Pinning down the Standard Model
- Precision phenomenology at the LHC -

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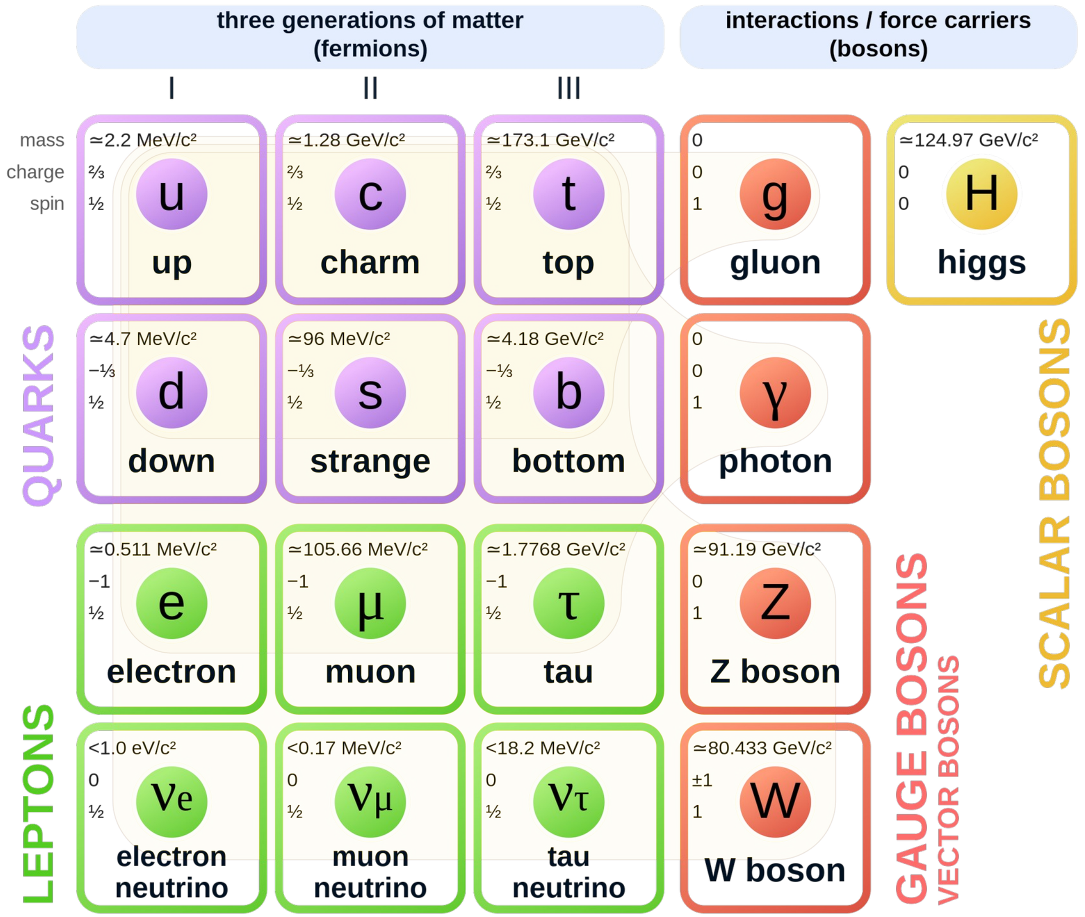
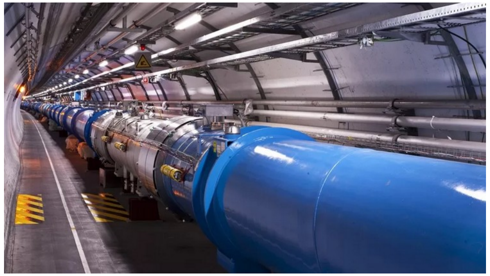
Standard Model of Elementary Particles



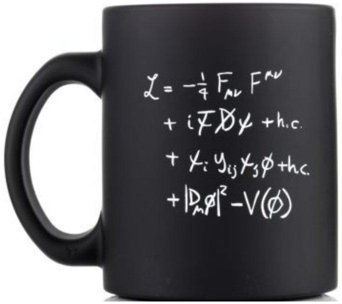
What are the fundamental building blocks of matter?

Scattering experiments

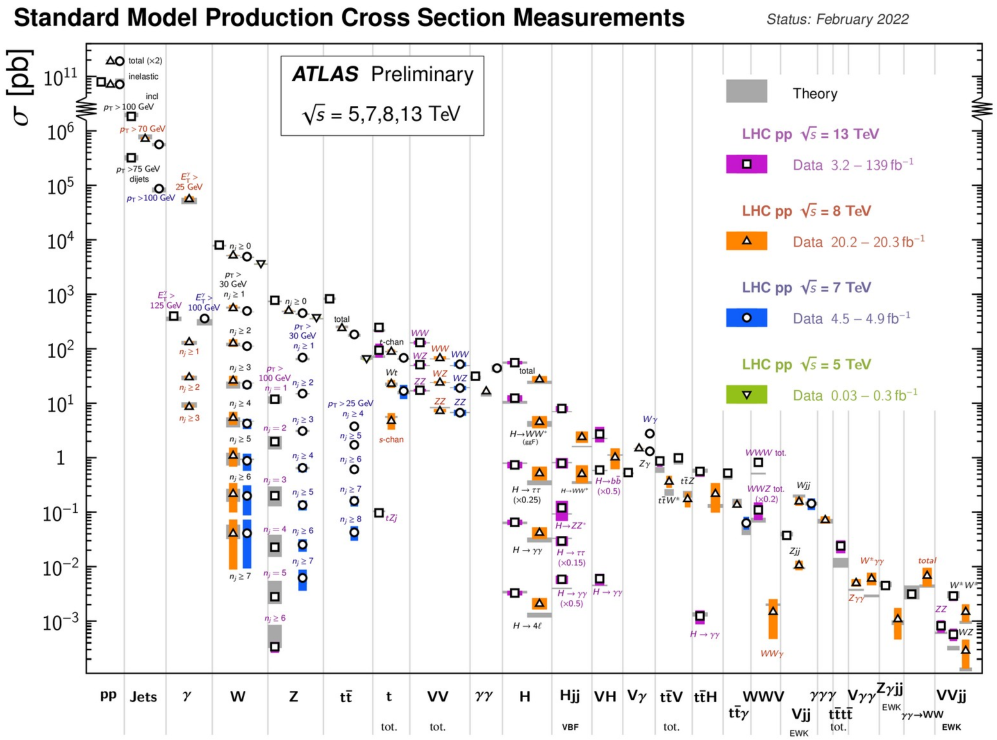
Large Hadron Collider (LHC)



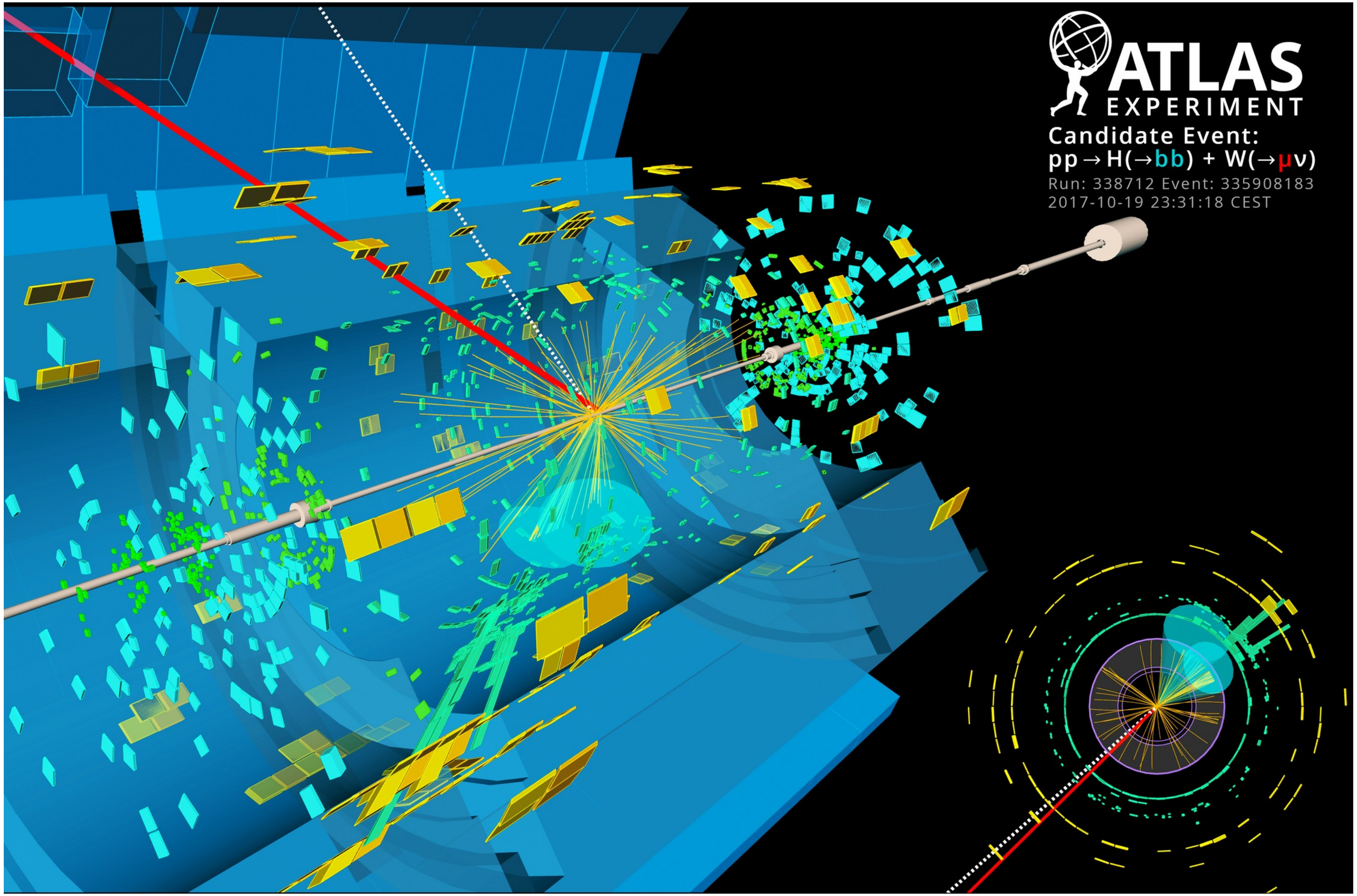
Credit: CERN



Theory/ Standard Model



Collision events



Theory picture of hadron collision events

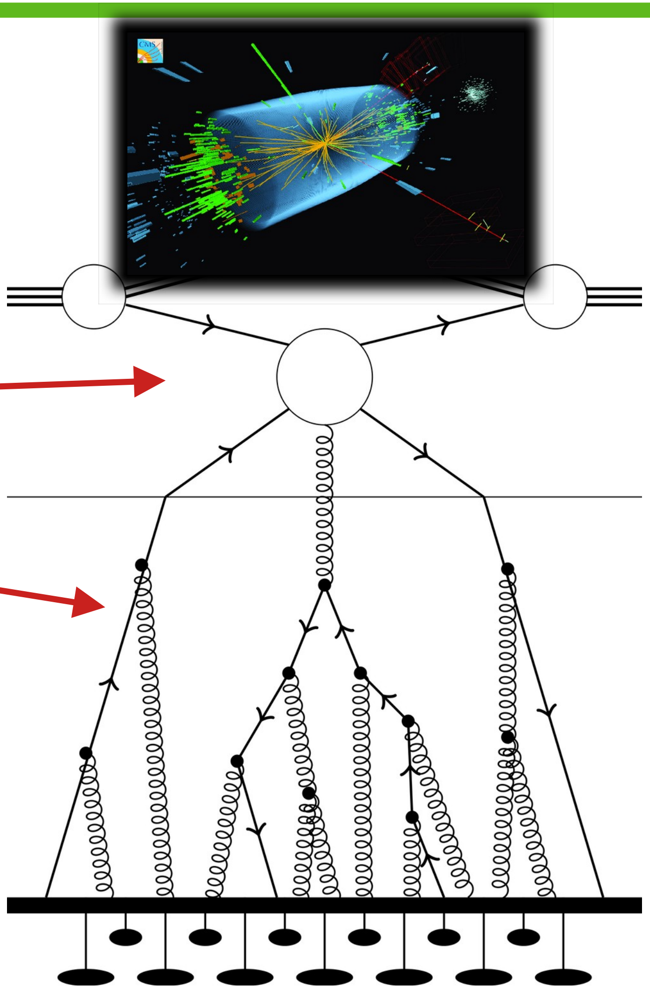
Guiding principle: factorization
"What you see depends on the energy scale"

In Quantum Chromodynamics (QCD):

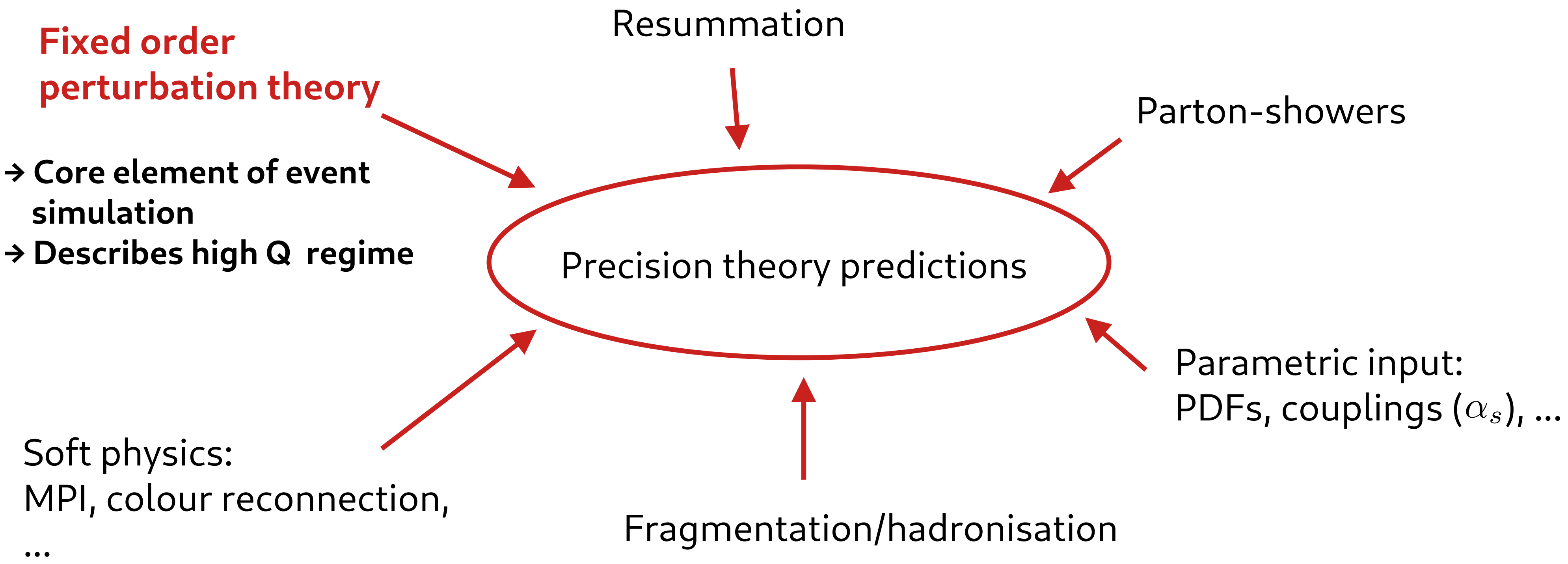
$Q \gg \Lambda_{\text{QCD}}$ **Fixed-order perturbation theory**
scattering of individual partons

$Q \gtrsim \Lambda_{\text{QCD}}$ **Parton-shower/Resummation**
all-order bridge between perturbative and non-perturbative physics

$Q \sim \Lambda_{\text{QCD}}$ **"Hadronization"/MPI/...**
non-perturbative physics



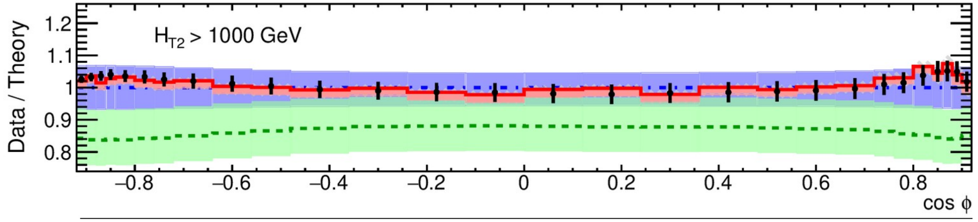
Precision predictions

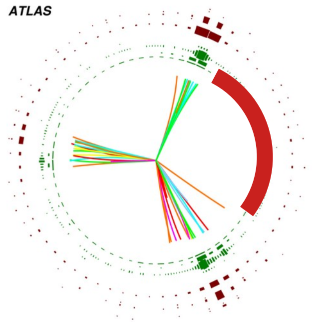


Precision through higher-order perturbation theory

Example: ATLAS
multi-jet measurements

[ATLAS 2301.09351]





$\cos\phi$

Cross section = **LO** + **NLO** + **NNLO** + **O(α_s^3)**

$\sim (\alpha_s)^1$ $\sim (\alpha_s)^2$

Theory uncertainty: **Order of magnitude** **O(10%)** **O(1%)**

Fixed-order expansion in the strong coupling $\alpha_s(m_Z) \approx 0.118$

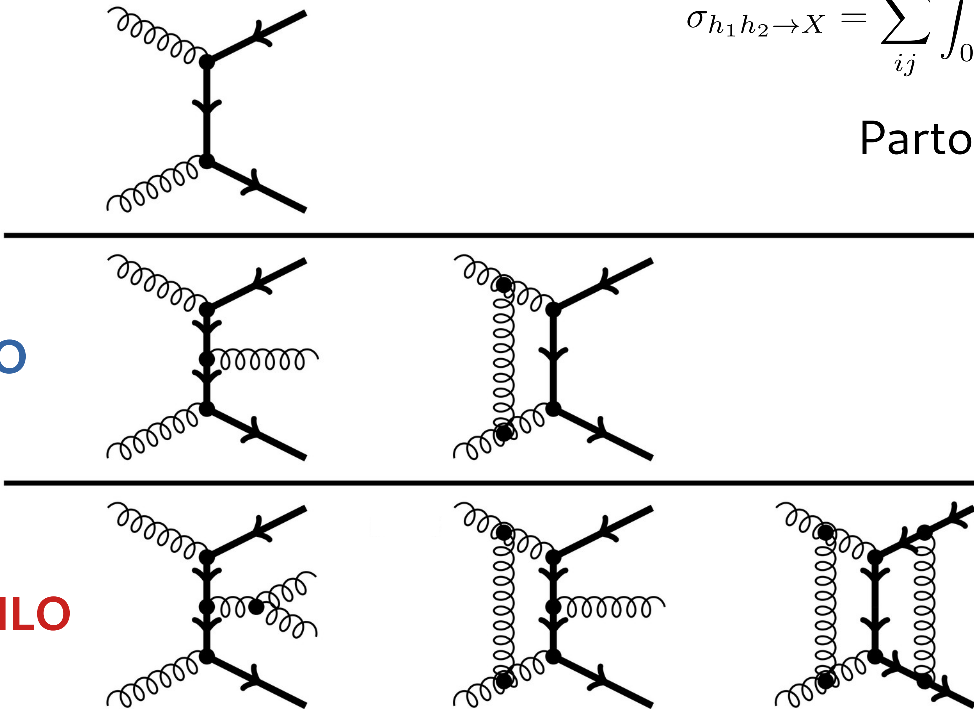
Experimental precision reaches percent-level already at LHC
next-to-next-to-leading order QCD needed on theory side!

NNLO QCD challenges

LO

NLO

NNLO



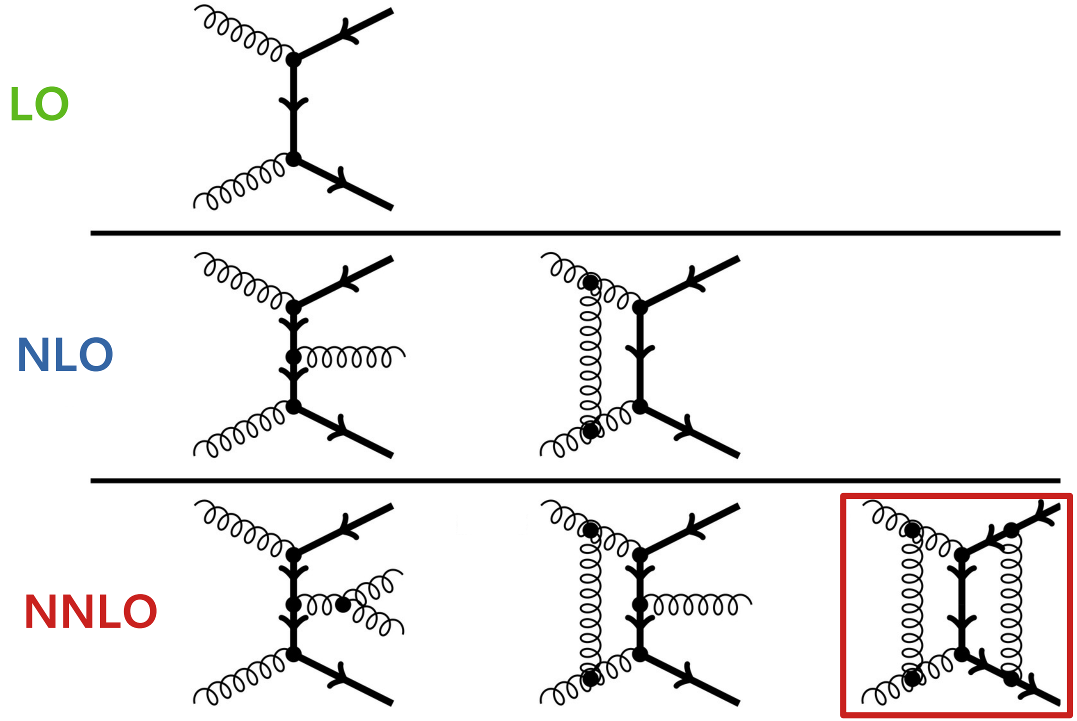
$$\sigma_{h_1 h_2 \to X} = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{\phi_{i,h_1}(x_1, \mu_F^2)}\underbrace{\phi_{j,h_2}(x_2, \mu_F^2)}\underbrace{\hat{\sigma}_{ij \to X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)}$$

Parton distribution functions

Partonic cross section:

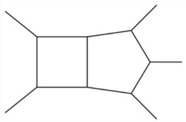
$$\hat{\sigma}_{ab \to X} = \hat{\sigma}^{(0)}_{ab \to X} + \hat{\sigma}^{(1)}_{ab \to X} + \hat{\sigma}^{(2)}_{ab \to X} + \mathcal{O}(\alpha_s^3)$$

NNLO QCD challenges

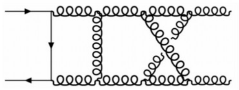


1) How to compute **multi-scale two-loop amplitudes**?
   → **fast growing complexity: rational** and **transcendental**
   → deeper understanding of the analytical properties
   → refinement of computational tools

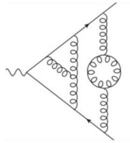
Two-loop 5-point
[Abreu, Agarwal, Badger, Buccioni, Chawdhry, Chicherin,Czakon, de Laurenties, Febres-Cordero, Gambuti, Gehrmann, Henn, Lo Presti, Manteuffel, Ma, Mitov, Page, Peraro, Poncelet, Schabiner Sotnikov, Tancredi, Zhang,...]



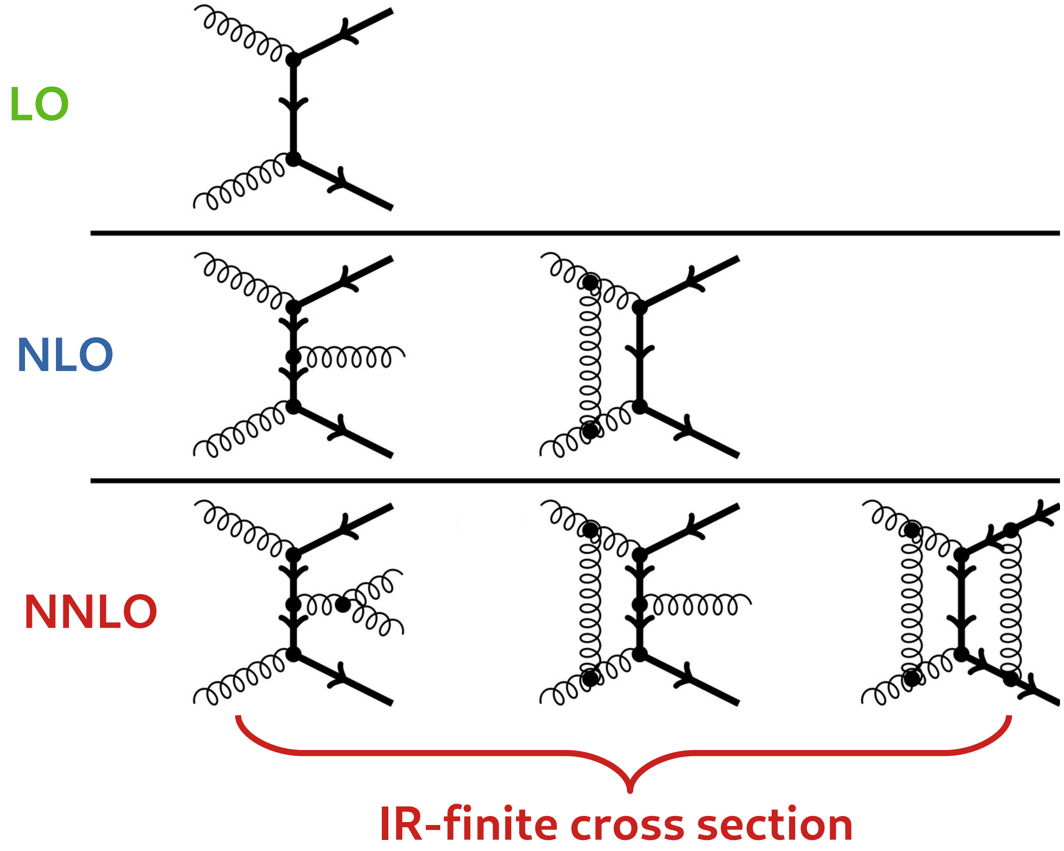
Three-loop 4-point
[Bargiela, Dobadilla, Canko, Caola, Jakubcik, Gambuti, Gehrmann, Henn, Lim, Mella, Mistleberger, Wasser, Manteuffel, Syrrakow, Smirnov, Trancredi, ...]



Four-loop 3-point
[Henn, Lee, Manteuffel, Schabinger, Smirnov, Steinhauser,...]



NNLO QCD challenges



qT-slicing [Catani’07], N-jettiness slicing [Gaunt‘15/Boughezal‘15], Antenna [Gehrmann’05-’08], Colorful [DelDuca‘05-’15], Projetction [Cacciari‘15], Geometric [Herzog‘18], Unsubtraction [Aguilera-Verdugo’19], Nested collinear [Caola’17], Local Analytic [Magnea’18], **Sector-improved residue subtraction** [Czakon’10-’14,’19]

2) How to achieve **infrared finite differential** cross sections at NNLO QCD?
~20 years to solve this problem
→ highly non-trivial IR structure
→ plethora of subtraction schemes

Multi-jet observables

Test of pQCD and extraction of strong coupling constant

NLO theory unc. > experimental unc.

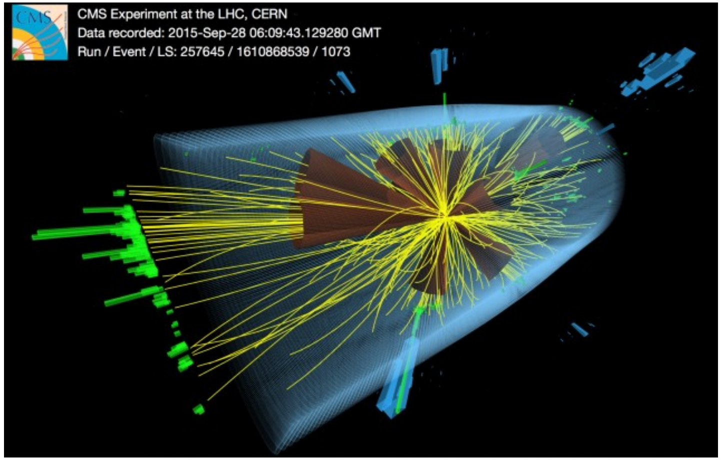
- NNLO QCD needed for precise theory-data comparisons
  → Restricted to two-jet data [Currie'17+later][Czakon'19]
- New NNLO QCD three-jet → access to more observables
  - Jet ratios

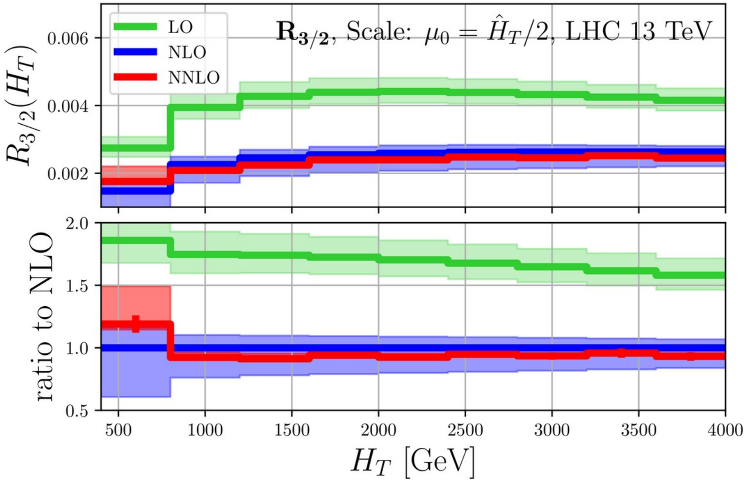
    Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC
    Czakon, Mitov, **Poncelet** [2106.05331]

$$R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$$

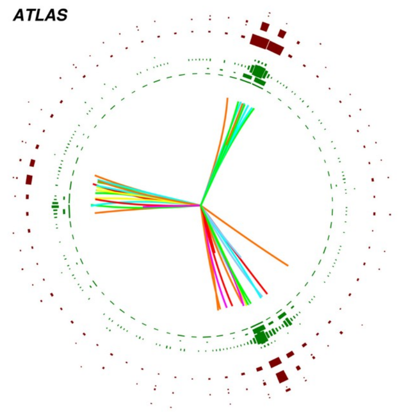
  - Event shapes

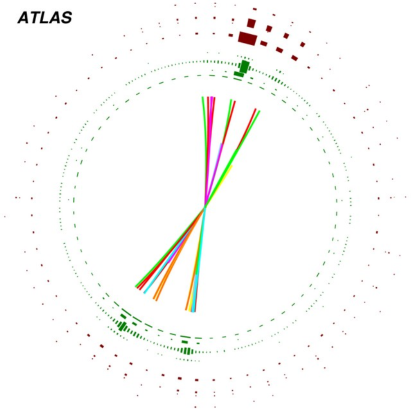
    NNLO QCD corrections to event shapes at the LHC
    Alvarez, Cantero, Czakon, Llorente, Mitov, **Poncelet** [2301.01086]





Encoding QCD dynamics in event shapes





Using (global) event information to separate different regimes of QCD event evolution:

- **Thrust & Thrust-Minor**

$$T_\perp = \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}\,, \quad \text{and} \quad T_m = \frac{\sum_i |\vec{p}_{T,i} \times \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}\,.$$

- **Energy-energy correlators**

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$

Separation of energy scales: $H_{T,2} = p_{T,1} + p_{T,2}$

Ratio to 2-jet:

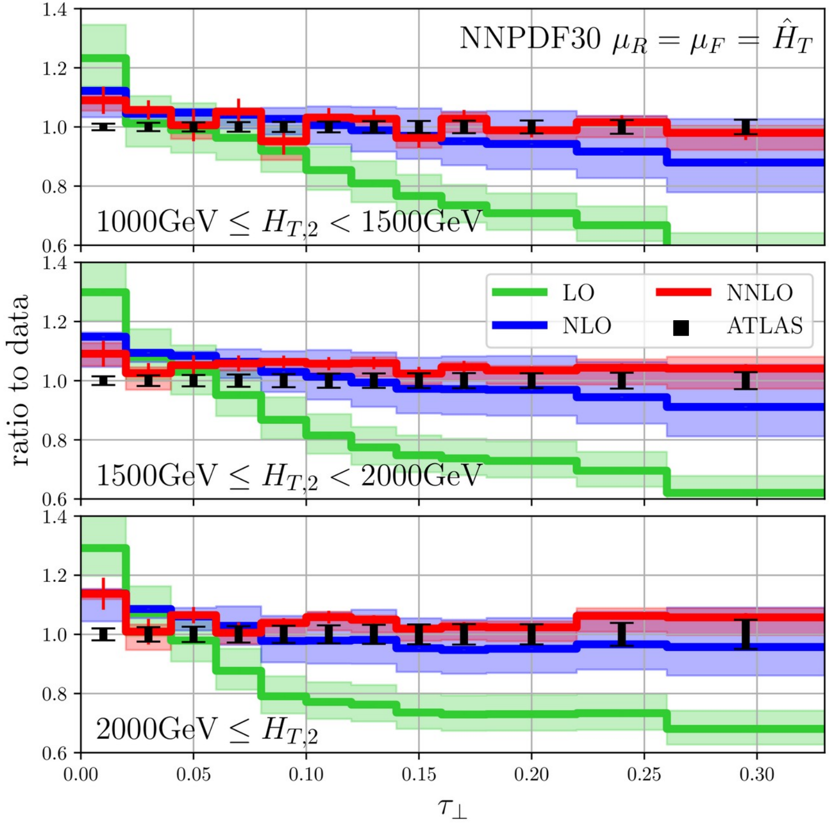
$$R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$$

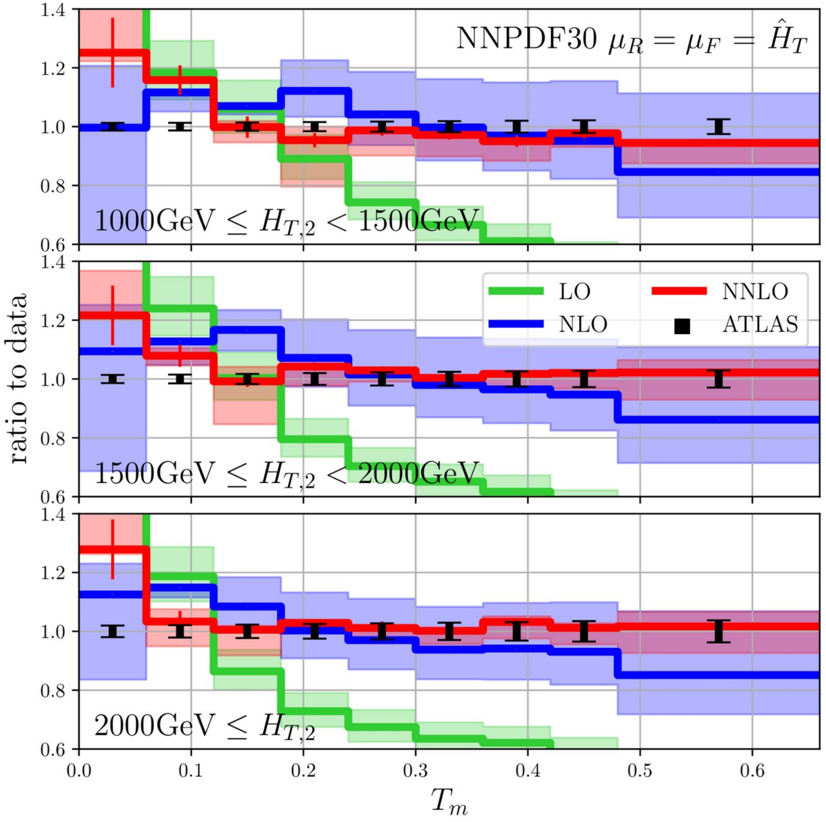
Here: use jets as input → experimentally advantageous (better calibrated, smaller non-pert.)

Transverse Thrust @ NNLO QCD

NNLO QCD corrections to event shapes at the LHC
Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet [2301.01086]

ATLAS [2007.12600]

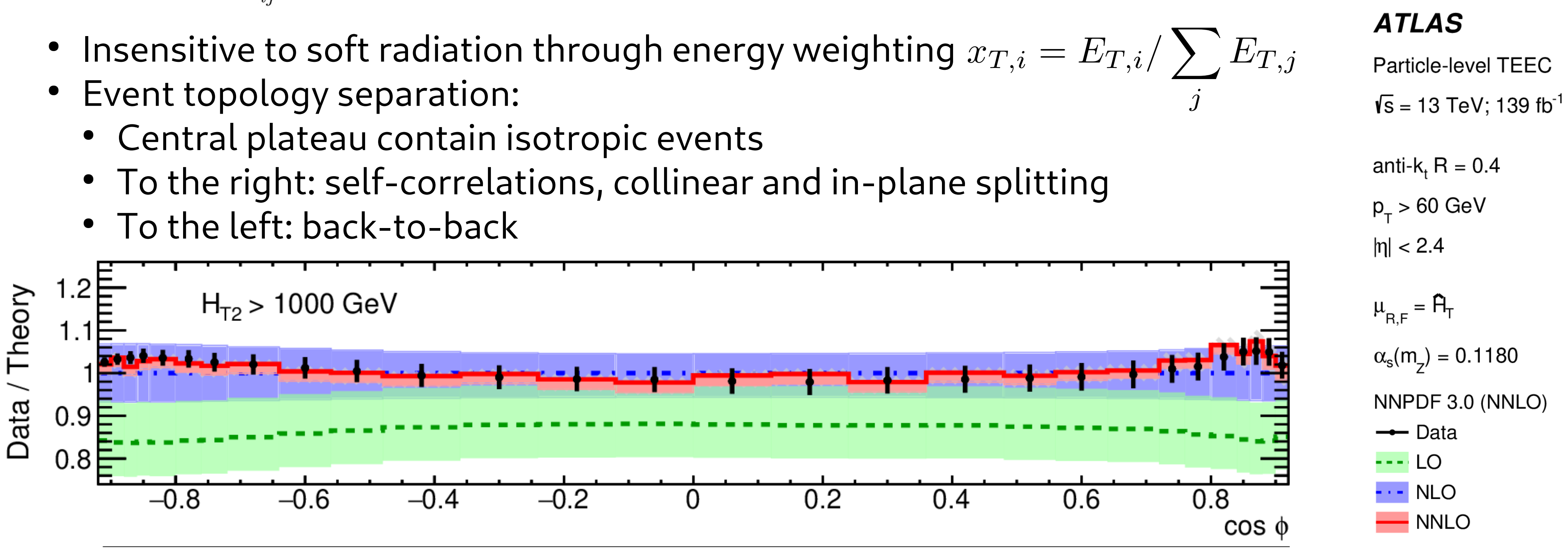




The transverse energy-energy correlator

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$

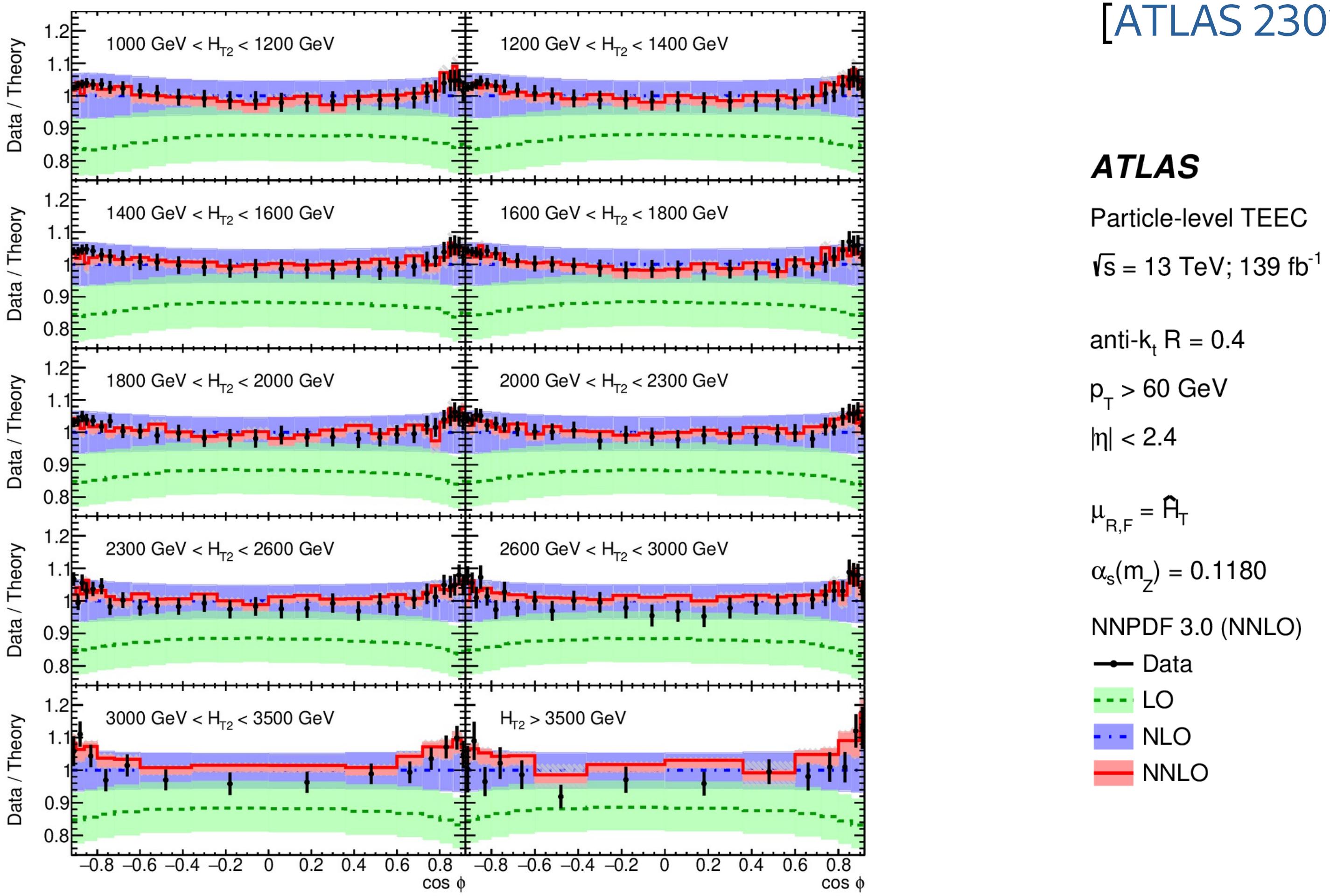
- Insensitive to soft radiation through energy weighting $x_{T,i} = E_{T,i}/\sum_j E_{T,j}$
- Event topology separation:
  - Central plateau contain isotropic events
  - To the right: self-correlations, collinear and in-plane splitting
  - To the left: back-to-back



[ATLAS 2301.09351]

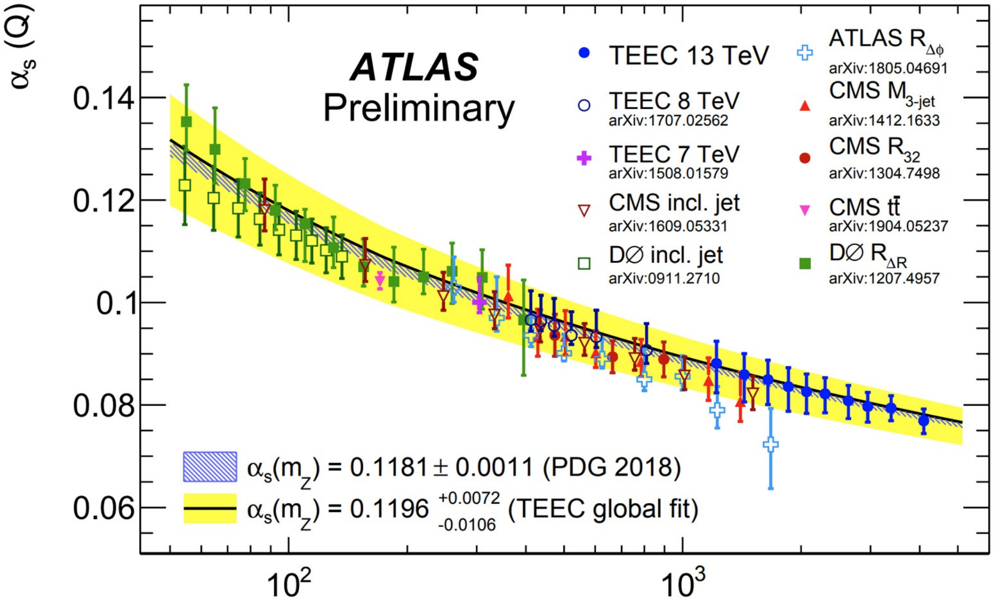
Double differential TEEC

[ATLAS 2301.09351]



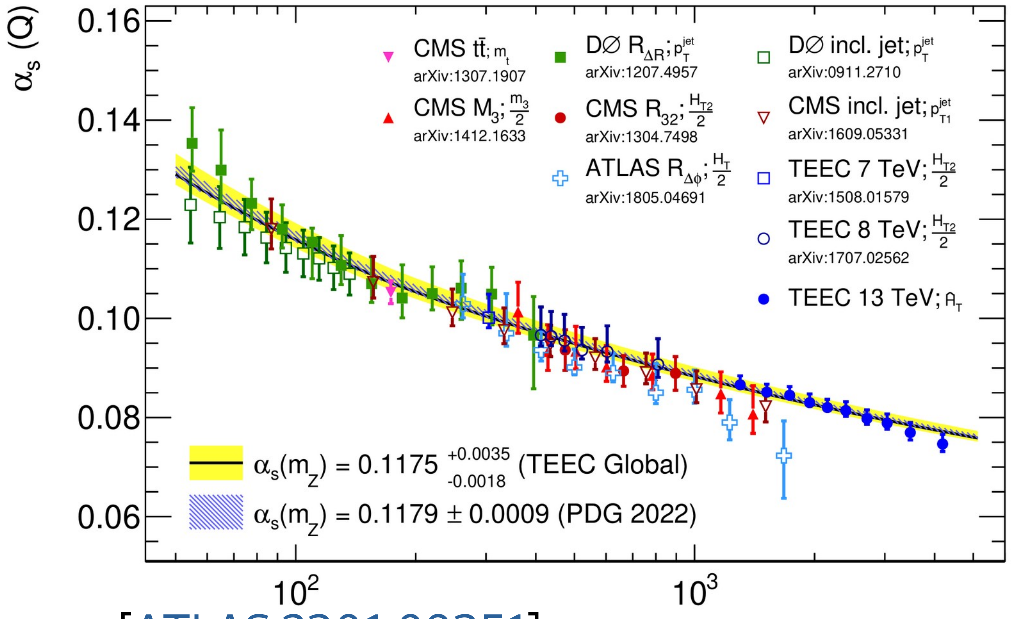
Running of α_S

NLO QCD



[ATLAS-CONF-2020-025]

NNLO QCD



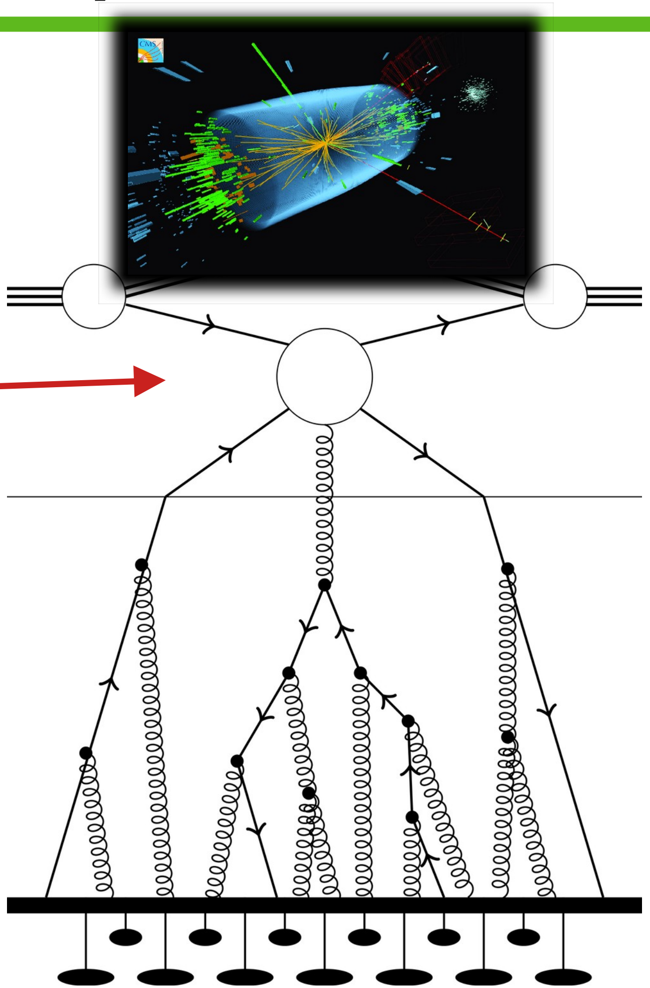
[ATLAS 2301.09351]

Beyond fixed-order perturbation theory

Guiding principle: factorization
"What you see depends on the energy scale"

In Quantum Chromodynamics (QCD):

$Q \gg \Lambda_{\mathrm{QCD}}$ **Fixed-order perturbation theory**
scattering of individual partons



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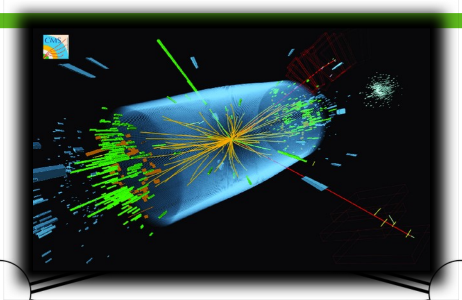
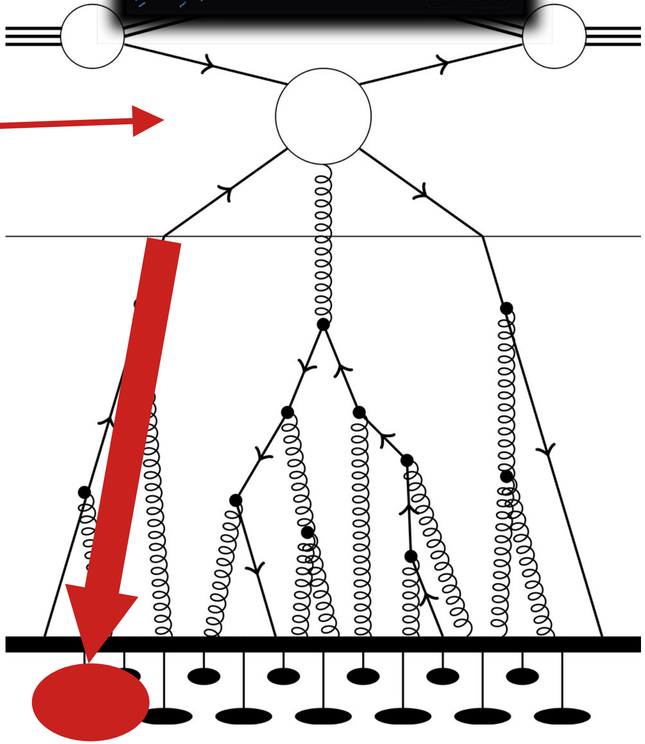
Parton to identified object transition "**Fragmentation**"
→ Resummation of collinear logs through 'DGLAP'
→ Non perturbative fragmentation functions
Example: B-hadrons in e+e-

$$\frac{d\sigma_B(m_b,z)}{dz} = \sum_i \left\{ \frac{d\sigma_i(\mu_{Fr},z)}{dz} \otimes D_{i\to B}(\mu_{Fr},m_b,z) \right\}(z) + \mathcal{O}(m_b^2)$$

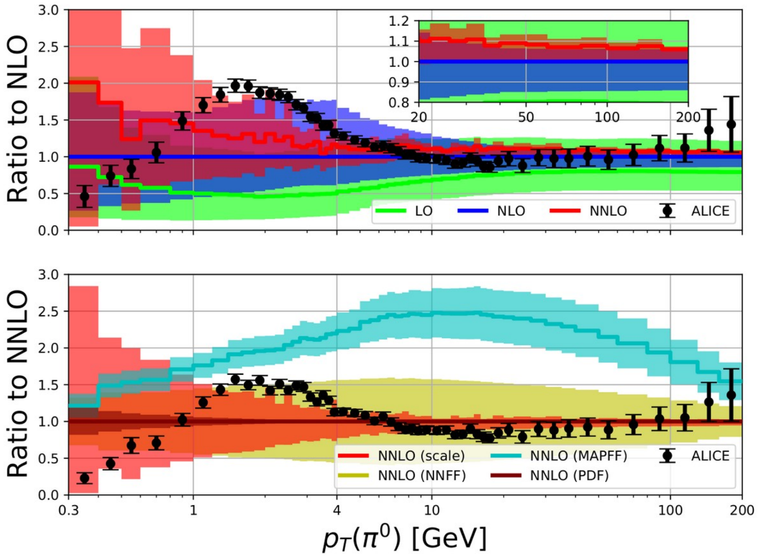




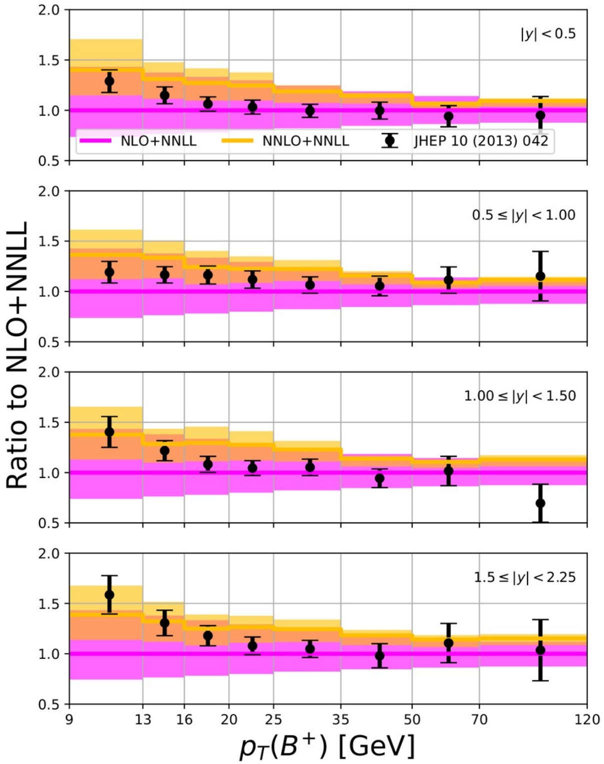
Identified hadrons

Inclusion of fragmentation through NNLO QCD:
[Czakon, Generet, Mitov, Poncelet]
- B-hadrons in top-decays [2210.06078,2102.08267]
- Open-bottom [2411.09684] → accepted in PRL
- Identified hadrons [2503.11489] → accepted in PRL

$$d\sigma_{pp\to h}(p) = \sum_i \int dz \; d\hat{\sigma}_{pp\to i}\left(\frac{p}{z}\right) D_{i\to h}(z)$$



Pion production



Open-bottom
@FONNLL:

$$\sigma(p_T) = \sigma(m, p_T) + G(p_T)(\sigma(0, p_T) - \sigma(0, p_T)|_{\mathrm{FO}})$$

Jet substructure

Semi-inclusive jet function [1606.06732, 2410.01902]

$$\frac{d\sigma_{\mathrm{LP}}}{dp_T d\eta} = \sum_{i,j,k} \boxed{\int_{x_{i,\min}}^{1} \frac{dx_i}{x_i} f_{i/P}(x_i,\mu) \int_{x_{j,\min}}^{1} \frac{dx_j}{x_j} f_{j/P}(x_j,\mu) \int_{z_{\min}}^{1} \frac{dz}{z} \mathcal{H}_{ij}^{k}(x_i,x_j,p_T/z,\eta,\mu)}$$
$$\times \boxed{J_k\left(z, \ln\frac{p_T^2 R^2}{z^2\mu^2},\mu\right)},$$

The same hard function as for identified hadrons!

Modified RGE:

[2402.05170,2410.01902]

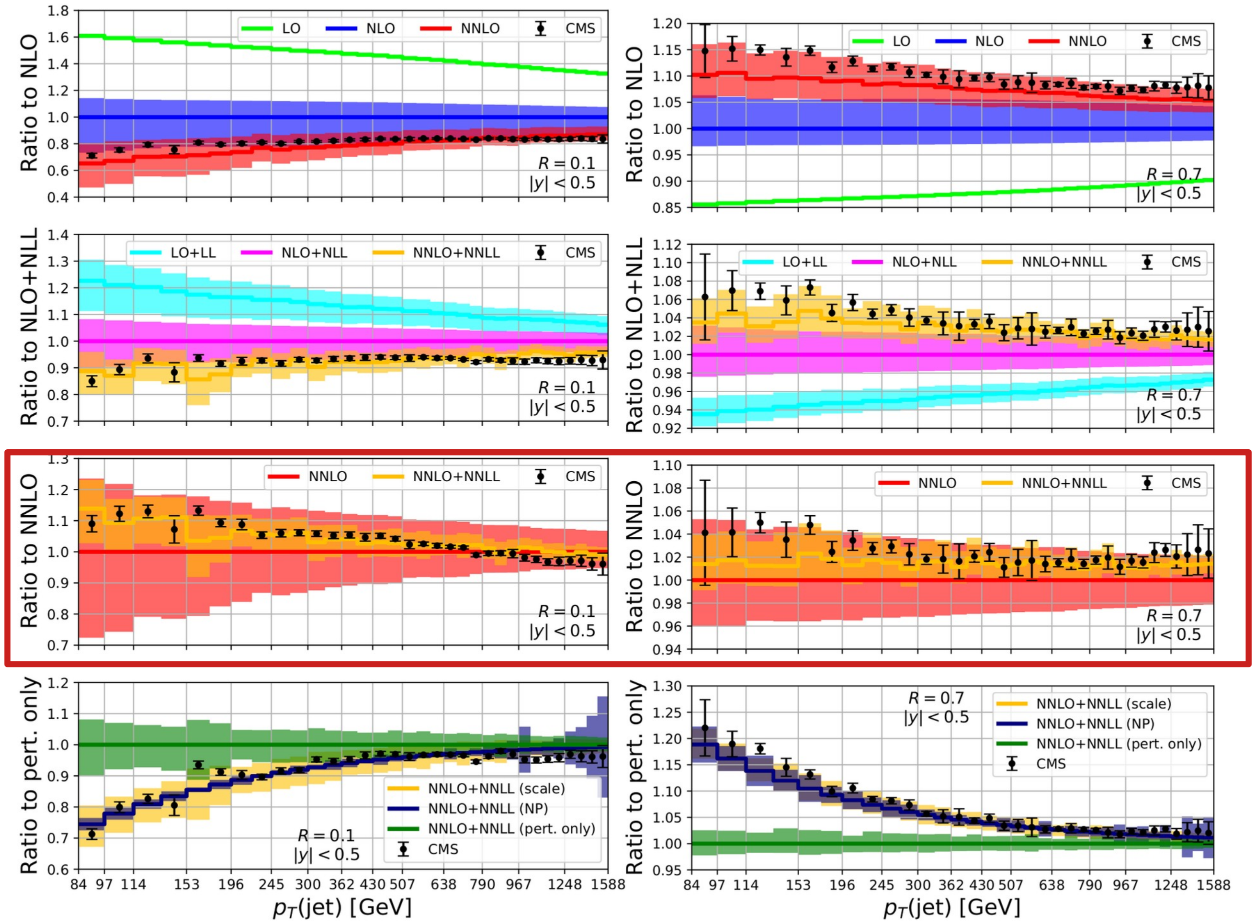
$$\frac{d\vec{J}\left(z,\ln\frac{p_T^2R^2}{z^2\mu^2},\mu\right)}{d\ln\mu^2} = \int_z^1 \frac{dy}{y} \vec{J}\left(\frac{z}{y},\ln\boxed{\frac{y^2p_T^2R^2}{z^2\mu^2}},\mu\right)\cdot \widehat{P}_T(y)$$

Side note: energy-energy correlators obey similar factorization!

Small-R jets

Application to small-R jets
[Generet, Lee, Moult, Poncelet, Zhang]
[2503.21866]

‘Triple’ differential measurement by CMS:
Y, pT, R [2005.05159]



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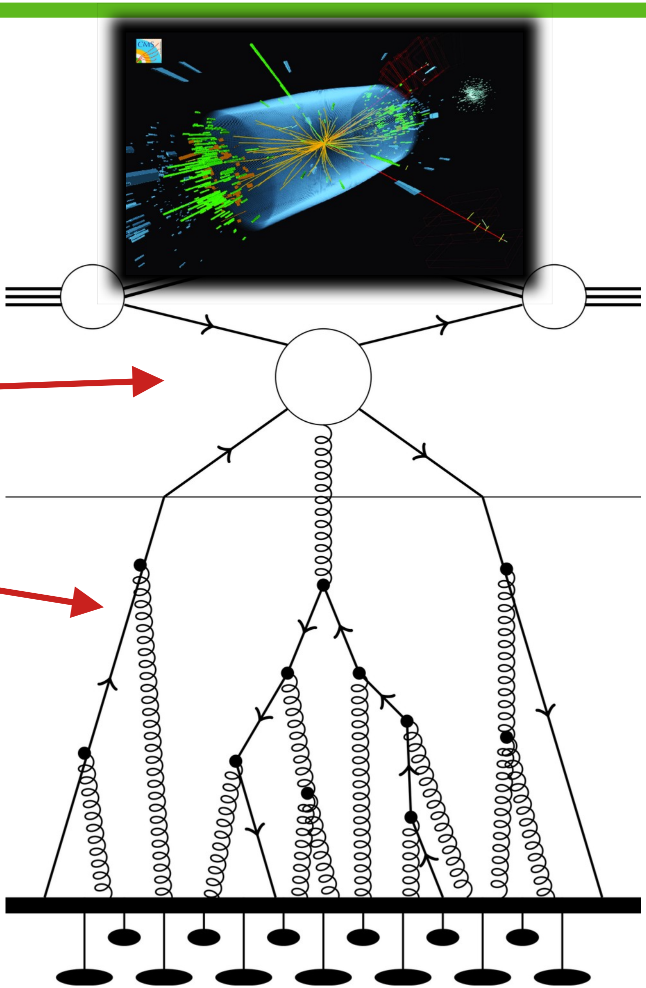
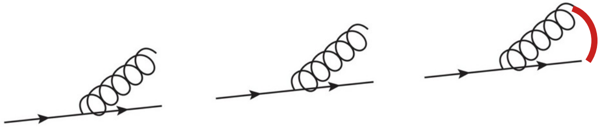


Fixed-order matching to parton-showers

The challenge

Combine fixed-order with parton shower evolution
while **preserving** the precision/accuracy of both!

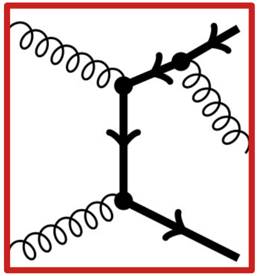


v ← shower scale

parton shower approximation: $\alpha_S^i(v)\ln^j(v)$
but to all orders!

accuracy: LL, NLL, NNLL, ...

"Hard" scattering scale
Q



Fixed-order: expansion in $\alpha_S(Q)$

→ cross section ~ $\underbrace{e^{\sum_{ij}\alpha_S^i(v)\ln^j(v)} \times (\mathrm{LO} + \mathrm{NLO} + \mathrm{NNLO} + \ldots)}$

A matching scheme needs to avoid double counting
of the logarithmic contributions!

Matching parton showers

At NLO QCD a solved problem → a breakthrough for LHC phenomenology

Local matching NLO+PS: MC@NLO, Powheg, Nagy-Soper, …

(core of event generators Madgraph_aMC@NLO, Sherpa, Powheg+Pythia, Herwig)

>80% of all exp. LHC papers cite at least one these!

Core idea: using subtractions schemes to construct showers & matching

(subtraction terms ⟷ parton shower kernels)

This is the **big challenge at NNLO QCD** for the theory community!

Some NNLO+PS matching approaches appeared recently but are either

- non-local → resummation/slicing based (for example: MiNNLOPS, Geneva) → limited generality
- or work only for simple cases like e+e- → jets (for example: Vincia) → work only where NNLO is known analytically

No scheme so far is based on a general local subtraction.

A general matching scheme at NNLO would be the next big breakthrough for precision collider physics!

This is what I want to achieve with

STAPLE!

Funded by the European Union

European Research Council
Established by the European Commission

Two core aspects:

1) **preserving the precision/accuracy** of the fixed-order & parton shower

2) achieving a parton shower with **high logarithmic accuracy**

Theory uncertainties

Uncertainties in precision phenomenology

Experiments are getting more precise → theory uncertainties matter!

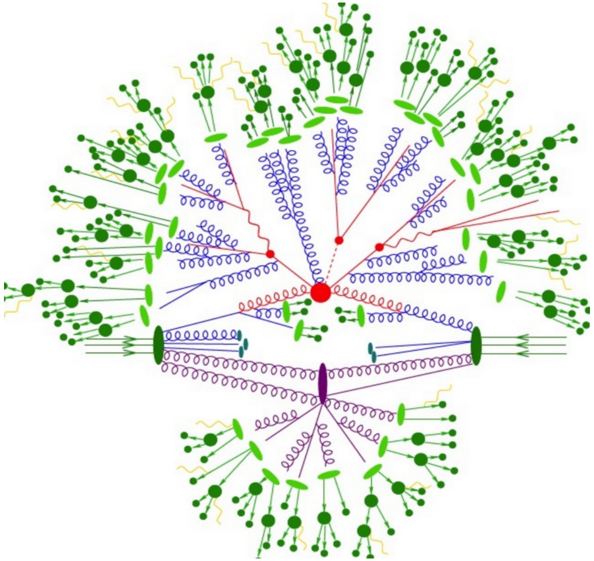
Sources of theory uncertainties:

- parametric (values of coupling parameters etc.)
→ variation of parameters within their uncertainties

- parton distribution functions (PDFs)
→ different error propagation methods (fit parameter, replicas,...)

- non-perturbative parameters in Monte Carlo simulations.
→ needs data constraints by definition. Problematic if dominant effect...

- missing higher orders in fixed-order and resummed predictions (MHOU)
→ tricky because we are trying to estimate the unknown….



[Credit: SHERPA]

Missing higher orders

Notation from: [Tackmann 2411.18606]
Beyond Scale Variations: Perturbative Theory Uncertainties from Nuisance Parameters

Generic perturbative expansion:

$$f(\alpha) = f_0 + f_1\alpha + f_2\alpha^2 + f_3\alpha^3 + \dots$$

f_i : the coefficient of the series, potentially unknown

We can compute the truncated series: $\hat{f}_i$: the true value, i.e. a value we actually computed

$$f^{\rm LO}(\alpha) = \hat{f}_0 \qquad f^{\rm NLO}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha \qquad f^{\rm NNLO}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha + \hat{f}_2\alpha^2$$

The missing terms are the source of uncertainty.
(assume convergence → the first missing is the dominant one)

$$f^{\rm LO+1}(\alpha) = \hat{f}_0 + f_1\alpha \quad f^{\rm NLO+1}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha + f_2\alpha^2 \quad f^{\rm NNLO+1}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha + \hat{f}_2\alpha^2 + f_3\alpha^3$$

Challenge: how to estimate f_1, f_2, f_3, … without computing them?

Theory uncertainties from scale variations

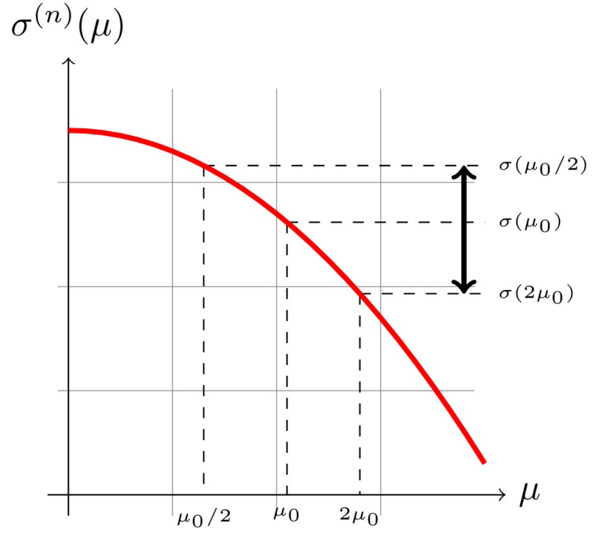
Lets focus on QCD as an example: $\alpha = \alpha_s(\mu_0)$

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(0)} + \alpha_s \mathrm{d}\sigma^{(1)} + \alpha_s^2 \mathrm{d}\sigma^{(2)} + \ldots \xrightarrow{\text{RGE}} \mu \frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\mu} = \mathcal{O}\left(\alpha_s^{(n_0+n+1)}\right)$$

Of same order as the next dominant term → exploiting this to estimate size of $\mathrm{d}\sigma^{(n+1)}$

Scale variation prescription (ad-hoc and heuristic choice)

- choose 'sensible' μ_0
  - → principle of fasted apparent convergence: $\sigma^{(n)}(\mu_{\mathrm{FAC}}) = 0$
  - → principle of minimal sensitivity: $\mu \frac{\mathrm{d}\sigma(\mu)}{\mathrm{d}\mu}\Big|_{\mu=\mu_{\mathrm{PMC}}} = 0$
  - → ...
- vary with a factor (typically 2)
- take envelope as uncertainty



Short comings of scale variations

- not always reliable … however in most cases issues are understood/expected:
  new channels, phase space constraints, etc. → often we can design workarounds
- however, some issues are more fundamental:
  → how to choose the central scale? → not a physical parameter, no 'true' value
  (Principle of fasted apparent convergence, principle of minimal sensitivity,…)
  → how to propagate the estimated uncertainty, no statistical interpretation!
  → what about correlations? Based on 'fixed form' of the lower orders and RGE.

(At the moment) two alternative approaches under investigation:

"Bayesian"

[Cacciari,Houdeau 1105.5152]
[Bonvini 2006.16293]
[Duhr, Huss, Mazeliauskas, Szafron 2106.04585]

"Theory Nuisance Parameter"

[Tackmann 2411.18606]
→ W mass extraction: [CMS 2412.13872]
[Cal,Lim, Scott,Tackmann Waalewijn 2408.13301]
[Lim, Poncelet, 2412.14910]

Introducing theory nuisance parameters (TNPs)

Generic perturbative expansion:

$$f(\alpha) = f_0 + f_1\alpha + f_2\alpha^2 + f_3\alpha^3 + \dots$$

Beyond Scale Variations: Perturbative Theory Uncertainties from Nuisance Parameters, Frank Tackmann [2411.18606]

$$f^{\mathrm{LO}+1}(\alpha) = \hat{f}_0 + f_1(\theta)\alpha$$

$$f^{\mathrm{NLO}+1}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha + f_2(\theta)\alpha^2$$

$$f^{\mathrm{NNLO}+1}(\alpha) = \hat{f}_0 + \hat{f}_1\alpha + \hat{f}_2\alpha^2 + f_3(\theta)\alpha^3$$

Introduce a parametrisation of unknown coefficients in terms of

"Theory nuisance parameters" θ

- The parametrization such that there is a true value: $f_i(\hat{\theta}) = \hat{f}_i$
- Distributions of θ "known" (for example from already existing computations)

→ Expert knowledge to construct such a parametrisation

Example: TNPs in resummed cross sections

[Tackman 2411.18606]

Transverse momentum resummation:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = [\underbrace{H \times B_a \times B_b \times S}](\alpha_s, \ln\left(\frac{p_T}{Q}\right)) + \mathcal{O}\left(\frac{p_T}{Q}\right)$$

$$X(\alpha_s, L) = X(\alpha_s)\exp\int_0^L \mathrm{d}L'\{\Gamma(\alpha_s(L'))L' + \gamma_X(\alpha_s(L'))\}$$

Perturbative expansion of resummation ingredients:

$$X(\alpha_s) = X_0 + \alpha_s X_1 + \alpha_s^2 X_2 + \cdots \boxed{+ \alpha_s^n X_n(\theta)}$$

$$\Gamma(\alpha_s) = \alpha_s[\Gamma_0 + \alpha_s\Gamma_1 + \alpha_s^2\Gamma_2 + \cdots \boxed{+ \alpha_s^n\Gamma_n(\theta)]}$$

$$\gamma(\alpha_s) = \alpha_s[\gamma_0 + \alpha_s\gamma_1 + \alpha_s^2\gamma_2 + \cdots \boxed{+ \alpha_s^n\gamma_n(\theta)]}$$

Task: find suitable parametrization and variation range

These are numbers for simple processes → only need normalisation

TNP parametrisations for resummation

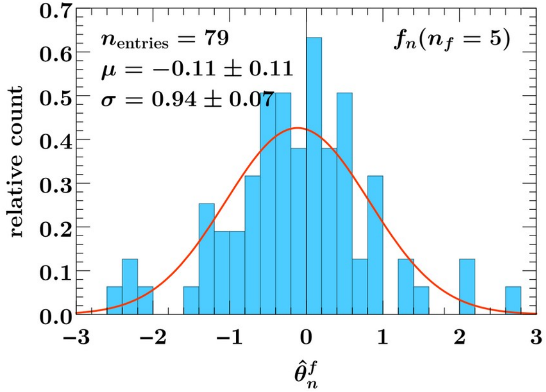
[Tackman 2411.18606]

$\gamma(\alpha_s)$	N_n	$\hat{\gamma}_0/N_0$	$\hat{\gamma}_1/N_1$	$\hat{\gamma}_2/N_2$	$\hat{\gamma}_3/N_3$	$\hat{\gamma}_4/N_4$
β	1	-15.3	-77.3	-362	-9652	-30941
	4^{n+1}	-3.83	-4.83	-5.65	-37.7	-30.2
	$4^{n+1}C_F C_A^n$	$\mathbf{-1.28}$	$\mathbf{-0.54}$	$\mathbf{-0.21}$	$\mathbf{-0.47}$	$\mathbf{-0.12}$
γ_m	1	-8.00	-112	-950	-5650	-85648
	4^{n+1}	-2.00	-7.028	-14.8	-22.1	-83.6
	$4^{n+1}C_F C_A^n$	$\mathbf{-1.50}$	$\mathbf{-1.76}$	$\mathbf{-1.24}$	$\mathbf{-0.61}$	$\mathbf{-0.77}$
$2\Gamma^q_{\mathrm{cusp}}$	1	$+10.7$	$+73.7$	$+478$	$+282$	$(+140000)$
	4^{n+1}	$+2.67$	$+4.61$	$+7.48$	$+1.10$	$(+137)$
	$4^{n+1}C_F C_A^n$	$\mathbf{+2.00}$	$\mathbf{+1.15}$	$\mathbf{+0.62}$	$\mathbf{+0.03}$	$\mathbf{(+1.27)}$

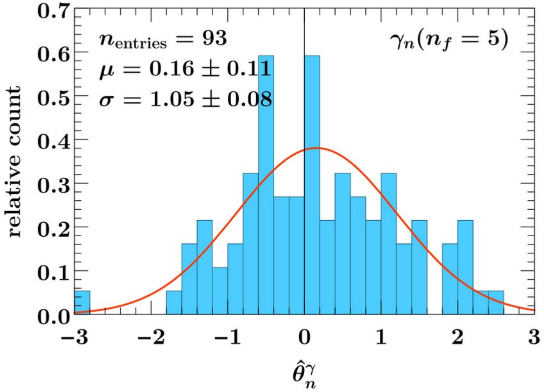
⋮

"Statistics over many computations"

Matrix elements



Anomalous dimensions



Some remarks on TNPs in resummation

Picture: simple ingredients that enter different computations/processes etc.

→ ideal situation

But actually not that simple:

- Scheme dependence of ingredients? E.g. scales, IR subtraction, …
  → might need modified parametrisations
- Some TNPs represent directly numbers: $\Gamma,\ \gamma,\ H$ for simple processes
  but others are functions → Beam functions, hard functions for more complicated processes
- Parametrisation works for the resummed spectrum. What about other observables?
- "Easy to implement" for use-cases so far
  → might be really expensive if each variation needs a full computation (Monte Carlos,...)

Is there a simpler, say "effective", way to do this for a general computation?

TNP approach for fixed-order computations

[Lim, Poncelet, 2412.14910]

$$\mathrm{d}\sigma = \alpha_s^n N_c^m \,\mathrm{d}\bar{\sigma}^{(0)} + \alpha_s^{n+1} N_c^{m+1}\,\mathrm{d}\bar{\sigma}^{(1)} + \alpha_s^{n+2} N_c^{m+2}\,\mathrm{d}\bar{\sigma}^{(2)} + \ldots$$

$$= \alpha_s^n N_c^m \,\mathrm{d}\bar{\sigma}^{(0)} \left[1 + \alpha_s N_c \left(\frac{\mathrm{d}\bar{\sigma}^{(1)}}{\mathrm{d}\bar{\sigma}^{(0)}}\right) + \alpha_s^2 N_c^2 \left(\frac{\mathrm{d}\bar{\sigma}^{(2)}}{\mathrm{d}\bar{\sigma}^{(0)}}\right) + \ldots\right]$$

Observation, i.e. "expert knowledge": $\frac{\mathrm{d}\bar{\sigma}^{(i)}}{\mathrm{d}\bar{\sigma}^{(0)}} \sim \mathcal{O}(1)$

Use some knowledge about lower orders but introduce parametric dependence:

$$\frac{\mathrm{d}\bar{\sigma}^{(N+1)}_{\mathrm{TNP}}}{\mathrm{d}\bar{\sigma}^{(0)}} = \sum_{j=1}^{N} f_k^{(j)}\left(\vec{\theta}, x\right)\left(\frac{\mathrm{d}\bar{\sigma}^{(j)}}{\mathrm{d}\bar{\sigma}^{(0)}}\right)$$

x → mapped kinematic variable

Approximation of original TNP philosophy

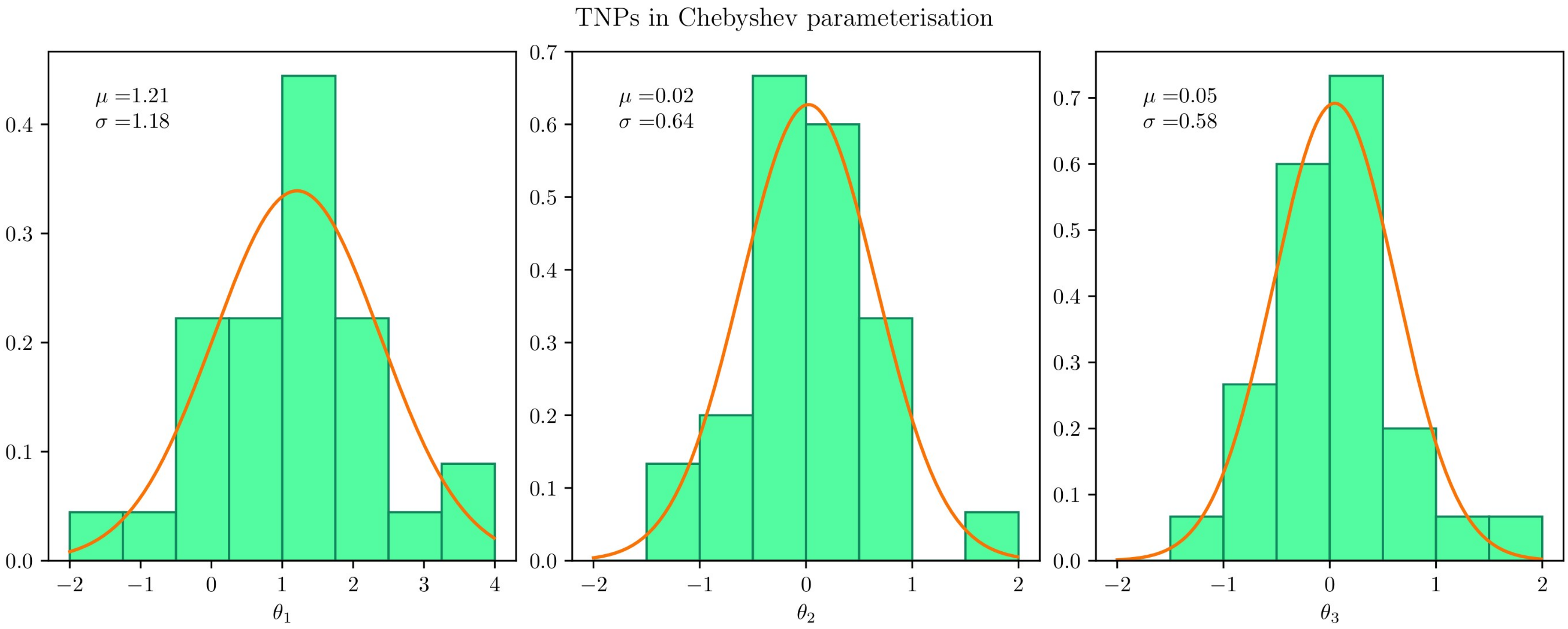
→ there is only $f_i(\hat{\theta}) \approx \hat{f}_i$

Chebyshev: $f_k^C\left(\vec{\theta}, x\right) = \frac{1}{2}\sum_{i=0}^{k} \theta_i T_i(x)$ $\qquad x \in [-1, 1]$

Process meta study

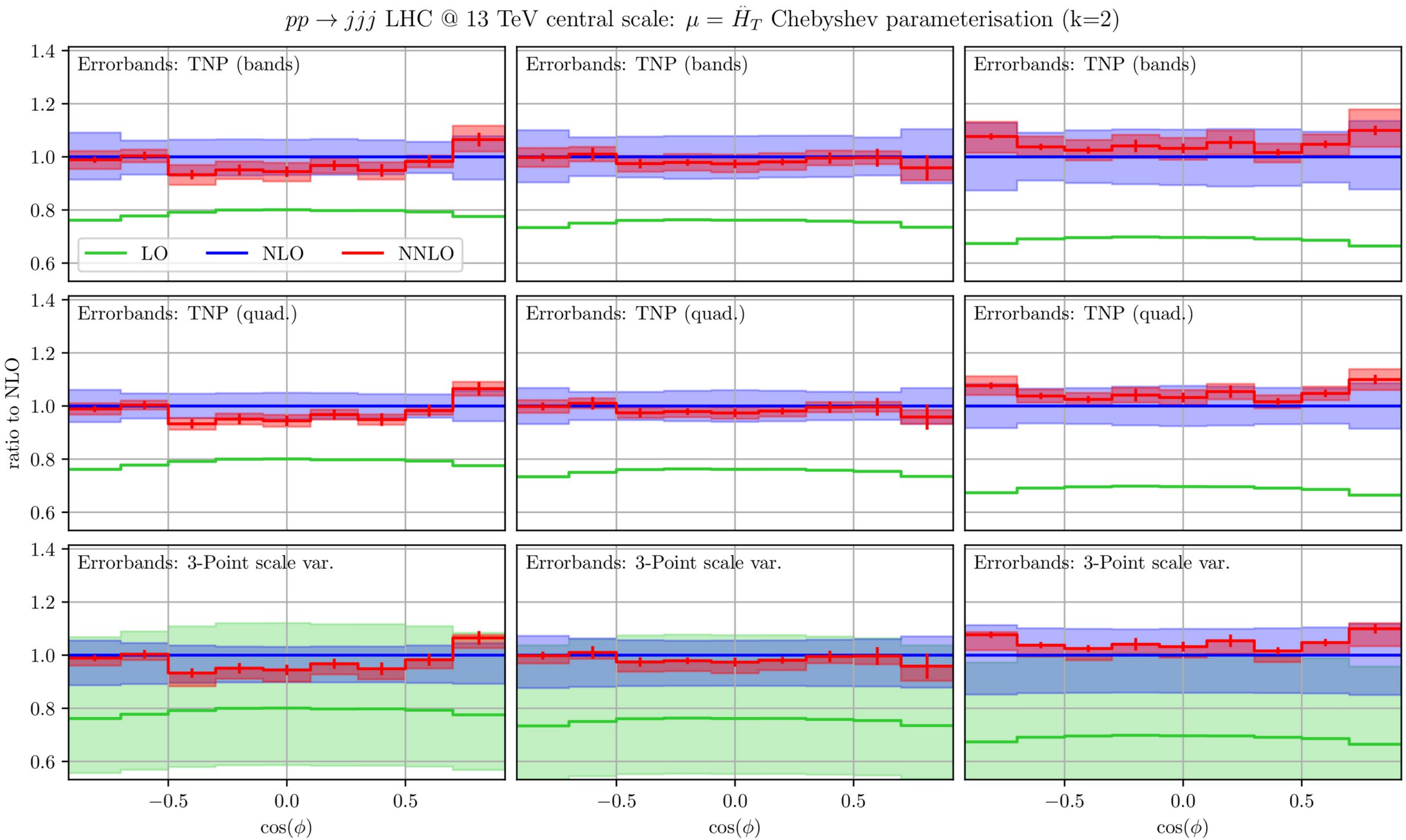
Process	$\sqrt{s}$/TeV	Scale	PDF	Distributions
$pp \to H$ (full theory)	13	$m_H/2$	`NNPDF3.1`	y_H
$pp \to ZZ^* \to e^+e^-\mu^+\mu^-$	13	M_T	`NNPDF3.1`	$M_{e^+\mu^+}$, $p_T^{e^+e^-}$, y_{e^+}
$pp \to WW^* \to e\nu_e\mu\nu_\mu$	13	m_W	`NNPDF3.1`	M_{WW}, $p_T^{\mu^-}$, y_{W^-}
$pp \to (W \to \ell\nu) + c$	13	$E_T + p_T^c$	`NNPDF3.1`	p_T^ℓ ,$\lvert y_\ell \rvert$,
$pp \to t\bar{t}$	13	$H_T/4$	`NNPDF3.1`	$M_{t\bar{t}}$, p_T^t, y_t
$pp \to t\bar{t} \to b\bar{b}\ell\bar{\ell}$	13	$H_T/4$	`NNPDF3.1`	$M_{\ell\bar{\ell}}$, $p_T^{b_1}$, $p_{T,\ell_1}/p_{T,\ell_2}$
$pp \to \gamma\gamma$	8	$M_{\gamma\gamma}$	`NNPDF3.1`	$M_{\gamma\gamma}$, $p_T^{\gamma_1}$, $y_{\gamma\gamma}$
$pp \to \gamma\gamma j$	13	$H_T/2$	`NNPDF3.1`	$M_{\gamma\gamma}$, $p_T^{\gamma\gamma}$, $\cos\phi_{\mathrm{CS}}$, $\lvert y_{\gamma_1} \rvert$, $\Delta\phi_{\gamma\gamma}$
$pp \to jjj$	13	$\hat{H}_T$	`MMHT2014`	TEEC with $H_{T,2} \in [1000, 1500), [1500, 2000), [3500, \infty)$ GeV
$pp \to \gamma jj$	13	H_T	`NNPDF3.1`	$M_{\gamma jj}$, p_T^j, $\lvert y_{\gamma-\mathrm{jet}} \rvert$, $E_{T,\gamma}$

Fits - Chebyshev parametrisation



$$f_k^C\left(\vec{\theta}, x\right) = \frac{1}{2}\sum_{i=0}^{k} \theta_i T_i(x)$$

Example: TEEC

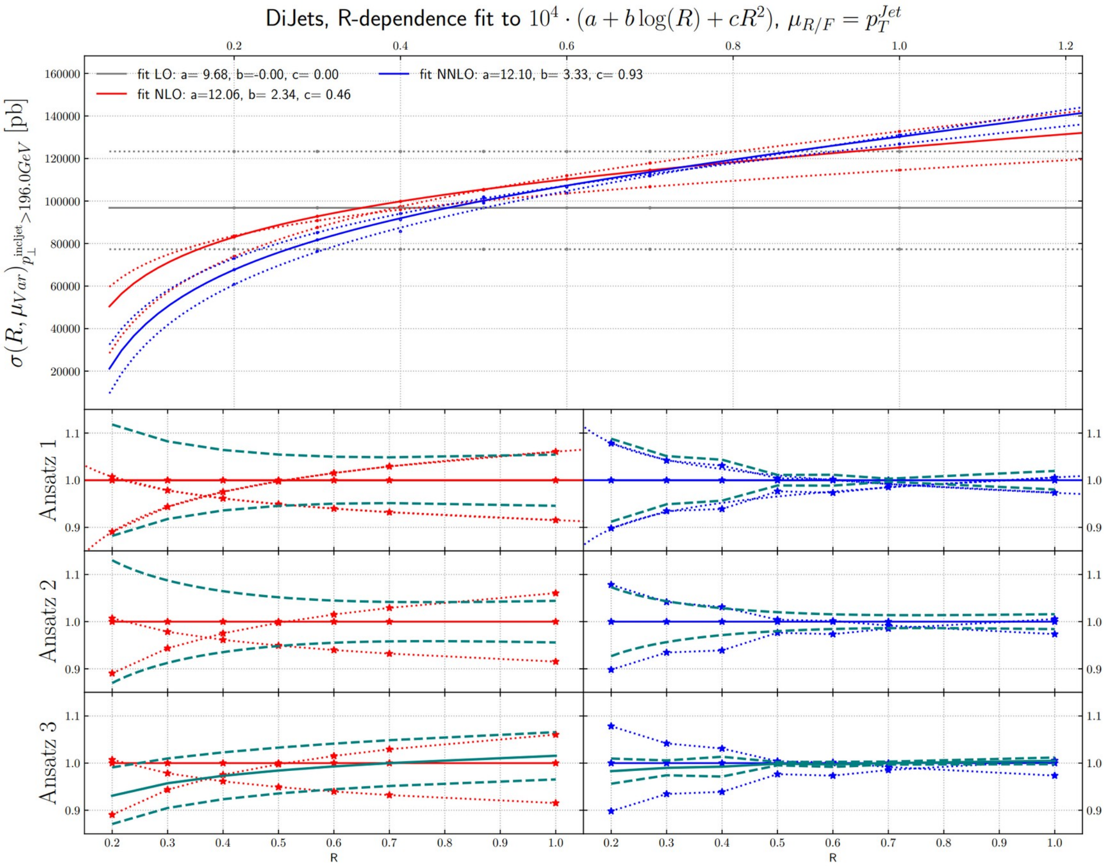


Example: inclusive jet production

→ Important process for PDF fits: sensitivity to gluon PDF at large-x

→ NNLO QCD corrections imply very small theory uncertainty

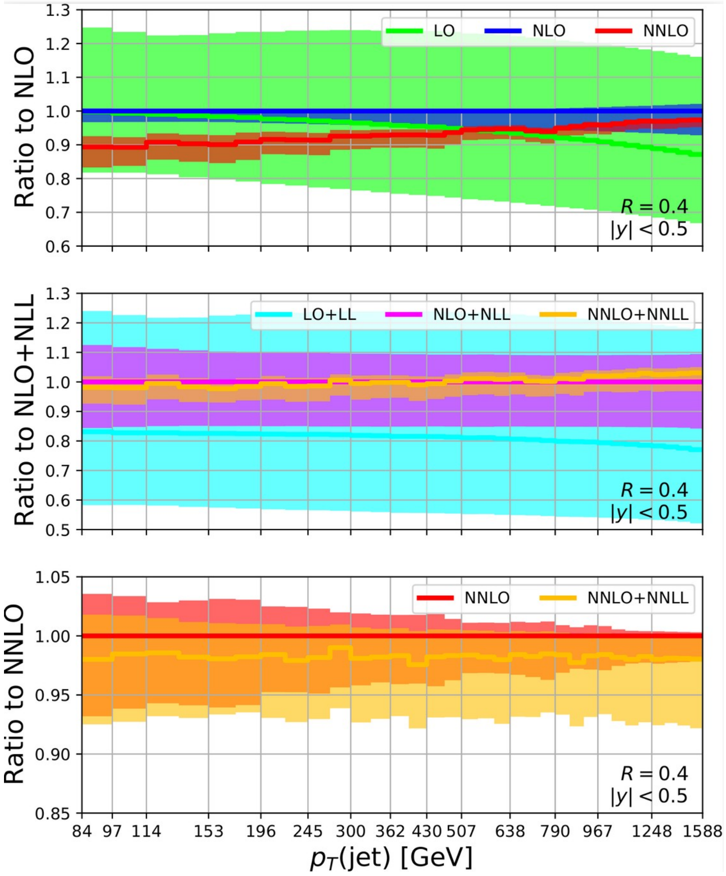
→ Significant jet radius dependence of uncertainties from scale variations



[1903.12563 Bellm et al]

Inclusive jet production: small-R resummation NNLO+NNLL

[Generet, Lee, Moult, Poncelet, Zhang'25]



FO scale variations

R=0.4

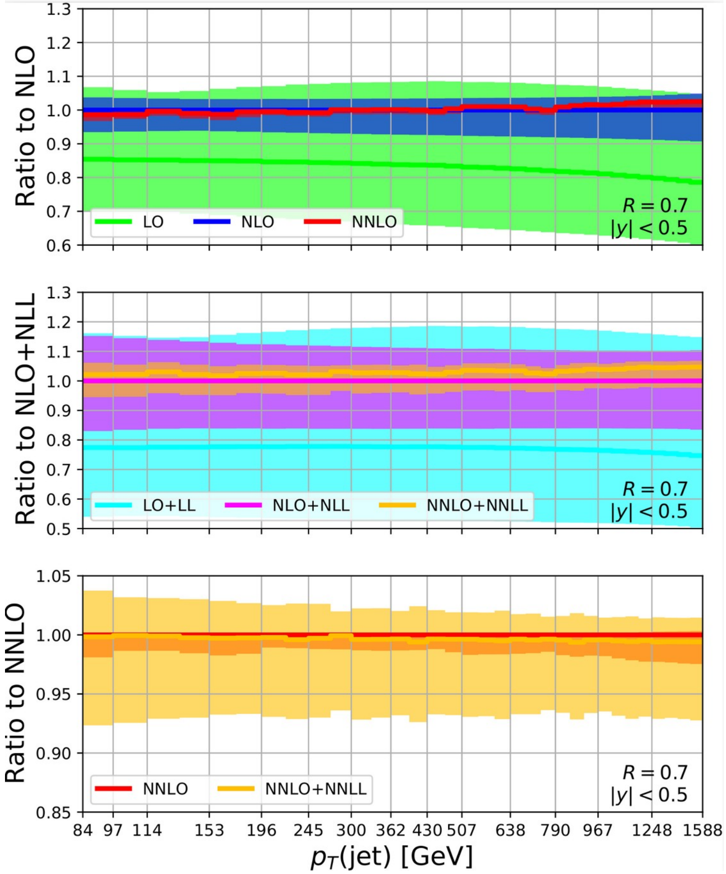
→ underestimation of NNLO correction

R=0.7

→ very small NNLO uncertainty

Resummation

→ stabilization of pert. series and uncertainties.

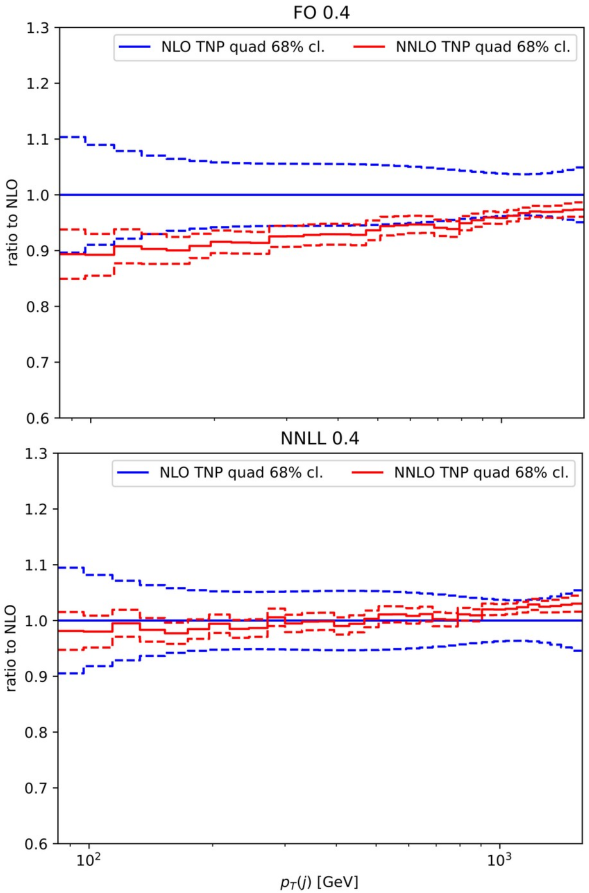


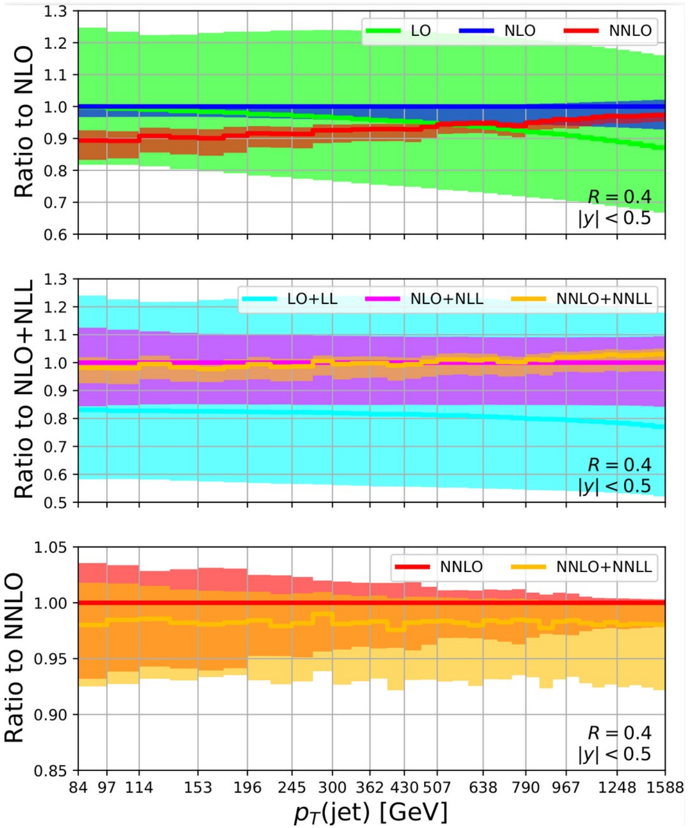
TNP uncertainties for inclusive jet production

R = 0.4

TNP uncertainties

- More sensible NLO uncertainties
- Similar to resummed scale variation



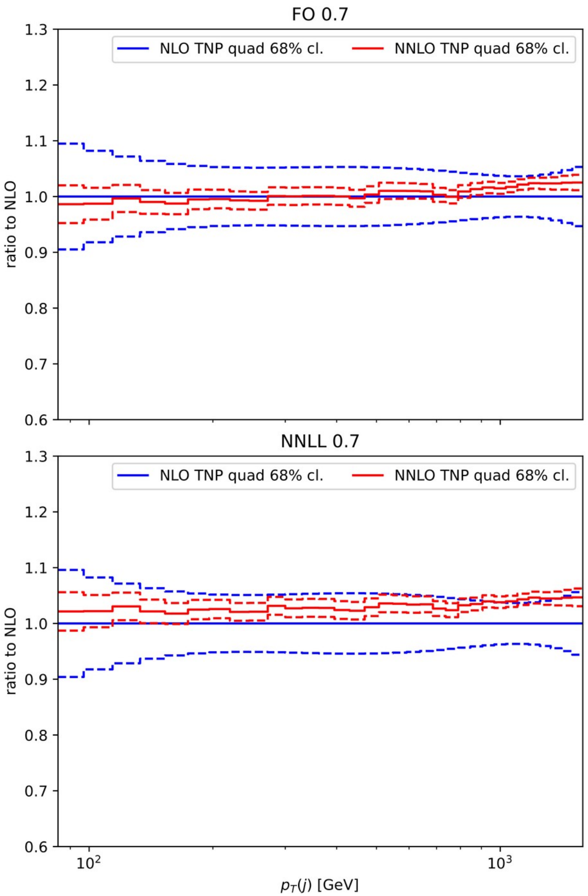


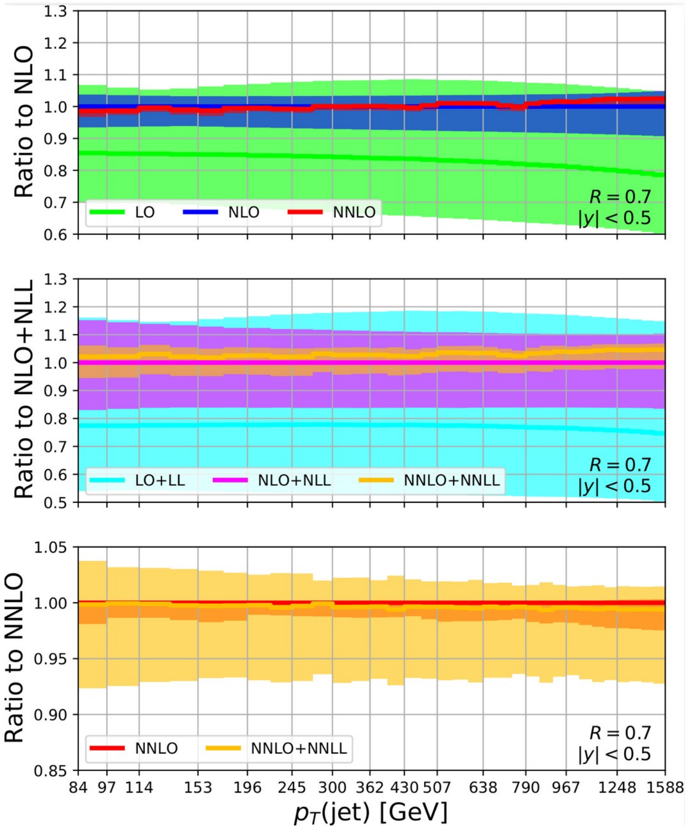
TNP uncertainties for inclusive jet production

R = 0.7

TNP uncertainties

- More sensible NNLO QCD uncertainties
- Similar to resummed scale variation





A more realistic approach

Thanks to Terry Generet to put this together!

The pT spectrum is a steeply falling function → effectively only few Mellin moments contribute

$$\frac{d\sigma}{dp_T} \approx \sum_{a,b} L_{ab}(\hat{E}/E = 2p_T/E)\frac{d\hat{\sigma}_{ab}}{dp_T}(N = \tilde{n}(2p_T/E))$$

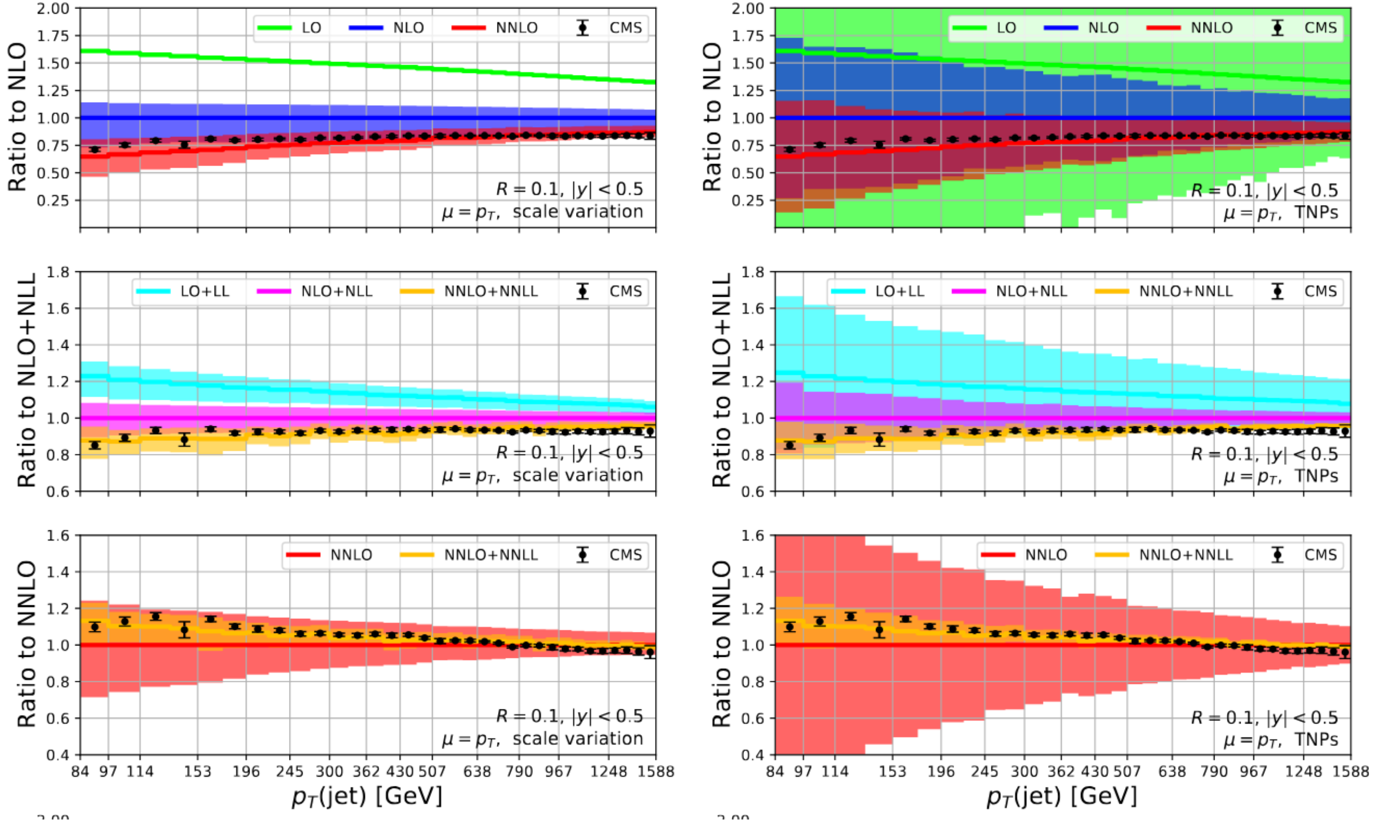
$$\begin{aligned} d\hat{\sigma}_{ab\to cd}(N) = {} & J_{\rm in}^{(a)}\left(\frac{\hat{s}}{N_{0a}^2\mu^2}, \alpha_s(\mu)\right) J_{\rm in}^{(b)}\left(\frac{\hat{s}}{N_{0b}^2\mu^2}, \alpha_s(\mu)\right) \\ & \times J_{\rm fr}^{(c)}\left(\frac{\hat{s}}{N_0^2\mu^2}, \alpha_s(\mu)\right) J_{\rm rec}^{(d)}\left(\frac{\hat{s}}{N_0\mu^2}, \frac{\hat{s}}{N_0^2\mu^2}, \alpha_s(\mu)\right) \\ & \times \mathrm{Tr}\left[\mathbf{H}_{ab\to cd}\left(\frac{\hat{s}}{\mu^2}, \alpha_s(\mu)\right)\mathbf{S}_{ab\to cd}\left(\frac{\hat{s}}{N_0^2\mu^2}, \alpha_s(\mu)\right)\right] + \mathcal{O}\left(\frac{1}{N}\right) \end{aligned}$$

These then can be broken down into scalar series:
(soft+hard functions require approx. of
colour matrix → error on the error)

$$\begin{aligned} J_{\rm in}^{(i)}\left(\frac{\hat{s}}{N_{0i}^2\mu^2}, \alpha_s(\mu)\right) = J_{\rm fr}^{(i)}\left(\frac{\hat{s}}{N_{0i}^2\mu^2}, \alpha_s(\mu)\right) = R_i(\alpha_s(\mu)) \\ \times \exp\left[\int_{\sqrt{\hat{s}}/N_{0i}}^{\mu} \frac{d\mu'}{\mu'}\left(A_i(\alpha_s(\mu'))\ln\left(\frac{\mu'^2 N_{0i}^2}{\hat{s}}\right) - \frac{1}{2}D_i(\alpha_s(\mu'))\right)\right] \end{aligned}$$

Theory uncertainties from TNPs for jets

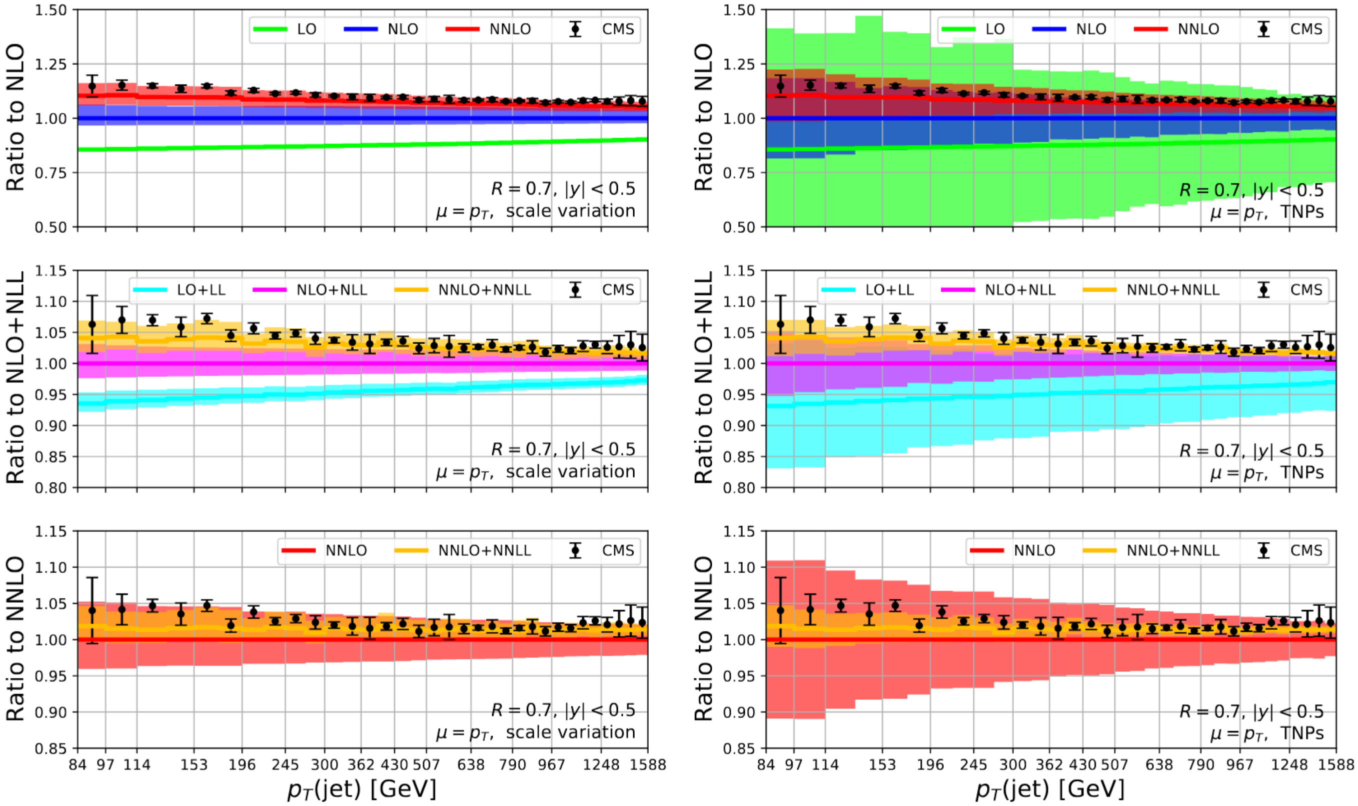
Small R: expect fixed-order to fail and resummation to be stable



side note
these are ratios (R/R=0.4),
TNPs allow
correct correlation!

Theory uncertainties from TNPs for jets

Intermediate R: observed small scale dependence → TNPs more realistic



Summary/Outlook

Higher-order (NNLO) QCD corrections are an important corner stone of LHC phenomenology

- Many phenomenological applications
  - **Precision tests of the SM**
  - PDF + SM parameter extractions: masses + couplings
  - Fragmentation processes start to appear → **application to jet substructure observables**
- **Theory uncertainties** move into focus
- **Multi-loop amplitudes** are **again** the **main bottleneck** to compute new NNLO proc.
- **Local matching to PS is next big step**