NNLO QCD corrections for ttbar+jet production

Rene Poncelet

CMS top-quark workshop
30th January 2026

Higher-order QCD corrections to top-quark pair production in association with a jet,
Badger, Becchetti, Brancaccio, Czakon, Hartanto, Poncelet, Zoia
[arxiv:2511.11431]

Double virtual QCD corrections to $t\bar{t}$+jet production at the LHC,
Badger, Becchetti, Brancaccio, Czakon, Hartanto, Poncelet, Zoia
[arxiv:2511.11424]

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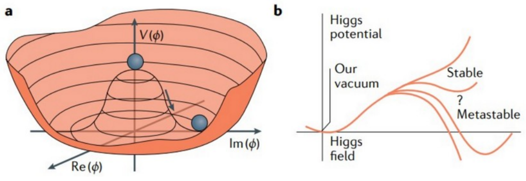
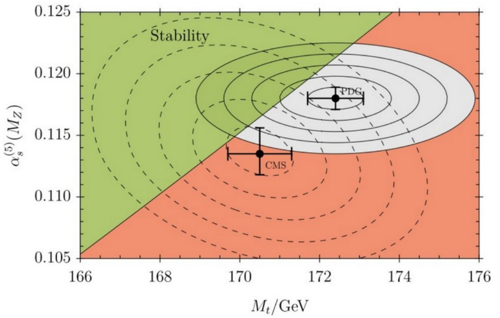
Why ttbar + jet?

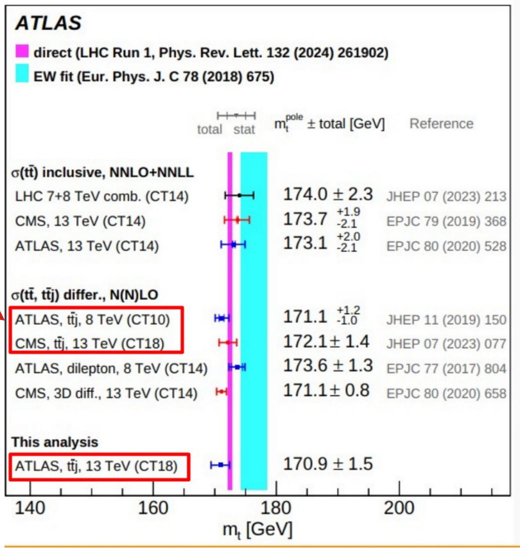
- 40% LHC top-quark pairs are produced in association with jets
  → many ttbar differential distributions are **directly sensitive** to ttbar+jet
- Additional opportunity to measure the **top-quark mass**, e.g. rho observable
  → precision determinations are crucial for SM tests
- QCD final state → important background for various rare SM processes or BSM searches

[Bass,DeRoeck,Kado
Nat Rev Phys 3,608–624(2021)]





[Hiller,Hohne,Litim,Steudtner 2401.08811]



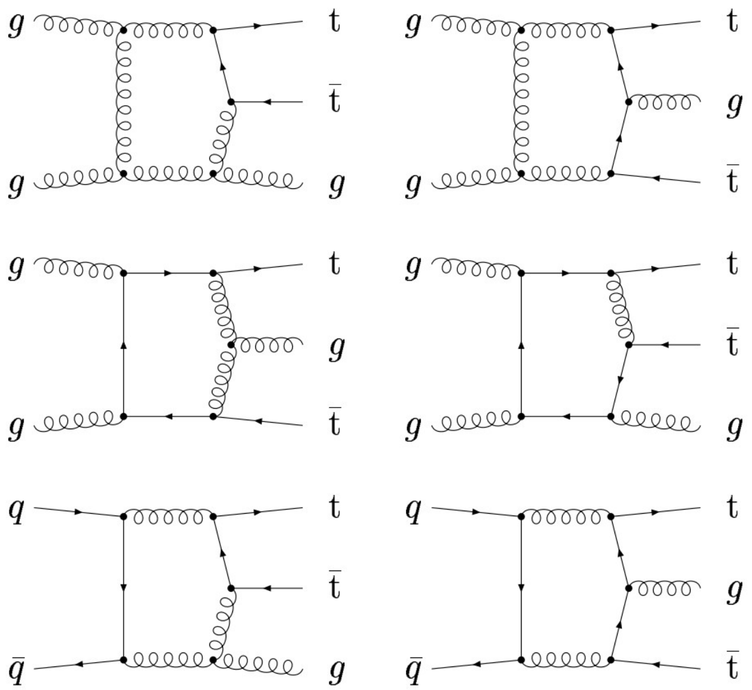
[ATLAS 2507.02632]

Theory predictions for top-quark pair + jet production I

NLO QCD (stable top)
- [Dittmaier,Uwer,Weinzierl (2007,2008)]

Merged NLO QCD+EW (stable top)
- [Gutschow,Lindert,Schonherr (2018)]

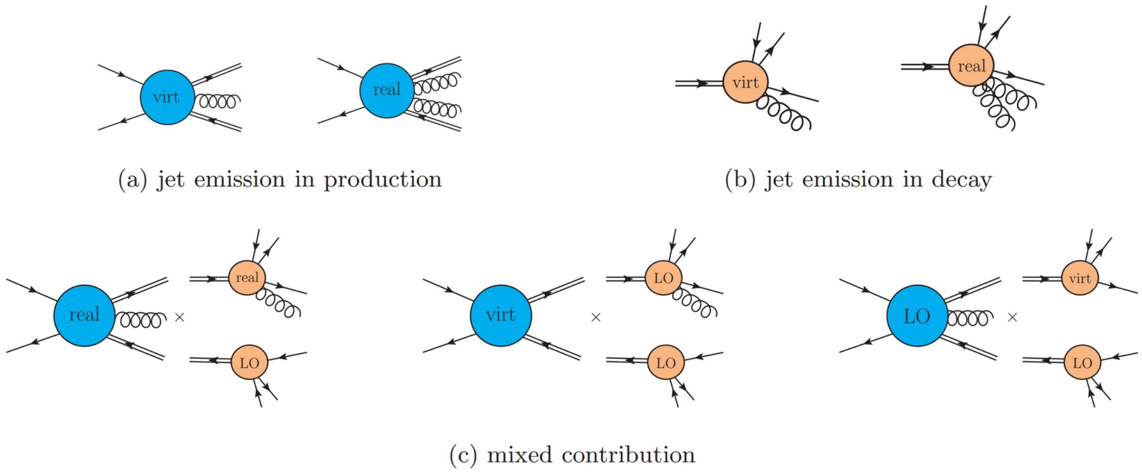
NLO QCD + PS
(+shower decays)
- [Kardos,Papadopoulos,Trocsanyi (2011)]
- [Alioli,Moch,Uwer (2011)]
- [Czakon,Hartanto,Kraus,Worek (2015)]

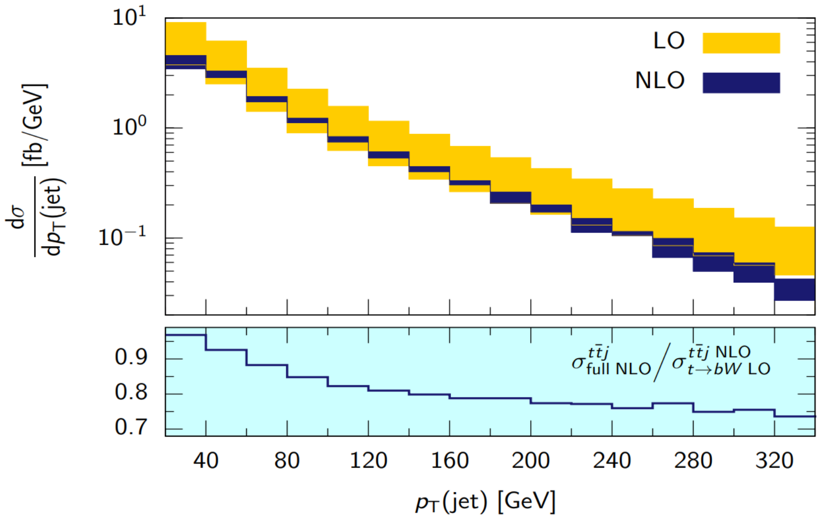


Theory predictions for top-quark pair + jet production II

NLO QCD (NWA)

- [Melnikov,(Scharf),Schulze (2010,2011)]

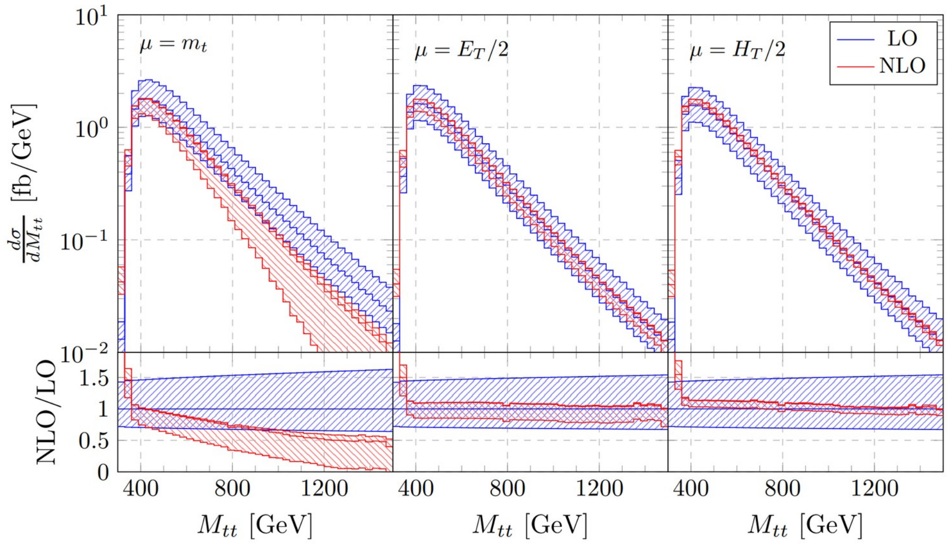




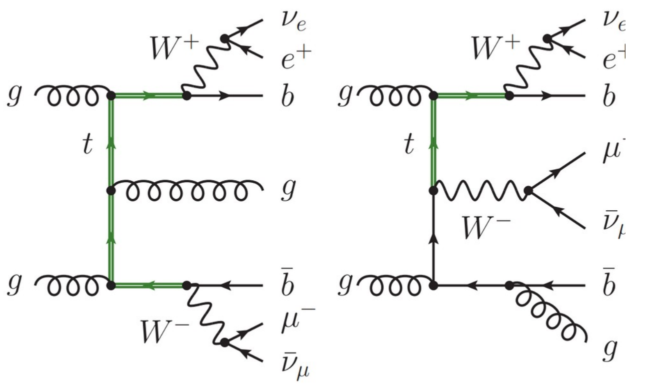
Theory predictions for top-quark pair + jet production III

NLO QCD (off-shell)

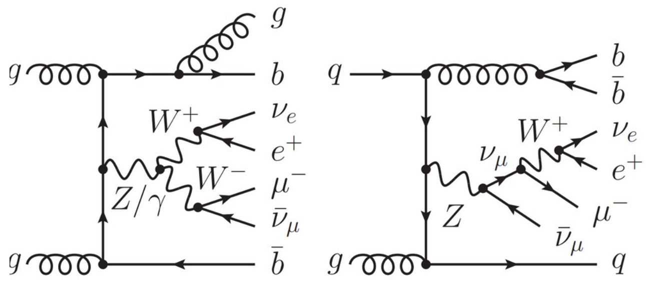
- [Bevilacqua,Hartanto,Kraus,Worek (2015,2016)]



double/single resonant



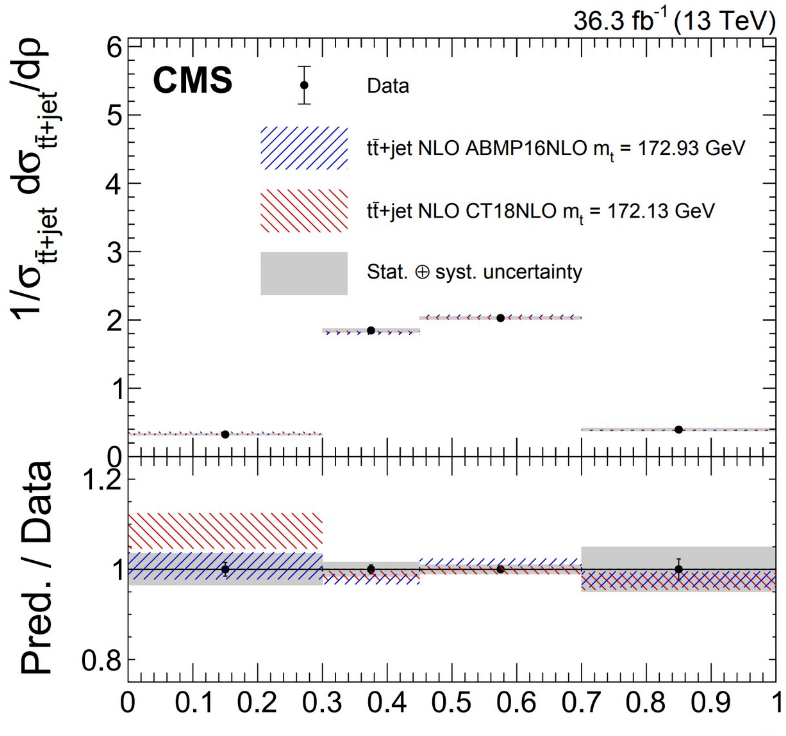
non resonant



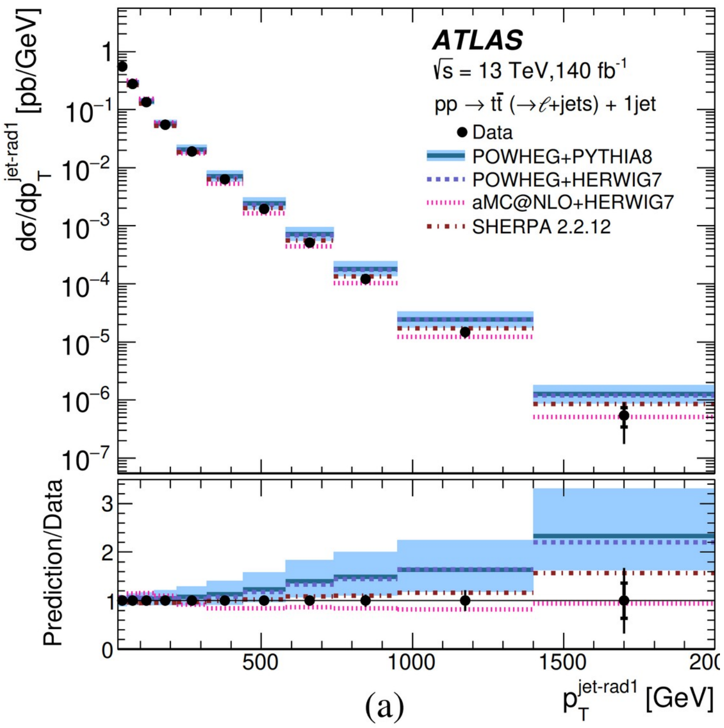
Why we need to go beyond NLO QCD?

Simple answer: the LHC experiments are doing an amazing job!

NLO QCD scale dependence sizeable compared to data uncertainties



[CMS 2207.02270]



[ATLAS 2406.19701]

Perturbative QCD through NNLO QCD

$$\sigma_{h_1 h_2 \to X} = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \phi_{i,h_1}(x_1, \mu_F^2) \phi_{j,h_2}(x_2, \mu_F^2) \underline{\hat{\sigma}_{ij \to X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)}$$

LO

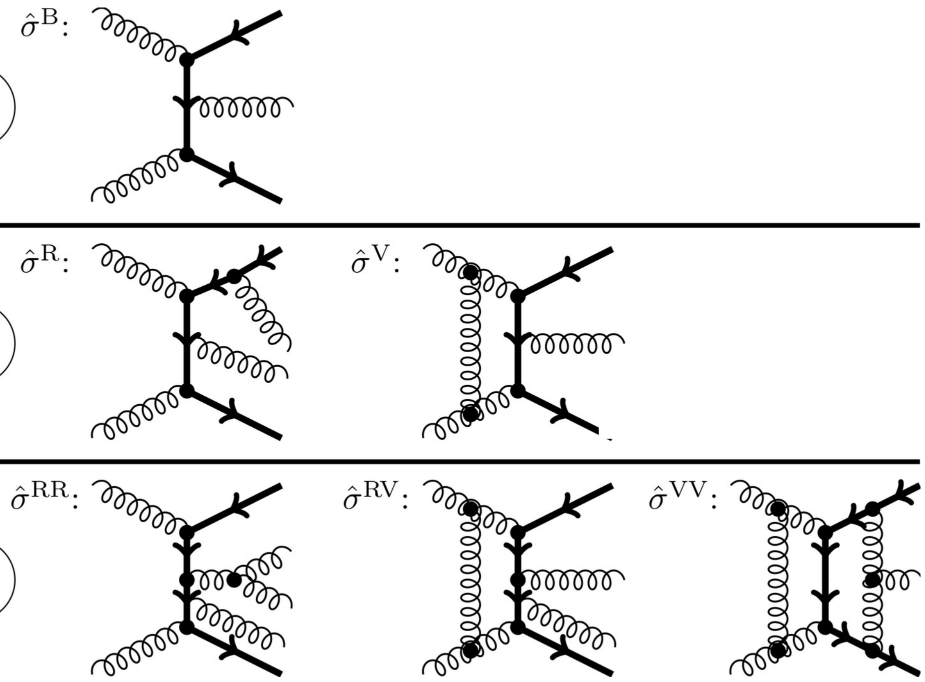
α_s^3

NLO

α_s^4

NNLO

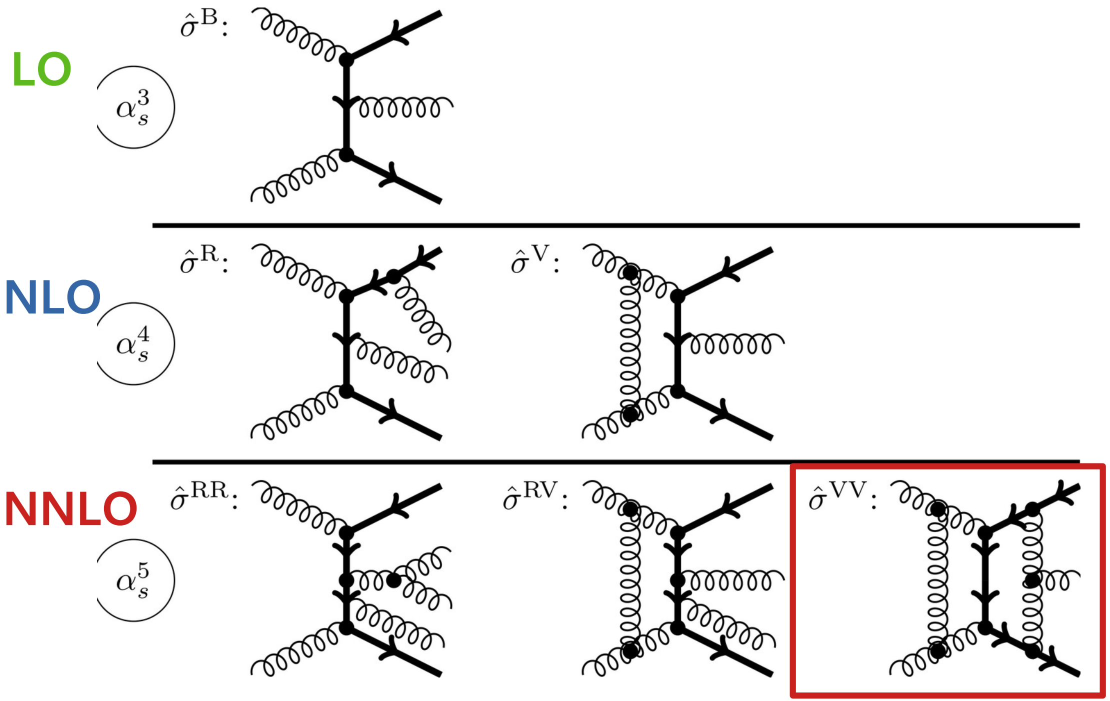
α_s^5



Partonic cross section:

$$\hat{\sigma}_{ab \to X} = \boxed{\hat{\sigma}^{(0)}_{ab \to X}} + \boxed{\hat{\sigma}^{(1)}_{ab \to X}} + \boxed{\hat{\sigma}^{(2)}_{ab \to X}} + \mathcal{O}(\alpha_s^3)$$

Perturbative QCD through NNLO QCD



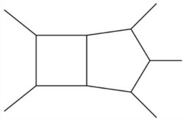
Multi-scale two-loop QCD amplitudes

→ **fast growing complexity:**
rational coef. and **special functions**

→ deeper understanding of the analytical properties

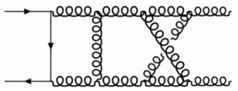
→ refinement of computational tools

Two-loop 5-point



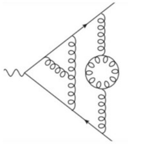
[Abreu, Agarwal, Badger, Buccioni, Chawdhry, Chicherin,Czakon, de Laurenties, Febres-Cordero, Gambuti, Gehrmann, Henn, Lo Presti, Manteuffel, Ma, Mitov, Page, Peraro, Poncelet, Schabiner Sotnikov, Tancredi, Zhang,...]

Three-loop 4-point



[Bargiela, Dobadilla, Canko, Caola, Jakubcik, Gambuti, Gehrmann, Henn, Lim, Mella, Mistleberger, Wasser, Manteuffel, Syrrakow, Smirnov, Trancredi, ...]

Four-loop 3-point

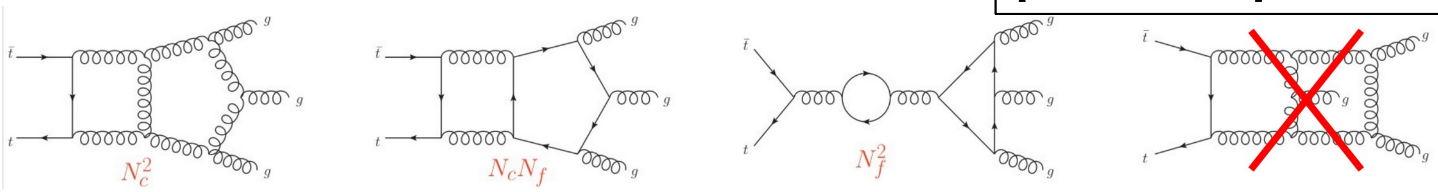


[Henn, Lee, Manteuffel, Schabinger, Smirnov, Steinhauser,...]

Virtual amplitudes

Sample diagrams

Double virtual QCD corrections to tt̄+jet production at the LHC,
Badger, Becchetti, Brancaccio, Czakon, Hartanto, **Poncelet**, Zoia
[arxiv:2511.11424]



Colour & spin structures

$$\mathcal{M}^{(L)}(1_{\bar{t}}, 2_t, 3_g, 4_g, 5_g) = \sqrt{2}\,\bar{g}_s^3 \left[(4\pi)^\epsilon e^{-\epsilon\gamma_E} \frac{\bar{\alpha}_s}{4\pi}\right]^L \times \sum_{\sigma \in Z_3} (t^{a_{\sigma(3)}} t^{a_{\sigma(4)}} t^{a_{\sigma(5)}})_{i_2}^{\bar{i}_1} \; A_g^{(L)}(1_{\bar{t}}, 2_t, \sigma(3)_g, \sigma(4)_g, \sigma(5)_g)$$

Retaining full spin information
→ allows to incorporate
spin-correlations in NWA decays

$$A_x^{(L)h_3h_4h_5} = \sum_{i=1}^{4} \Psi_i \, \mathcal{G}_{x;i}^{(L)h_3h_4h_5}$$

Leading colour expansion of
partial amplitudes:

$$\mathcal{F}_x^{(1)} = N_c \, F_x^{(1),N_c} + n_f \, F_x^{(1),n_f} \,,$$

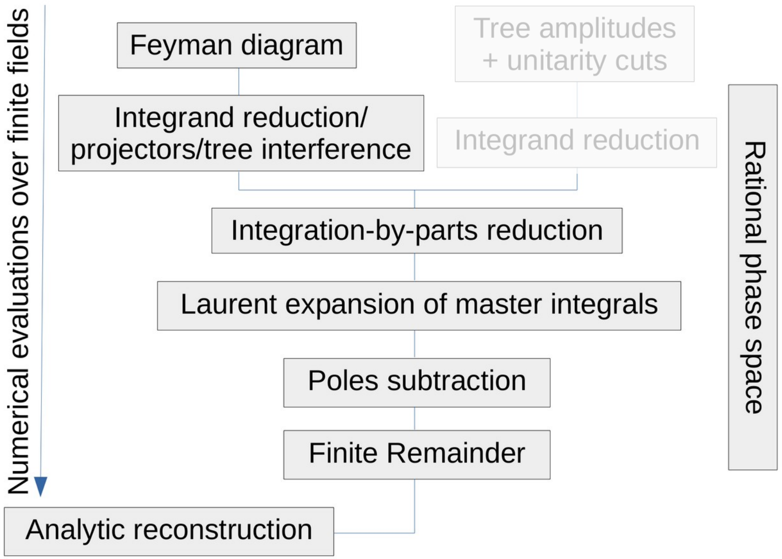
$$\mathcal{F}_x^{(2)} = N_c^2 \, F_x^{(2),N_c^2} + N_c n_f \, F_x^{(2),N_c n_f} + n_f^2 \, F_x^{(2),n_f^2} \,.$$

Partial amplitudes: masters expressed in basis functions

$$\tilde{\mathcal{F}}_{x,i}^{(L)h_1h_2h_3} = \sum_j d_j(\{p\},\epsilon)\mathrm{MI}_j(\{p\},\epsilon) \longrightarrow \tilde{\mathcal{F}}_{x;i}^{(L)h_3h_4h_5} = \sum_k \underbrace{\boxed{r_{x;i,k}^{(L)h_3h_4h_5}}}_{\text{rational}} \overbrace{\boxed{m_k(\vec{f})}}^{\text{algebraic}}$$

Reconstruction of rational parts

Workflow



QGRAF[Nogueira], FORM[Vermaseren,etal]
MATHEMATICA, SPINNEY[Cullen,etal]

Credit: Bayu

finite field framework: FINITEFLOW[Peraro(2019)]

IBP identities generated using LITERED[Lee(2012)]
solved numerically in FINITEFLOW using
Laporta algorithm[Laporta(2000)]

Mature technology + new optimizations

- Syzygy and module intersection techniques to simplify IBPs [NeatIBP]
- Exploitation of Q-linear relations
- Denominator Ansaetze
- on-the-fly univariate partial fraction

	1	2	3	4
master integral coefficients of mass-renormalised amplitude (full ϵ dependence)	404/393	398/389	411/402	421/411
special function coefficients of finite remainder	314/303	305/296	321/312	326/317
linear relations	291/280	287/278	299/293	304/299
denominator matching #1	291/0	287/0	299/0	304/0
partial fraction decomposition in x_{5123}	44/40	55/51	57/54	58/54
denominator matching #2	44/0	54/0	54/0	56/0
number of sample points (1 prime field)	137076	89624	161482	179838

Algebraic part → special functions

Differential Equations: $\mathrm{d}\vec{\mathrm{MI}} = \mathrm{d}A(\{p\},\epsilon)\vec{\mathrm{MI}}$

Canonical basis: $\mathrm{d}\vec{\mathrm{MI}} = \epsilon\mathrm{d}\tilde{A}(\{p\})\vec{\mathrm{MI}}$

[Remiddi, 97]
[Gehrmann, Remiddi, 99]
[Henn, 13]

→ not available in ttj case (elliptic integrals):
However, still translates into a system of DEQs for basis functions: $\mathrm{d}G(\vec{d}) = M(\vec{d})\cdot G(\vec{d})$

$$G(\vec{d}) = \begin{pmatrix} f_i^{(4^*)} \\ f_i^{(4)} \\ \text{weight-3} \\ \text{weight-2} \\ f_i^{(1)} \\ 1 \end{pmatrix}, \qquad M(\vec{d}) = \begin{pmatrix} Y_{4^*,4^*} & 0 & Y_{4^*,3} & Y_{4^*,2} & 0 & 0 \\ 0 & 0 & X_{4,3} & 0 & 0 & 0 \\ 0 & 0 & 0 & X_{3,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & X_{2,1} & 0 \\ 0 & 0 & 0 & 0 & 0 & X_{1,0} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Solve numerically from pre-computed boundary points [AMFlow]

Numerical evaluation of master integrals

→ Solve DEQ by direct integration
(Bulirsch-Stoer algorithm)

→ Error control based on each function's contribution to squared matrix element

$$\mathcal{Q} \equiv \frac{1}{(4\pi)^4} \frac{2 \overline{\sum}_{\text{colour}} \overline{\sum}_{\text{pol.}} \mathcal{R}^{(0)*}\mathcal{R}^{(2)}}{\overline{\sum}_{\text{colour}} \overline{\sum}_{\text{pol.}} \left|\mathcal{R}^{(0)}\right|^2}$$

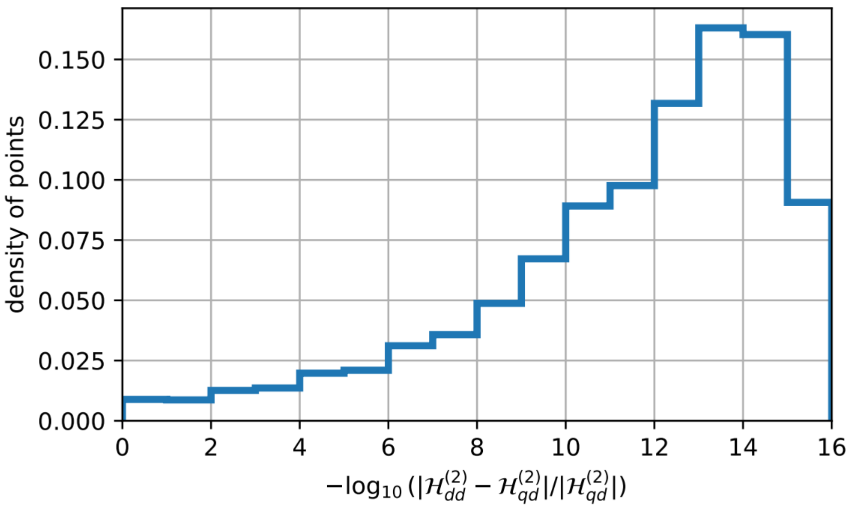
$$\Delta\mathcal{Q}(t) = \left[\sum_{i=1}^{947} \left| \frac{\partial Q}{\partial f_i} \right|^2_{\vec{f}=\vec{f}(t)} \left| \Delta f_i(t) \right|^2 \right]^{1/2}$$

→ directly control numerical error of the final result

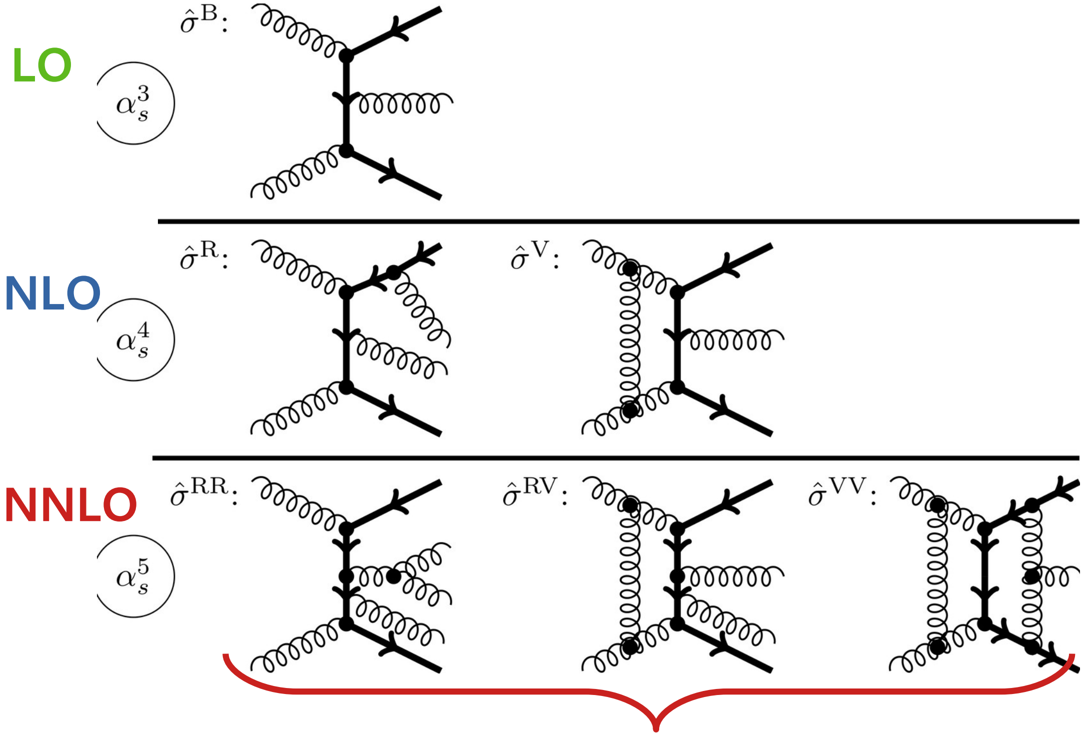
→ ready for phenomenology

C++ code publicly available

Channel	Functions [s]	Coefficents [s]	Assembly [s]	total [s]
$gg \to \bar{t}tg$	2.69	30.90	1.58	35.17
$\bar{q}g \to \bar{t}t\bar{q}$	2.16	9.40	0.18	11.74
$qg \to \bar{t}tq$	2.50	9.62	0.21	12.33
$q\bar{q} \to \bar{t}tg$	2.12	9.30	0.18	11.60



Perturbative QCD through NNLO QCD



IR-finite cross section

Infrared finite differential cross sections at NNLO QCD:

~20 years to solve this problem

→ highly non-trivial IR structure

→ plethora of subtraction schemes

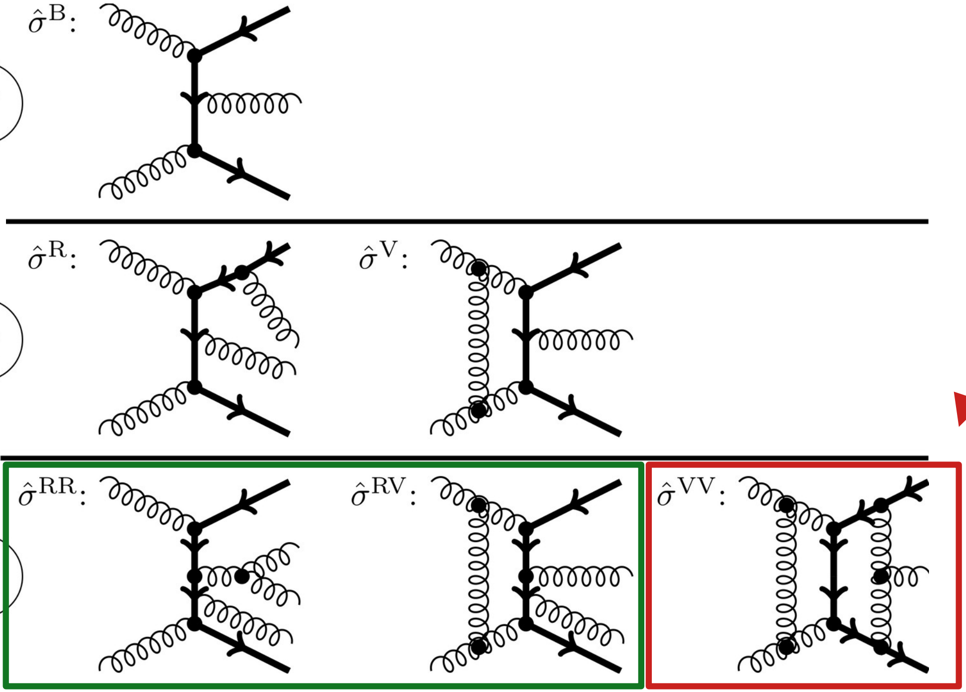
qT-slicing [Catani'07]
N-jettiness slicing [Gaunt'15/Boughezal'15]
Antenna [Gehrmann'05-'08]
Colorful [DelDuca'05-'15]
Projetction [Cacciari'15]
Geometric [Herzog'18]
Unsubtraction [Aguilera-Verdugo'19]
Nested collinear [Caola'17]
Local Analytic [Magnea'18]
Sector-improved residue subtraction [Czakon'10-'14,'19]

Perturbative QCD through NNLO QCD

LO α_s^3

NLO α_s^4

NNLO α_s^5



Complete colour dependence

Leading color approximation for two-loop finite remainder

Log-dependence (only lower loops) in **full color**

$$F^{(2)}_{\mathrm{l.c.\ resc.}}(\mu^2) = \frac{F^{(0)}}{F^{(0)}_{\mathrm{l.c.}}} F^{(2)}_{\mathrm{l.c.}}(Q^2) + \sum_{i=0}^{4} c_i \log^i(\mu^2/Q^2)$$

Rescaling captures some of the effect

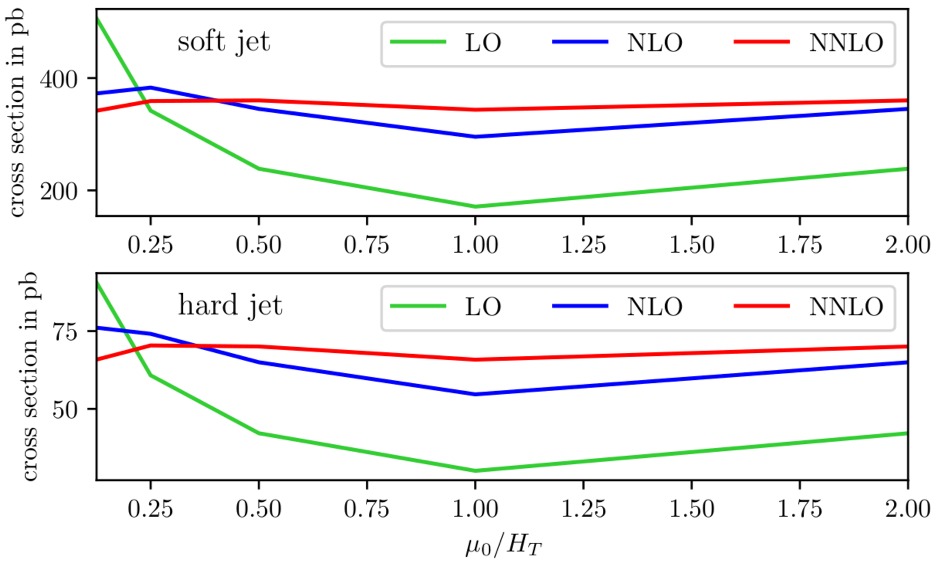
Results for total cross section

LHC phase spaces at 13 TeV

anti-kT R = 0.4
PDF4LHC21

Soft-jet: $p_T(j) > 30\,\text{GeV},\ |y(j)| < 2.5$

Hard-jet: $p_T(j) \geq 120\,\text{GeV},\ p_T(j) \geq 0.4(p_T(t) + p_T(\bar{t})),\ |y(j)| < 2.4$

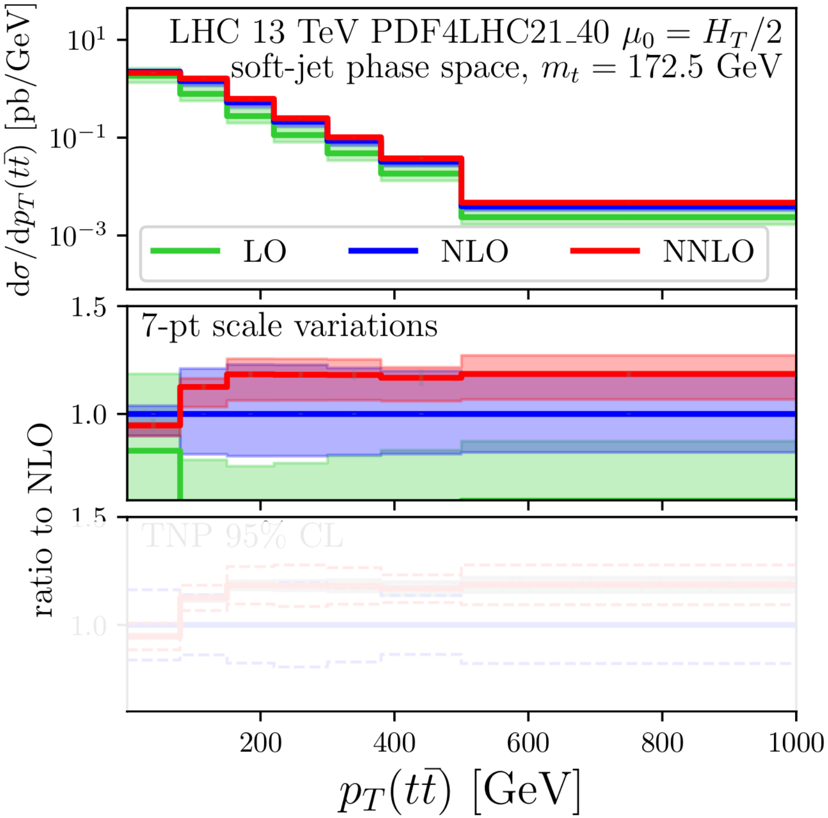


Dynamical scale choice:

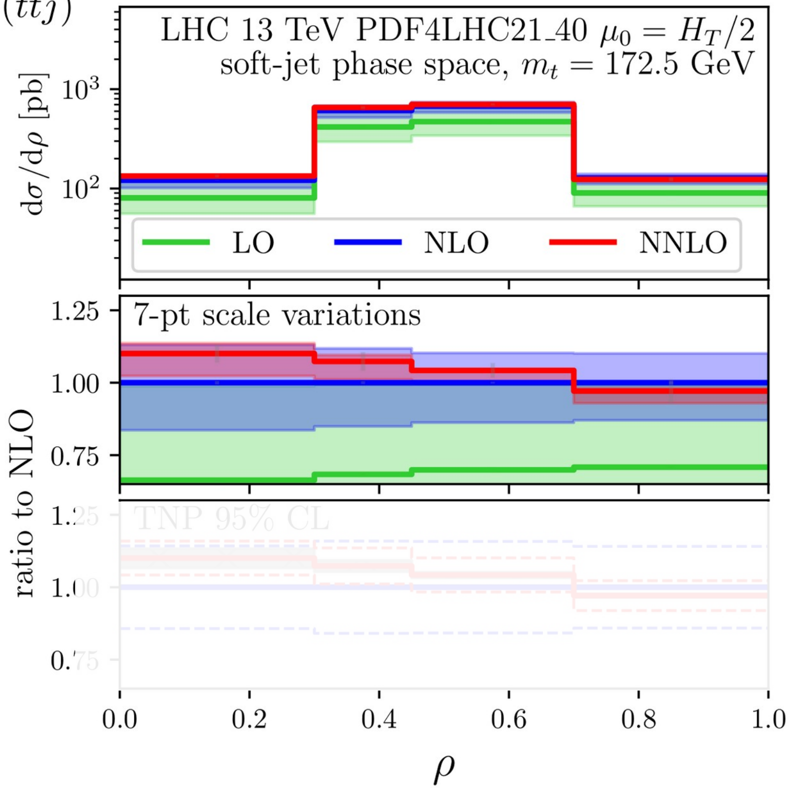
$$\mu_R = \mu_F = \mu_0 = H_T/n$$

$$H_T = M_T(t) + M_T(\bar{t}) + p_T(j_1)$$

Differential cross sections

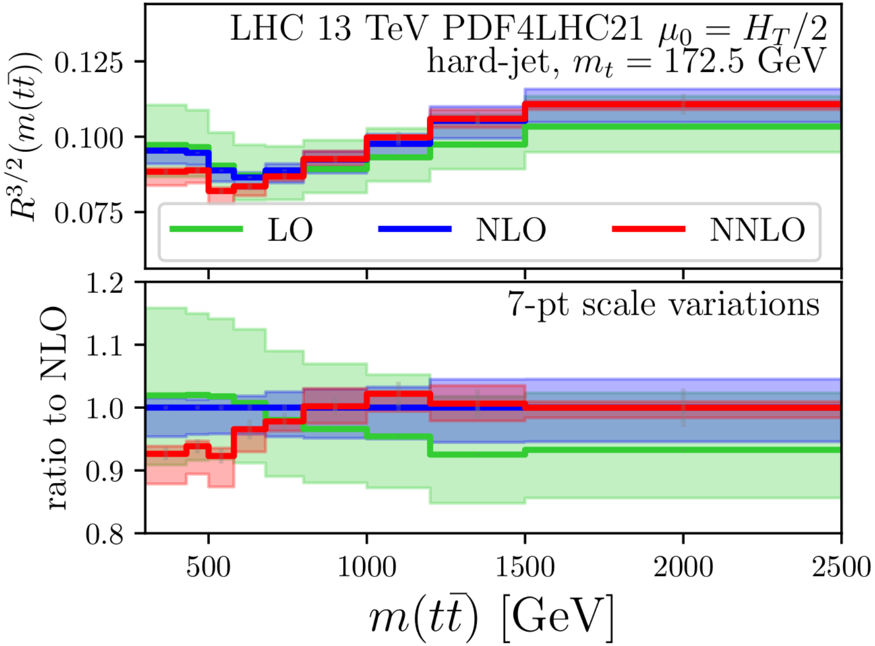


$$\rho = \frac{2m_0}{m(t\bar{t}j)}$$

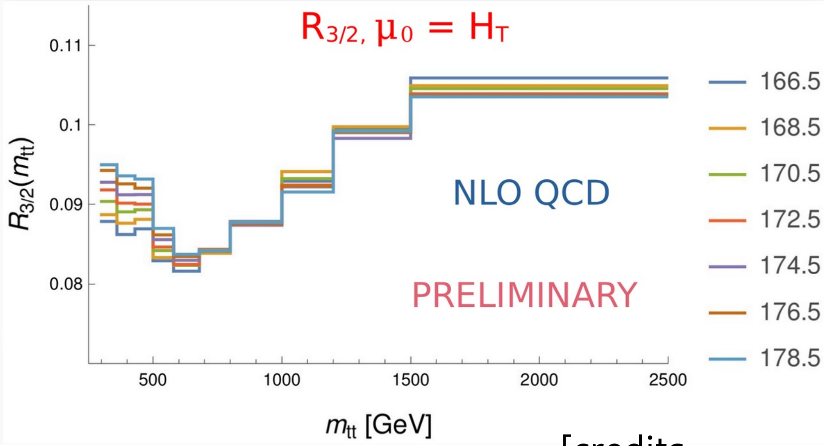


Coloured bands from 7-point scale variations

R32 ratios



$$R^{3/2}(X) = \frac{d\sigma^{t\bar{t}j}(X)}{d\sigma^{t\bar{t}}(X)}$$ → Sensitivity to α_s & m_t



[credits Bayu Hartanto]

Theory uncertainties

Various sources of theory uncertainties: scales, PDFs, alphaS, …

→ zoom in on two sources on theory uncertainties:

1: **missing-higher-order-uncertainties (MHOU)**

2: **missing sub-leading colour contributions**

Theory uncertainties from scale variations

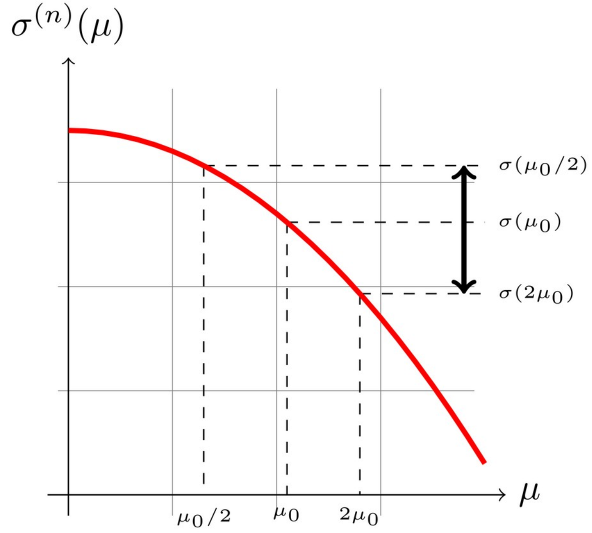
Lets focus on QCD as an example: $\alpha = \alpha_s(\mu_0)$

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(0)} + \alpha_s \mathrm{d}\sigma^{(1)} + \alpha_s^2 \mathrm{d}\sigma^{(2)} + \dots \xrightarrow{\text{RGE}} \mu \frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\mu} = \mathcal{O}\left(\alpha_s^{(n_0+n+1)}\right)$$

Of same order as the next dominant term → exploiting this to estimate size of $\mathrm{d}\sigma^{(n+1)}$

Scale variation prescription (ad-hoc and heuristic choice)

- choose 'sensible' μ_0
  - → principle of fasted apparent convergence: $\sigma^{(n)}(\mu_{\mathrm{FAC}}) = 0$
  - → principle of minimal sensitivity: $\mu \frac{\mathrm{d}\sigma(\mu)}{\mathrm{d}\mu}\Big|_{\mu=\mu_{\mathrm{PMC}}} = 0$
  - → ...
- vary with a factor (typically 2)
- take envelope as uncertainty



Short comings of scale variations

- not always reliable … however in most cases issues are understood/expected:
  new channels, phase space constraints, etc. → often we can design workarounds
- however, some issues are more fundamental:
  → how to choose the central scale? → not a physical parameter, no ‘true’ value
    (Principle of fasted apparent convergence, principle of minimal sensitivity,…)
  → how to propagate the estimated uncertainty, no statistical interpretation!
  → what about correlations? Based on ‘fixed form’ of the lower orders and RGE.

(At the moment) two alternative approaches under investigation:

“Bayesian”

[Cacciari,Houdeau 1105.5152]
[Bonvini 2006.16293]
[Duhr, Huss, Mazeliauskas, Szafron 2106.04585]

“Theory Nuisance Parameter”

[Tackmann 2411.18606]
→ W mass extraction: [CMS 2412.13872]
[Cal,Lim, Scott,Tackmann Waalewijn 2408.13301]
[Lim, Poncelet, 2412.14910]
[Cridge,Marinelli,Tackmann,2506.13874]

TNP approach for fixed-order computations

[Lim, Poncelet, 2412.14910]

$$\begin{aligned} \mathrm{d}\sigma &= \alpha_s^n N_c^m \,\mathrm{d}\bar{\sigma}^{(0)} + \alpha_s^{n+1} N_c^{m+1}\,\mathrm{d}\bar{\sigma}^{(1)} + \alpha_s^{n+2} N_c^{m+2}\,\mathrm{d}\bar{\sigma}^{(2)} + \dots \\ &= \alpha_s^n N_c^m \,\mathrm{d}\bar{\sigma}^{(0)} \left[1 + \alpha_s N_c \left(\frac{\mathrm{d}\bar{\sigma}^{(1)}}{\mathrm{d}\bar{\sigma}^{(0)}}\right) + \alpha_s^2 N_c^2 \left(\frac{\mathrm{d}\bar{\sigma}^{(2)}}{\mathrm{d}\bar{\sigma}^{(0)}}\right) + \dots\right] \end{aligned}$$

Observation, i.e. "expert knowledge": $\dfrac{\mathrm{d}\bar{\sigma}^{(i)}}{\mathrm{d}\bar{\sigma}^{(0)}} \sim \mathcal{O}(1)$

Use some knowledge about lower orders but introduce parametric dependence:

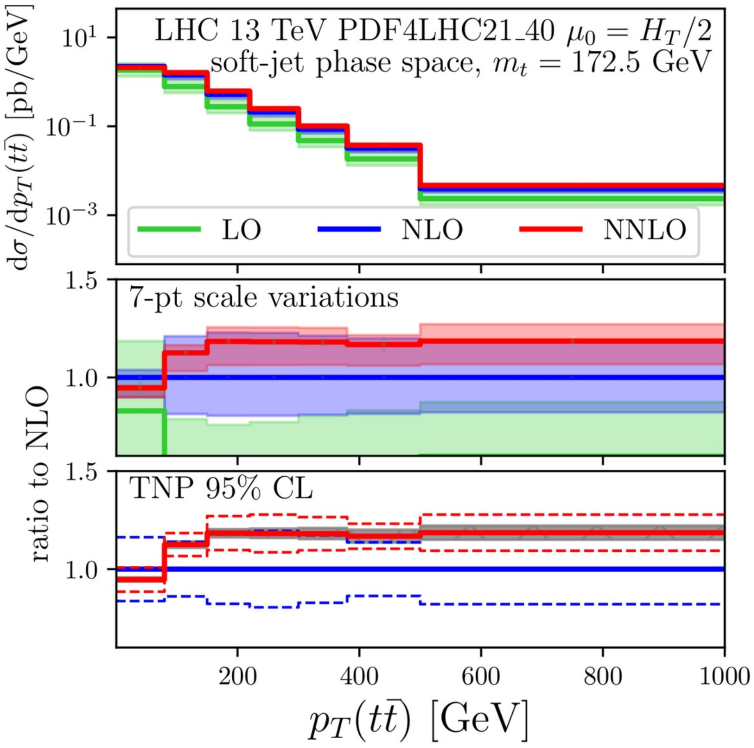
$$\frac{\mathrm{d}\bar{\sigma}^{(N+1)}_{\mathrm{TNP}}}{\mathrm{d}\bar{\sigma}^{(0)}} = \sum_{j=1}^{N} f_k^{(j)}\left(\vec{\theta}, x\right) \left(\frac{\mathrm{d}\bar{\sigma}^{(j)}}{\mathrm{d}\bar{\sigma}^{(0)}}\right)$$

x → mapped kinematic variable

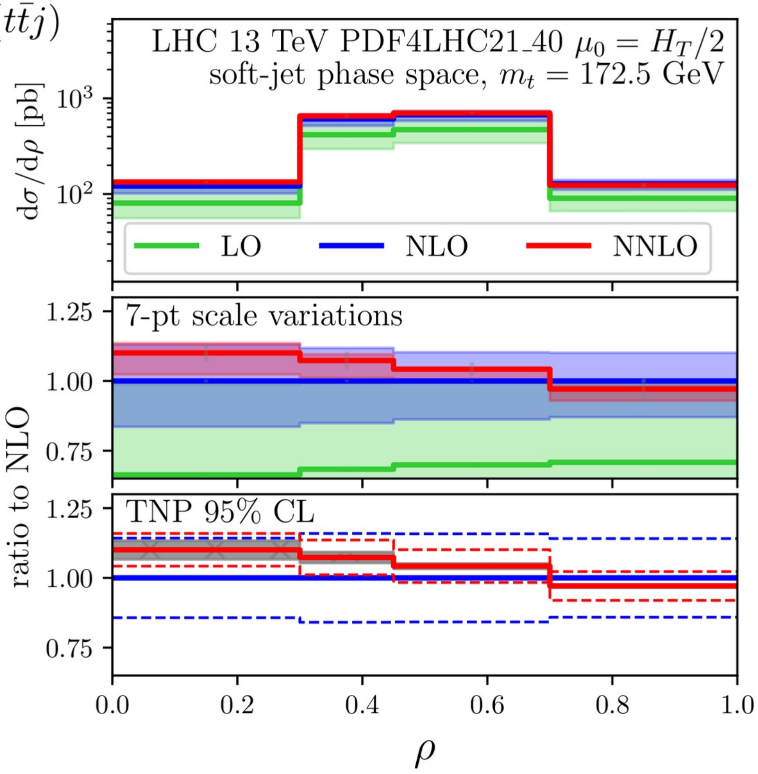
Approximation of original TNP philosophy
→ there is only $f_i(\hat{\theta}) \approx \hat{f}_i$

Chebyshev: $f_k^C\left(\vec{\theta}, x\right) = \dfrac{1}{2}\sum_{i=0}^{k} \theta_i T_i(x)$ $\qquad x \in [-1, 1]$

TNP uncertainty estimates



$$\rho = \frac{2m_0}{m(t\bar{t}j)}$$



Good agreement between TNP and scale variations!

Small interlude: sub-leading colour in 2 → 3 processes

How much color do we really need? Two-loop subleading-color effects in photon and jet physics [Czakon, Poncelet 2512.17591]

	Leading colour amplitudes	Full colour amplitudes
$pp \to \gamma\gamma\gamma$	[1911.00479][2010.15834] [2012.13553]	[2305.17056]
$pp \to \gamma\gamma j$	[2102.01820][2103.04319]	[2105.04585]
$pp \to \gamma j j$		[2304.06682]
$pp \to jjj$	[1904.00945] [2102.13609]	[2311.09870] [2311.10086] [2311.18752]

Approximation made in 2 → 3 calculations:

$$\mathcal{F}^{(2)}(\mu_R^2) = 2\,\mathrm{Re}\left[\mathcal{M}^{\dagger(0)}\mathcal{F}^{(2)}\right](\mu_R^2) + \left|\mathcal{F}^{(1)}\right|^2(\mu_R^2) \equiv \mathcal{F}^{(2)}(s_{12}) + \sum_{i=1}^{4} c_i \ln^i\left(\frac{\mu_R^2}{s_{12}}\right)$$

$$F^{(2)}_{\text{l.c. resc.}}(\mu^2) = \frac{F^{(0)}}{F^{(0)}_{\text{l.c.}}} F^{(2)}_{\text{l.c.}}(Q^2) + \sum_{i=0}^{4} c_i \log^i(\mu^2/Q^2)$$

Parametrise the sub-leading colour terms

For example $pp \to \gamma\gamma j$

$$\frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)} | \mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu = \hat{E}_{\mathrm{CMS}})\rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)}\rangle}$$

$$= \boxed{\begin{aligned} &N_c^2\Bigg[c_{1,0} + \frac{1}{N_c^2} c_{1,1} + \frac{1}{N_c^4} c_{1,2} + \frac{n_f}{N_c}\left(c_{2,0} + \frac{1}{N_c^2} c_{2,1}\right)\Bigg] \\ &+ \sum_{q'} \frac{Q_{q'}}{Q_q} N_c\left(c_{3,0} + \frac{1}{N_c^2} c_{3,1}\right) + \sum_{q'} \frac{Q_{q'}^2}{Q_q^2} N_c\left(c_{4,0} + \frac{1}{N_c^2} c_{4,1} + \frac{n_f}{N_c}\, c_{5,0}\right)\Big)\Big] \end{aligned}}$$

full colour structure

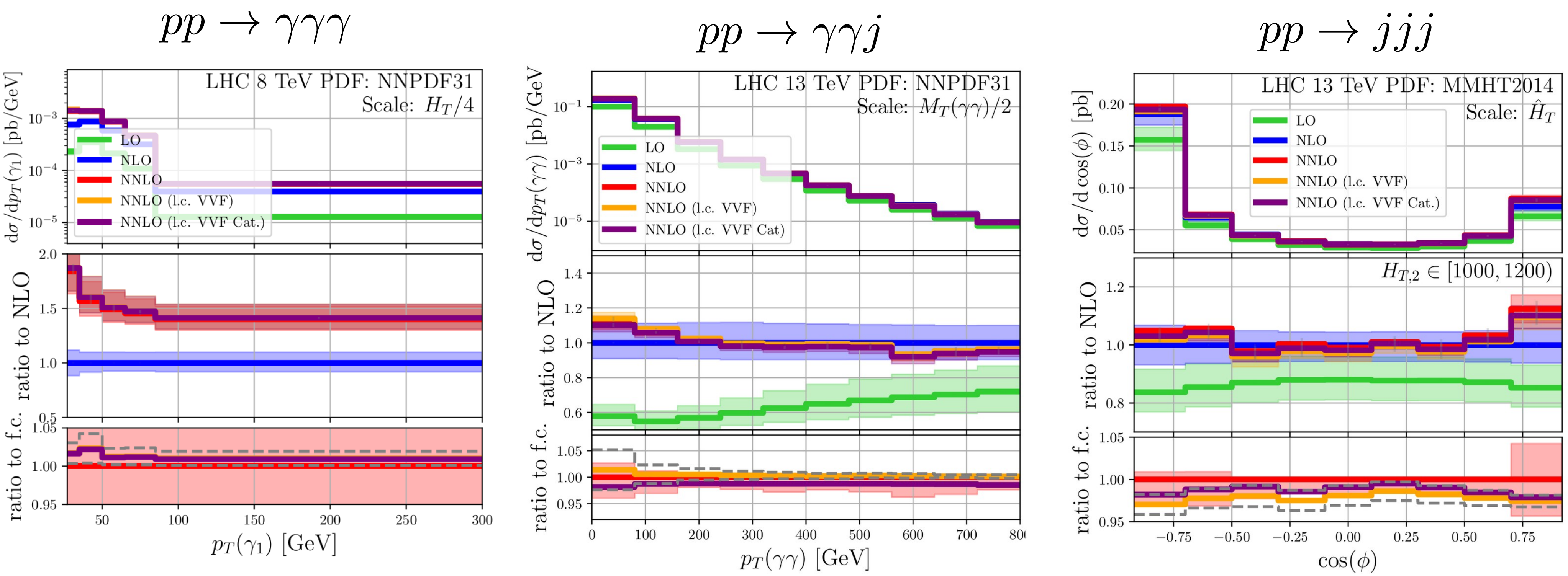
$$\approx \boxed{N_c^2\left[c_{1,0} + \frac{n_f}{N_c} c_{2,0}\right] + \sum_{q'} \frac{Q_{q'}^2}{Q_q^2}\, n_f\, c_{5,0}}\,.$$

Leading-colour / planar approximation

Parametrise the leading missing term:

$$\left.\frac{2\,\mathrm{Re}\,\langle \mathcal{M}^{(0)} | \mathcal{F}^{(\overline{\mathrm{MS}},2)}(\mu = \hat{E}_{\mathrm{CMS}})\rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)}\rangle}\right|_{\mathrm{l.c.}}^{\theta} = N_c^2\left[c_{1,0} + \frac{n_f}{N_c} c_{2,0}\right] + \sum_{q'} \frac{Q_{q'}^2}{Q_q^2}\, n_f\, c_{5,0} + \sum_{q'} \frac{Q_{q'}^2}{Q_q^2} N_c \theta\, c_{1,0} \qquad \theta \in [-1, 1]$$

Sub-leading colour estimates vs. reality



Grey dashed lines: envelopes of variation

→ considering the simplicity of the Ansatz **satisfying results**!

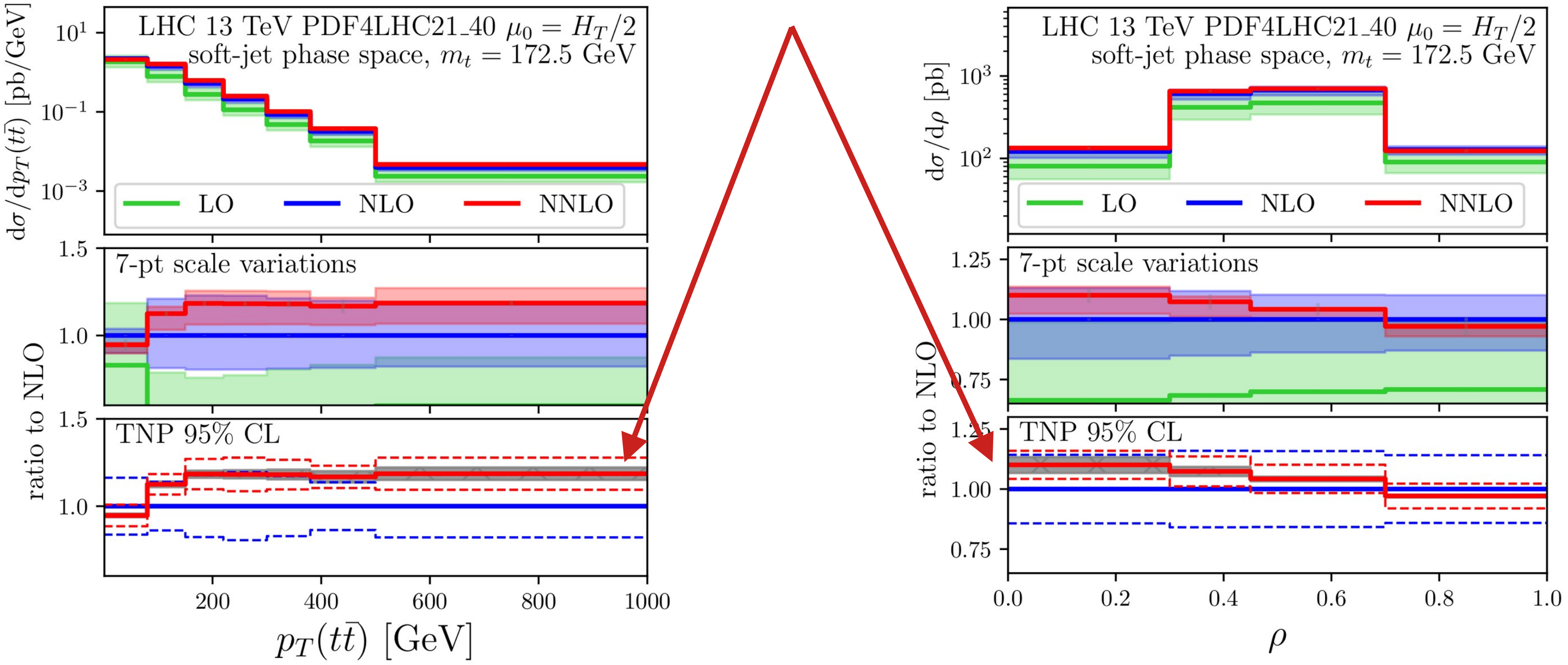
ttj ‘sub-leading colour’ uncertainty

Full NNLO QCD except for two-loop virtuals: $$F^{(2)}_{\text{l.c. resc.}}(\mu^2) = \frac{F^{(0)}}{F^{(0)}_{\text{l.c.}}} F^{(2)}_{\text{l.c.}}(Q^2) + \sum_{i=0}^{4} c_i \log^i(\mu^2/Q^2)$$

Naive ‘TNP‘-parametrisation of sub-leading colour: $$\frac{F^{(2)}_{l.c.}}{F^{(0)}_{l.c.}} \to \frac{F^{(2)}_{l.c.}}{F^{(0)}_{l.c.}} + \frac{\theta}{N_c^2} \frac{F^{(2)}_{l.c.}}{F^{(0)}_{l.c.}} \qquad \theta \in [-1, 1]$$

Top-quark pair+jet production: differential observables

Sub-leading colour unc. → grey band



Caveat: might fail close to threshold due to octet contributions! → needs some refinement

Summary & outlook

- New NNLO QCD* calculations **improve perturbative uncertainties for ttj drastically**
  → precision phenomenology for ttj
  → improved top-quark mass extractions from differential cross sections: rho, R32

  *only approximation made is the sub-leading double virtual corrections
  → error estimates based on colour factor structures

- Crucial next steps:
  - **Top-quark mass variations** → just running … wip
  - **FastNLO grids** (? … computationally challenging at this point)
  - **Incorporation of decays** (at least in the NWA) → re-use technology from ttbar