1 + \alpha_s^2 \Gamma_2 + \cdots \boxed{+ \alpha_s^n \Gamma_n(\theta)]}$$

$$\gamma(\alpha_s) = \alpha_s[\gamma_0 + \alpha_s \gamma_1 + \alpha_s^2 \gamma_2 + \cdots \boxed{+ \alpha_s^n \gamma_n(\theta)]}$$

Task: find suitable parametrization and variation range

These are numbers here
→ only need normalisation

Example: TNPs in resummed cross sections

[Tackman 2411.18606]

Transverse momentum resummation at leading power ($\mu = Q$):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = \underbrace{[H \times B_a \times B_b \times S]}_{}(\alpha_s, \ln\left(\frac{p_T}{Q}\right)) + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right)$$

+ non pert.
+ quark mass
+ QED/EW

$$X(\alpha_s, L) = X(\alpha_s) \exp \int_0^L \mathrm{d}L' \{\Gamma(\alpha_s(L'))L' + \gamma_X(\alpha_s(L'))\}$$

Perturbative expansion of resummation ingredients:

→ beam functions B are bit trickier

→ x – dependence determined by PDFs and matching kernels (eff. splitting functions)

$$\tilde{b}_{i,n}(x) = \sum_j \int \frac{\mathrm{d}z}{z} \tilde{I}_{ij,n}(z)\, f_j\left(\frac{x}{z}\right)$$

→ if we don't need correct correlations in the boson rapidity → simplified ansatz:

$$\tilde{I}_{ij,n}(z, \theta_n^{B_{ij}}) = \frac{3}{2}\, \theta_n^{B_{ij}}\, \hat{\tilde{I}}_{ij,n}(z)$$

Priors for TNP parameters: example qT-resummation

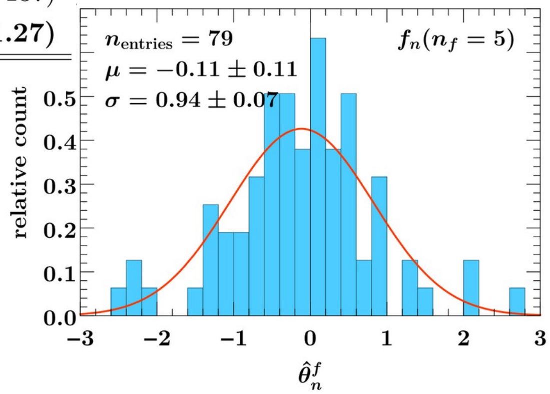
[Tackman 2411.18606]

$\gamma(\alpha_s)$	N_n	$\hat{\gamma}_0/N_0$	$\hat{\gamma}_1/N_1$	$\hat{\gamma}_2/N_2$	$\hat{\gamma}_3/N_3$	$\hat{\gamma}_4/N_4$
β	1	-15.3	-77.3	-362	-9652	-30941
	4^{n+1}	-3.83	-4.83	-5.65	-37.7	-30.2
	$4^{n+1}C_F C_A^n$	$\mathbf{-1.28}$	$\mathbf{-0.54}$	$\mathbf{-0.21}$	$\mathbf{-0.47}$	$\mathbf{-0.12}$
γ_m	1	-8.00	-112	-950	-5650	-85648
	4^{n+1}	-2.00	-7.028	-14.8	-22.1	-83.6
	$4^{n+1}C_F C_A^n$	$\mathbf{-1.50}$	$\mathbf{-1.76}$	$\mathbf{-1.24}$	$\mathbf{-0.61}$	$\mathbf{-0.77}$
$2\Gamma^q_{\mathrm{cusp}}$	1	$+10.7$	$+73.7$	$+478$	$+282$	$(+140000)$
	4^{n+1}	$+2.67$	$+4.61$	$+7.48$	$+1.10$	$(+137)$
	$4^{n+1}C_F C_A^n$	$\mathbf{+2.00}$	$\mathbf{+1.15}$	$\mathbf{+0.62}$	$\mathbf{+0.03}$	$\mathbf{(+1.27)}$

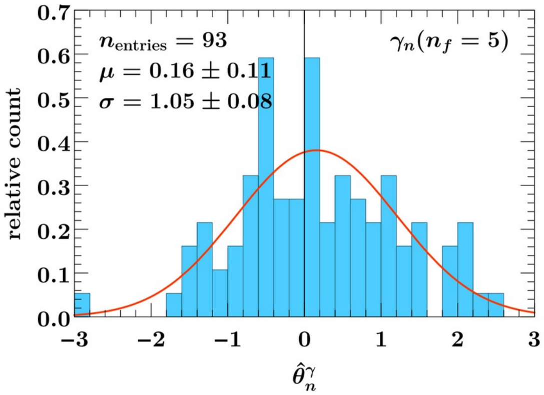
⋮

"Statistic of many computations/loop orders"

Matrix elements

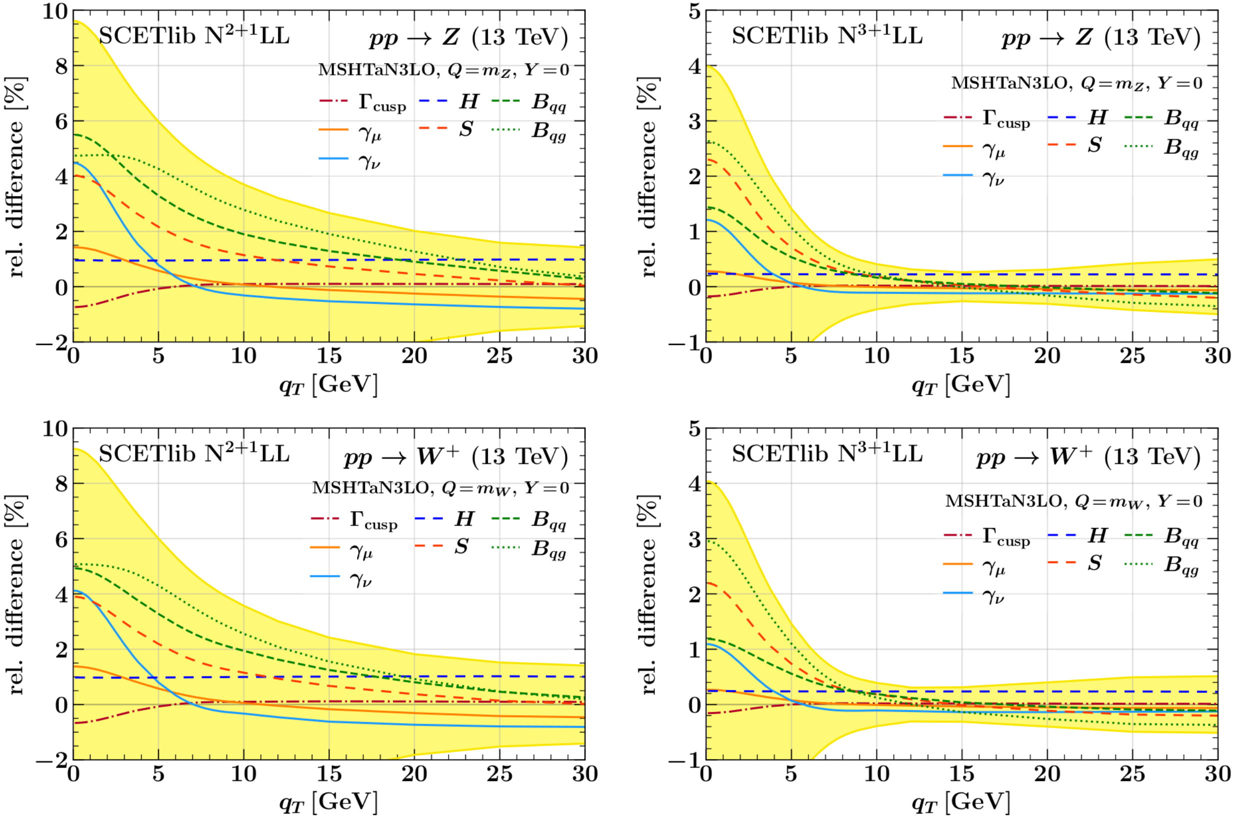


Anomalous dimensions



Turn this observation around to construct a **prior** for the **unknown** parameters
→ use variation as a priori estimates of MHOU

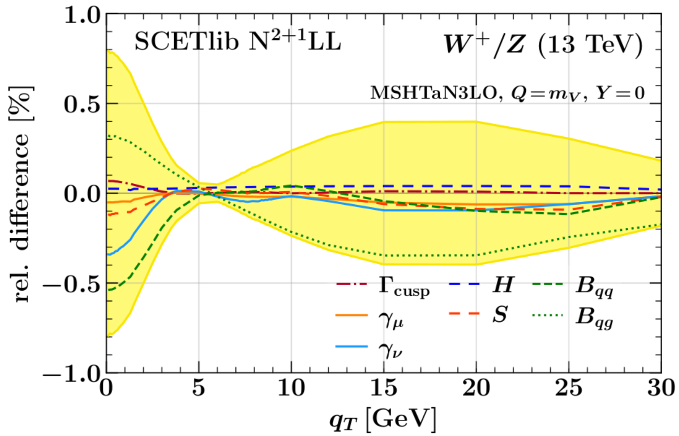
Point-wise correlations from TNP variations

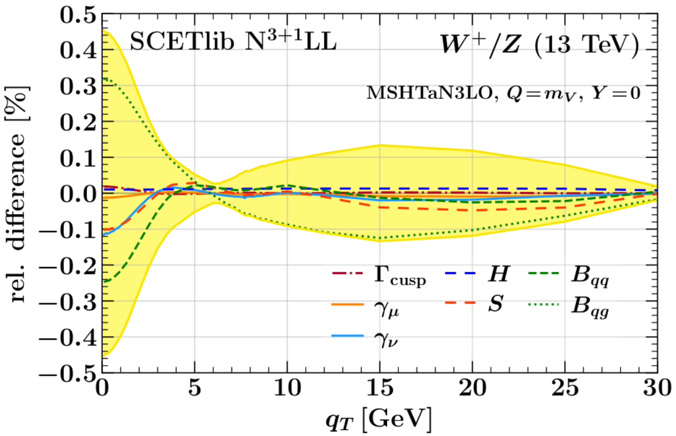


Lines: individual +- 1 variation

Yellow band:
quadrature sum of all components (sym)

W/Z ratio

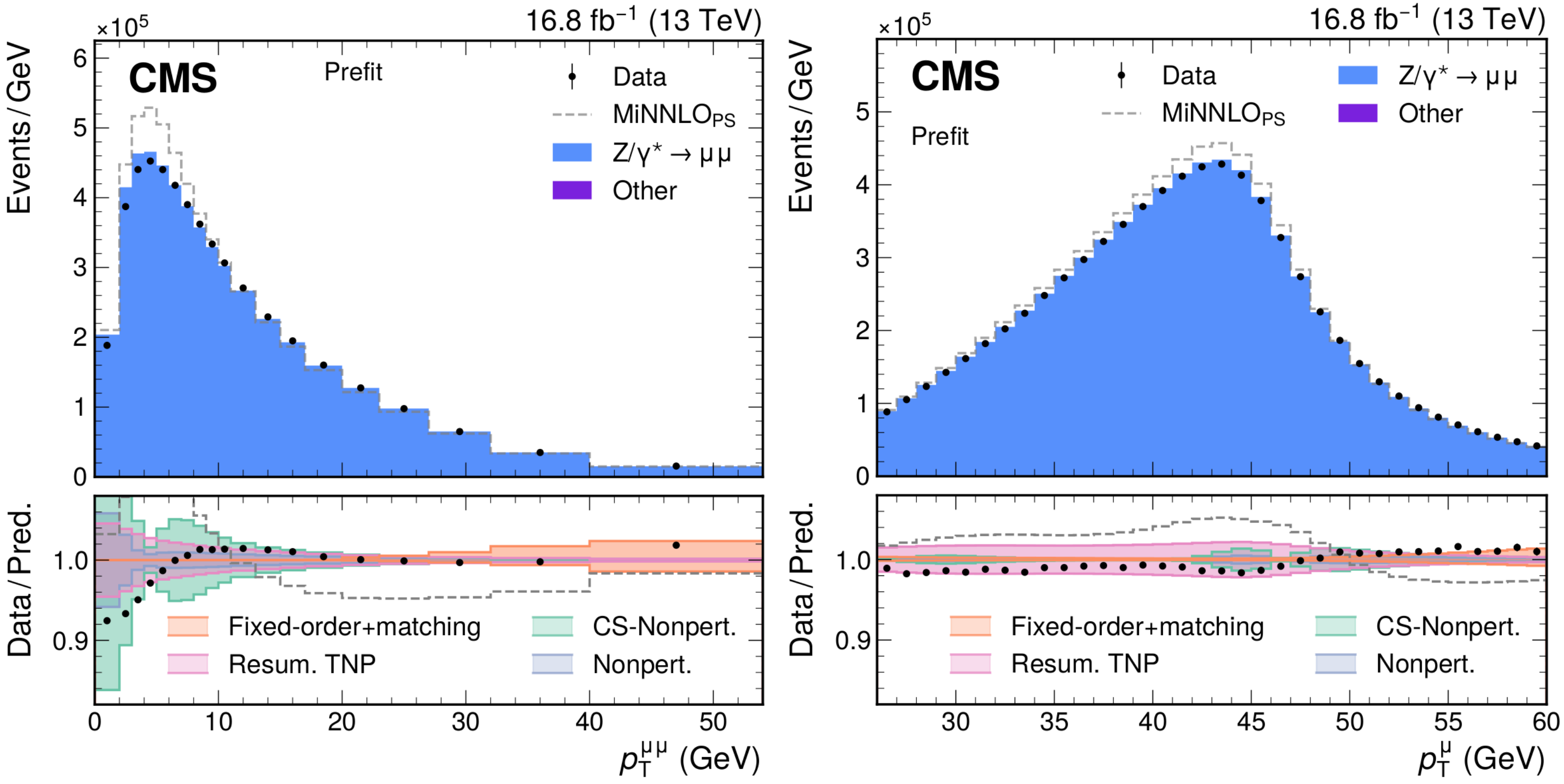




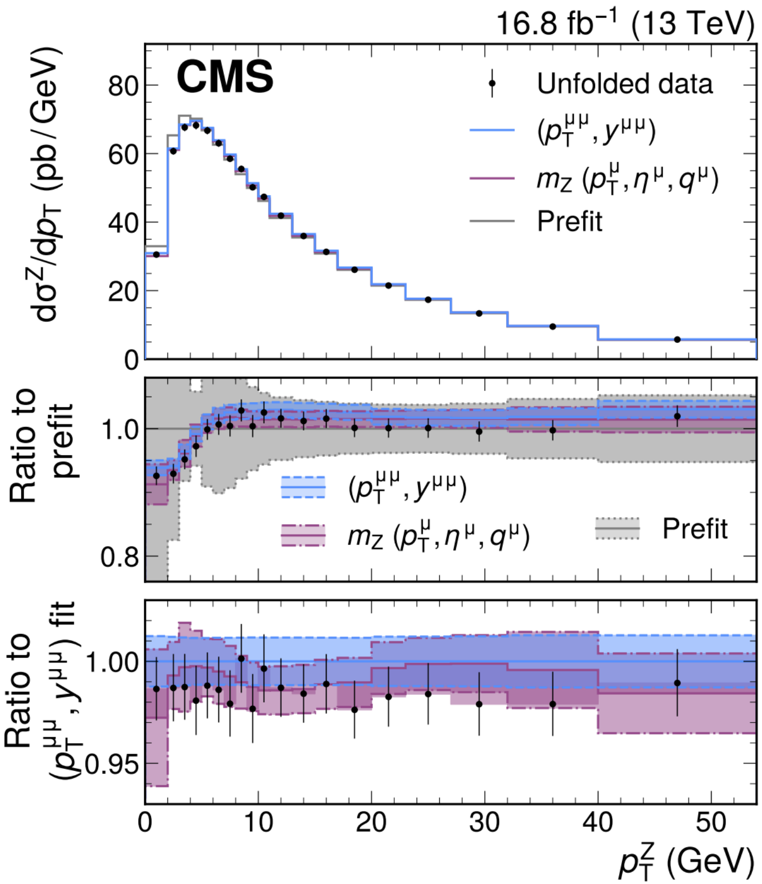
Many TNP variations are strongly correlated
→ cancellation in the ratio (similar to what on sees with scale variations)

Caveat: this holds for the perturbative bits, non-perturbative effects might be different

W-mass extraction: using Z as a test



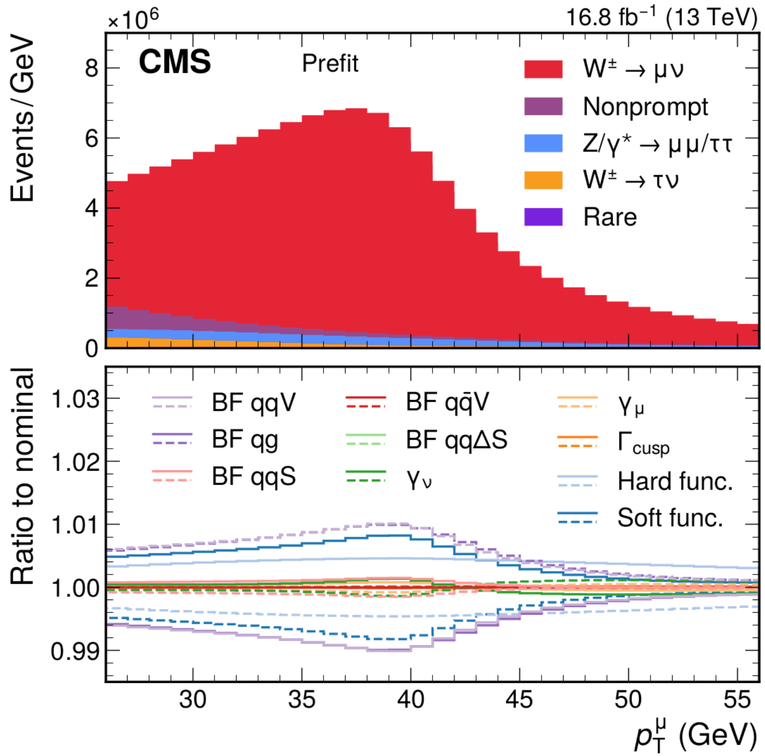
W-mass extraction: using Z as a test



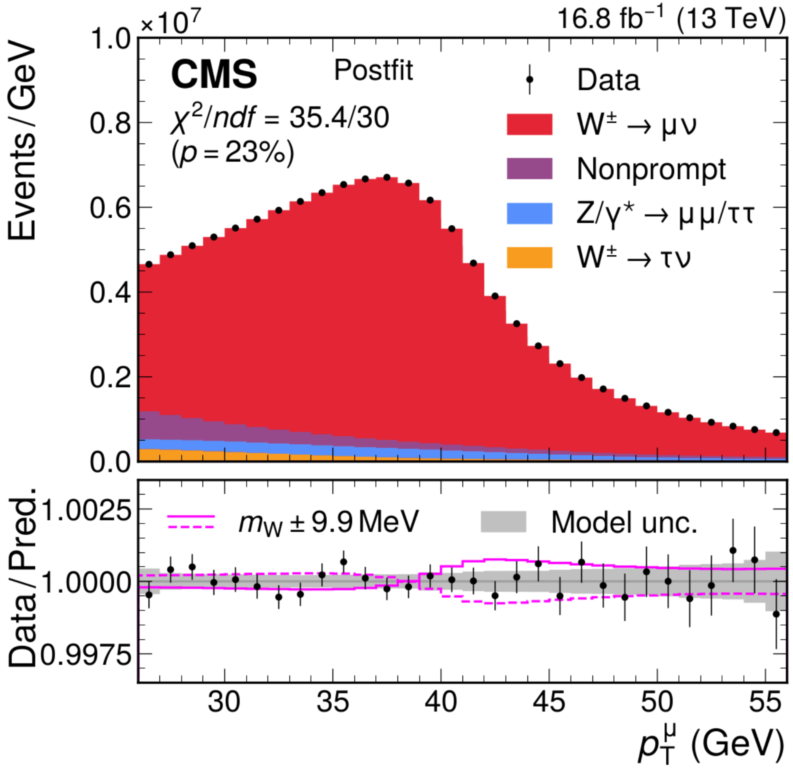
Using pT(Z) vs. W-like mZ fit
→ good consistency
→ demonstrates that pT(l)xeta(l)xq is sensitive enough for a competitive constraint vs pT(V)xy(V)
→ mZ consistent with PDG → internal consistency check

→ checked also different TNP schemes, i.e. ways to parameterize the higher order terms

W result

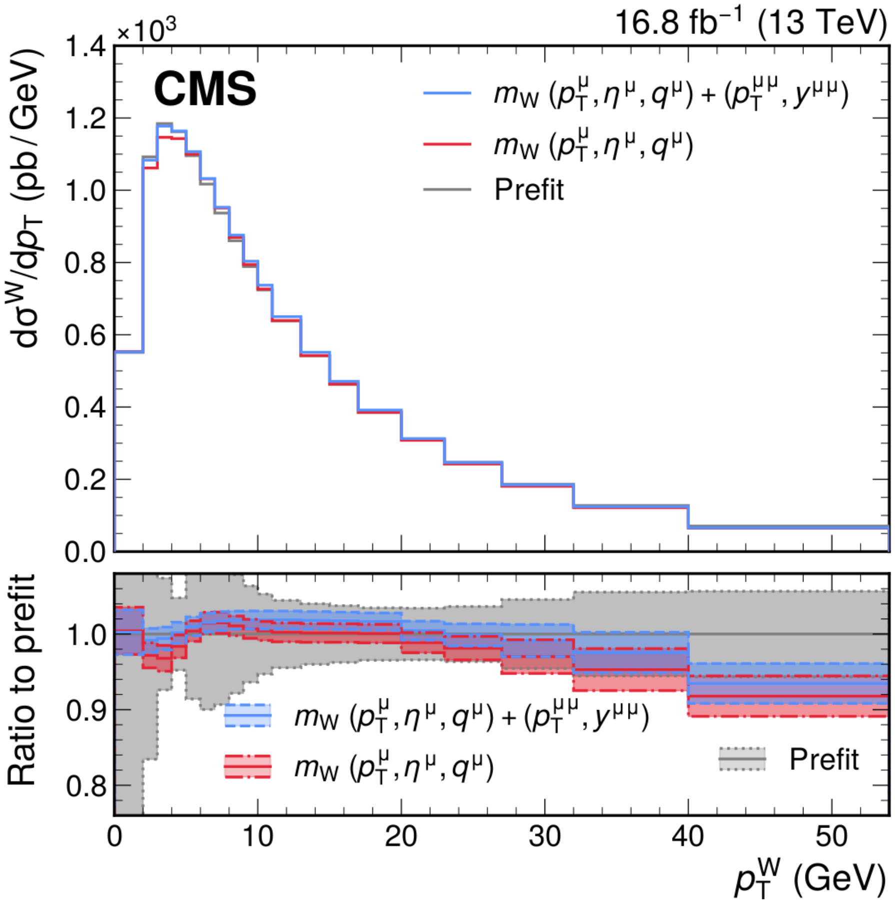


×10^6
16.8 fb^-1 (13 TeV)
CMS
Prefit
W± → μν
Nonprompt
Z/γ* → μμ/ττ
W± → τν
Rare
Events/GeV
Ratio to nominal
BF qqV
BF qg
BF qqS
BF qq̄V
BF qqΔS
γν
γμ
Γcusp
Hard func.
Soft func.
pT^μ (GeV)



×10^7
16.8 fb^-1 (13 TeV)
CMS
Postfit
χ²/ndf = 35.4/30
(p = 23%)
Data
W± → μν
Nonprompt
Z/γ* → μμ/ττ
W± → τν
Events/GeV
Data/Pred.
mW ± 9.9 MeV
Model unc.
pT^μ (GeV)

Combining W+Z?



Adding Z data:

→ only mildly impacts results

→ results are consistent: hints at consistent QCD modelling between W and Z

→ consistent constraints on non-pert. par.

Some thoughts...

Picture that emerges from the application of TNPs to qT-resummation:
simple ingredients that enter different computations/processes etc. → ideal situation

But actually not that simple:

- Scheme dependence of ingredients (canonical choices make life easier)?
  e.g. scales, IR subtraction, … → needs modified parametrisations
- Some TNPs represent directly numbers: $\Gamma,\ \gamma,\ H$ for simple processes
  but others are functions → Beam functions, hard functions for more complicated processes
- "Easy to implement" for use-cases so far
  → might be really expensive if each variation needs a expensive computation (Monte Carlos,...)

TNP approach for fixed-order computations

[Lim, Poncelet, 2412.14910]

$$\mathrm{d}\sigma = \alpha_s^n N_c^m \,\mathrm{d}\bar{\sigma}^{(0)} + \alpha_s^{n+1} N_c^{m+1} \,\mathrm{d}\bar{\sigma}^{(1)} + \alpha_s^{n+2} N_c^{m+2} \,\mathrm{d}\bar{\sigma}^{(2)} + \dots$$
$$= \alpha_s^n N_c^m \,\mathrm{d}\bar{\sigma}^{(0)} \left[1 + \alpha_s N_c \left(\frac{\mathrm{d}\bar{\sigma}^{(1)}}{\mathrm{d}\bar{\sigma}^{(0)}} \right) + \alpha_s^2 N_c^2 \left(\frac{\mathrm{d}\bar{\sigma}^{(2)}}{\mathrm{d}\bar{\sigma}^{(0)}} \right) + \dots \right]$$

Observation, i.e. “expert knowledge”: $\frac{\mathrm{d}\bar{\sigma}^{(i)}}{\mathrm{d}\bar{\sigma}^{(0)}} \sim \mathcal{O}(1)$

Use some knowledge about lower orders but introduce parametric dependence:

$$\frac{\mathrm{d}\bar{\sigma}^{(N+1)}_{\mathrm{TNP}}}{\mathrm{d}\bar{\sigma}^{(0)}} = \sum_{j=1}^{N} f_k^{(j)}\left(\vec{\theta}, x\right) \left(\frac{\mathrm{d}\bar{\sigma}^{(j)}}{\mathrm{d}\bar{\sigma}^{(0)}} \right)$$

Approximation → there is only $f_i(\hat{\theta}) \approx \hat{f}_i$
→ correspond to the extreme case with
all other dependencies are integrated out

See also MSHT [2207.04739]

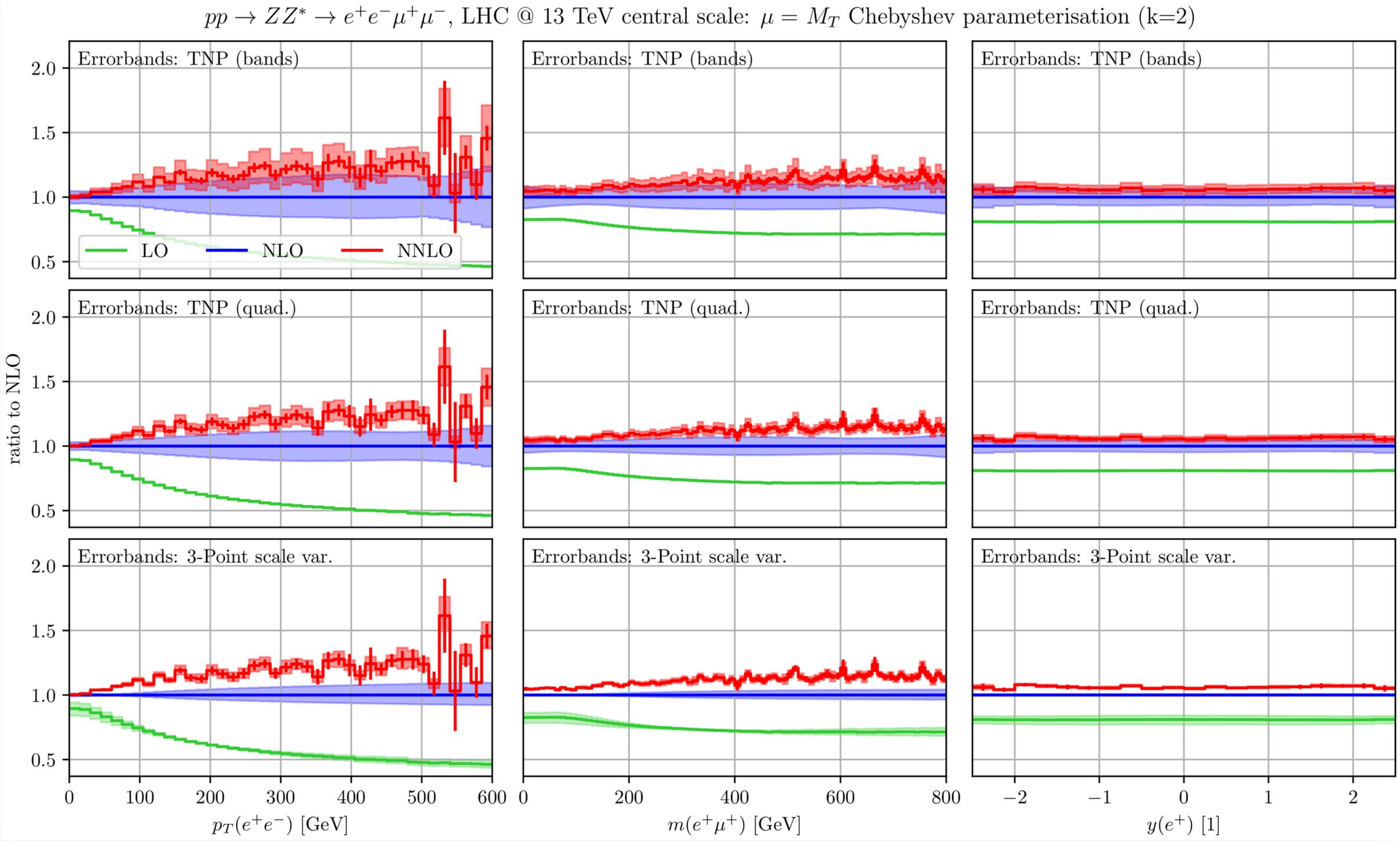
Bernstein: $f_k^B\left(\vec{\theta}, x\right) = \sum_{i=0}^{k} \theta_i \binom{k}{i} x^{k-i} (1-x)^i$

$x \in [0,1]$

Chebyshev: $f_k^C\left(\vec{\theta}, x\right) = \frac{1}{2} \sum_{i=0}^{k} \theta_i T_i(x)$

$x \in [-1,1]$

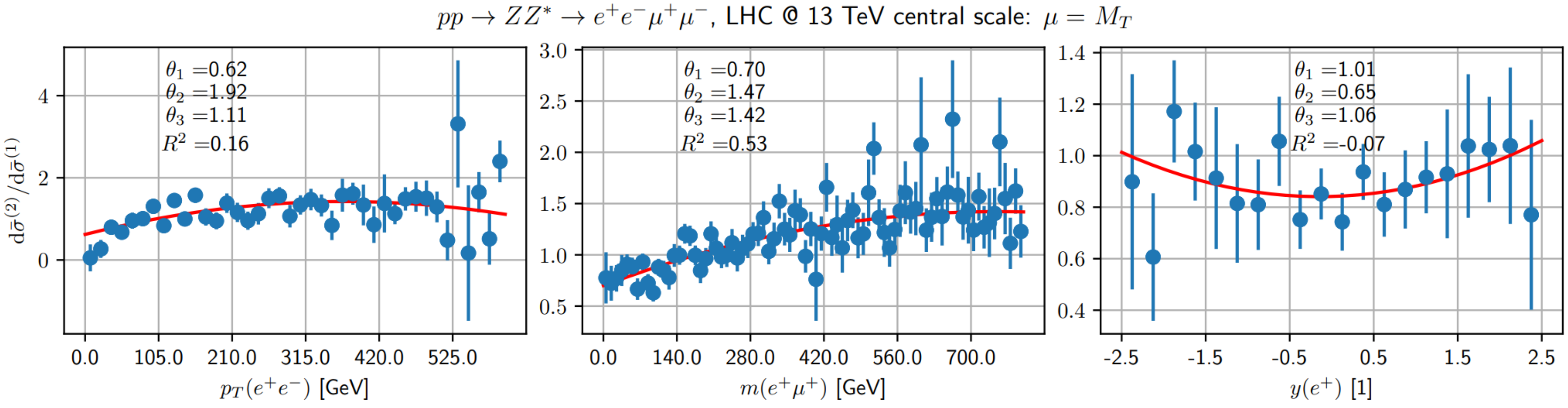
Uncertainties from TNPs - ZZ



Band: sample
$\theta \in [-1, 1]$
Quad: add individual
$\theta = \pm 1$
in quadrature
(→ 65% CL)

Example of TNP fit: pp → ZZ

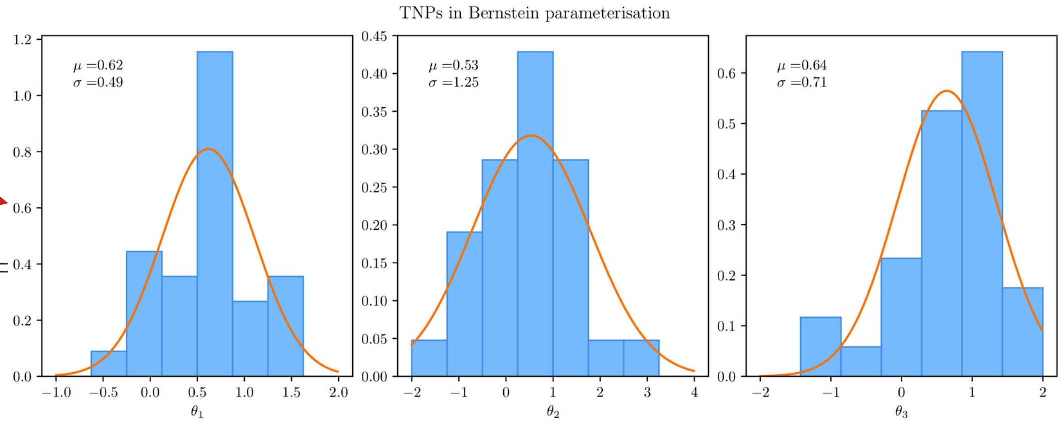
$$\frac{d\bar{\sigma}^{(N+1)}_{\text{TNP}}}{d\bar{\sigma}^{(0)}} = \sum_{j=1}^{N} f_k^{(j)}\left(\vec{\theta}, x\right)\left(\frac{d\bar{\sigma}^{(j)}}{d\bar{\sigma}^{(0)}}\right) \qquad f_k^B\left(\vec{\theta}, x\right) = \sum_{i=0}^{k} \theta_i \begin{pmatrix} k \\ i \end{pmatrix} x^{k-i}(1-x)^i$$

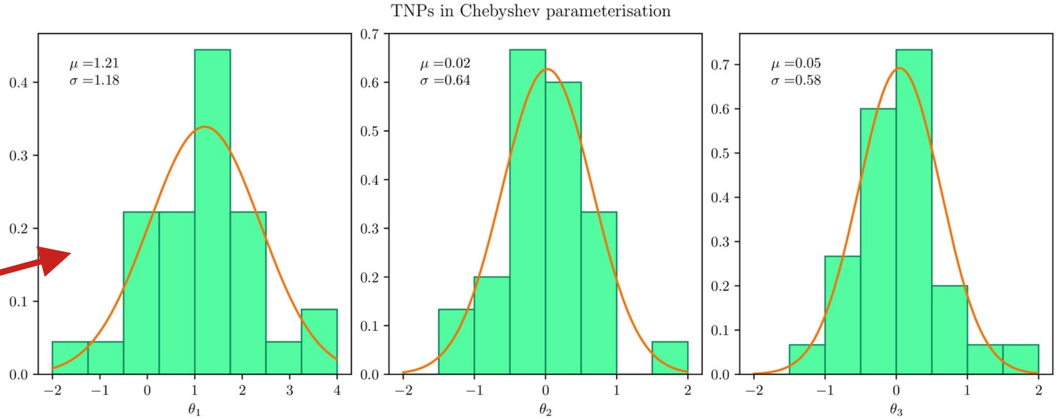


Fits - meta-study

$$f_k^B\left(\vec{\theta},x\right) = \sum_{i=0}^{k} \theta_i \begin{pmatrix} k \\ i \end{pmatrix} x^{k-i}(1-x)^i$$

Process	Distributions
$pp \to H$ (full theory)	y_H
$pp \to ZZ^* \to e^+e^-\mu^+\mu^-$	$M_{e^+\mu^+}$, $p_T^{e^+e^-}$, y_{e^+}
$pp \to WW^* \to e\nu_e\mu\nu_\mu$	M_{WW}, $p_T^{\mu^-}$, y_{W^-}
$pp \to (W \to \ell\nu) + c$	p_T^ℓ ,$\lvert y_\ell \rvert$,
$pp \to t\bar{t}$	$M_{t\bar{t}}$, p_T^t, y_t
$pp \to t\bar{t} \to b\bar{b}\ell\bar{\ell}$	$M_{\ell\bar{\ell}}$, $p_T^{b_1}$, $p_{T,\ell_1}/p_{T,\ell_2}$
$pp \to \gamma\gamma$	$M_{\gamma\gamma}$, $p_T^{\gamma_1}$, $y_{\gamma\gamma}$
$pp \to \gamma\gamma j$	$M_{\gamma\gamma}$, $p_T^{\gamma\gamma}$, $\cos\phi_{\mathrm{CS}}$, $\lvert y_{\gamma_1}\rvert$, $\Delta\phi_{\gamma\gamma}$
$pp \to jjj$	EEC with $H_{T,2} \in [1000,1500), [1500,2000), [3500,\infty)$
$pp \to \gamma jj$	$M_{\gamma jj}$, p_T^j, $\lvert y_{\gamma-\mathrm{jet}}\rvert$, $E_{T,\gamma}$

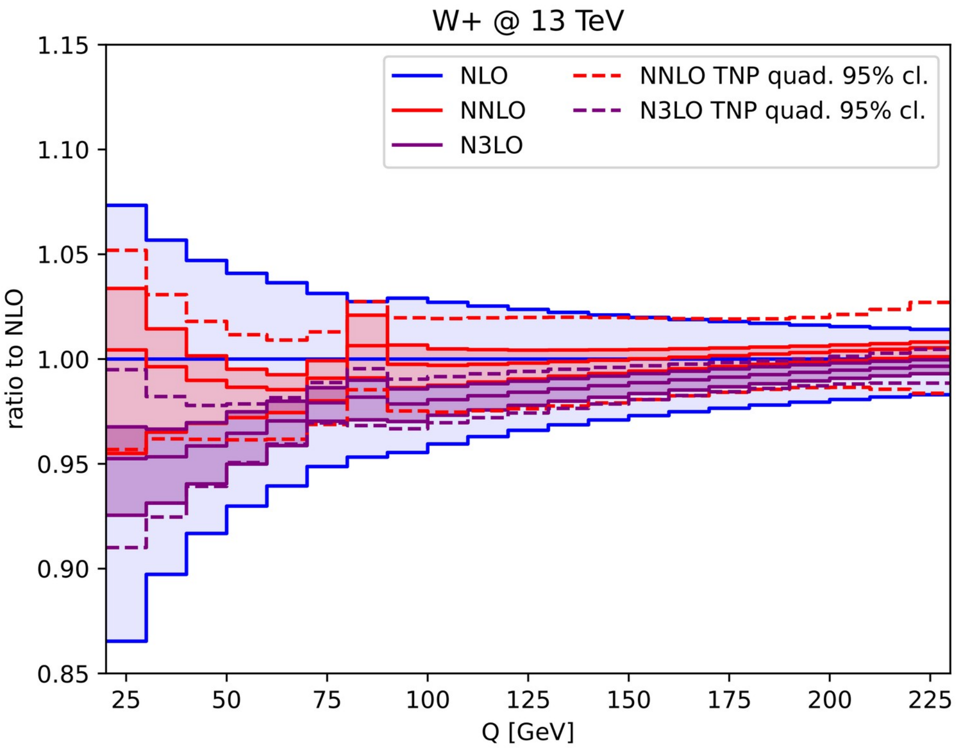


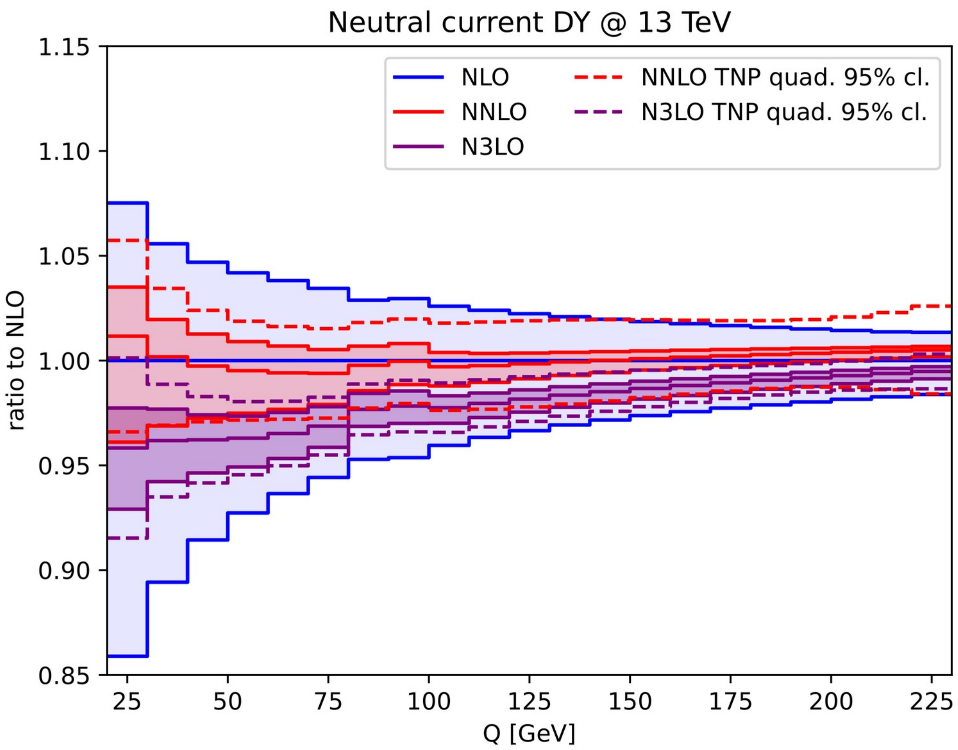


$$f_k^C\left(\vec{\theta},x\right) = \frac{1}{2}\sum_{i=0}^{k} \theta_i T_i(x)$$

N3LO example: Drell-Yan

Numbers from n3loxs [Baglio, Duhr, Mistlberger, Szafron'22]





Points for discussion, caveats and open questions

Some arising questions regarding fixed-order model: $\frac{\mathrm{d}\bar{\sigma}^{(N+1)}_{\mathrm{TNP}}}{\mathrm{d}\bar{\sigma}^{(0)}} = \sum_{j=1}^{N} f_k^{(j)}\left(\vec{\theta}, x\right)\left(\frac{\mathrm{d}\bar{\sigma}^{(j)}}{\mathrm{d}\bar{\sigma}^{(0)}}\right)$

- How does the uncertainty estimate depend on the central scale choice?
  → bad scale choices lead to large uncertainties by construction due to large corrections.
- What about NLO uncertainty if $\mathrm{d}\bar{\sigma}^{(1)} = 0$ for given scale?
  → amend parametrisation by j = 0 term.
- Each parametrisation is for one observable at a time:
  How to deal with higher dimensional distributions? Consistency upon integration?
  → WIP
- What about EW corrections?
  → Sudakov logs should work well! → Radiation from resonances more difficult.
- **How to correlate different processes at fixed-order**?
  → ???, would require something like: $\mathrm{d}\bar{\sigma}^{(n)}(\theta) = \mathrm{d}\Phi \left\langle M^0 \middle| \mathcal{P}(\theta) \middle| M^0 \right\rangle$ $\mathcal{P}(\theta)$ → process-independent “operator”
- How sensitive are we to the parametrisation? How many terms?
  → two quite general parametrisations tested, increase degree by demand.

Take home message

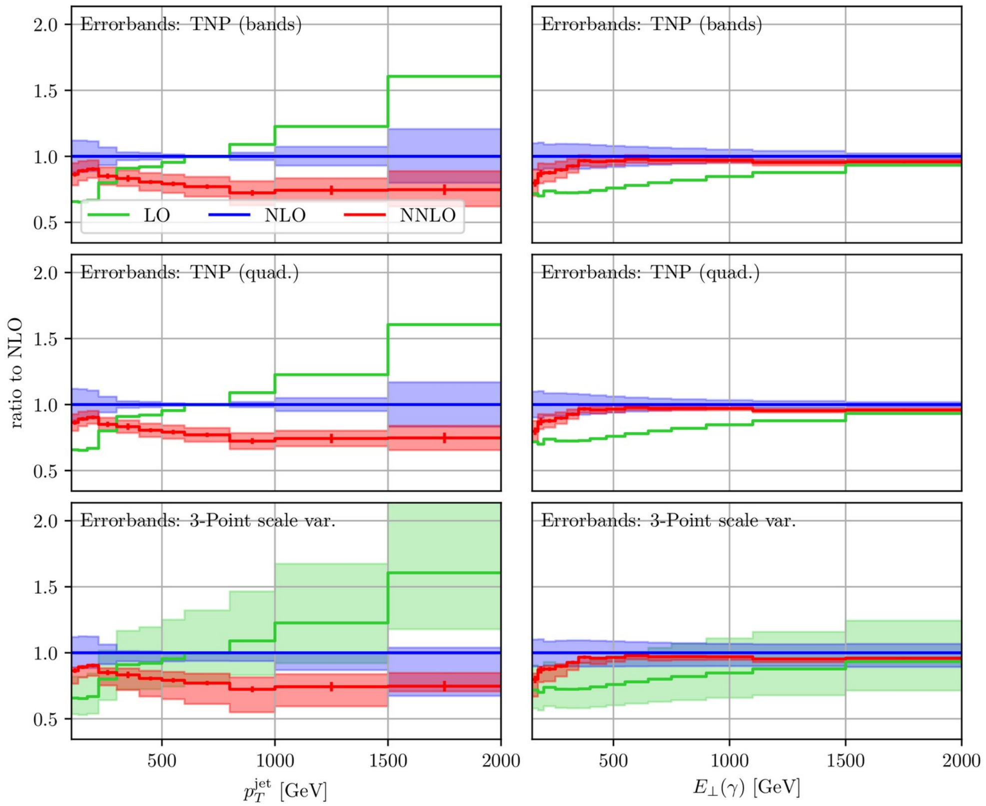
- Increasing precision demands accurate theory uncertainty estimates
- De-facto standard: scale variations
  → various short-comings: robustness, no statistical interpretation, correlations,...
- Alternative approaches to scale variations: Bayesian and TNP approach
- **Theory Nuisance Parameters**
  - In principle less biased, better correlations → does not depend on any "known" orders … however needs "expert knowledge"
  - Allows for a statistical interpretation and constraints from data!
  - Fixed-order tricky, not much knowledge about higher-order terms
    Proposed approx. TNP parametrisation of differential cross sections shows promising first results → next step: application to an actual parameter fit

Is this the ultimate answer? Surely not, but a step in the right direction!

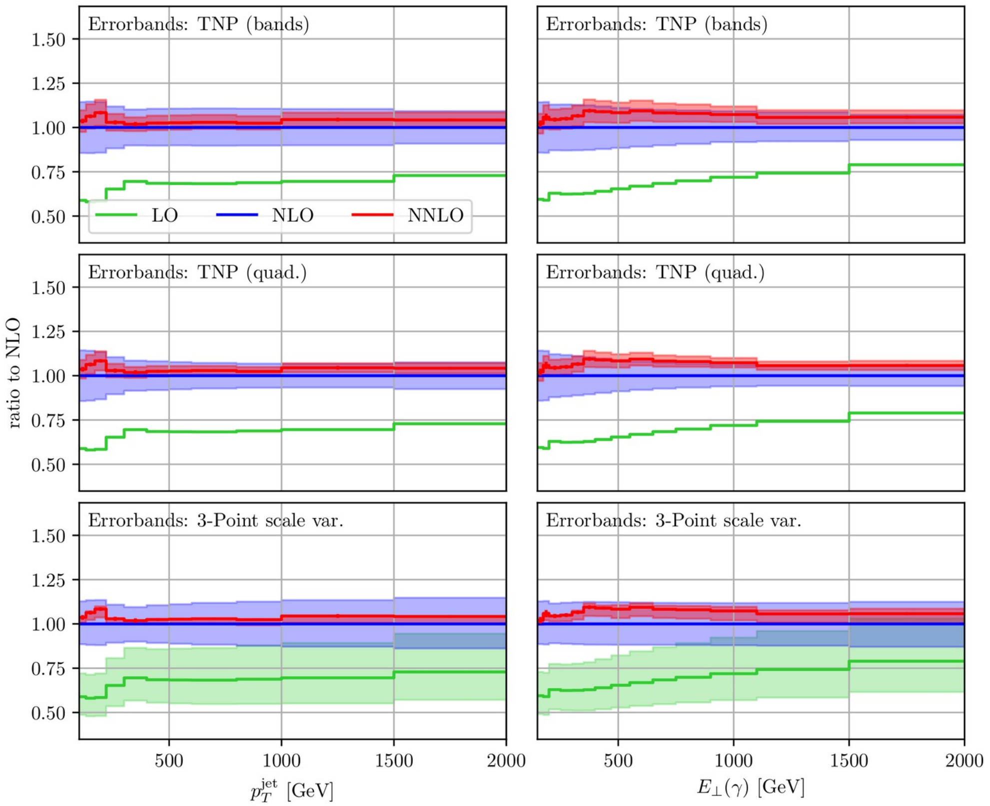
Backup

Challenging case

“Bad” scale choice $\mu = E_T$ no j=0 term

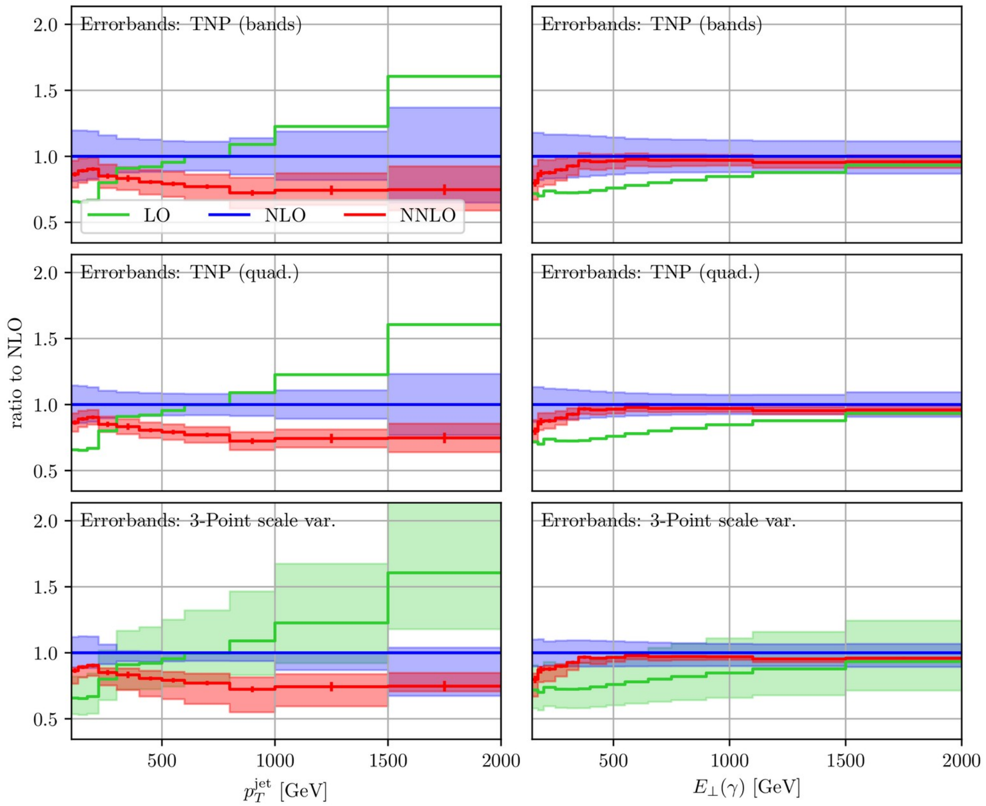


“Good” choice $\mu = H_T$ no j=0 term

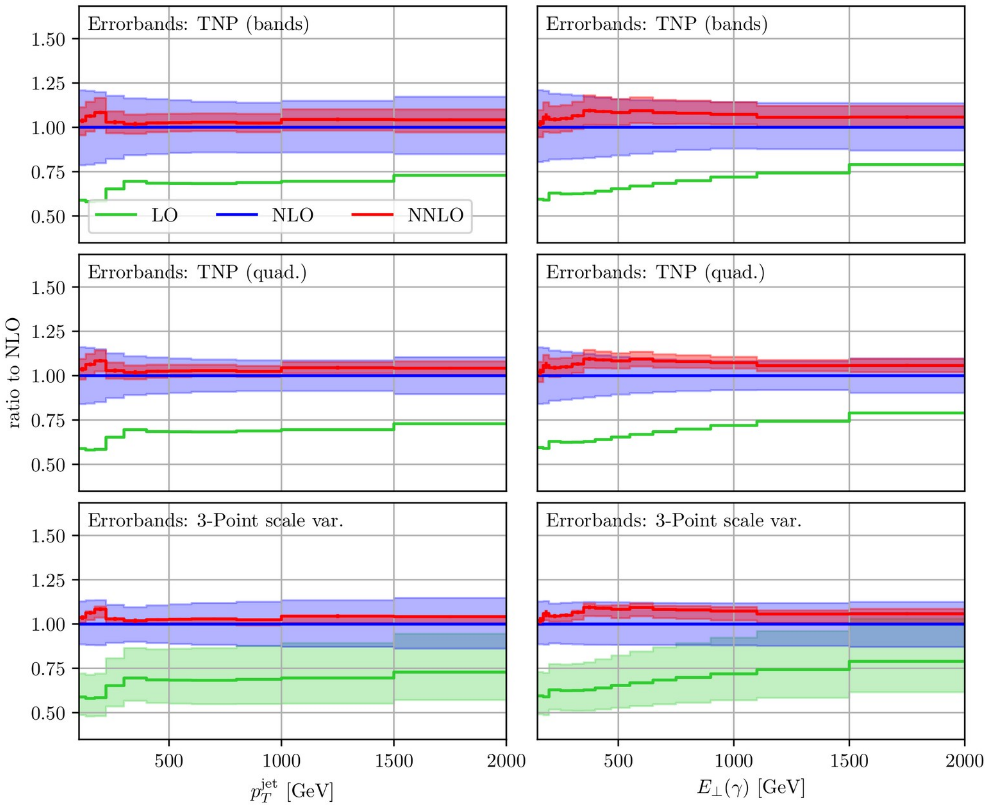


Challenging case → extended parametrisations

“Bad” scale choice $\mu = E_T$ with j=0 term

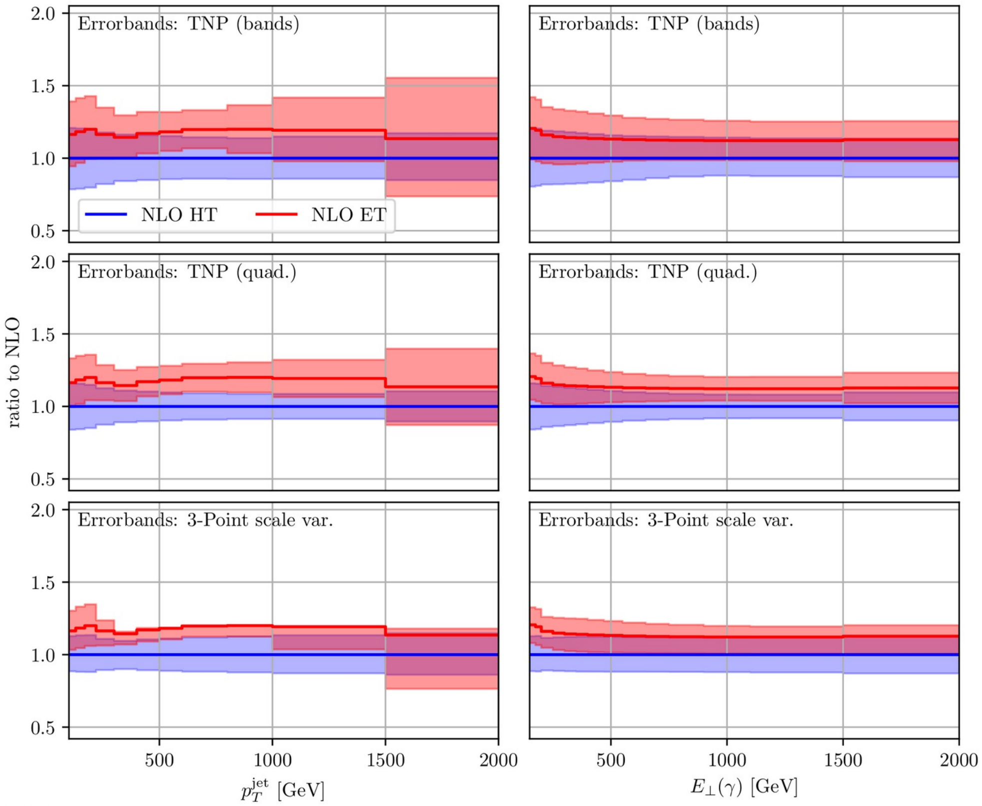


“Good” choice $\mu = H_T$ with j=0 term

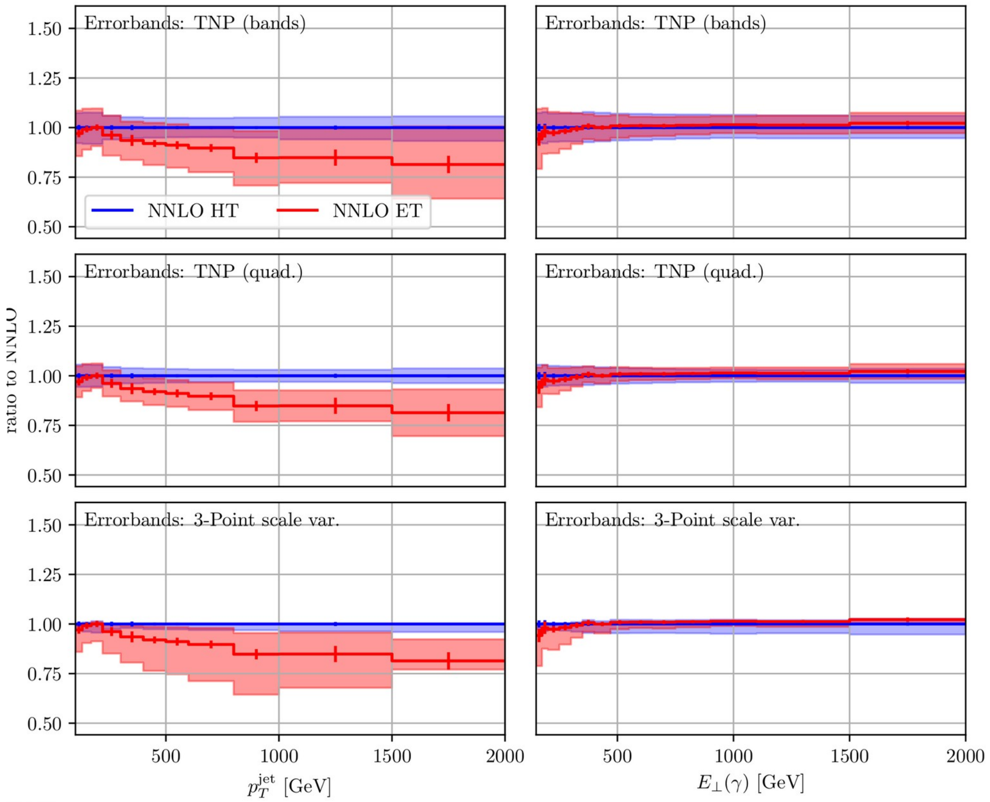


Challenging case → comparisons

NLO QCD



NNLO QCD



Bayesian approach I

→ Instead of ad-hoc fixed variation try to give some probabilistic interpretation

$$d\sigma = d\sigma^{(0)}(1 + \delta^{(1)} + \delta^{(2)} + \dots)$$

Probability to find coefficient $\delta^{(n+1)}$ given $\delta^{(n)}$: [Cacciari,Houdeau 1105.5152]

$$P(\delta^{(n+1)}|\delta^{(n)}) = \frac{P(\delta^{(n+1)})}{P(\delta^{(n)})} = \frac{\int da P(\delta^{(n+1)}|a)P_0(a)}{\int da P(\delta^{(n)}|a)P_0(a)}$$

Need to provide model and prior

Bayes: $P(A|B) = P(B|A)P(A)/P(B)$ with: $P(\delta^{(n)}|\delta^{(n+1)}) = 1$

CH model: $\delta_k = c_k \alpha_s^k$ c_k come from geometric series: $|c_k| \le \bar{c} \quad \forall k$

Geometric model: $|\delta_k| \le c a^k \quad \forall k$ [Bonvini 2006.16293]

abc model: $b - c \le \frac{\delta_k}{a^k} \le b + c \quad \forall k$ [Duhr, Huss, Mazeliauskas, Szafron 2106.04585]

Bayesian approach II

Inclusion of scale dependence: $$P(\delta_{n+1}|\delta_n) = \int \mathrm{d}\mu P(\delta_{n+1}|\delta_n;\mu)P(\mu|\delta_n)$$

Scale marginalisation (the scale becomes a model parameter)

$$\mathcal{P}_{\rm sm}(\Sigma|\boldsymbol{\Sigma}_n) \approx \frac{\int d\mu\, P(\Sigma - \Sigma_n(\mu)|\boldsymbol{\Sigma}_n(\mu))\, P(\boldsymbol{\Sigma}_n(\mu))\, P_0(\mu)}{\int d\mu'\, P(\boldsymbol{\Sigma}_n(\mu'))\, P_0(\mu')}\,.$$

$\mu_{\rm FAC}$

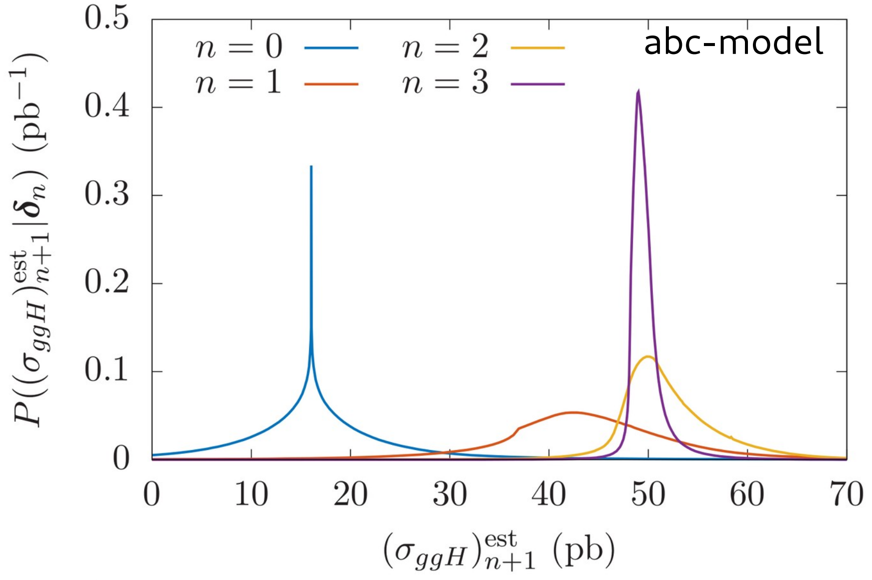
Scale average (the results are averaged with weight function)

$$\mathcal{P}_{\rm sa}(\Sigma|\boldsymbol{\Sigma}_n) \approx \int d\mu\, w(\mu)\, P(\Sigma - \Sigma_n(\mu)|\boldsymbol{\Sigma}_n(\mu))$$

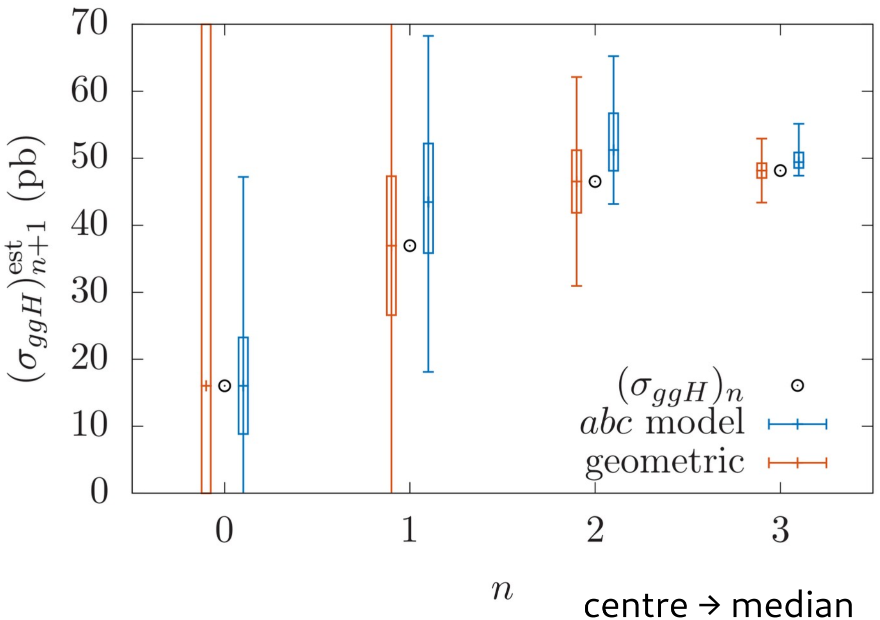
$\mu_{\rm PMS}$

Bayesian approach III

Example: Higgs production in gluon - fusion



Comparison of different unc. estimates:



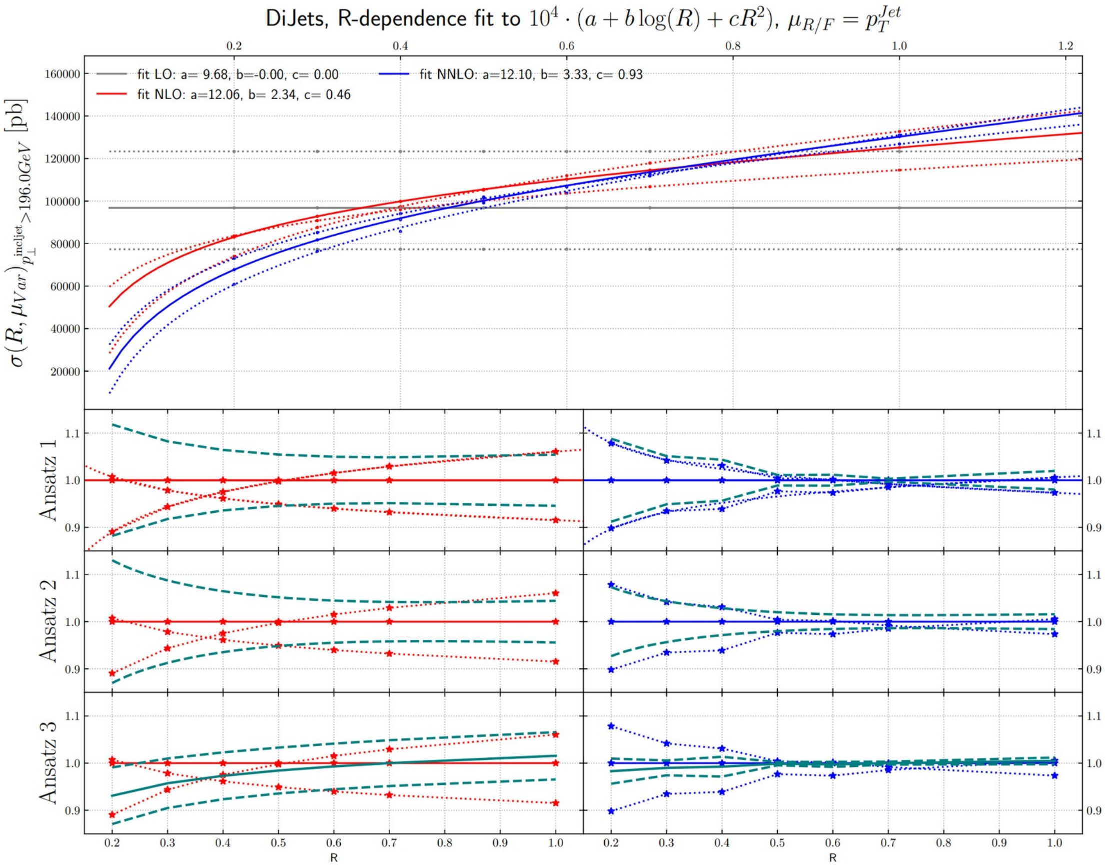
centre → median
error box → 68% CI
error bar → 95% CI

Example: inclusive jet production

→ Important process for PDF fits: sensitivity to gluon PDF at large-x

→ NNLO QCD corrections imply very small theory uncertainty

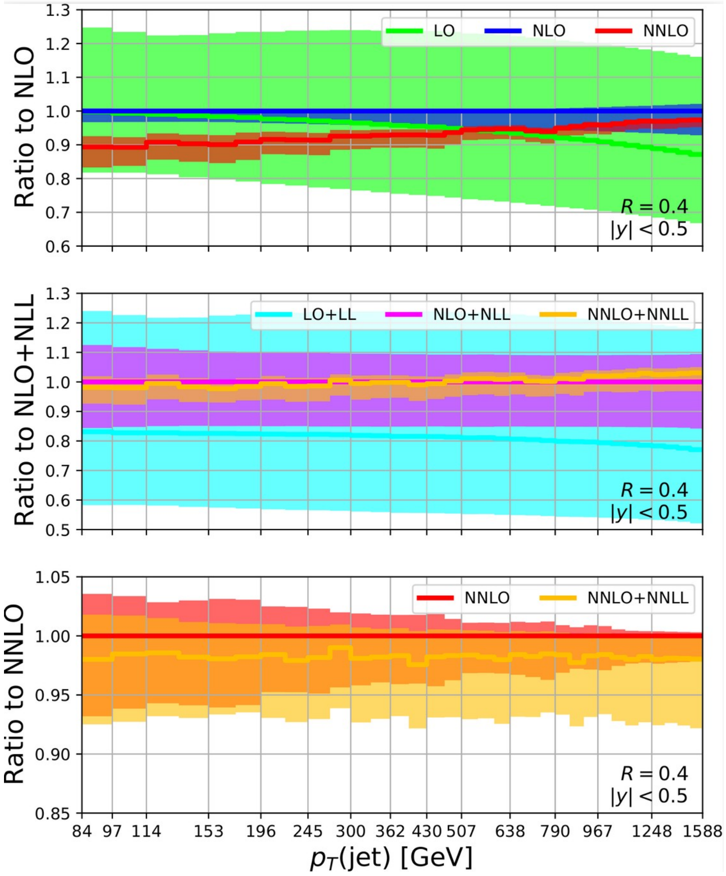
→ Significant jet radius dependence of uncertainties from scale variations



[1903.12563 Bellm et al]

Inclusive jet production: small-R resummation NNLO+NNLL

[Generet, Lee, Moult, Poncelet, Zhang'25]



FO scale variations

R=0.4

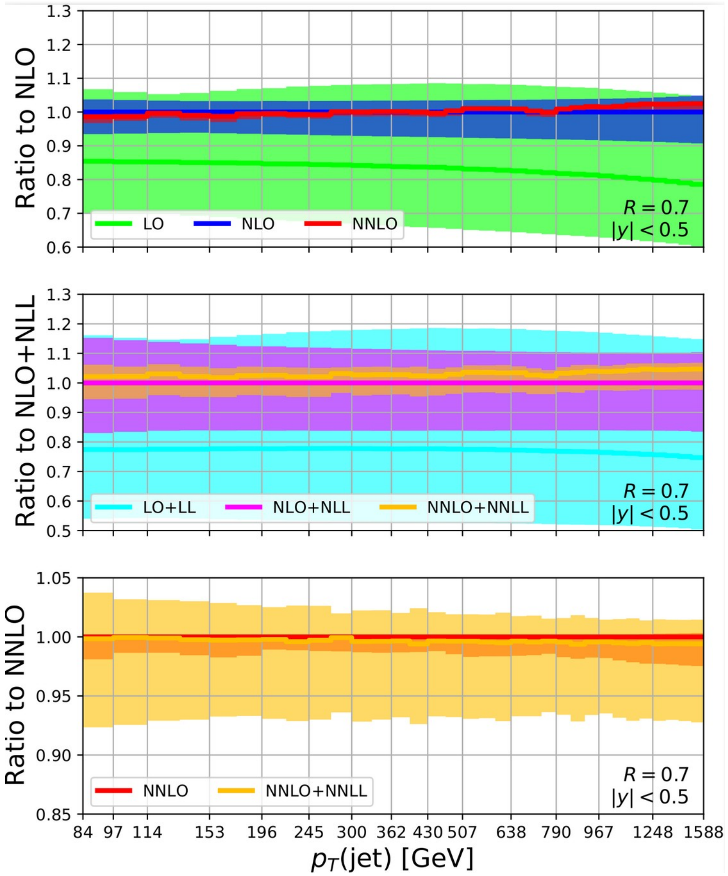
→ underestimation of NNLO correction

R=0.7

→ very small NNLO uncertainty

Resummation

→ stabilization of pert. series and uncertainties.

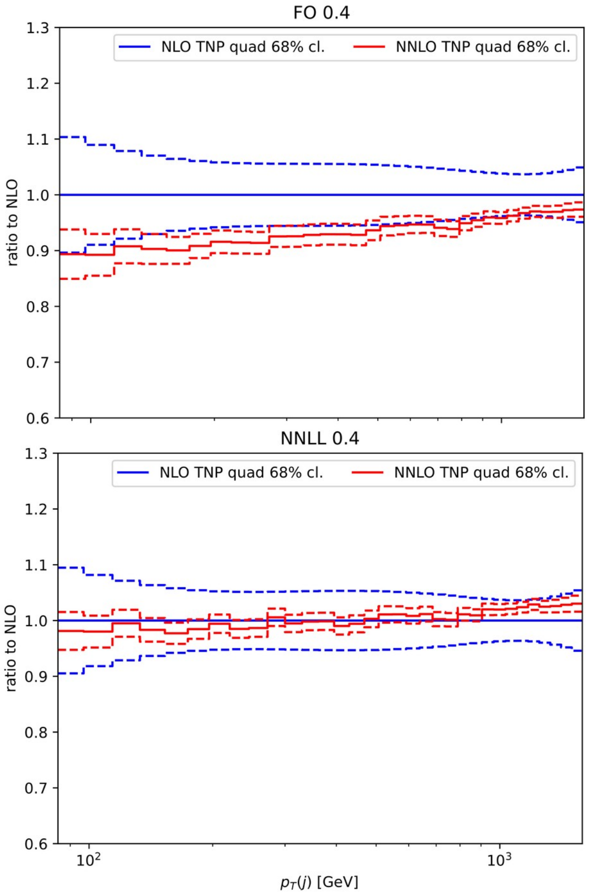


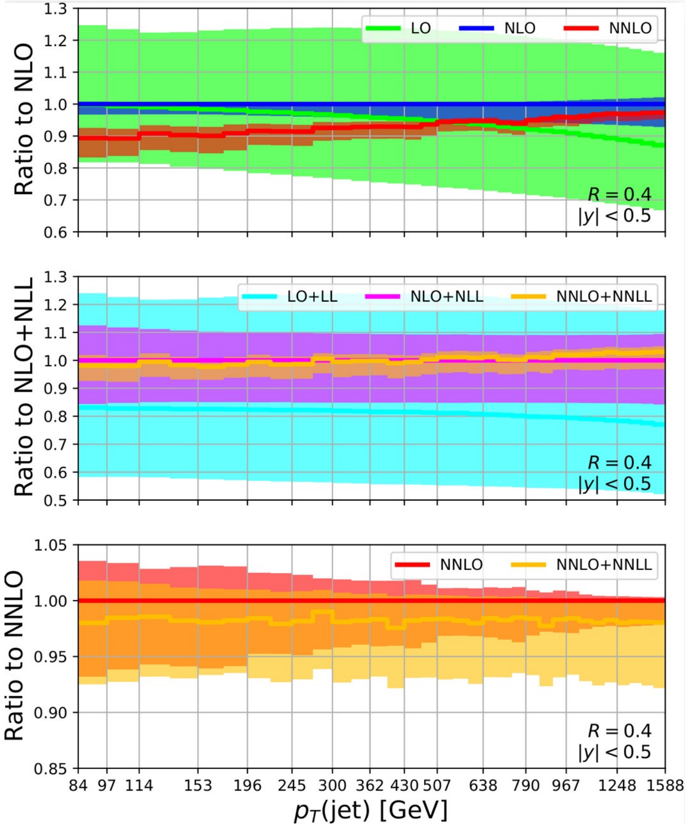
TNP uncertainties for inclusive jet production

R = 0.4

TNP uncertainties

- More sensible NLO uncertainties
- Similar to resummed scale variation



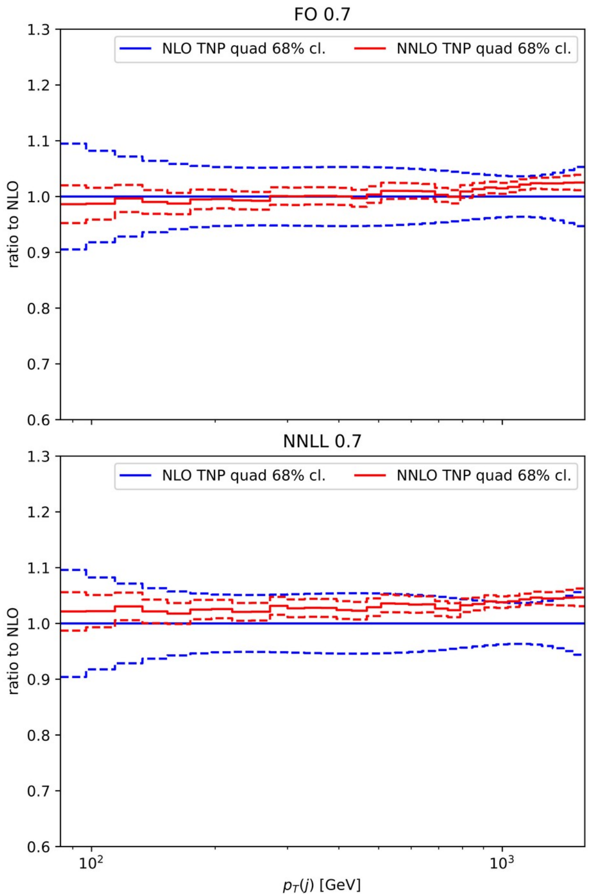


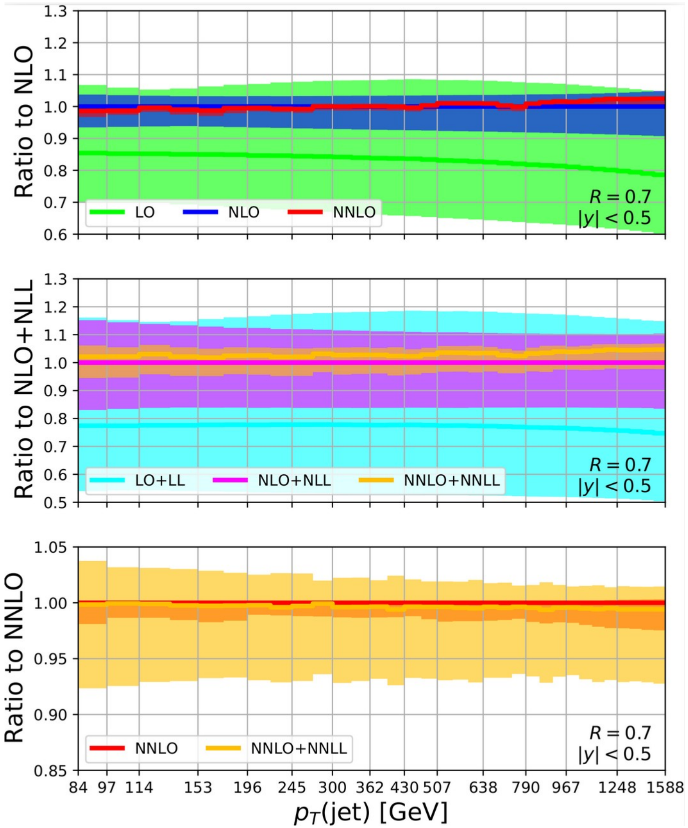
TNP uncertainties for inclusive jet production

R = 0.7

TNP uncertainties

- More sensible NNLO QCD uncertainties
- Similar to resummed scale variation





A more realistic approach

Thanks to Terry Generet to put this together!

The pT spectrum is a steeply falling function → effectively only few Mellin moments contribute

$$\frac{d\sigma}{dp_T} \approx \sum_{a,b} L_{ab}(\hat{E}/E = 2p_T/E)\frac{d\hat{\sigma}_{ab}}{dp_T}(N = \tilde{n}(2p_T/E))$$

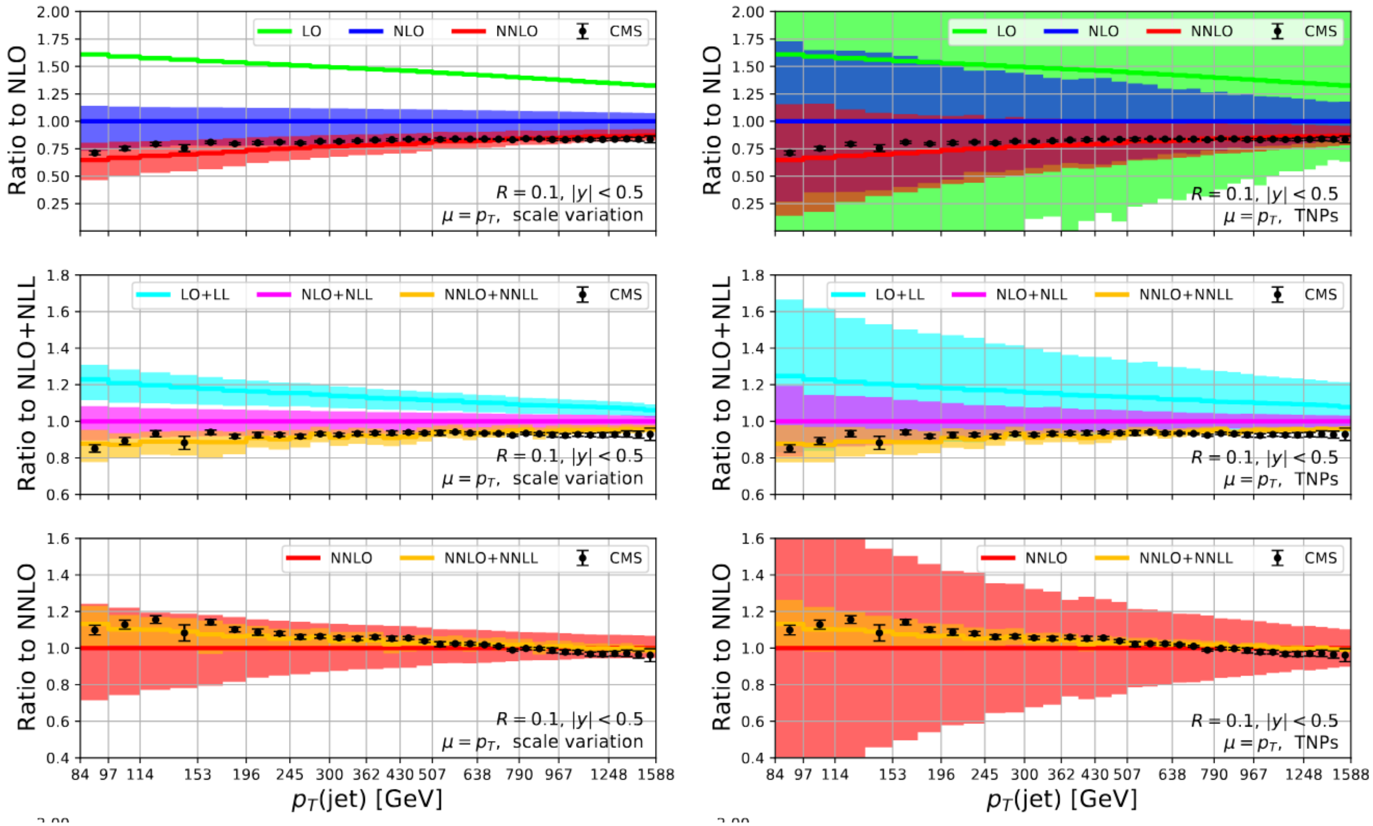
$$\begin{aligned} d\hat{\sigma}_{ab\to cd}(N) = {} & J_{\text{in}}^{(a)}\left(\frac{\hat{s}}{N_{0a}^2\mu^2}, \alpha_s(\mu)\right) J_{\text{in}}^{(b)}\left(\frac{\hat{s}}{N_{0b}^2\mu^2}, \alpha_s(\mu)\right) \\ & \times J_{\text{fr}}^{(c)}\left(\frac{\hat{s}}{N_0^2\mu^2}, \alpha_s(\mu)\right) J_{\text{rec}}^{(d)}\left(\frac{\hat{s}}{N_0\mu^2}, \frac{\hat{s}}{N_0^2\mu^2}, \alpha_s(\mu)\right) \\ & \times \text{Tr}\left[\mathbf{H}_{ab\to cd}\left(\frac{\hat{s}}{\mu^2}, \alpha_s(\mu)\right)\mathbf{S}_{ab\to cd}\left(\frac{\hat{s}}{N_0^2\mu^2}, \alpha_s(\mu)\right)\right] + \mathcal{O}\left(\frac{1}{N}\right) \end{aligned}$$

These then can be broken down into scalar series:
(soft+hard functions require approx. of
colour matrix → error on the error)

$$\begin{aligned} J_{\text{in}}^{(i)}\left(\frac{\hat{s}}{N_{0i}^2\mu^2}, \alpha_s(\mu)\right) &= J_{\text{fr}}^{(i)}\left(\frac{\hat{s}}{N_{0i}^2\mu^2}, \alpha_s(\mu)\right) = R_i(\alpha_s(\mu)) \\ & \times \exp\left[\int_{\sqrt{\hat{s}}/N_{0i}}^{\mu} \frac{d\mu'}{\mu'}\left(A_i(\alpha_s(\mu'))\ln\left(\frac{\mu'^2 N_{0i}^2}{\hat{s}}\right) - \frac{1}{2}D_i(\alpha_s(\mu'))\right)\right] \end{aligned}$$

Theory uncertainties from TNPs for jets

Small R: expect fixed-order to fail and resummation to be stable



side note
these are ratios
(R/R=0.4),
TNPs allow
correct correlation!

Theory uncertainties from TNPs for jets

Intermediate R: observed small scale dependence → TNPs more realistic