Precision QCD phenomenology for multi-scale processes at the Large Hadron Collider

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2018 - 2023	**Research Associate** Cavendish Laboratory, Cambridge, UK
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Awards:	**Leverhulme Early Career Fellowship 2021** **Guido Altarelli Prize 2025** **ERC Starting Grant 2025**

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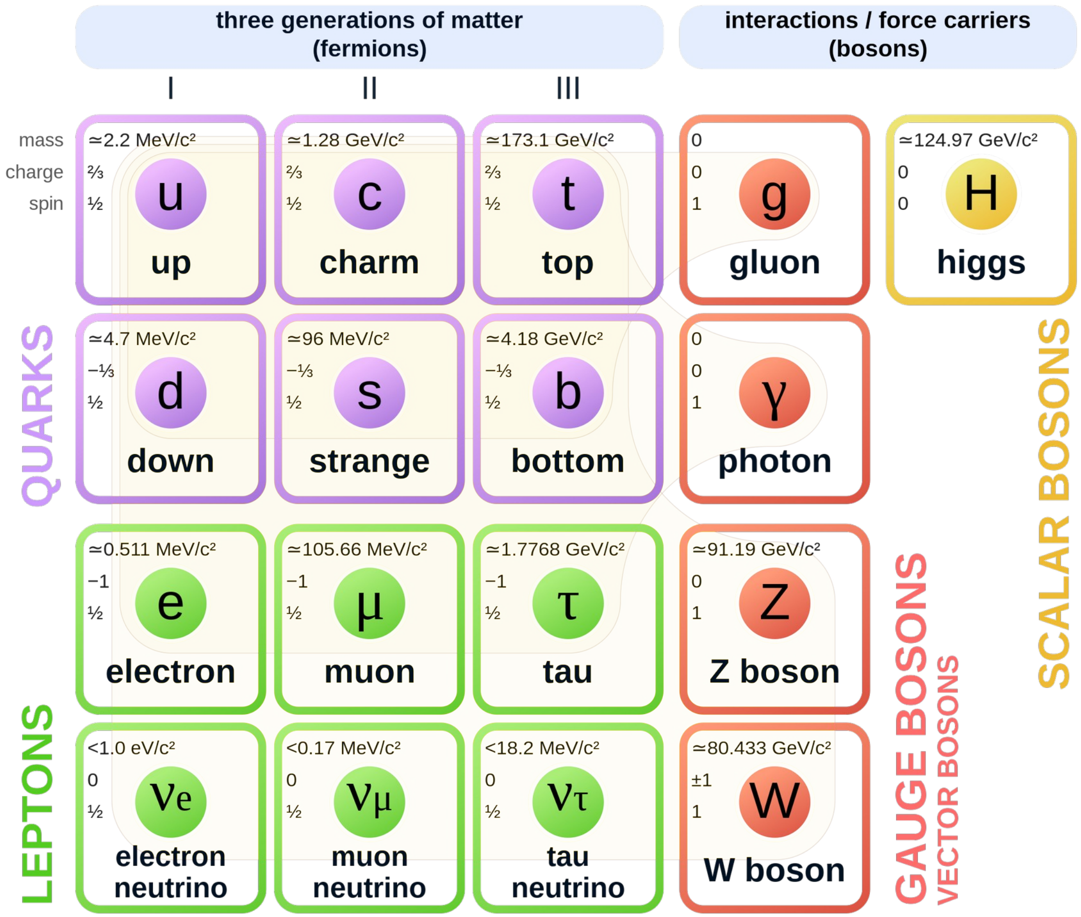
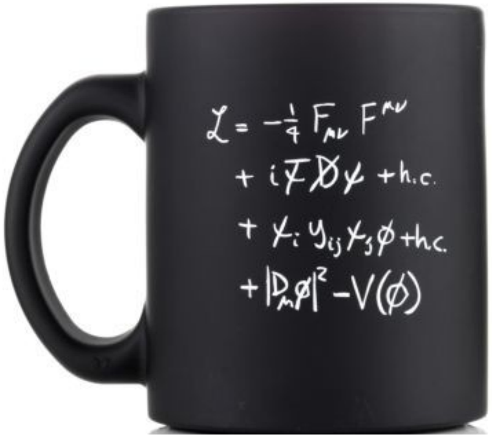
Standard Model of Elementary Particles



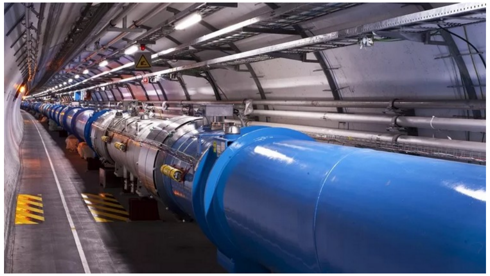


Credit: Wikipedia/CERN

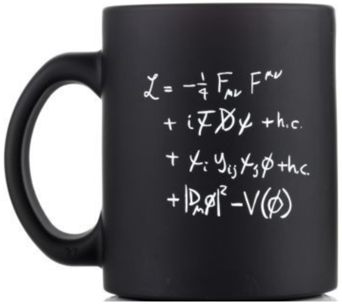
What are the fundamental building blocks of matter?

Scattering experiments

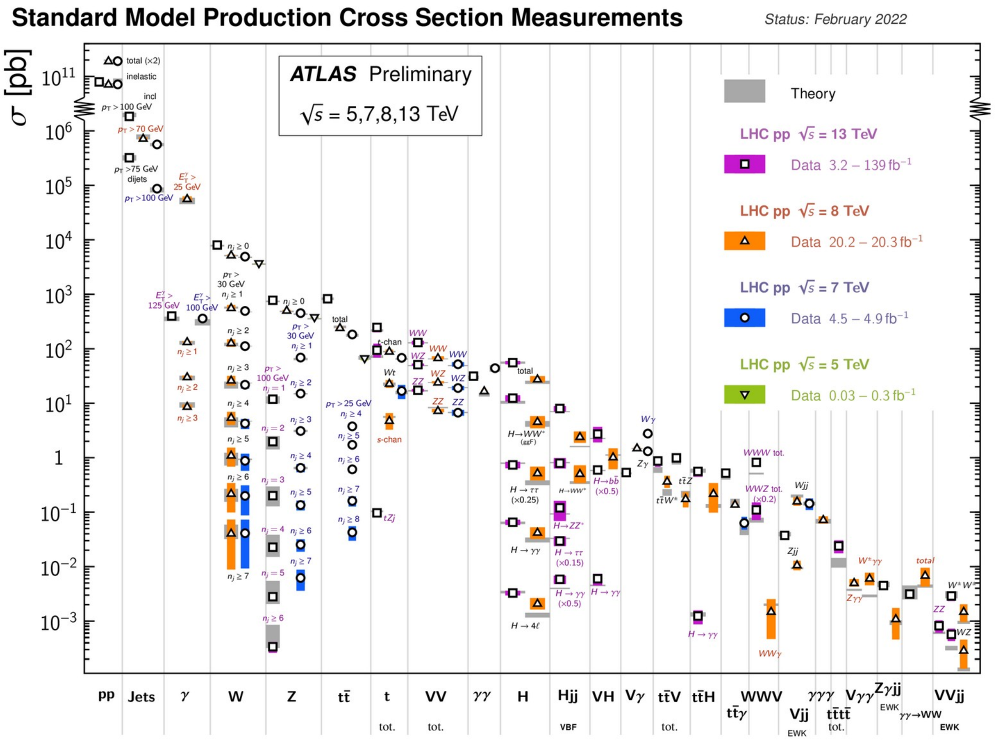
Large Hadron Collider (LHC)



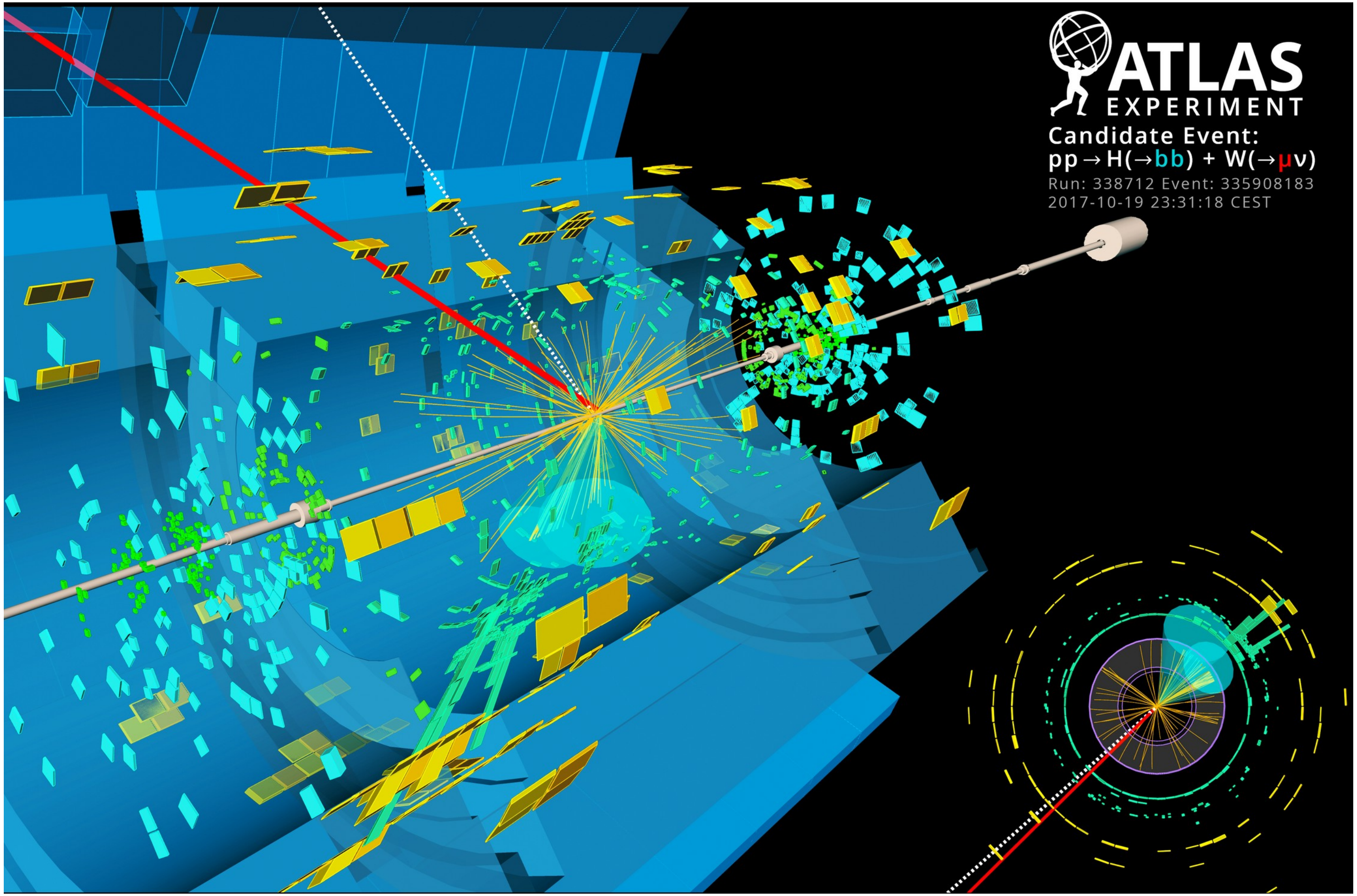
Credit: CERN



Theory/ Standard Model



Collision events



Theory picture of hadron collision events

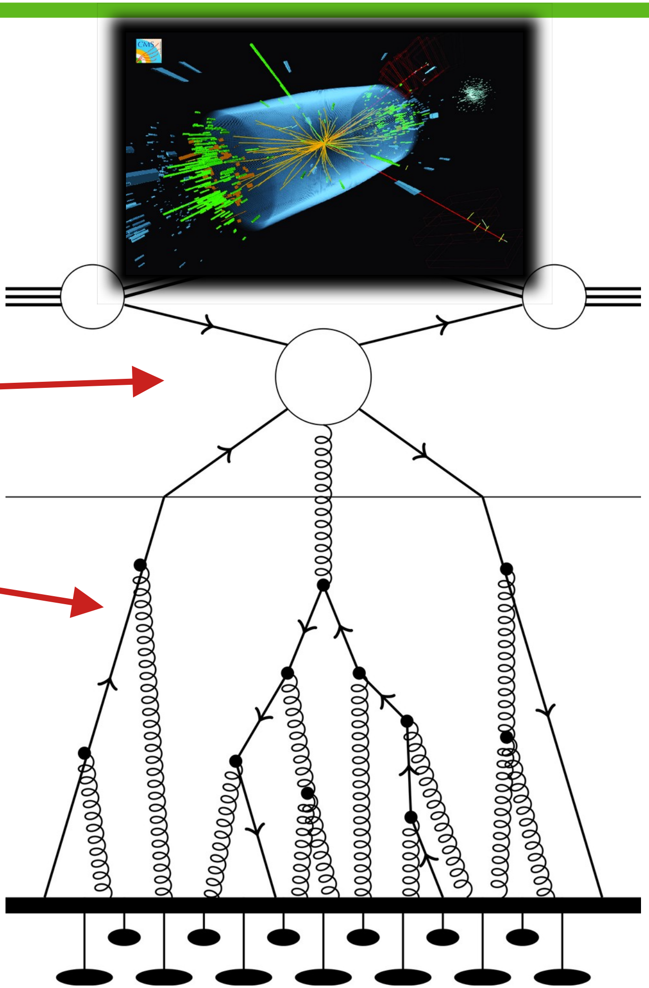
Guiding principle: factorization
"What you see depends on the energy scale"

In Quantum Chromodynamics (QCD):

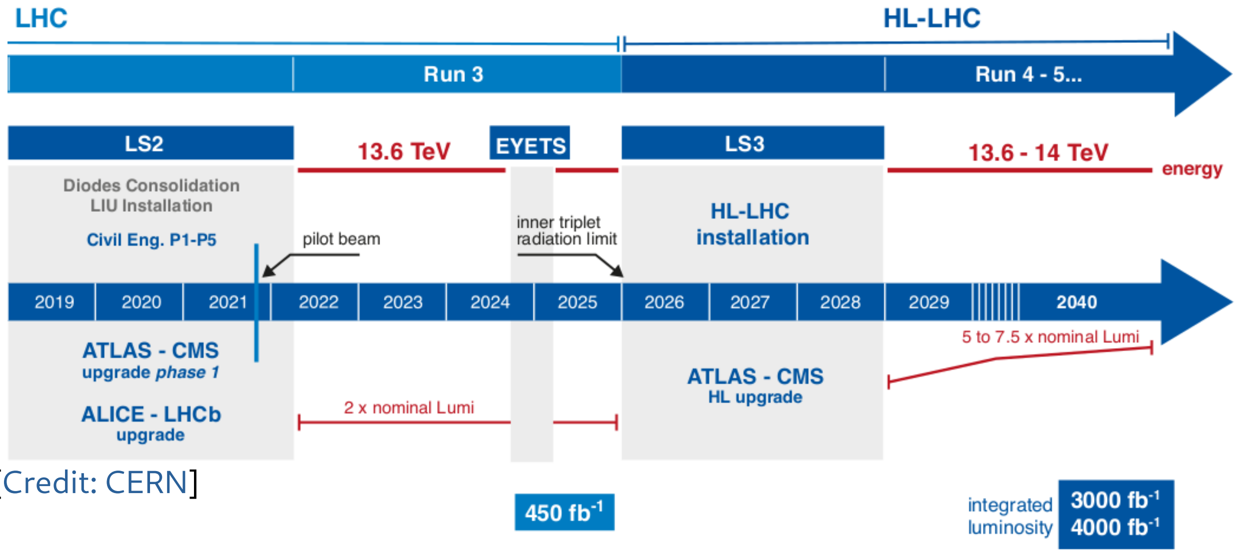
$Q \gg \Lambda_{\mathrm{QCD}}$ **Fixed-order perturbation theory**
scattering of individual partons

$Q \gtrsim \Lambda_{\mathrm{QCD}}$ **Parton-shower/Resummation**
all-order bridge between perturbative and non-perturbative physics

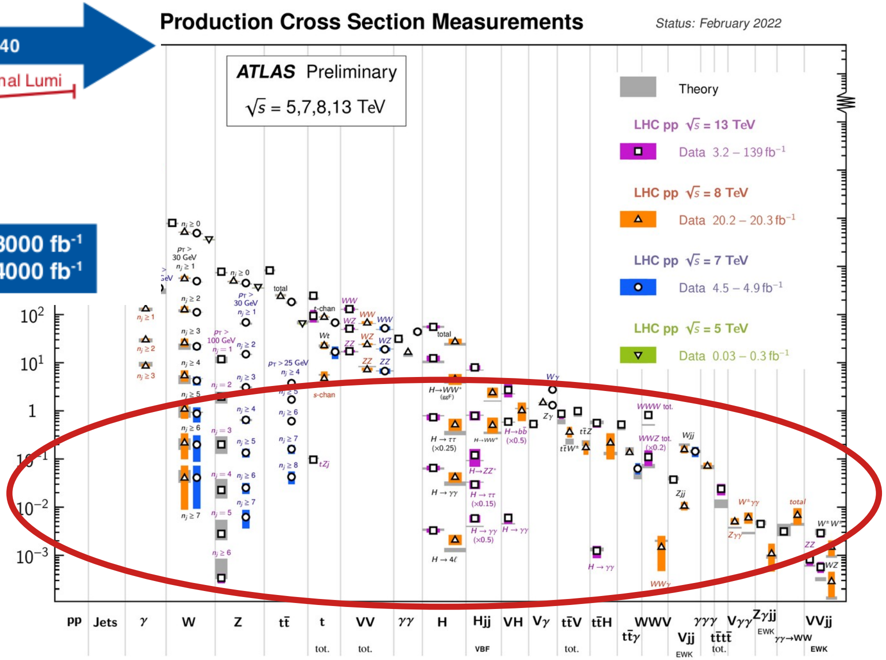
$Q \sim \Lambda_{\mathrm{QCD}}$ **"Hadronization"/MPI/...**
non-perturbative physics



LHC Precision era and future experiments



[Credit: CERN]



More precision for higher multiplicity processes:
→ theory needs to keep up!

Precision predictions

Fixed order perturbation theory

→ **Core element of event simulation**
→ **Describes high Q regime**

Resummation

Parton-showers

Precision theory predictions

Parametric input:
PDFs, couplings (α_s), …

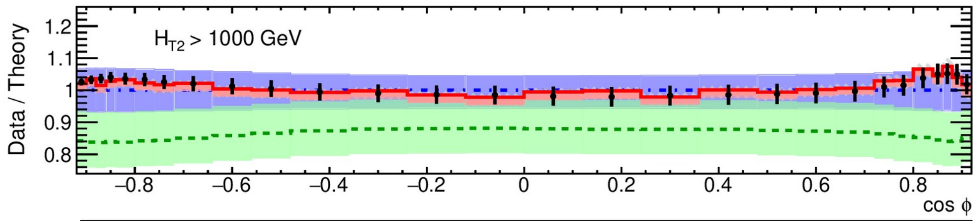
Fragmentation/hadronisation

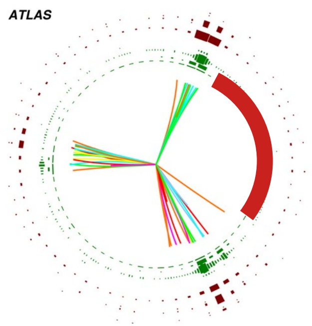
Soft physics:
MPI, colour reconnection, …

Precision through higher-order perturbation theory

Example: ATLAS
multi-jet measurements

[ATLAS 2301.09351]





$\cos\phi$

Cross section = **LO** + **NLO** + **NNLO** + **O(α_s^3)**

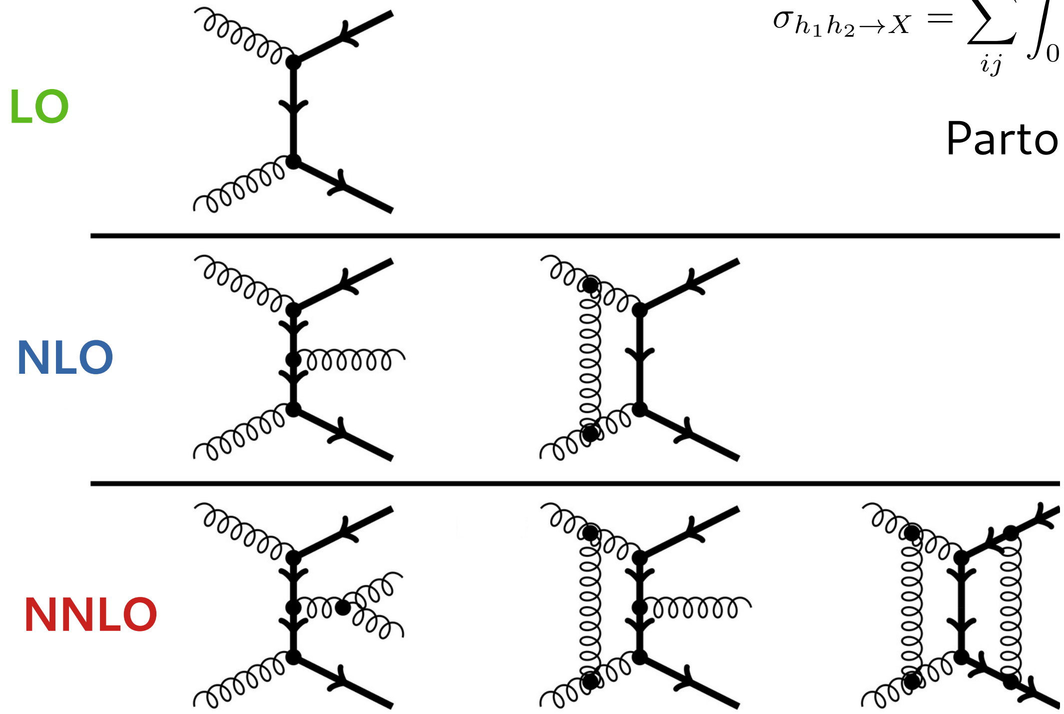
$\sim (\alpha_s)^1$ $\sim (\alpha_s)^2$

Theory uncertainty: **Order of magnitude** **O(10%)** **O(1%)**

Fixed-order expansion in the strong coupling $\alpha_s(m_Z) \approx 0.118$

Experimental precision reaches percent-level already at LHC
next-to-next-to-leading order QCD needed on theory side!

NNLO QCD in collinear factorization



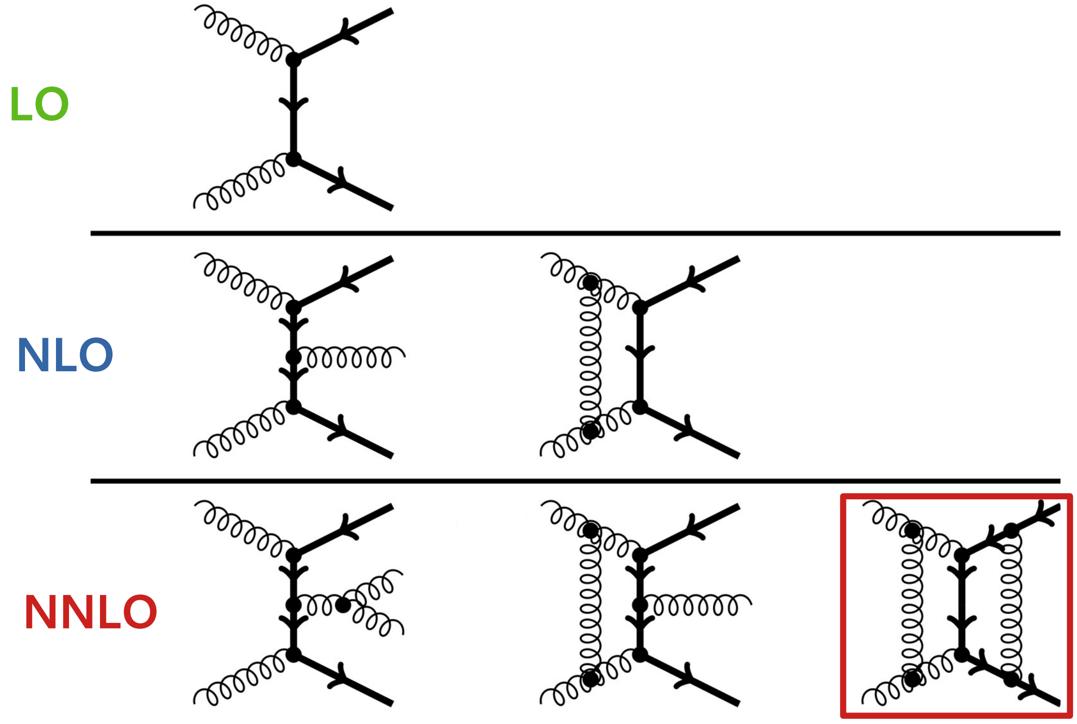
$$\sigma_{h_1 h_2 \to X} = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \phi_{i,h_1}(x_1, \mu_F^2) \phi_{j,h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij \to X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

Parton distribution functions

Partonic cross section:

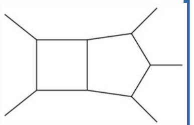
$$\hat{\sigma}_{ab \to X} = \hat{\sigma}^{(0)}_{ab \to X} + \hat{\sigma}^{(1)}_{ab \to X} + \hat{\sigma}^{(2)}_{ab \to X} + \mathcal{O}(\alpha_s^3)$$

NNLO QCD challenges: two-loop amplitudes

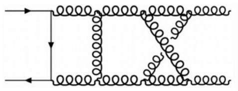


How to compute
multi-scale two-loop QCD amplitudes?
→ **fast growing complexity:**
rational and **transcendental**
→ deeper understanding of the analytical properties
→ refinement of computational tools

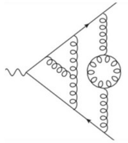
Two-loop 5-point
[Abreu, Agarwal, Badger, Buccioni, Chawdhry, Chicherin,Czakon, de Laurenties, Febres-Cordero, Gambuti, Gehrmann, Henn, Lo Presti, Manteuffel, Ma, Mitov, Page, Peraro, Poncelet, Schabiner Sotnikov, Tancredi, Zhang,...]



Three-loop 4-point
[Bargiela, Dobadilla, Canko, Caola, Jakubcik, Gambuti, Gehrmann, Henn, Lim, Mella, Mistleberger, Wasser, Manteuffel, Syrrakow, Smirnov, Trancredi, ...]



Four-loop 3-point
[Henn, Lee, Manteuffel, Schabinger, Smirnov, Steinhauser,...]



Overview 2 → 3 massless amplitudes

$pp \to \gamma\gamma\gamma$

NNLO QCD corrections to three-photon production at the LHC,
Chawdhry, Czakon, Mitov, **Poncelet**
[JHEP 02 (2020) 057]

Also [2010.15834]

$pp \to \gamma\gamma j$

Two-loop leading-colour QCD helicity amplitudes for two-photon plus jet production at the LHC
Chawdhry, Czakon, Mitov, **Poncelet**
[JHEP 07, 164 (2021)]

Also [2102.01820,2105.04585]

$pp \to jjj$

[1904.00945]
[2102.13609]
[2306.15431]

Integral representation in terms of **"Pentagon functions"** [2009.07803]

Amplitudes

More recently:

$pp \to Vjj$
$pp \to t\bar{t}j$

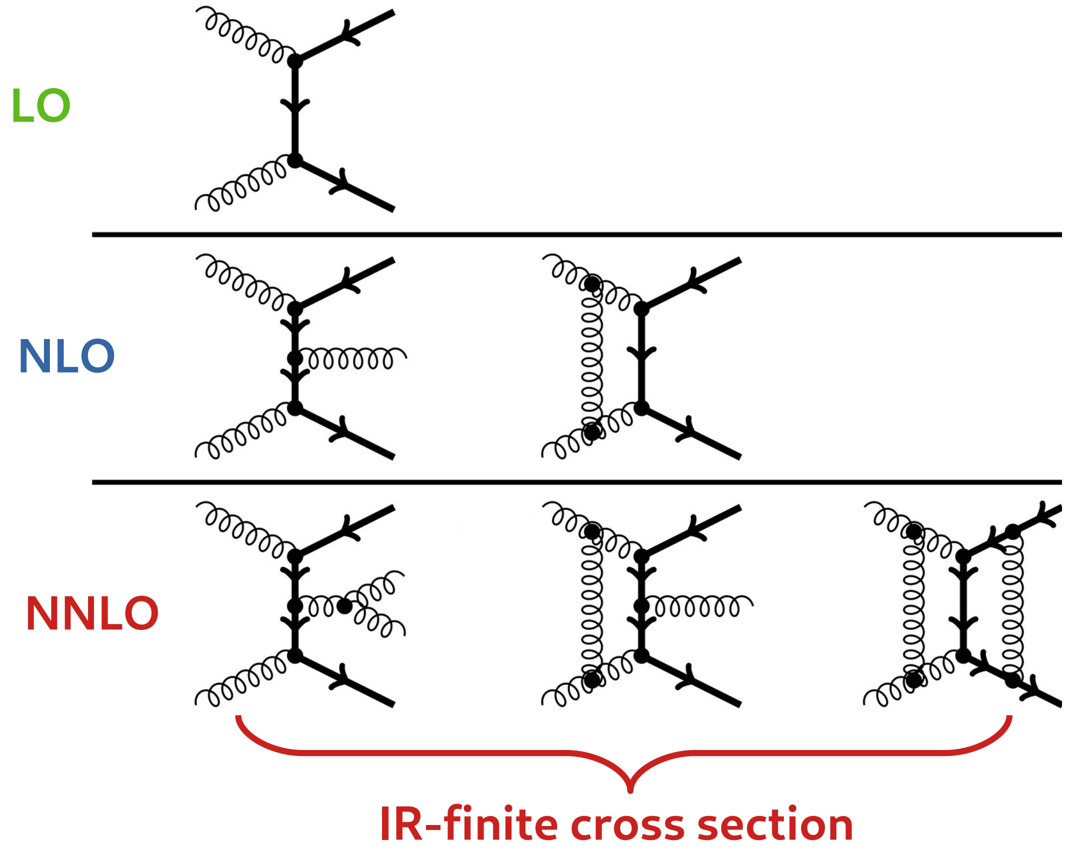
$pp \to Wb\bar{b}$

Next-to-next-to-leading order QCD corrections to Wbb production at the LHC,
Hartanto, **Poncelet**, Popescu, Zoia
[Phys. Rev. D 106, 074016 (2022)]

$pp \to \gamma jj$

Isolated photon production in association with a jet pair through next-to-next-to-leading order in QCD
Badger, Czakon, Hartanto, Moodie, Peraro, **Poncelet**, Zoia [JHEP 10, 071 (2023)]

NNLO QCD challenges: real radiation



How to achieve **infrared (IR) finite differential** cross sections at NNLO QCD?

~20 years to solve this problem

qT-slicing [Catani'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projetction [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17], Local Analytic [Magnea'18]

A complete NNLO QCD scheme:

Sector-improved residue subtraction

[Czakon et al.'10-'14,'19]

→ Improvements:

(+ first practical test of **all** NNLO IR limits)

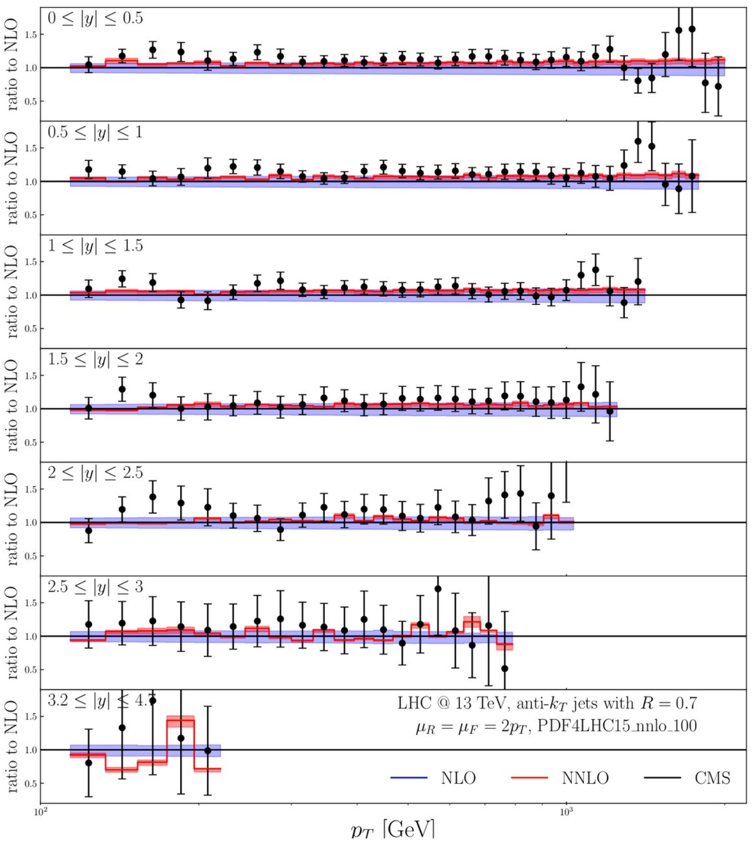
Single-jet inclusive rates with exact color at $O(a_s^4)$
Czakon,Hameren,Mitov,**Poncelet,** JHEP 10 (2019), 262

Minimal sector-improved residue subtraction

Single-jet inclusive rates with exact color at $\mathcal{O}(\alpha_s^4)$
Czakon,Hameren,Mitov,**Poncelet**, JHEP 10 (2019), 262

Refined formulation of the sector-improved residue subtraction

- New phase space parametrisation
  → minimization of subtraction kinematics
  → improved computational efficiency/stability
- Improved sector decomposition
- New 4 – dimensional formulation
- First application: inclusive jet production
  → demonstrates that the **scheme is complete**
  → no approximations



The fixed-order NNLO QCD revolution

Phenomenological applications of the **Sector-improved residue subtraction**

- Top-quark pair + decays [...**Poncelet**...'19'20']
- W+jet/ Z+jet [...**Poncelet**...'20'21'22]
- Inclusive jet production [...**Poncelet**...'19]
- Three-jet [...**Poncelet**...'21'23]
- Photon+jet-pair [...**Poncelet**...'23]
- Photon-pair+jet [...**Poncelet**...'21]
- Three photon [...**Poncelet**...'19]
- Higgs [...**Poncelet**...'24]
- Identified hadrons [...**Poncelet**...'21'23'24'25]
- Open-bottom [...**Poncelet**...'24]
- + many collaborations with ATLAS + CMS experiments

Independent works at NNLO QCD:
- Polarised bosons [...**Poncelet**...'21'25]
- W+2b-jets [...**Poncelet**...'22]
- Jets at NNLO+NNLL (small-R) [...**Poncelet**...'25]
- Theory uncertainty models [...**Poncelet**...'24]

- first-evers
- not achieved by any other scheme

My PhD

2002-03	2004-05	2006-07	2008-09	2010-11	2012-13	2014-15	2016-17	2018-19	2020-21	2022-23	2024-25
						Hj Caola et al.	HH De Florian et al.				
						ZZ Grazzini et al.	MATRIX Grazzini et al.				
						Zj Gehrmann-De Ridder et al.	jj l.c. Currie et al.				
						VBF Cacciari et al.	γ j Campbell et al.				
						Hj Boughezal et al.	γ Campbell et al.				
						Wj Boughezal et al.	Z p_T Gehrmann-De Ridder et al.				
						Hj Boughezal et al.	MCFM at NNLO Boughezal et al.				Z bb (mass.) Mazzitelli al.
						Zγ Wγ Grazzini et al.	WW Grazzini et al.	HH w. top-mass Grazzini et al.	$b\bar{b}$ Catani et al.	ttH (soft-H) Catani et al.	H bb (mass.) Biello al.
WH tot. Brein et al.					jj (partial) Currie et al.	$t\bar{t}$ Czakon et al.	WZ Grazzini et al.	ZZ+decays Kallweit et al.	Z+b Gauld et al.	VH+j Gauld et al.	ttH (mass.) Devoto al.
H tot. Ravindran et al.		H, Catani et al.		$\gamma\gamma$ Catani et al.	Zγ Grazzini et al.	WW Gehrmann et al.	$\gamma\gamma$ Campbell et al.	jj f.c. Poncelet et al.	jjj Poncelet et al.	Z+c Gauld et al.	ttW (soft-W) Buonocore al.
H tot. Anastasiou et al.	H Anastasiou et al.	W/Z, Melnikov et al.		WH Ferrera et al.	$t\bar{t}$ tot. Czakon et al.	ZH Ferrera et al.	WH/ZH Campbell et al.	$\gamma\gamma\gamma$ Poncelet et al.	$\gamma\gamma$ j Poncelet et al.	γ jj Poncelet et al.	id. hadrons Poncelet et al.
W/Z/H tot. Harlander et al.	H Anastasiou et al.	W, Melnikov et al.	W/Z, Catani et al.	VBF tot. Bolzoni et al.	Hj (partial) Boughezal et al.	ZZ Cascioli et al.	Zj Boughezal et al.	$t\bar{t}$+decays Poncelet et al.	W+c Poncelet et al.	W b b Poncelet et al.	H (t/b int.) Poncelet et al.

Overview 2 → 3 cross sections

$pp \to \gamma\gamma\gamma$

NNLO QCD corrections to three-photon production at the LHC,
Chawdhry, Czakon, Mitov, **Poncelet**
[JHEP 02 (2020) 057]

also MATRIX [2010.04681]

$pp \to \gamma\gamma j$

NNLO QCD corrections to diphoton production with an additional jet at the LHC
Chawdhry, Czakon, Mitov, **Poncelet**
[JHEP 09 (2021) 093]

also NNLOJET [2501.14021]

Sector-improved residue-subtraction

$pp \to jjj$

Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC
Czakon, Mitov, **Poncelet**
Phys.Rev.Lett. 127 (2021) 15, 152001

NNLO QCD corrections to event shapes at the LHC Alvarez, Cantero, Czakon, Llorente, Mitov, **Poncelet**
JHEP 03 (2023) 129

also NNLOJET (gluons only) [2203.13531]

$pp \to \gamma jj$

Isolated photon production in association with a jet pair through next-to-next-to-leading order in QCD
Badger, Czakon, Hartanto, Moodie, Peraro, **Poncelet**, Zoia [JHEP 10, 071 (2023)]

$pp \to Wb\bar{b}$

Next-to-next-to-leading order QCD corrections to Wbb production at the LHC,
Hartanto, **Poncelet**, Popescu, Zoia
[Phys. Rev. D 106, 074016 (2022)]

Multi-jet observables

Test of pQCD and extraction of strong coupling constant

NLO theory unc. (MHO) > experimental unc.

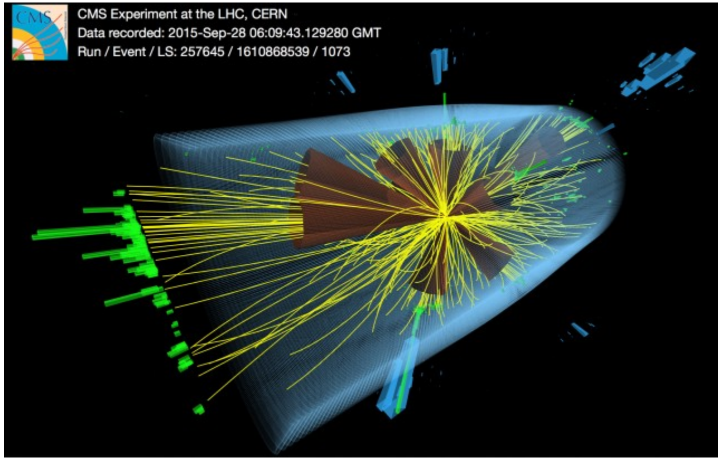
- NNLO QCD needed for precise theory-data comparisons
  → Restricted to two-jet data [Currie'17+later][Czakon'19]
- New NNLO QCD three-jet → access to more observables
  - Jet ratios

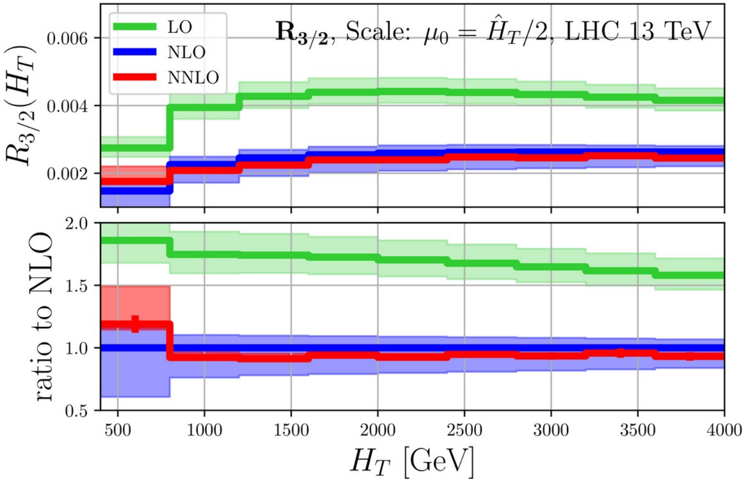
Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC
Czakon, Mitov, **Poncelet** Phys.Rev.Lett. 127 (2021) 15, 152001

$$R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$$

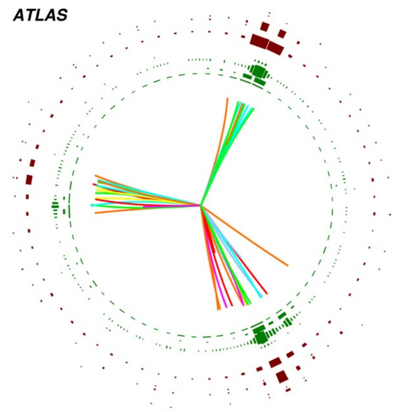
  - Event shapes

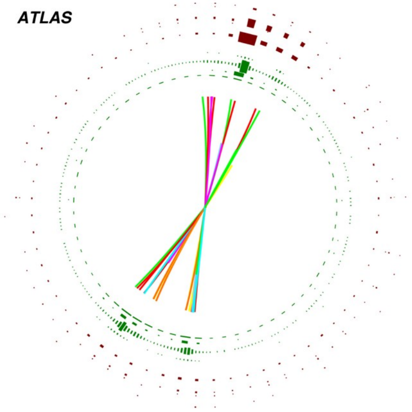
NNLO QCD corrections to event shapes at the LHC
Alvarez, Cantero, Czakon, Llorente, Mitov, **Poncelet** JHEP 03 (2023) 129





Encoding QCD dynamics in event shapes





Using (global) event information to separate different regimes of QCD event evolution:

Energy-energy correlators

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$

Separation of energy scales: $H_{T,2} = p_{T,1} + p_{T,2}$

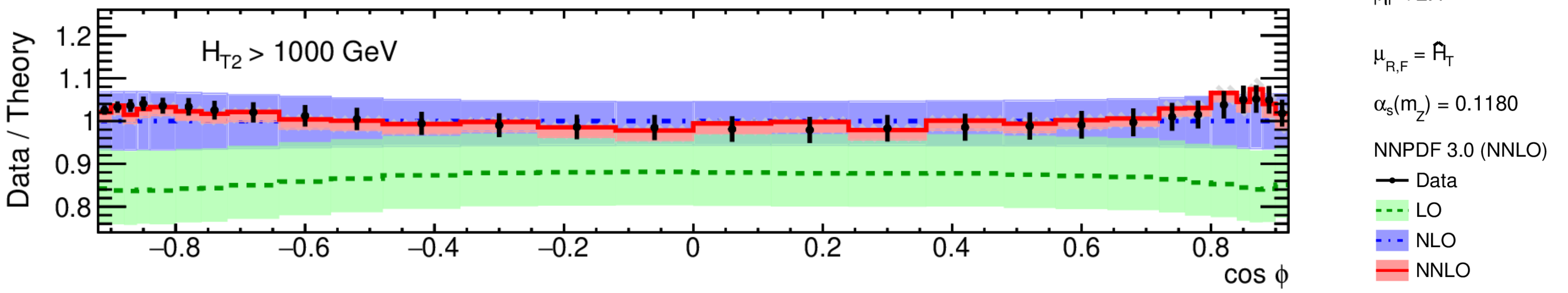
Ratio to 2-jet: $R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \dfrac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$

Here: jets as input → experimentally advantageous (better calibrated, smaller non-pert.)

The transverse energy-energy correlator

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$

- Insensitive to soft radiation through energy weighting $x_{T,i} = E_{T,i}/\sum_j E_{T,j}$
- Event topology separation:
  - Central plateau contain isotropic events
  - To the right: self-correlations, collinear and in-plane splitting
  - To the left: back-to-back

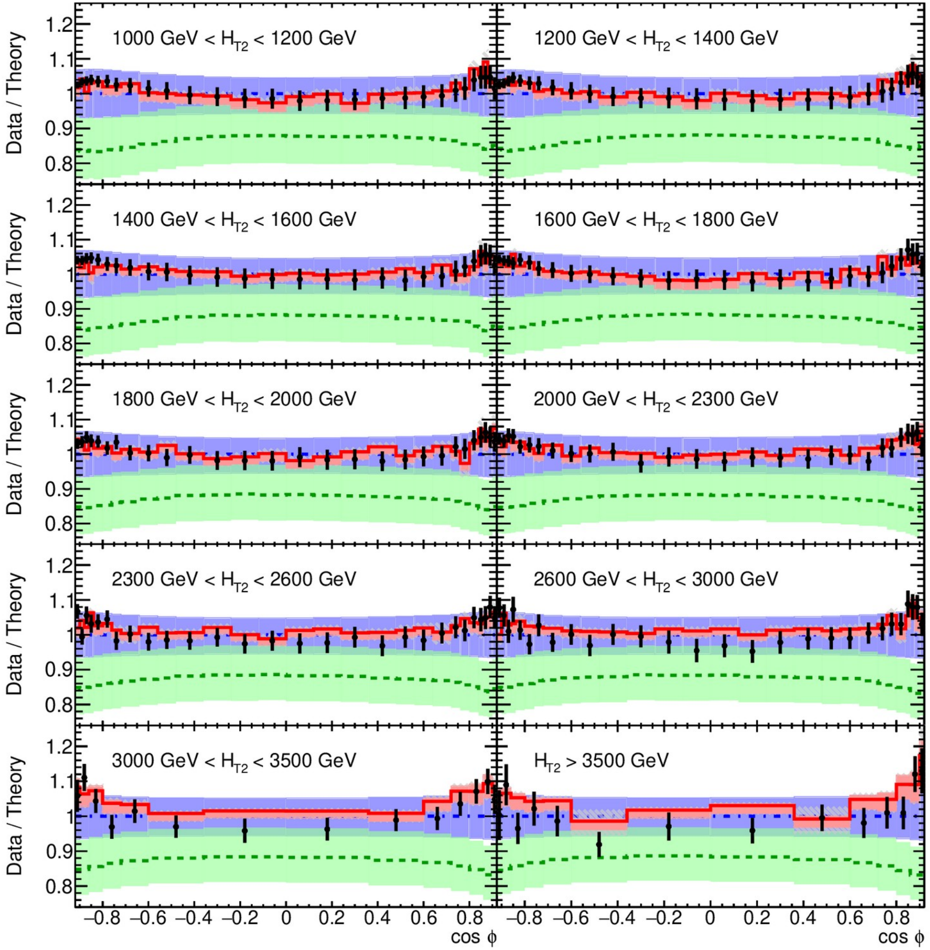


[ATLAS 2301.09351]

Predictions:

NNLO QCD corrections to event shapes at the LHC
Alvarez, Cantero, Czakon, Llorente, Mitov, **Poncelet** JHEP 03 (2023) 129

Double differential TEEC



[ATLAS 2301.09351]

ATLAS

Particle-level TEEC

$\sqrt{s}$ = 13 TeV; 139 fb^{-1}

anti-k_t R = 0.4

p_T > 60 GeV

$|\eta|$ < 2.4

$\mu_{R,F} = \hat{H}_T$

$\alpha_s(m_Z)$ = 0.1180

NNPDF 3.0 (NNLO)

Data

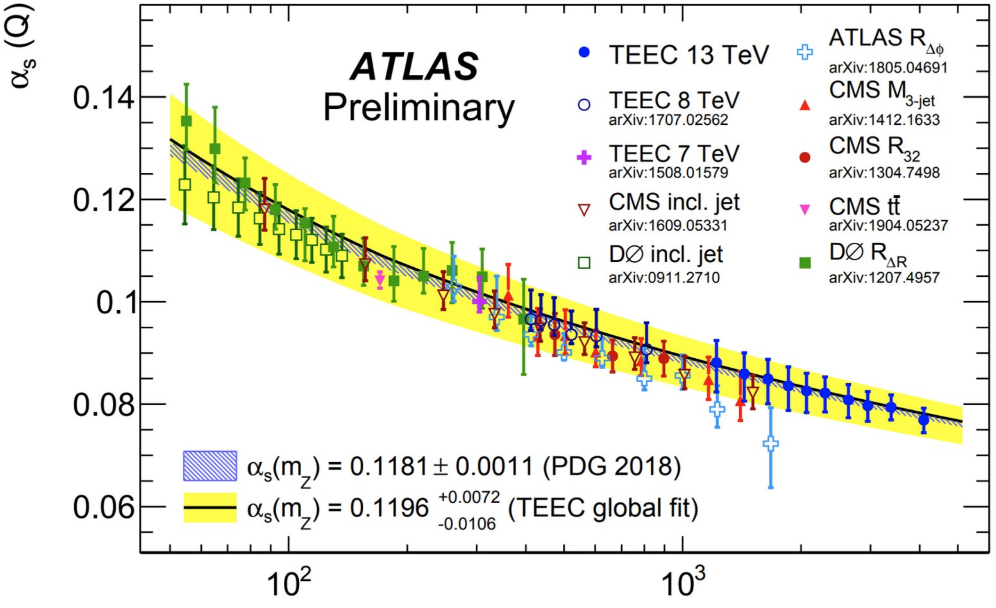
LO

NLO

NNLO

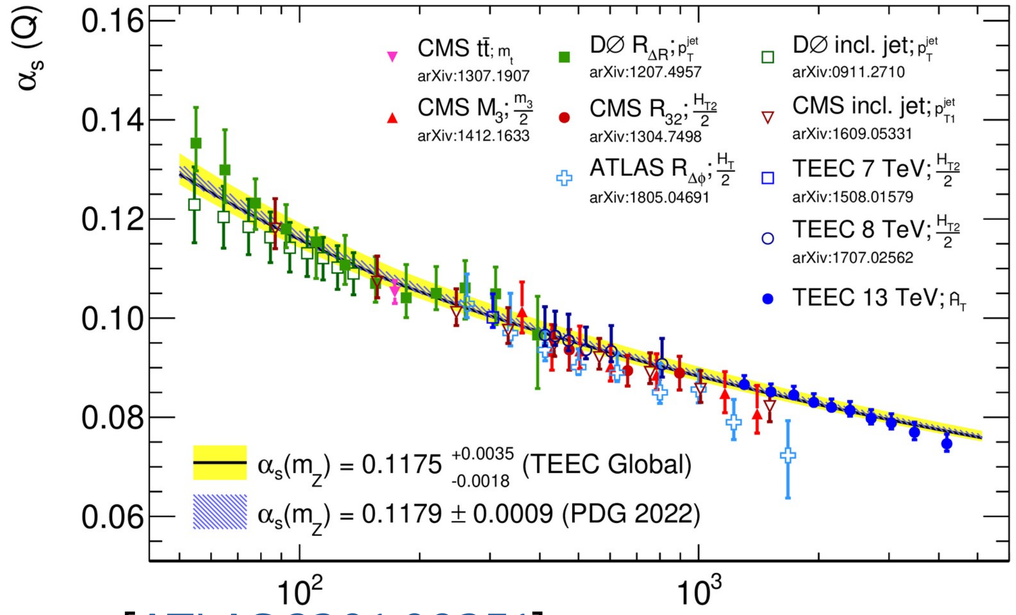
Running of α_S

NLO QCD



[ATLAS-CONF-2020-025]

NNLO QCD

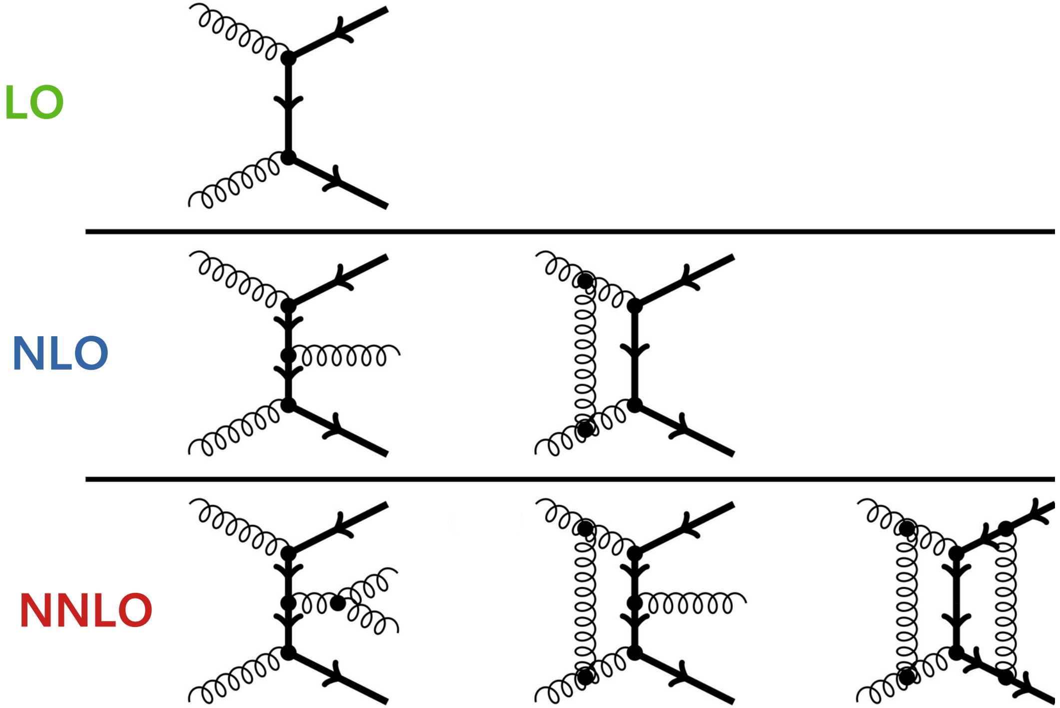


[ATLAS 2301.09351]

Summary

Higher-order (NNLO) QCD corrections are an important corner stone of LHC phenomenology

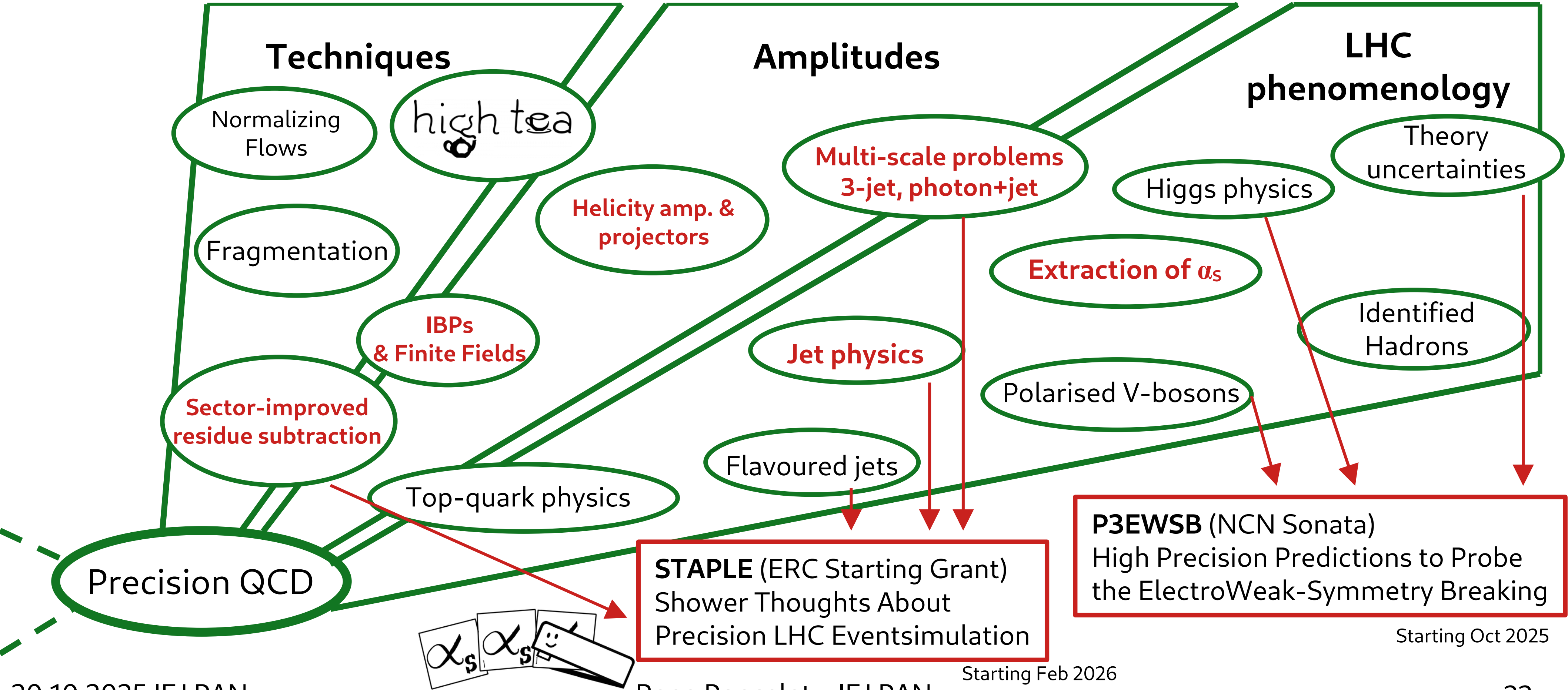
- Underpins SM phenomenological applications
  - **Precision tests**
  - **Measuring** SM parameters (PDFs, masses + couplings, FF)



- General NNLO QCD scheme
  +
  five-point two-loop amplitudes

  → **Precision predictions for multi-scale processes**

Pioneering research on higher-order calculations on multi-scale processes, including the first computations of next-to-next-to-leading order corrections in Quantum Chromodynamics to all massless two-to-three processes relevant at the Large Hadron Collider.



Backup

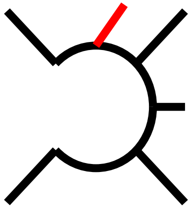
Next-to-leading order case

$$\hat{\sigma}^{(1)}_{ab} = \hat{\sigma}^{\mathrm{R}}_{ab} + \hat{\sigma}^{\mathrm{V}}_{ab} + \hat{\sigma}^{\mathrm{C}}_{ab}$$

KLN theorem
sum is finite for sufficiently inclusive observables and regularization scheme independent

Each term separately infrared (IR) divergent:

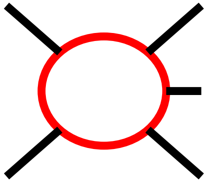
Real corrections:



$$\hat{\sigma}^{\mathrm{R}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

Phase space integration over unresolved configurations

Virtual corrections:



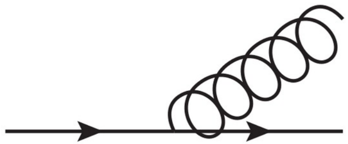
$$\hat{\sigma}^{\mathrm{V}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n}\, 2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n} \middle| \mathcal{M}^{(1)}_{n} \right\rangle \mathrm{F}_{n}$$

Integration over loop-momentum
(UV divergences cured by renormalization)

IR singularities in real radiation

$$\hat{\sigma}^{\mathrm{R}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

Finite function

$\ni \mathrm{d}\Phi_1$ 

$$\sim \int_0 \mathrm{d}E\mathrm{d}\theta \frac{1}{E(1-\cos\theta)} f(E, \cos(\theta))$$

Divergent

Regularization in Conventional Dimensional Regularization (CDR) $d = 4 - 2\epsilon$

$$\to \int_0 \mathrm{d}E\mathrm{d}\theta \frac{1}{E^{1-2\epsilon}(1-\cos\theta)^{1-\epsilon}} f(E, \cos(\theta)) \sim \frac{1}{\epsilon^2} + \ldots$$

Cancellation against similar divergences in

$$\hat{\sigma}^{\mathrm{V}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_n \, 2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_n \middle| \mathcal{M}^{(1)}_n \right\rangle \mathrm{F}_n$$

How to extract these poles? Slicing and Subtraction

Central idea: Divergences arise from infrared (IR, soft/collinear) limits → Factorization!

Slicing

$$\hat{\sigma}^{\mathrm{R}}_{ab} = \frac{1}{2\hat{s}} \int_{\delta(\Phi)\geq\delta_c} \mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1} + \frac{1}{2\hat{s}} \int_{\delta(\Phi)<\delta_c} \mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

$$\approx \frac{1}{2\hat{s}} \int_{\delta(\Phi)\geq\delta_c} \mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1} + \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_n \, \tilde{M}(\delta_c) \mathrm{F}_n + \mathcal{O}(\delta_c)$$

$$... + \hat{\sigma}^{\mathrm{V}}_{ab} = \text{finite}$$

Subtraction

$$\hat{\sigma}^{\mathrm{R}}_{ab} = \frac{1}{2\hat{s}} \int \left(\mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1} - \mathrm{d}\tilde{\Phi}_{n+1} \, \mathcal{S} \mathrm{F}_n \right) + \frac{1}{2\hat{s}} \int \mathrm{d}\tilde{\Phi}_{n+1} \, \mathcal{S} \mathrm{F}_n$$

$$\frac{1}{2\hat{s}} \int \mathrm{d}\tilde{\Phi}_{n+1} \, \mathcal{S} \mathrm{F}_n = \frac{1}{2\hat{s}} \int \underline{\mathrm{d}\Phi_n \, \mathrm{d}\Phi_1} \, \mathcal{S} \mathrm{F}_n$$

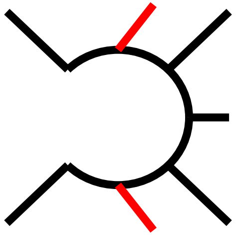
Phase space factorization
→ momentum mappings

Most popular NLO QCD schemes:
CS [hep-ph/9605323],
FKS [hep-ph/9512328]

→ Basis of modern event simulation

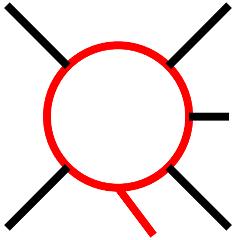
Partonic cross section beyond NLO

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\mathrm{VV}} + \hat{\sigma}_{ab}^{\mathrm{RV}} + \hat{\sigma}_{ab}^{\mathrm{RR}} + \hat{\sigma}_{ab}^{\mathrm{C2}} + \hat{\sigma}_{ab}^{\mathrm{C1}}$$



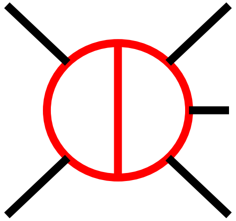
Real-Real

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$



Real-Virtual

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$



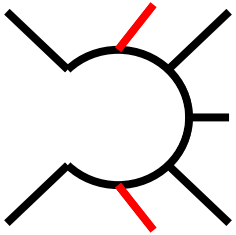
Virtual-Virtual

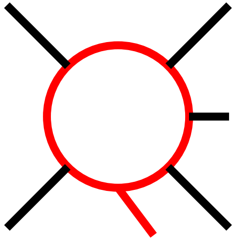
$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

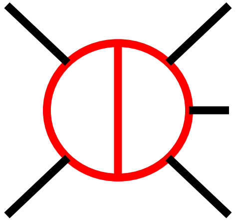
$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_n$ $\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$

Partonic cross section beyond NLO

$$\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$$







Technically substantially more complicated!

Main bottlenecks:

- Real - real → overlapping singularities
  Many possible limits → good organization principle needed
- Real - virtual → stable matrix elements
- Virtual - virtual → complicated case-by-case analytic treatment

Sector decomposition I

Considering working in CDR:

→ Virtuals are usually done in this regularization: $\hat{\sigma}_{ab}^{VV} = \sum_{i=-4}^{0} c_i \epsilon^i + \mathcal{O}(\epsilon)$

→ Can we write the real radiation as such expansion?

→ Difficult integrals, analytical impractical (except very simple observables)!

→ Numerics not possible, integrals are divergent → ε-poles!

How to extract these poles? → Sector decomposition!

Divide and conquer the phase space:

$$1 = \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \quad \Rightarrow \quad \hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

Sector decomposition II

Divide and conquer the phase space

- Each $\mathcal{S}_{i,k}$ (NLO), $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$ (NNLO) has simpler divergences:
  - Soft limits of partons i and j
  - Collinear w.r.t partons k (and l) of partons i and j

$$\mathcal{S}_{i,k} = \frac{1}{D_1 d_{i,k}} \qquad D_1 = \sum_{ik} \frac{1}{d_{i,k}} \qquad d_{i,k} = \frac{E_i}{\sqrt{s}}(1 - \cos\theta_{ik})$$

- Parametrization w.r.t. reference parton makes divergences explicit

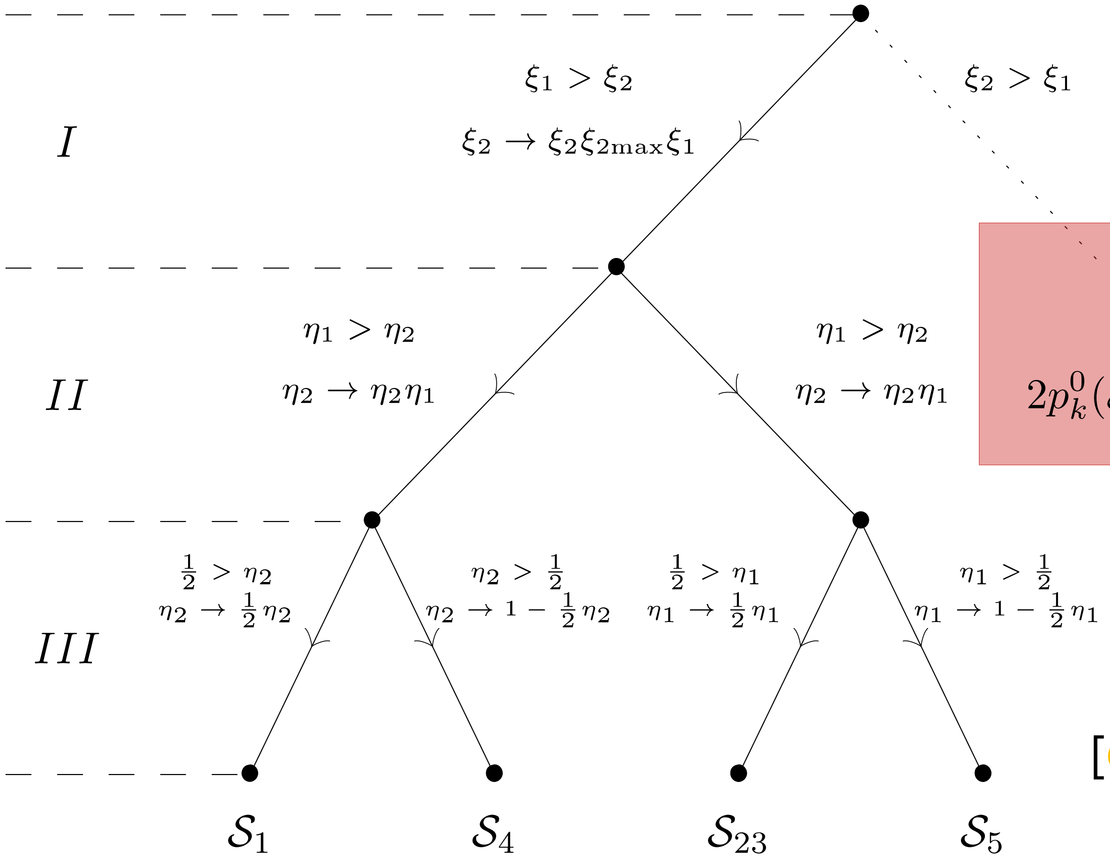
$$\hat{\eta}_i = \tfrac{1}{2}(1 - \cos\theta_{ik}) \in [0,1] \qquad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$$

- Example: Splitting function

$$\sim \frac{1}{s_{ik}} P(z) \qquad s_{ik} = (p_i + p_k)^2 = 2 p_k^0 u_{\max}^0 \xi_i \eta_i \qquad \sim \frac{1}{\eta_i \xi_i}$$

Sector decomposition II – triple collinear factorization

Three particle invariant does not factorize trivially:



Double soft factorization:

$$\theta(u_i^0 - u_j^0) + \theta(u_i^0 - u_j^0)$$

$$(p_k + u_i + u_j)^2 =$$

$$2p_k^0(\xi_i \eta_i u_{\max}^{0,i} + \xi_j \eta_j u_{\max}^{0,j} + \xi_i \xi_j \frac{u_{max}^{0,i} u_{max}^{0,j}}{p_k^0} \angle(u_i, u_j))$$

[Czakon'10,Caola'17]

Sector decomposition III

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\,\mathcal{S}_{kl,m}\left\langle \mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2} = \sum_{\text{sub-sec.}}\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i\, \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}}\, \mathrm{d}\tilde{\mu}(\{x_i\}) \underbrace{\prod x_i^{a_i+1}\left\langle \mathcal{M}_{n+2}\middle|\mathcal{M}_{n+2}\right\rangle}_{\text{regular}} \mathrm{F}_{n+2}$$

$$x_i \in \{\eta_1,\xi_1,\eta_2,\xi_2\}$$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. }+\text{ sub.}}$$

$$\int_0^1 \mathrm{d}x\,\left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x)-f(0)}{x^{1+b\epsilon}}$$

Finite NNLO cross section

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_{n}$$

sector decomposition and master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. } + \text{ sub.}}$$

$$\left(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}\right) \quad \left(\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}\right) \quad \left(\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}\right) \quad \left(\sigma_{SU}^{C1}, \sigma_{DU}^{C1}\right) \quad \left(\sigma_{DU}^{C2}, \sigma_{FR}^{C2}\right)$$

re-arrangement of terms → 4-dim. formulation [Czakon'14,Czakon'19]

$$\underline{\left(\sigma_F^{RR}\right)} \quad \underline{\left(\sigma_F^{RV}\right)} \quad \underline{\left(\sigma_F^{VV}\right)} \quad \underline{\left(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}\right)} \quad \underline{\left(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2}\right)} \quad \underline{\left(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}\right)}$$

separately finite: ε poles cancel

Improved phase space generation

Phase space cut and differential observable introduce

mis-binning : mismatch between kinematics in subtraction terms

→ leads to increased variance of the integrand

→ slow Monte Carlo convergence

New phase space parametrization [Czakon'19]:

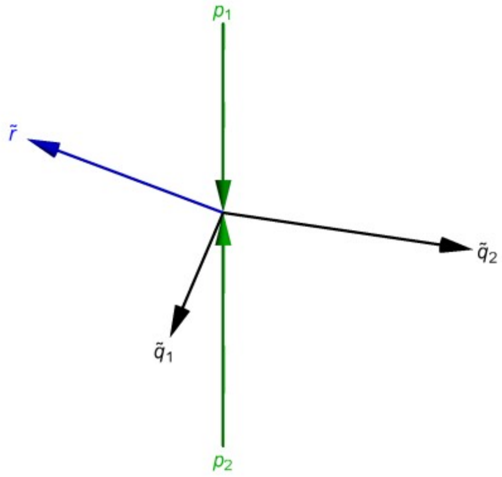
Minimization of # of different subtraction kinematics in each sector

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i : $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,,\;\; \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

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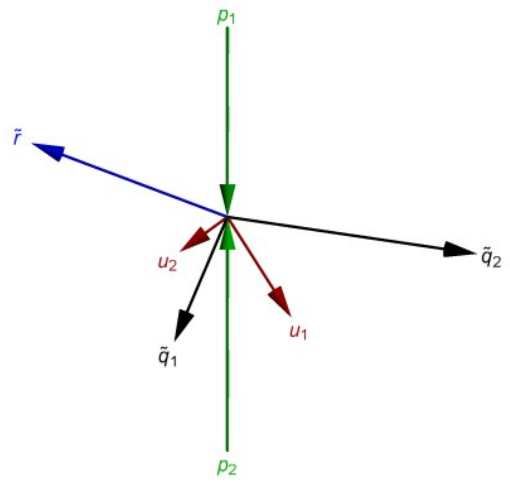

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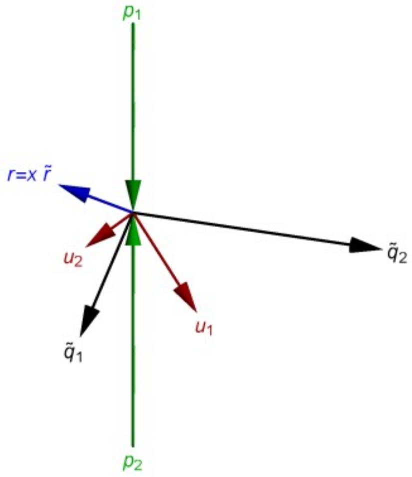

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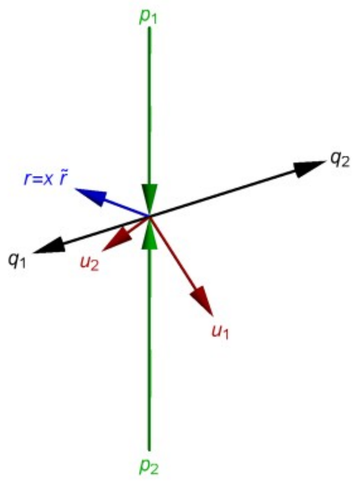


Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i : $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2 \,, \quad \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

t'HV corrections

Observables: Implemented by infrared safe measurement function (MF) F_m

Infrared property in STRIPPER context:

- $\{x_i\} \to 0 \leftrightarrow$ single unresolved limit
  $\Rightarrow \mathrm{F}_{n+2} \to \mathrm{F}_{n+1}$
- $\{x_i\} \to 0 \leftrightarrow$ double unresolved limit
  $\Rightarrow \mathrm{F}_{n+2} \to \mathrm{F}_n$
  $\Rightarrow \mathrm{F}_{n+1} \to \mathrm{F}_n$

Tool for new formulation in the 't Hooft Veltman scheme:

Parameterized MF F^α_{n+1}

- $\mathrm{F}^\alpha_n \equiv 0$ for $\alpha \neq 0$
  (NLO MF)
- 'arbitrary' F^0_n
  (NNLO MF)
- $\alpha \neq 0 \Rightarrow \mathrm{DU} = 0$ and SU separately finite

Example: $F^\alpha_{n+1} = F_{n+1}\Theta_\alpha(\{\alpha_i\})$
with $\Theta_\alpha = 0$ if some $\alpha_i < \alpha$

t'HV corrections

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \qquad \text{where} \qquad \sigma_{SU}^{c} = \int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{c}} F_{n+1} + I_{n}^{\mathrm{c}} F_{n} \right)$$

NLO measurement function ($\alpha \neq 0$):

$$\int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{RR}} + I_{n+1}^{\mathrm{RV}} + I_{n+1}^{\mathrm{C1}} \right) F_{n+1}^{\alpha} = \text{finite in 4 dim.}$$

All divergences cancel in d-dimensions:

$$\sum_{c} \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1}^{\alpha} \equiv \sum_{c} \mathcal{I}^{c} = 0$$

t'HV corrections

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \underbrace{-\mathcal{I}^{RR} - \mathcal{I}^{RV} - \mathcal{I}^{C1}}_{=0}$$

$$\begin{aligned}
\sigma_{SU}^{c} - \mathcal{I}^{c} = & \int \mathrm{d}\Phi_{n+1} \left\{ \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} + I_{n+1}^{c,(0)} \right] F_{n+1} + \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} + I_{n}^{c,(0)} \right] F_{n} \right\} \\
& - \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1} \Theta_\alpha(\{\alpha_i\}) \\
= & \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)} F_{n+1} + I_{n}^{c,(-2)} F_{n}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)} F_{n+1} + I_{n}^{c,(-1)} F_{n}}{\epsilon} \right] (1 - \Theta_\alpha(\{\alpha_i\})) \\
& + \int \mathrm{d}\Phi_{n+1} \left[I_{n+1}^{c,(0)} F_{n+1} + I_{n}^{c,(0)} F_{n} \right] + \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} \right] F_{n} \Theta_\alpha(\{\alpha_i\})
\end{aligned}$$

$$=: \underbrace{Z^c(\alpha)}_{\text{integrable, zero volume for } \alpha\to 0} + \underbrace{C^c}_{\text{no divergencies}} + \underbrace{N^c(\alpha)}_{\text{only } F_n \to \text{DU}}$$

t'HV corrections

Looks like slicing, but it is slicing *only* for divergences
$\rightarrow$ no actual slicing parameter in result

Powerlog-expansion:

$$N^c(\alpha) = \sum_{k=0}^{\ell_{\max}} \ln^k(\alpha) N_k^c(\alpha)$$

- all $N_k^c(\alpha)$ regular in α
- start expression independent of $\alpha \Rightarrow$ all logs cancel
- only $N_0^c(0)$ relevant

Putting parts together:

$$\sigma_{SU} - \sum_c N_0^c(0) \text{ and } \sigma_{DU} + \sum_c N_0^c(0)$$

are finite in 4 dimension

$$\Downarrow$$

SU contribution: $\sigma_{SU} - \sum_c N_0^c = \sum_c C^c$
original expression σ_{SU} in 4-dim without poles, no further ϵ pole cancellation

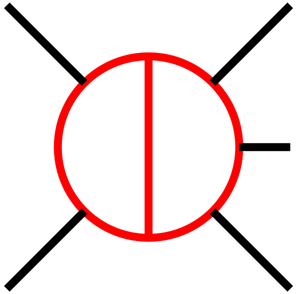
C++ framework

- Formulation allows efficient algorithmic implementation → STRIPPER
- **High degree of automation**:
  - Partonic processes (taking into account all symmetries)
  - Sectors and subtraction terms
  - Interfaces to Matrix-element providers + O(100) hardcoded:
    AvH, OpenLoops, Recola, NJET, HardCoded

    → In practice: Only two-loop matrix elements required
- **Broad range of applications** through additional facilities:
  - Narrow-Width & Double-Pole Approximation
  - Fragmentation
  - Polarised intermediate massive bosons
  - (Partial) Unweighting → Event generation for **HighTEA**
  - Interfaces: FastNLO, FastJet

Two-loop five-point amplitudes

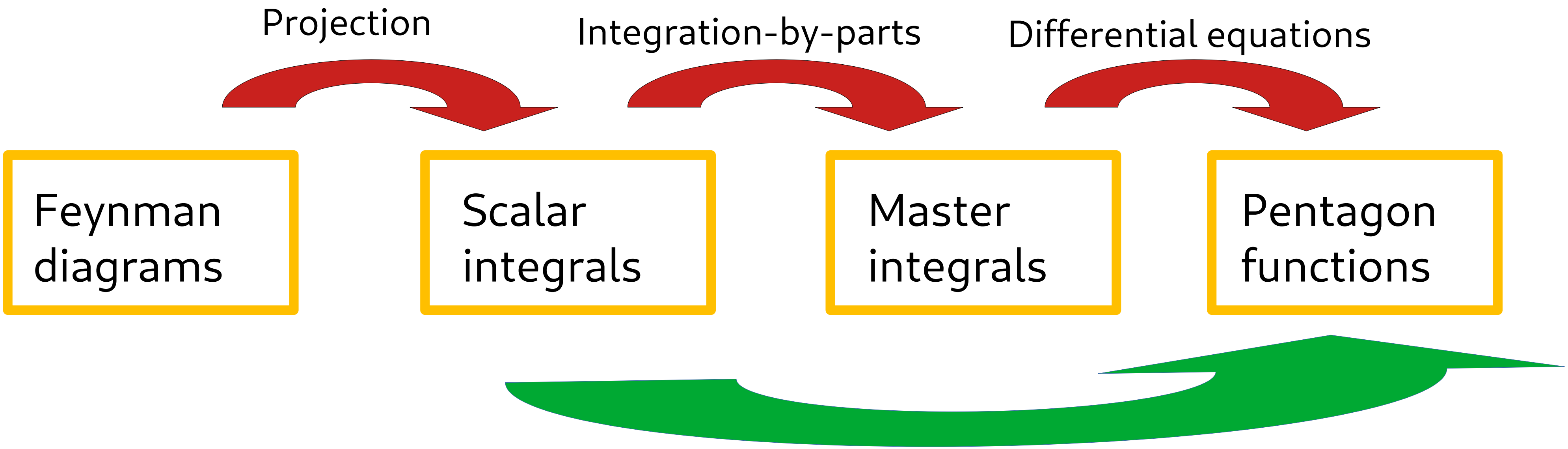
Massless:
[Chawdry’19’20’21] (3A+2j,2A+3j)
[Abreu’20’21] (3A+2j,5j)
[Agarwal’21] (2A+3j)
[Badger’21’23] (5j,gggAA,jjjjA)



1 external mass:
[Abreu’21] (W+4j)
[Badger’21’22] (Hqqgg,W4q,Wajjj)
[Hartanto’22] (W4q)

Overview

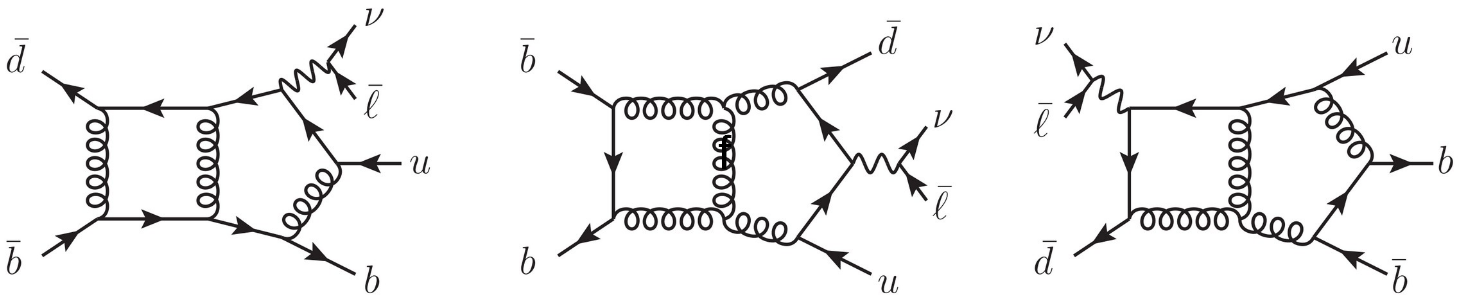
Old school approach:



Automated framework using finite fields
to avoid expression swell based on
FiniteFlow [Peraro'19]

Projection to scalar integrals

Generate diagrams (contributing to leading-colour) with QGRAF



Factorizing decay: $A_6^{(L)} = A_5^{(L)\mu} D_\mu P$ $\qquad$ $M_6^{2(L)} = \sum_{\text{spin}} A_6^{(0)^*} A_6^{(L)} = M_5^{(L)\mu\nu} D_{\mu\nu} |P|^2$

Projection on scalar functions (FORM+Mathematica):
→ anti-commuting γ_5 + Larin prescription

$$M_5^{(L)\mu\nu} = \sum_{i=1}^{16} a_i^{(L)} v_i^{\mu\nu}$$

$$a_i^{(L)} = a_i^{(L),\text{even}} + \text{tr}_5 a_i^{(L),\text{odd}} \qquad a_i^{(L),p} = \sum_i c_{j,i}(\{p\},\epsilon)\mathcal{I}(\{p\},\epsilon)$$

Integration-By-Parts reduction

$$a_i^{(L),p} = \sum_i c_{j,i}(\{p\},\epsilon)\mathcal{I}(\{p\},\epsilon)$$

→ prohibitively large number of integrals

$$\mathcal{I}_i(\{p\},\epsilon) \equiv \mathcal{I}(\vec{n_i},\{p\},\epsilon) = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d}\frac{\mathrm{d}^d k_2}{(2\pi)^d}\prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\},\{k\})$$

Integration-By-Parts identities connect different integrals → system of equations
→ only a small number of independent “master” integrals

$$0 = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d}\frac{\mathrm{d}^d k_2}{(2\pi)^d} l_\mu \frac{\partial}{\partial l^\mu}\prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\},\{k\}) \quad \text{with} \quad l \in \{p\} \cap \{k\}$$

LiteRed (+ Finite Fields)

$$a_i^{(L),p} = \sum_i d_{j,i}(\{p\},\epsilon)\mathrm{MI}(\{p\},\epsilon)$$

Master integrals & finite remainder

Differential Equations: $\mathrm{d}\vec{\mathrm{MI}} = \mathrm{d}A(\{p\},\epsilon)\vec{\mathrm{MI}}$

Canonical basis: $\mathrm{d}\vec{\mathrm{MI}} = \epsilon \mathrm{d}\tilde{A}(\{p\})\vec{\mathrm{MI}}$

[Remiddi, 97]
[Gehrmann, Remiddi, 99]
[Henn, 13]

Simple iterative solution

$$\mathrm{MI}_i = \sum_w \epsilon^w \tilde{\mathrm{MI}}_i^w \quad \text{with} \quad \tilde{\mathrm{MI}}_i^w = \sum_j c_{i,j} m_j$$

Chen-iterated integrals
"Pentagon"-functions

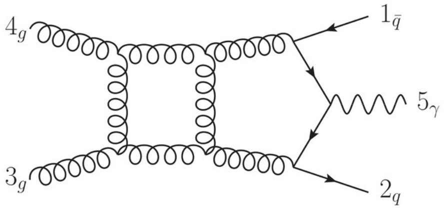
[Chicherin, Sotnikov, 20]
[Chicherin, Sotnikov, Zoia, 21]

Putting everything together (and removing of IR poles):

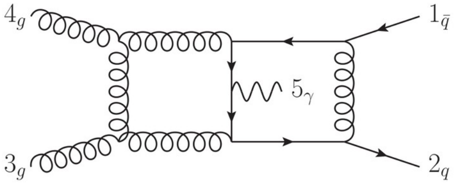
$$f_i^{(L),p} = a_i^{(L),p} - \text{poles} \qquad f_i^{(L),p} = \sum_j c_{i,j}(\{p\}) m_j + \mathcal{O}(\epsilon)$$

Reconstruction of Amplitudes

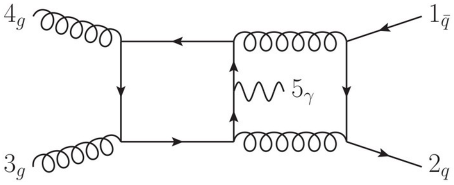
[Badger'21]



$A^{(2),N_c^2}_{34;q}, A^{(2),N_c}_{\delta;q}$



$A^{(2),1}_{34;q}, A^{(2),N_c}_{\delta;q}, A^{(2),1/N_c}_{\delta;q}$



$A^{(2),N_c}_{34;l}, A^{(2),1/N_c}_{34;l}, A^{(2),1/N_c^2}_{\delta;l}$

New optimizations

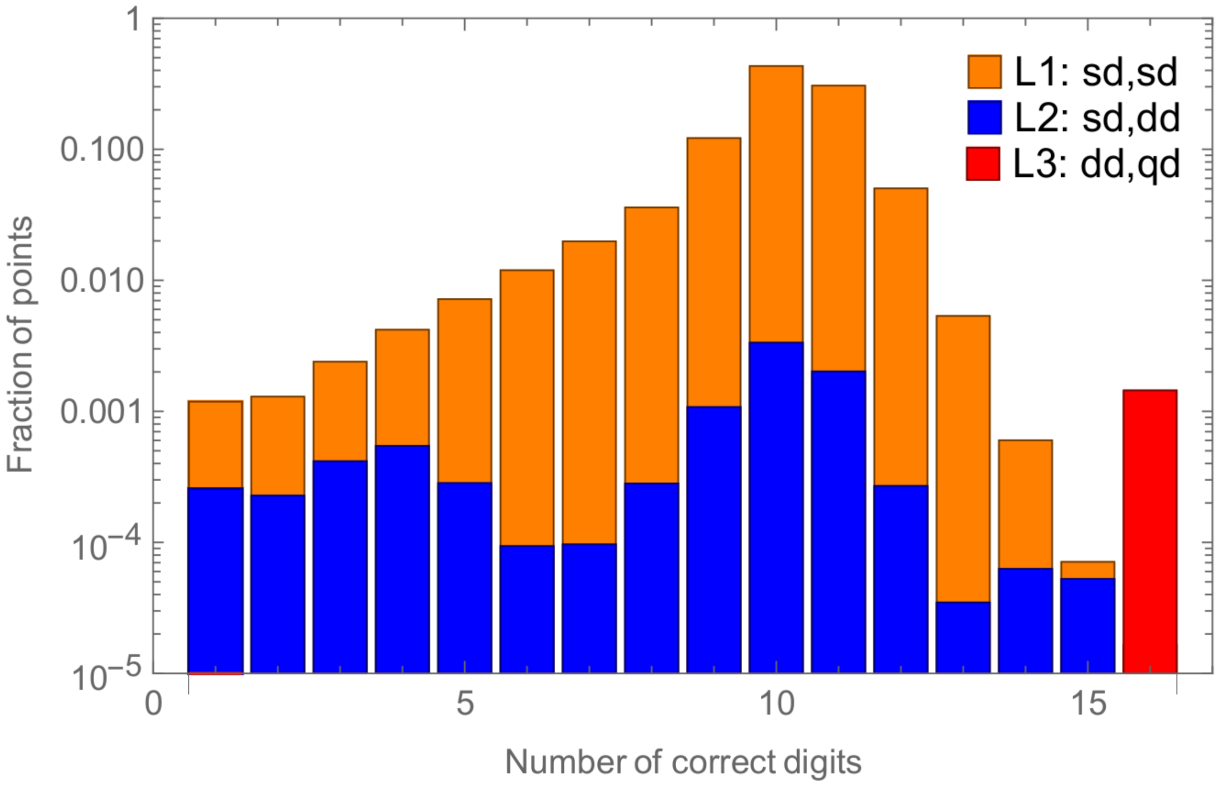
- Syzygy's to simplify IBPs
- Exploitation of Q-linear relations
- Denominator Ansaetze
- On-the-fly partial fractioning

amplitude	helicity	original	stage 1	stage 2	stage 3	stage 4
$A^{(2),1}_{34;q}$	$-++-+$	94/91	74/71	74/0	22/18	22/0
$A^{(2),1}_{34;q}$	$-+-++$	93/89	90/86	90/0	24/14	18/0
$A^{(2),1/N_c^2}_{34;q}$	$-++-+$	90/88	73/71	73/0	23/18	22/0
$A^{(2),1/N_c^2}_{34;q}$	$-+-++$	90/86	86/82	86/0	24/14	19/0
$A^{(2),1/N_c}_{34;l}$	$-+-++$	89/82	74/67	73/0	27/14	20/0
$A^{(2),1/N_c}_{34;l}$	$-++-+$	85/81	61/58	60/0	27/18	20/0
$A^{(2),N_c^2}_{34;q}$	$-+-++$	58/55	54/51	53/0	20/16	20/0

Massive reduction of complexity

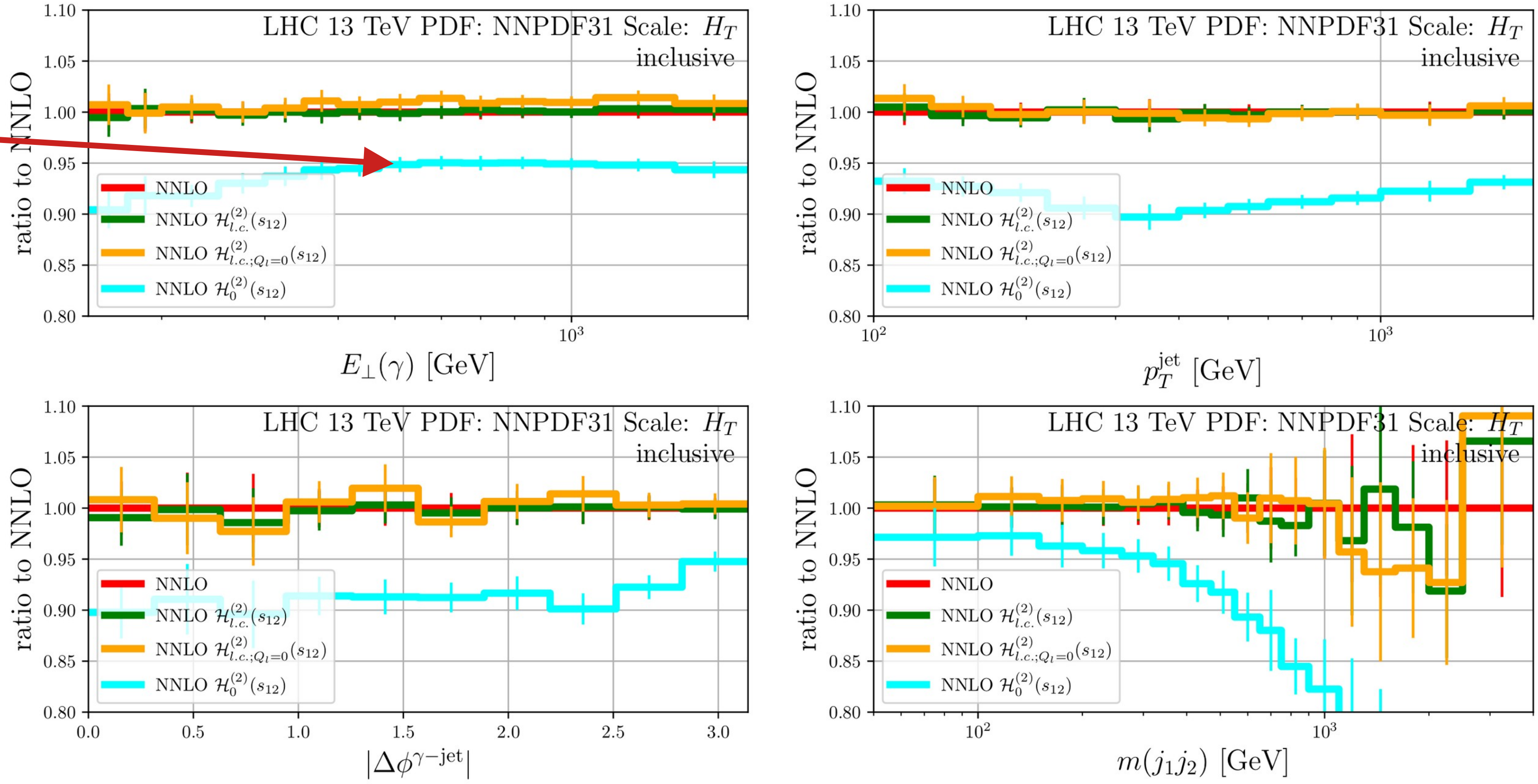
Two-loop matrix element stability

- Stable evaluation requires high floating point precision for rational functions
- In rarer cases higher precision "Pentagon" functions necessary
- 2.2 million events needed → fast evaluation essential



Quality of leading colour the approximation

Two-loop
contribution
~ 5-10%
wrt. full NNLO
(scheme dep.)



“Leading colour”
Approximation
”Error” = O(~1%)
wrt full NNLO

Slicing and Subtraction

Slicing

- Conceptually simple
- Recycling of lower computations
- Non-local cancellations/power-corrections
  → computationally expensive

NNLO QCD schemes

qT-slicing [Catani'07],
N-jettiness slicing [Gaunt'15/Boughezal'15]

Subtraction

- Conceptually more difficult
- Local subtraction → efficient
- Better numerical stability
- Choices:
  - Momentum mapping
  - Subtraction terms
  - Numerics vs. analytic

Antenna [Gehrmann'05-'08],
Colorful [DelDuca'05-'15],
Sector-improved residue subtraction [Czakon'10-'14'19]
Projection [Cacciari'15],
Nested collinear [Caola'17],
Geometric [Herzog'18],
Unsubtraction [Aguilera-Verdugo'19],
...

Theory picture of hadron collision events

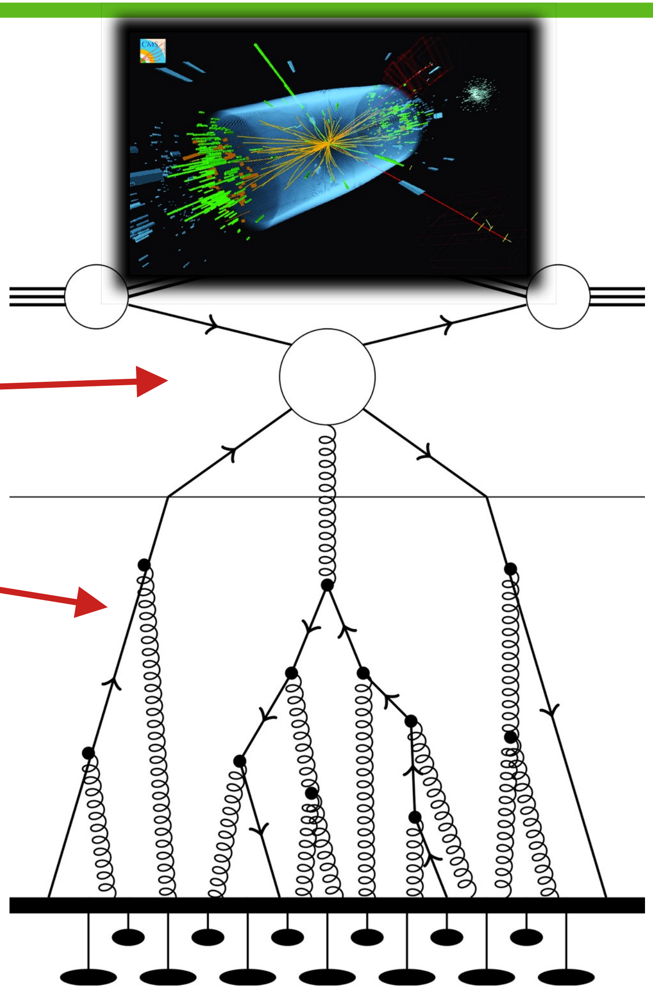
Guiding principle: factorization
"What you see depends on the energy scale"

In Quantum Chromodynamics (QCD):

$Q \gg \Lambda_{\mathrm{QCD}}$ **Fixed-order perturbation theory**
scattering of individual partons

$Q \gtrsim \Lambda_{\mathrm{QCD}}$ **Parton-shower/Resummation**
all-order bridge between perturbative and non-perturbative physics

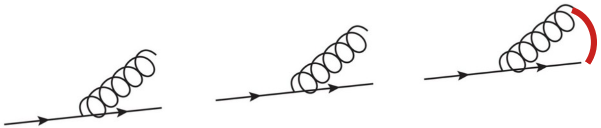
$Q \sim \Lambda_{\mathrm{QCD}}$ **"Hadronization"/MPI/...**
non-perturbative physics



Fixed-order matching to parton-showers

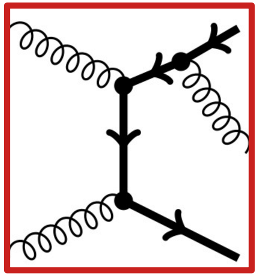
The challenge

Combine fixed-order with parton shower evolution
while **preserving** the precision/accuracy of both!



v ← shower scale

"Hard" scattering scale Q



parton shower approximation: $\alpha_S^i(v)\ln^j(v)$
but to all orders!

accuracy: LL, NLL, NNLL, ...

Fixed-order: expansion in $\alpha_S(Q)$

→ cross section ~ $\underbrace{e^{\sum_{ij}\alpha_S^i(v)\ln^j(v)} \times (\mathrm{LO} + \mathrm{NLO} + \mathrm{NNLO} + \ldots)}$

A matching scheme needs to avoid double counting
of the logarithmic contributions!

Matching parton showers

At NLO QCD a solved problem → a breakthrough for LHC phenomenology

Local matching NLO+PS: MC@NLO, Powheg, Nagy-Soper, …

(core of event generators Madgraph_aMC@NLO, Sherpa, Powheg+Pythia, Herwig)

>80% of all exp. LHC papers cite at least one these!

Core idea: using subtractions schemes to construct showers & matching

(subtraction terms ⟷ parton shower kernels)

This is the **big challenge at NNLO QCD** for the theory community!

Some NNLO+PS matching approaches appeared recently but are either

- non-local → resummation/slicing based (for example: MiNNLOPS, Geneva) → limited generality
- or work only for simple cases like e+e- → jets (for example: Vincia) → work only where NNLO is known analytically

No scheme so far is based on a general local subtraction.

A general matching scheme at NNLO would be the next big breakthrough for precision collider physics!

This is what I want to achieve with

STAPLE!

Funded by the European Union

European Research Council
Established by the European Commission

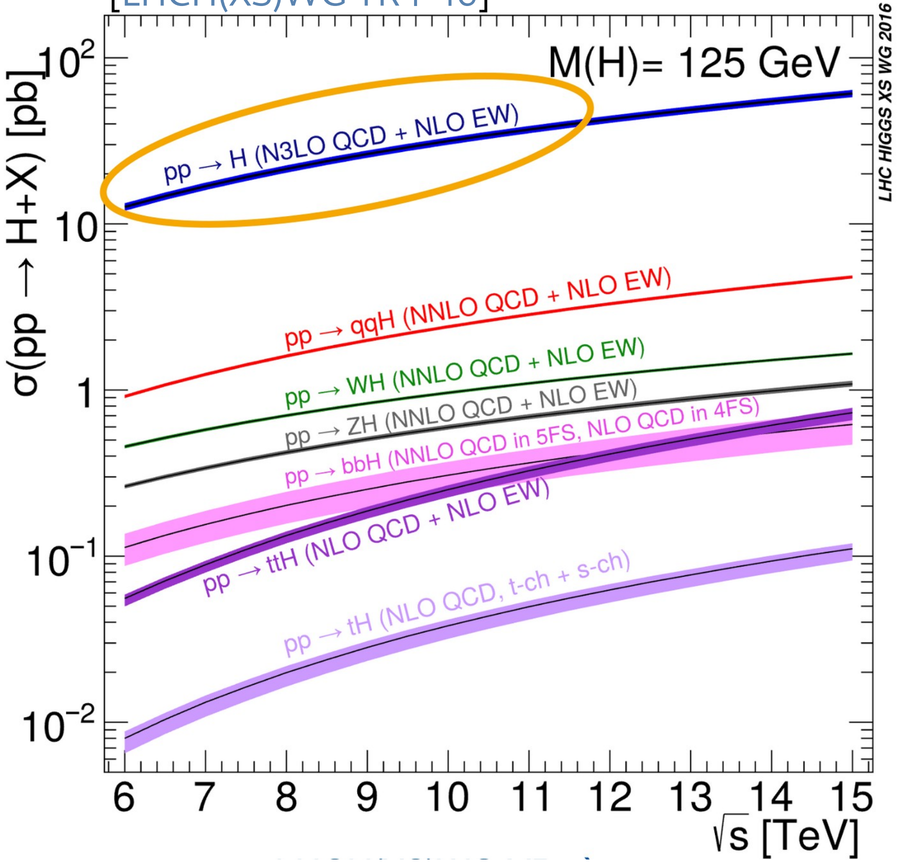
Two core aspects:

1) **preserving the precision/accuracy** of the fixed-order & parton shower

2) achieving a parton shower with **high logarithmic accuracy**

Higgs-production at hadron colliders

[LHCH(XS)WG YR4' 16]



- Higgs production is dominated through gluon-fusion
- Experimental measurement

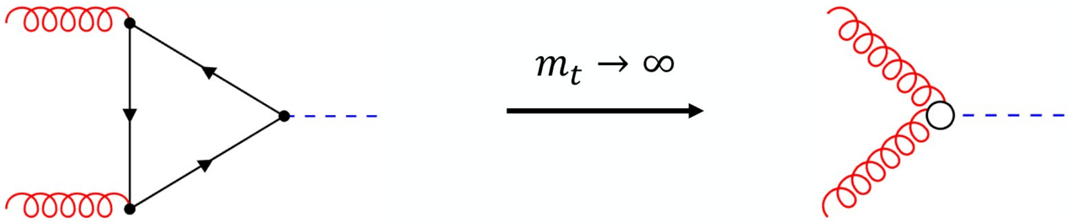
$$\sigma^{\mathrm{exp.}}_{gg\to H} = 47.1 \pm 3.8 \ \mathrm{pb}$$

[CMS'22]

- HL LHC expects 2 % uncertainty
- Theory predictions need to keep up → Higher-order predictions crucial!

HTL and HEFT

Heavy Top Limit (HTL or EFT):



$$\sigma_{gg\to H} = \sigma^{\mathrm{HTL}}_{gg\to H} + \mathcal{O}\left(\frac{m_H^2}{m_t^2}\right) \quad \text{for} \quad m_t \to \infty$$

Higgs Effective Field Theory (HEFT or rEFT): $\sigma^{\mathrm{N}^n\mathrm{LO}}_{\mathrm{HEFT}} = \frac{\sigma^{\mathrm{LO}}}{\sigma^{\mathrm{LO}}_{\mathrm{HTL}}} \sigma^{\mathrm{N}^n\mathrm{LO}}_{\mathrm{HTL}} \approx 1.064 \times \sigma^{\mathrm{N}^n\mathrm{LO}}_{\mathrm{HTL}}$

captures some of the top-quark mass effects for inclusive observables.
At higher loop-order questionable → needs full computation.
How to deal with other quark mass effects?

Precision predictions for Higgs production in gluon-fusion

[LHCH(XS)WG YR4' 16]

Immense community effort to achieve precise theory predictions

$$\boxed{\sigma = 48.58\,\mathrm{pb}^{+2.22\,\mathrm{pb}\,(+4.56\%)}_{-3.27\,\mathrm{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\mathrm{pb}\,(3.20\%)\ (\text{PDF+}\alpha_s)\,.}$$

$48.58\,\mathrm{pb} =$	$16.00\,\mathrm{pb}$	$(+32.9\%)$	(LO, rEFT)	[Georgi, Glashow, Machacek, Nanopoulos'78]
	$+\,20.84\,\mathrm{pb}$	$(+42.9\%)$	(NLO, rEFT)	[Dawson '91][Djouadi, Spira Zerwas '91]
	$-\ \ 2.05\,\mathrm{pb}$	(-4.2%)	$((t,b,c)$, exact NLO)	[Graudenz, Spira, Zerwas '93]
	$+\ \ 9.56\,\mathrm{pb}$	$(+19.7\%)$	(NNLO, rEFT)	[Ravindran, Smith, Van Neerven '02] [Harlander, Kilgore '02][Anastasiou, Melnikov '02]
	$+\ \ 0.34\,\mathrm{pb}$	$(+0.7\%)$	(NNLO, $1/m_t$)	[Harlander, Ozeren'09][Pak, Rogal, Steinhauser'10] [Harlander, Mantler, Marzani, Ozeren '10]
	$+\ \ 2.40\,\mathrm{pb}$	$(+4.9\%)$	(EW, QCD-EW)	[Aglietti, Bonciani, Degrassi,Vicini'04] [Actis, Passarino, Sturm, Uccirati'08] [Anastasiou, Boughezal, Petriello'09]
	$+\ \ 1.49\,\mathrm{pb}$	$(+3.1\%)$	(N^3LO, rEFT)	[Anastasiou, Duhr, Dulat, Herzog, Mistlberger'15]

Remaining theory uncertainties

[LHCH(XS)WG YR4' 16]

N4LO approximation
[Das, Moch, Vogt '20]

aN3LO PDFs
[MSHT'22,NNPDF'24]

Exact top-mass dependence through NNLO QCD
[Czakon, Harlander, Klappert, Niggetiedt'21]

Input parameters

$\sqrt{S}$	13 TeV
m_h	125 GeV
PDF	PDF4LHC15_nnlo_100
$\alpha_s(m_Z)$	0.118
$m_t(m_t)$	162.7 GeV ($\overline{\text{MS}}$)
$m_b(m_b)$	4.18 GeV ($\overline{\text{MS}}$)
$m_c(3GeV)$	0.986 GeV ($\overline{\text{MS}}$)
$\mu = \mu_R = \mu_F$	62.5 GeV ($= m_H/2$)

δ(scale)	δ(trunc)	δ(PDF-TH)	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$
$+0.10$ pb -1.15 pb	± 0.18 pb	± 0.56 pb	± 0.49 pb	± 0.40 pb	± 0.49 pb
$+0.21\%$ -2.37%	$\pm 0.37\%$	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	$\pm 1\%$

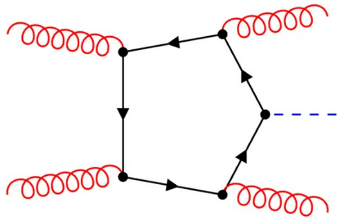
N3LO HEFT
[Mistlberger'18]

Improved QCD-EW predictions
[Bonetti, Melnikov, Trancredi'18] [Anastasiou et al '19]
[Bonetti et al. '20][Bechetti et al. '21] [Bonetti, Panzer, Trancredi '22]

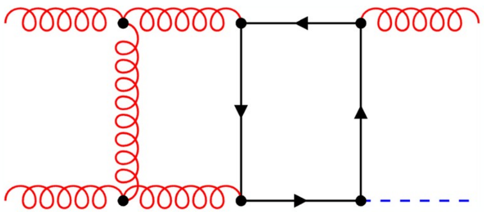
Bottom-top-interference
[Czakon, Eschment, Niggetiedt, Poncelet, Schellenberger, Phys.Rev.Lett. 132 (2024) 21, 211902, JHEP 10 (2024) 210, EurekAlert]

Bottom-top interference effects through NNLO QCD

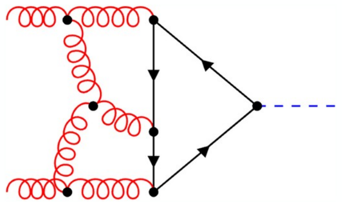
Double real (one-loop)



Real virtual (two-loop)



Double virtual (three-loop)



Renorm. scheme	$\overline{\text{MS}}$	on-shell
$\mathcal{O}(\alpha_s^2)$	-1.11	-1.98
LO	$-1.11^{+0.28}_{-0.43}$	$-1.98^{+0.38}_{-0.53}$
$\mathcal{O}(\alpha_s^3)$	-0.65	-0.44
NLO	$-1.76^{+0.27}_{-0.28}$	$-2.42^{+0.19}_{-0.12}$
$\mathcal{O}(\alpha_s^4)$	$+0.02$	$+0.43$
NNLO	$-1.74(2)^{+0.13}_{-0.03}$	$-1.99(2)^{+0.29}_{-0.15}$

Renormalisation scheme independence at NNLO

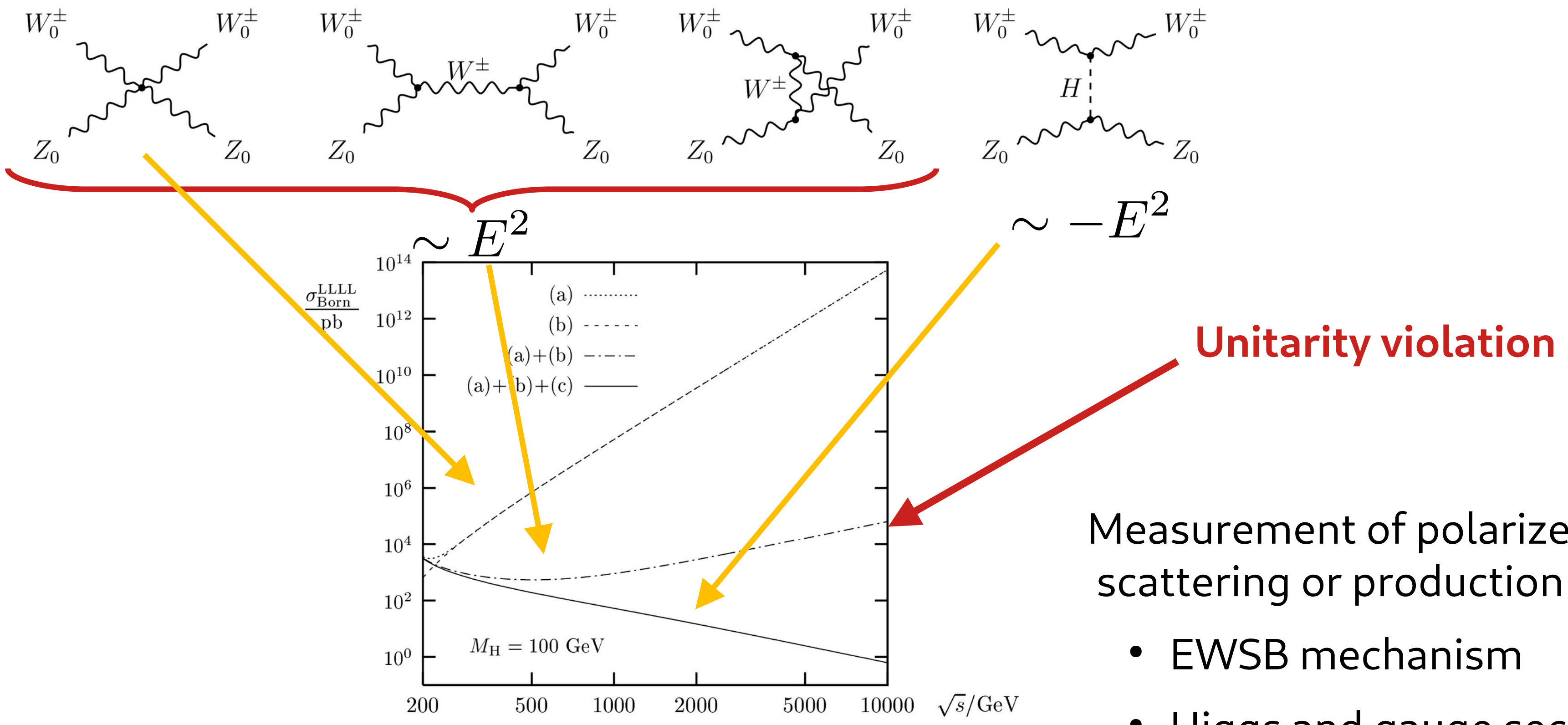
Pure top-quark mass effects

Order	σ_{HEFT} [pb]	$(\sigma_t - \sigma_{\text{HEFT}})$ [pb]
$\mathcal{O}(\alpha_s^2)$	$+16.30$	–
LO	$16.30^{+4.36}_{-3.10}$	–
$\mathcal{O}(\alpha_s^3)$	$+21.14$	-0.303
NLO	$37.44^{+8.42}_{-6.29}$	$-0.303^{+0.10}_{-0.17}$
$\mathcal{O}(\alpha_s^4)$	$+9.72$	$+0.147(1)$
NNLO	$47.16^{+4.21}_{-4.77}$	$-0.156(1)^{+0.13}_{-0.03}$

Bottom-top interference larger than top mass effect

Other ways to probe the Higgs? → Polarised bosons!

Longitudinal Vector-Boson-Scattering (VBS)



Radiative corrections to W+ W- → W+ W- in the electroweak standard model
A. Denner, T. Hahn hep-ph/9711302

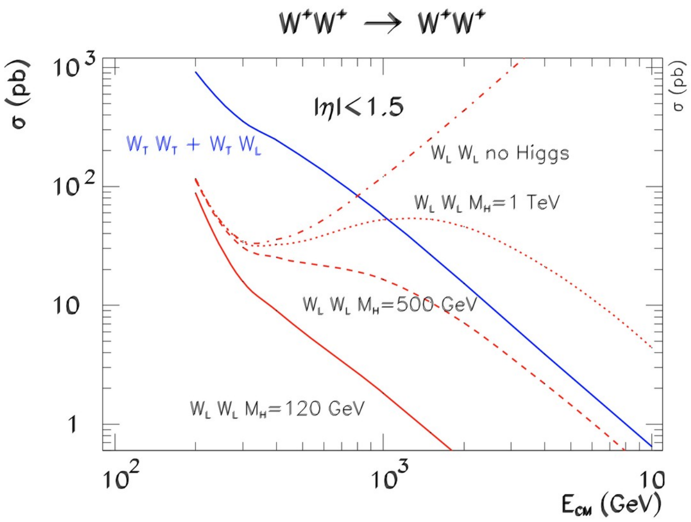
Measurement of polarized boson scattering or production probes:

- EWSB mechanism
- Higgs and gauge sector
- New physics models

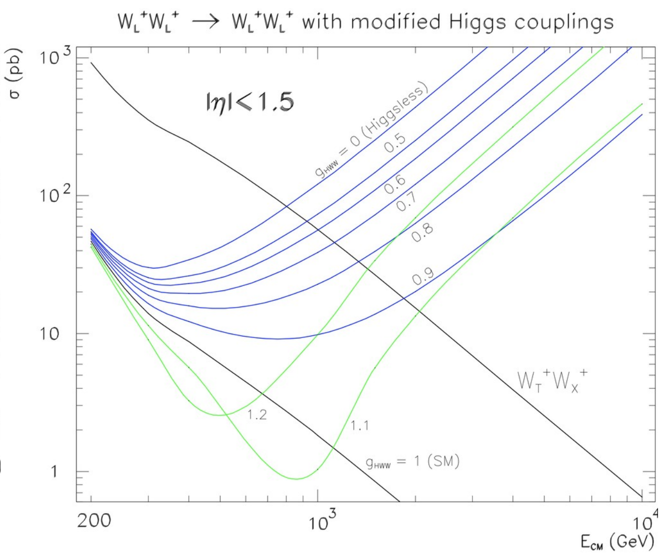
Longitudinal Vector-Boson-Scattering (VBS)

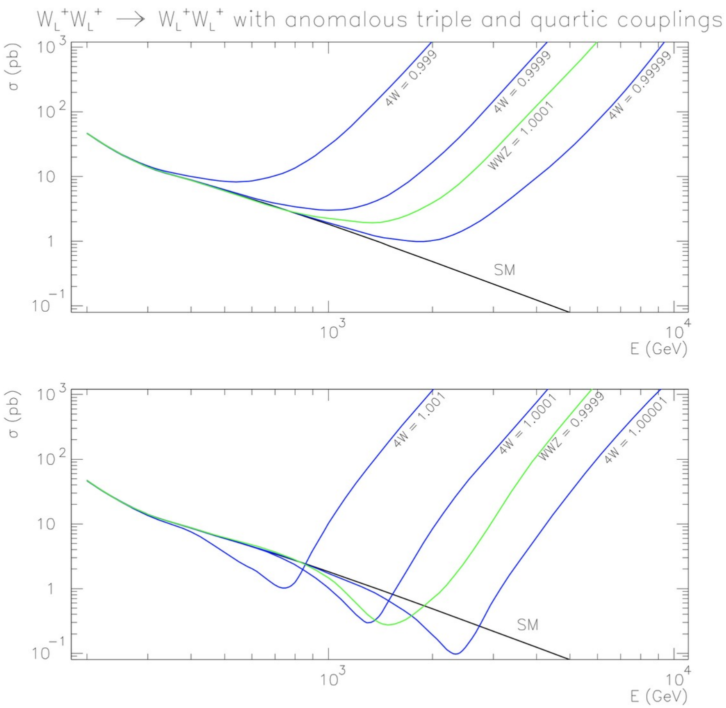
The Higgs boson and the physics of WW scattering before and after Higgs discovery
M. Szleper 1412.8367

Sensitivity to the Higgs mass



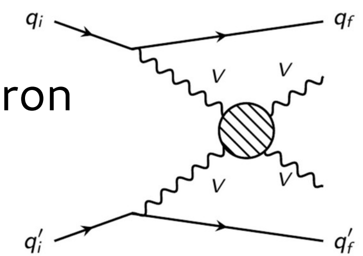
Modified HVV, VVV, VVVV couplings



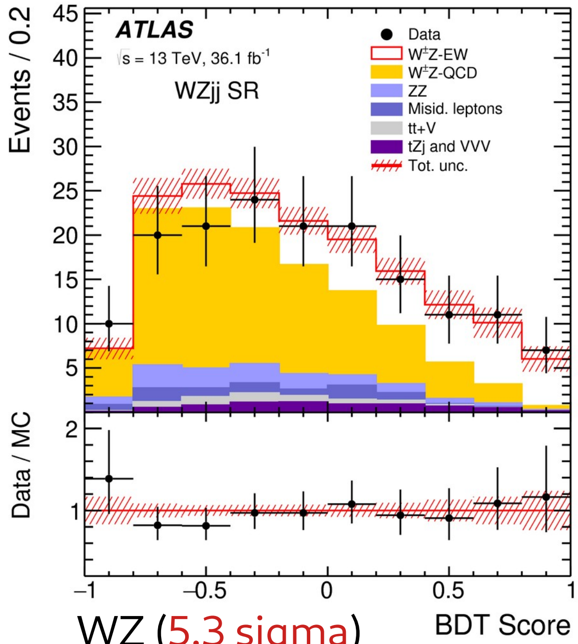


VBS at hadron colliders

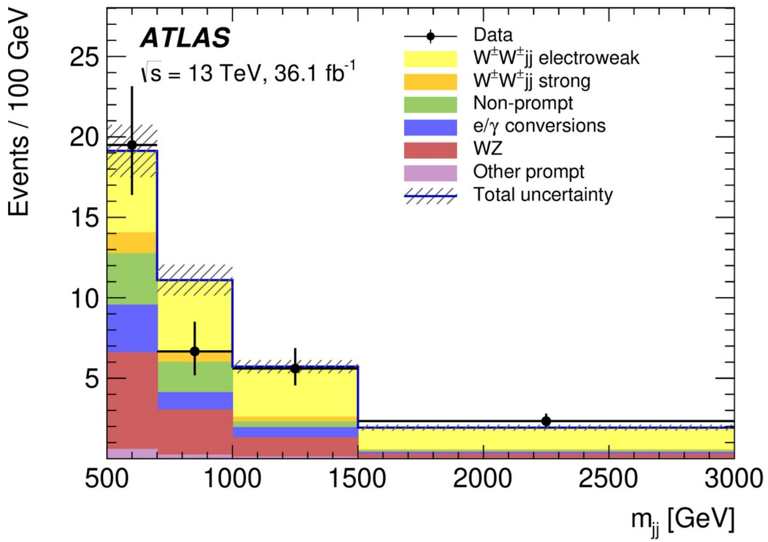
VBS at hadron colliders



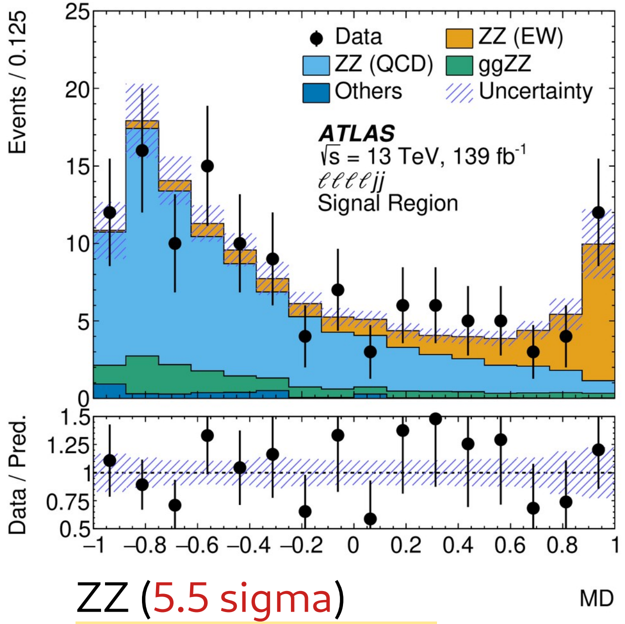
Separate from background processes through VBS topology
→ a rare process, but observed.



WZ (5.3 sigma)
ATLAS 1812.09740



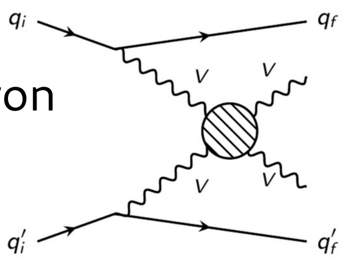
W+W+/W-W- (6.5 sigma)
ATLAS 1906.03203



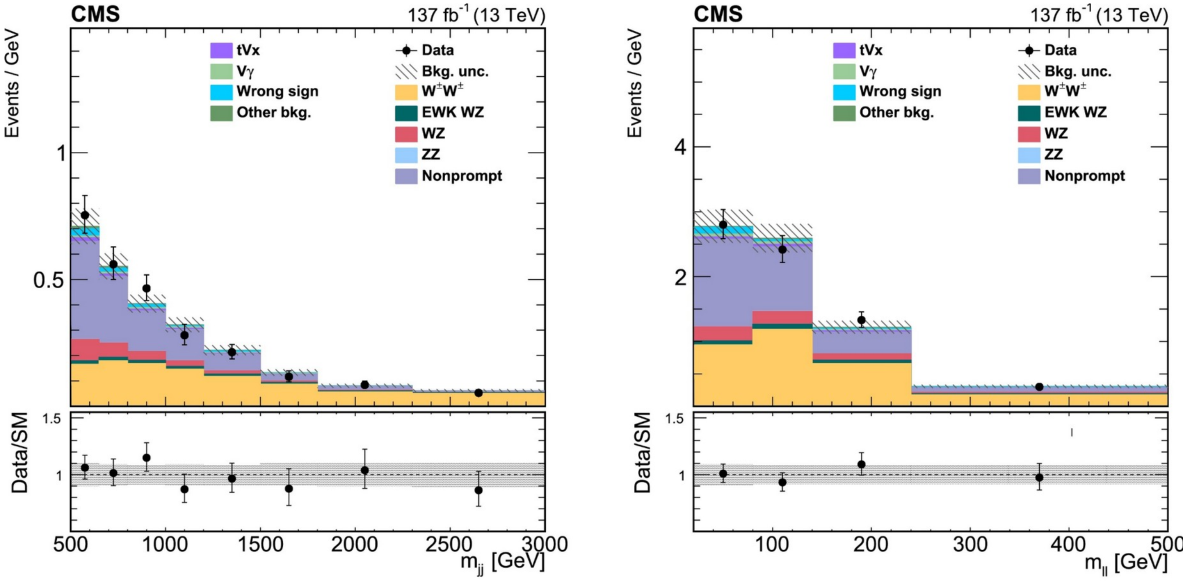
ZZ (5.5 sigma)
ATLAS 2004.10612

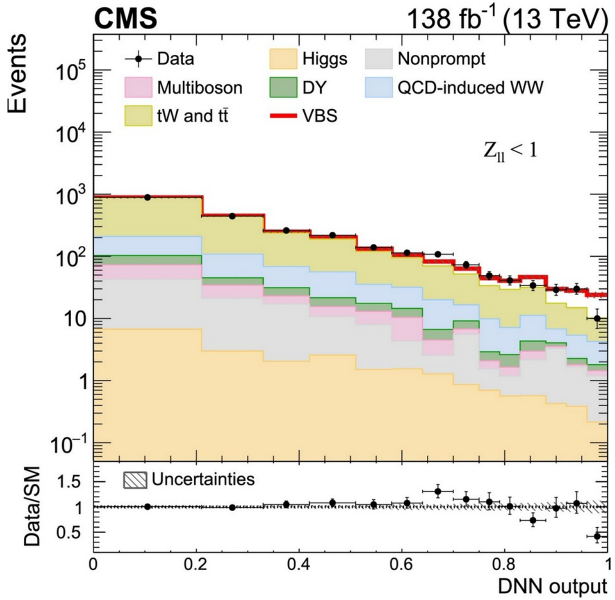
VBS at hadron colliders

VBS at hadron colliders



Separate from background processes through VBS topology
→ a rare process, but observed.

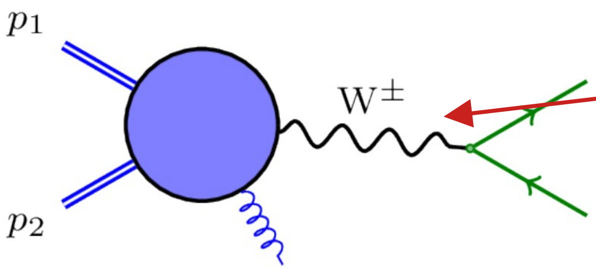




WZ (6.8 sigma) + W+W+/W-W- (diff. xsec)
CMS 2005.01173

W+W- (5.6 sigma)
CMS 2205.05711

Polarised boson production



$$\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}\right) \rightarrow \sum_\lambda \epsilon_\lambda^{*\mu} \epsilon_\lambda^\nu$$

λ = +/-/L

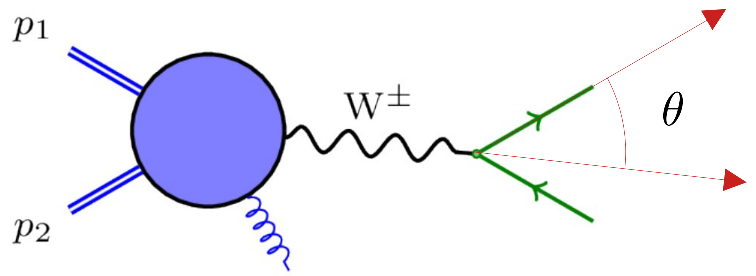
Can we extract the longitudinal component?

Measurements of longitudinal polarisation fractions:

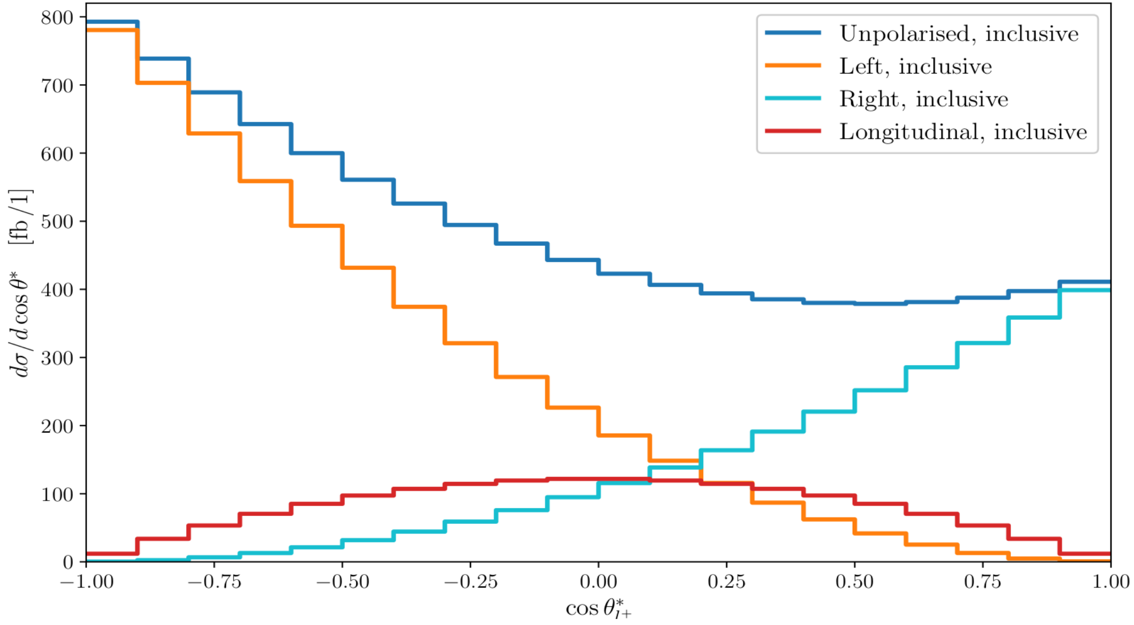
Measurement of the Polarization of W Bosons with Large Transverse Momenta in W+Jets Events at the LHC,
CMS 1104.3829
Measurement of the polarisation of W bosons produced with large transverse momentum in pp collisions at \sqrt{s}=7 TeV with the ATLAS experiment,
ATLAS 1203.2165
Measurement of WZ production cross sections and gauge boson polarisation in pp collisions at sqrt(s) = 13 TeV with the ATLAS detector,
ATLAS 1902.05759
Measurement of the inclusive and differential WZ production cross sections, polarization angles, and triple gauge couplings in pp collisions at sqrt(s) = 13 TeV,
CMS 2110.11231
Observation of gauge boson joint-polarisation states in WZ production from pp collisions at sqrt(s) = 13 TeV with the ATLAS detector
ATLAS 2211.09435
Evidence of pair production of longitudinally polarised vector bosons and study of CP properties in ZZ → 4ℓ events with the ATLAS detector at sqrt(s) = 13 TeV
ATLAS 2310.04350
Studies of the Energy Dependence of Diboson Polarization Fractions and the Radiation-Amplitude-Zero Effect in WZ Production with the ATLAS Detector
ATLAS 2402.16365

How to measure polarized bosons?

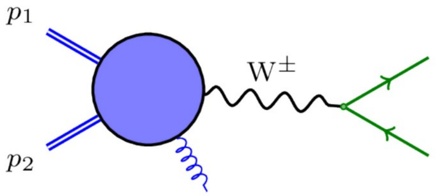
- We can’t measure boson polarization directly.
- Luckily decay products can be used as a “polarimeter”:



W+ decay (W- mirrored around 0)



Polarized cross sections



On-shell bosons:
(DPA or NWA)

$$\left(-g^{\mu\nu}+\frac{k^\mu k^\nu}{k^2}\right)\to\sum_\lambda \epsilon_\lambda^{*\mu}\epsilon_\lambda^\nu$$

$$M \;=\mathbf{P}_\mu\cdot\frac{-g_{\mu\nu}+\frac{k^\mu k^\nu}{k^2}}{k^2-M_V^2+iM_V\Gamma_V}\cdot\mathbf{D}_\nu$$

$$|M|^2=\underbrace{\sum_\lambda |M_\lambda|^2}_{\to\text{ polarised x-sections}}+\underbrace{\sum_{\lambda\neq\lambda'} M_\lambda^* M_{\lambda'}}_{\text{Interferences}}$$

Create samples of fixed polarisation:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X}=f_L\frac{\mathrm{d}\sigma_L}{\mathrm{d}X}+f_R\frac{\mathrm{d}\sigma_R}{\mathrm{d}X}+f_0\frac{\mathrm{d}\sigma_0}{\mathrm{d}X}\left(+f_{int.}\frac{\mathrm{d}\sigma_{int.}}{\mathrm{d}X}\right)$$

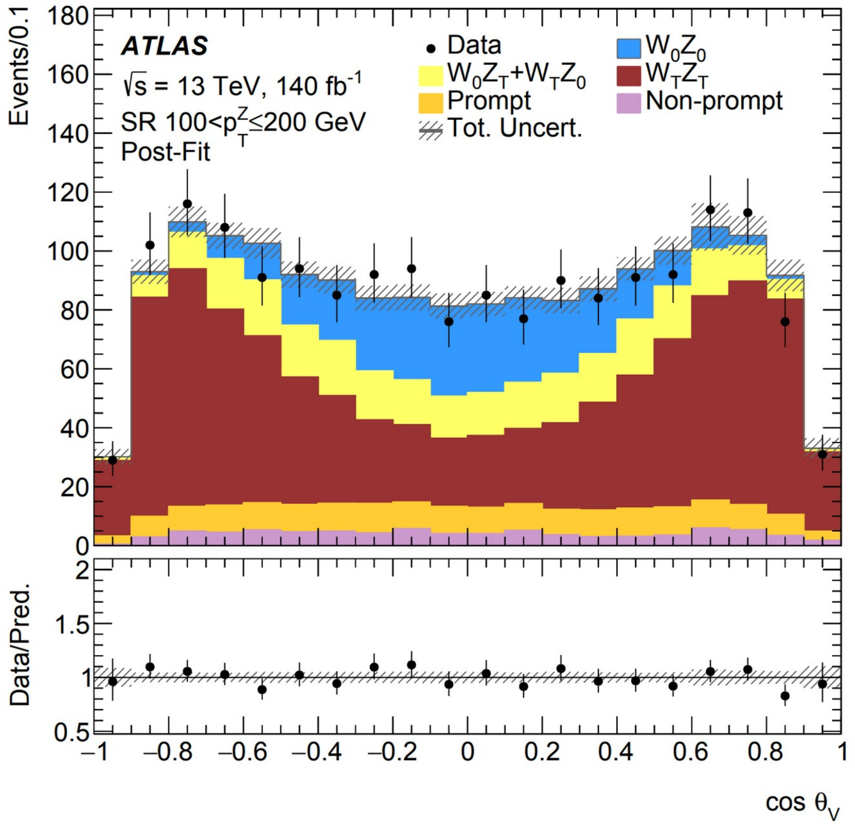
and fit f_L, f_R, f_0 to measured $\frac{\mathrm{d}\sigma^{exp.}}{\mathrm{d}X}$

Polarized cross sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = f_L \frac{\mathrm{d}\sigma_L}{\mathrm{d}X} + f_R \frac{\mathrm{d}\sigma_R}{\mathrm{d}X} + f_0 \frac{\mathrm{d}\sigma_0}{\mathrm{d}X} \left(+ f_{int.} \frac{\mathrm{d}\sigma_{int.}}{\mathrm{d}X} \right)$$

- Interferences can be handled
- Does not rely on extrapolations to the full phase space
  X can be any observable → lab frame observables
- $\frac{\mathrm{d}\sigma_i}{\mathrm{d}X}$ can be systematically improved

Example polarisation measurement in ATLAS



Studies of the Energy Dependence of Diboson Polarization Fractions and the Radiation-Amplitude-Zero Effect in WZ Production with the ATLAS Detector, ATLAS 2402.16365

	Measurement	
	$100 < p_T^Z \leq 200$ GeV	$p_T^Z > 200$ GeV
f_{00}	$0.19 \pm^{0.03}_{0.03}$ (stat) $\pm^{0.02}_{0.02}$ (syst)	$0.13 \pm^{0.09}_{0.08}$ (stat) $\pm^{0.02}_{0.02}$ (syst)
f_{0T+T0}	$0.18 \pm^{0.07}_{0.08}$ (stat) $\pm^{0.05}_{0.06}$ (syst)	$0.23 \pm^{0.17}_{0.18}$ (stat) $\pm^{0.06}_{0.10}$ (syst)
f_{TT}	$0.63 \pm^{0.05}_{0.05}$ (stat) $\pm^{0.04}_{0.04}$ (syst)	$0.64 \pm^{0.12}_{0.12}$ (stat) $\pm^{0.06}_{0.06}$ (syst)
f_{00} obs (exp) sig.	5.2 (4.3) σ	1.6 (2.5) σ

	Prediction	
	$100 < p_T^Z \leq 200$ GeV	$p_T^Z > 200$ GeV
f_{00}	0.152 ± 0.006	0.234 ± 0.007
f_{0T}	0.120 ± 0.002	0.062 ± 0.002
f_{T0}	0.109 ± 0.001	0.058 ± 0.001
f_{TT}	0.619 ± 0.007	0.646 ± 0.008

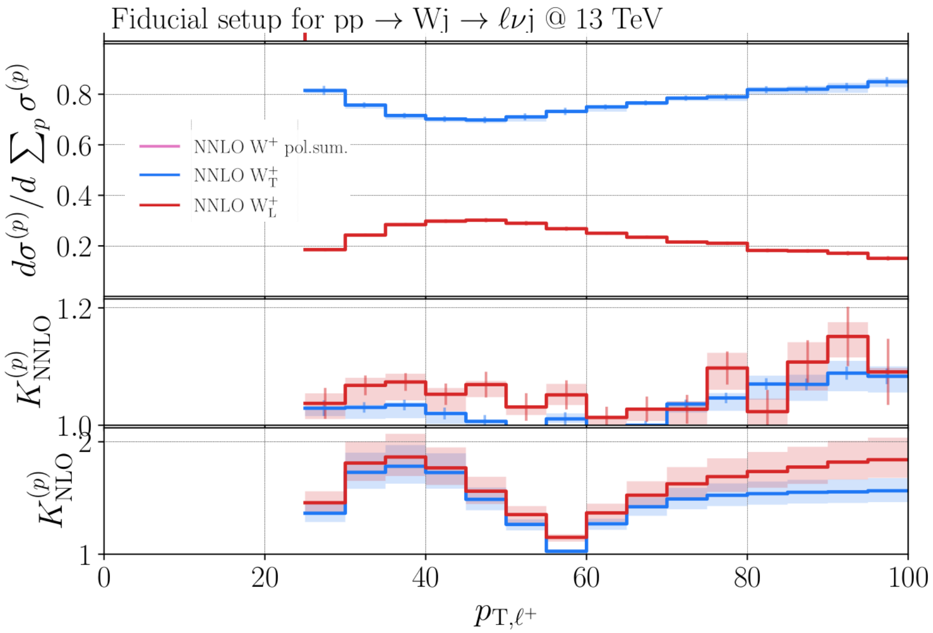
Polarized cross sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = f_L \frac{\mathrm{d}\sigma_L}{\mathrm{d}X} + f_R \frac{\mathrm{d}\sigma_R}{\mathrm{d}X} + f_0 \frac{\mathrm{d}\sigma_0}{\mathrm{d}X} \left(+ f_{int.} \frac{\mathrm{d}\sigma_{int.}}{\mathrm{d}X} \right)$$

- Interferences can be handled
- Does not rely on extrapolations to the full phase space
  X can be any observable → lab frame observables
- $\frac{\mathrm{d}\sigma_i}{\mathrm{d}X}$ can be systematically improved

Higher-order QCD/EW corrections + PS
to minimize uncertainties from missing higher orders (scale uncertainties)

Why do we need higher-order corrections?



Important observation:

Inclusive K-factors are not enough

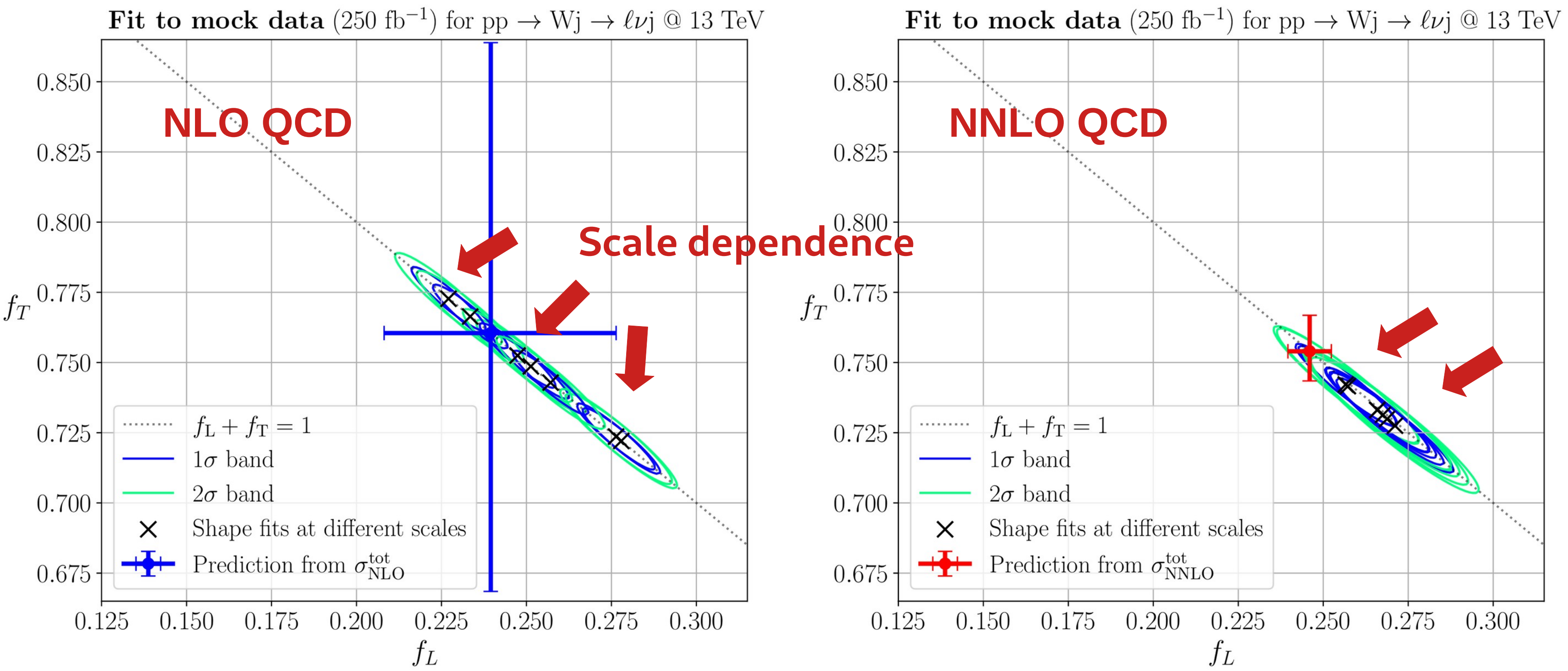
1) Differential polarization fraction have shapes
2) Higher-order corrections dependent on polarization! Just using unpolarized K-factor would lead to distortion of spectrum.
3) NNLO QCD needed to reach percent-level scale-dependence → MHOU

Polarised W+j production at the LHC: a study at NNLO QCD accuracy, Pellen, Poncelet, Popescu 2109.14336

W+jet: mock-data fit

Fit to mock-data (based on NNLO QCD and 250 fb^{-1} stats):
→ extreme case to see effect of scale dependence reduction

Observable: $\cos(\ell, j_1)$



COMETA polarisation study

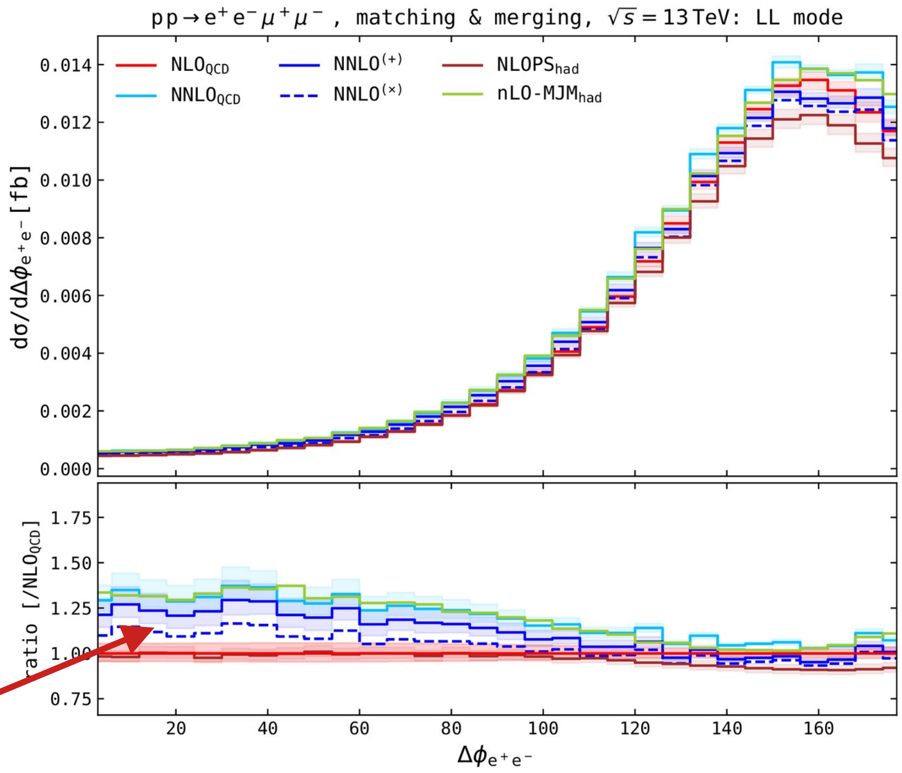
COMETA

Precise Standard-Model predictions for polarised Z-boson pair production and decay at the LHC

Costanza Carrivale,[a] Roberto Covarelli,[b] Ansgar Denner,[c] Dongshuo Du,[d] Christoph Haitz,[c] Mareen Hoppe,[e] Martina Javurkova,[f] Duc Ninh Le,[g] Jakob Linder,[h] Rafael Coelho Lopes de Sa,[f] Olivier Mattelaer,[i] Susmita Mondal,[j] Giacomo Ortona,[k] Giovanni Pelliccioli,[k,1] Rene Poncelet,[l,1] Karolos Potamianos,[m] Richard Ruiz,[l] Marek Schönherr,[n] Frank Siegert,[e] Lailin Xu,[d] Xingyu Wu,[d] Giulia Zanderighi[h]

Validation/comparisons of MC codes

Fixed order:
BBMC, Mocanlo, MulBos, Stripper
Event generators:
MadGraph, Sherpa, Powheg+Pythia

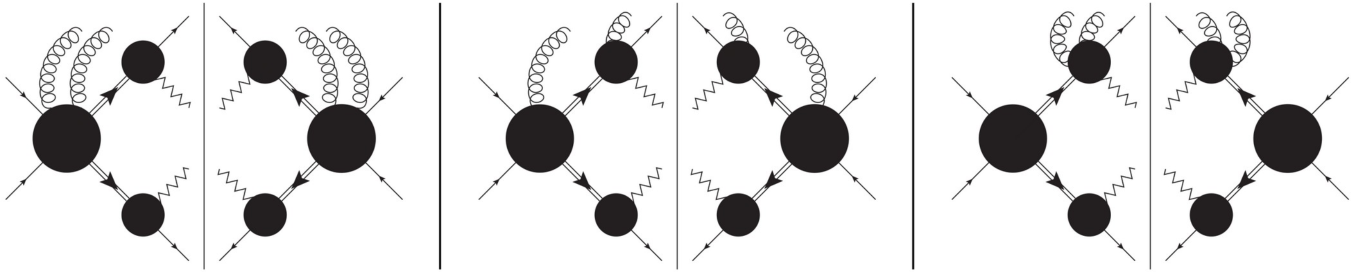


Largest QCD corrections come from the modelling of hard radiation (recoil) → not captured by PS

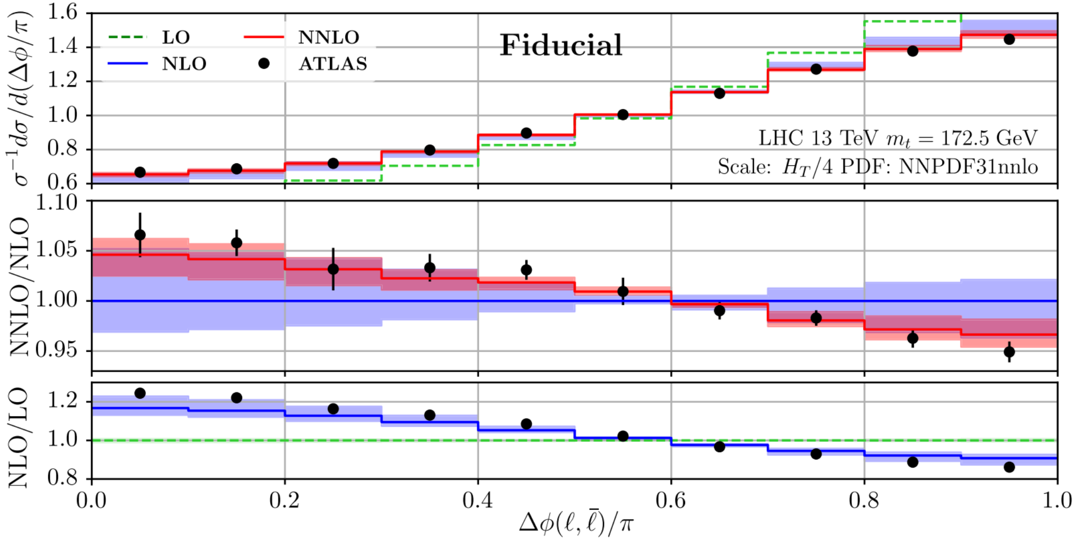
Spin-correlations in top-quark pair production

[Behring, Czakon, Mitov, Papanastasiou, Poncelet PRL 123 (2019) 8 082001]

This is not really a surprise…



Hard recoil in top-quark pair production and decay causes significant shape effects!



P3EWSB

[High **P**recision **P**redictions to **P**robe the **E**lectro**W**eak-**S**ymmetry **B**reaking]

Funded under SONATA 20 UMO-2024/55/D/ST2/00934

More holistic analysis of NNLO QCD corrections to spin-observables

→ more **polarised LHC processes**: top-quark production, Higgs-strahlung, ...

→ impact on **quantum information observables**

which are typically based of angular correlations

→ implementation in **HighTEA** for easy access

[2304.05993]

https://www.precision.hep.phy.cam.ac.uk/hightea

Comprehensive **M**ultiboson **E**xperiment-**T**heory **A**ction

- WG1 - Theoretical framework, precision calculations and simulation
- WG2 - Technological innovation in data analysis
- WG3 - Experimental Measurements
- WG4 - Management and Event Organization
- WG5 - Inclusiveness and Outreach

Further information:
https://www.cost.eu/actions/CA22130/ and https://cometa.web.cern.ch/

Polarised nLO+PS: SHERPA

Polarised cross sections for vector boson production with SHERPA
Hoppe, Schönherr, Siegert 2310.14803

- New bookkeeping of boson polarizations in SHERPA for LO MEs
- Approximate NLO corrections: nLO+PS
  → Reals+matching are treated exact
  → loop matrix elements unpolarised
- Comparison with multi-jet merged calculations

Comparison with literature
- nLO+PS approximation in fair agreement with full NLO
  → good for polarization fractions

W^+Z	σ^{NLO} [fb]	Fraction [%]	K-factor	$\sigma^{\text{nLO+PS}}_{\text{SHERPA}}$ [fb]	Fraction [%]	K-factor
full	35.27(1)		1.81	33.80(4)		
unpol	34.63(1)	100	1.81	33.457(26)	100	1.79
			Laboratory frame			
L-U	8.160(2)	23.563(9)	1.93	7.962(5)	23.796(25)	1.91
T-U	26.394(9)	76.217(34)	1.78	25.432(21)	76.01(9)	1.75
int	0.066(10) (diff)	0.191(29)	2.00	0.064(7)	0.191(22)	2.40(40)
U-L	9.550(4)	27.577(14)	1.73	9.275(16)	27.72(5)	1.72
U-T	25.052(8)	72.342(31)	1.83	24.156(18)	72.20(8)	1.81
int	0.028(10) (diff)	0.081(29)	-0.49	0.026(7)	0.079(22)	-0.471(34)

Polarized VV @ (N)NLO QCD / NLO EW

Fiducial polarization observables in hadronic WZ production: A next-to-leading order QCD+EW study,
Baglio, Le Duc 1810.11034
Anomalous triple gauge boson couplings in ZZ production at the LHC and the role of Z boson polarizations,
Rahama, Singh 1810.11657
Polarization observables in WZ production at the 13 TeV LHC: Inclusive case,
Baglio, Le Duc 1910.13746
Unravelling the anomalous gauge boson couplings in ZW+- production at the LHC and the role of spin-1 polarizations,
Rahama, Singh 1911.03111
Polarized electroweak bosons in W+W- production at the LHC including NLO QCD effects,
Denner, Pelliccioli 2006.14867
NLO QCD predictions for doubly-polarized WZ production at the LHC,
Denner, Pelliccioli 2010.07149
NNLO QCD study of polarised W+W- production at the LHC,
Poncelet, Popescu 2102.13583
NLO EW and QCD corrections to polarized ZZ production in the four-charged-lepton channel at the LHC,
Denner, Pelliccioli 2107.06579
Breaking down the entire spectrum of spin correlations of a pair of particles involving fermions and gauge bosons,
Rahama, Singh 2109.09345
Doubly-polarized WZ hadronic cross sections at NLO QCD+EW accuracy,
Duc Ninh Le, Baglio 2203.01470
Doubly-polarized WZ hadronic production at NLO QCD+EW: Calculation method and further results
Duc Ninh Le, Baglio, Dao 2208.09232
NLO QCD corrections to polarised di-boson production in semi-leptonic final states
Denner, Haitz, Pelliccioli 2211.09040
Polarised cross sections for vector boson production with SHERPA
Hoppe, Schönherr, Siegert 2310.14803
Polarised-boson pairs at the LHC with NLOPS accuracy
Pelliccioli, Zanderighi 2311.05220
NLO EW corrections to polarised W+W- production and decay at the LHC
Denner, Haitz, Pelliccioli 2311.16031
NLO electroweak corrections to doubly-polarized W+W- production at the LHC
Thi Nhung Dao, Duc Ninh 2311.17027
Polarized ZZ pairs in gluon fusion and vector boson fusion at the LHC
Javurkova, Ruiz, Coelho, Sandesara 2401.17365

Other polarized cross section calculations

- Polarised VBS (so far LO):
  W boson polarization in vector boson scattering at the LHC,
  Ballestrero, Maina, Pelliccioli 1710.09339
  Polarized vector boson scattering in the fully leptonic WZ and ZZ channels at the LHC,
  Ballestrero, Maina, Pelliccioli 1907.04722
  Automated predictions from polarized matrix elements
  Buarque Franzosi, Mattelaer, Ruiz, Shil 1912.01725
  Different polarization definitions in same-sign WW scattering at the LHC,
  Ballestrero,Maina, Pelliccioli 2007.07133

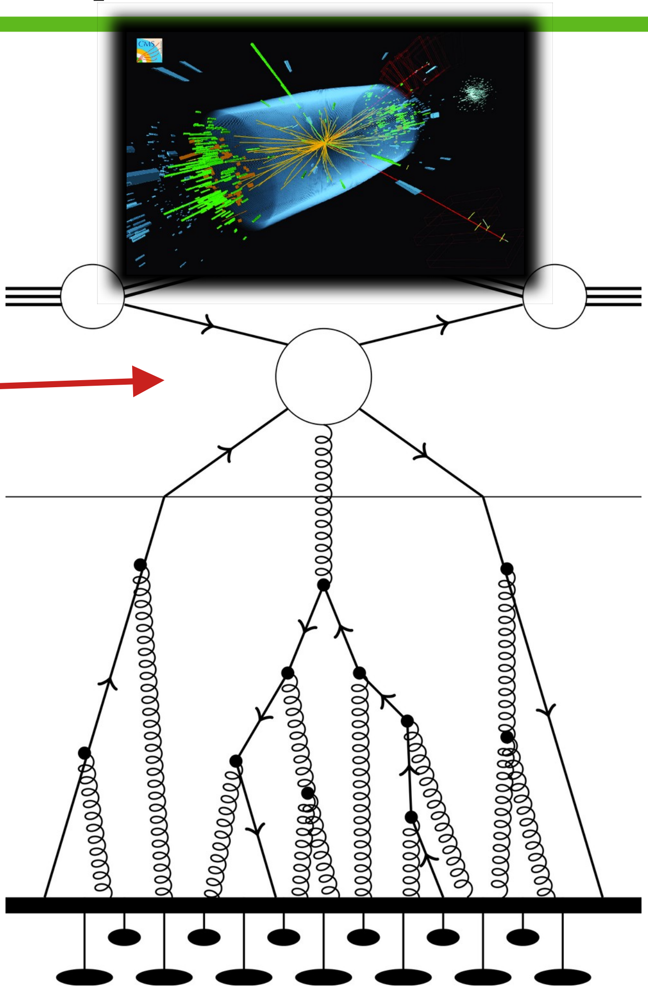
- Single boson production
  Left-Handed W Bosons at the LHC,
  Z. Bern et. al. 1103.5445
  Electroweak gauge boson polarisation at the LHC,
  Stirling, Vryonidou 1204.6427
  What Does the CMS Measurement of W-polarization Tell Us about the Underlying Theory of the Coupling of W-Bosons to Matter?,
  Belyaev, Ross 1303.3297
  Polarised W+j production at the LHC: a study at NNLO QCD accuracy,
  Pellen, Poncelet, Popescu 2109.14336

Beyond fixed-order perturbation theory

Guiding principle: factorization
"What you see depends on the energy scale"

In Quantum Chromodynamics (QCD):

$Q \gg \Lambda_{\mathrm{QCD}}$ **Fixed-order perturbation theory**
scattering of individual partons



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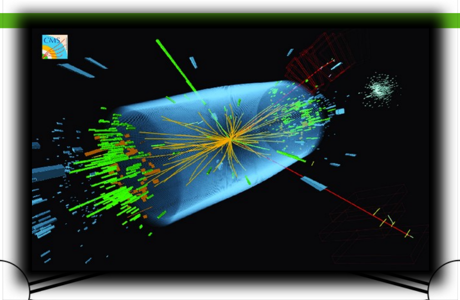
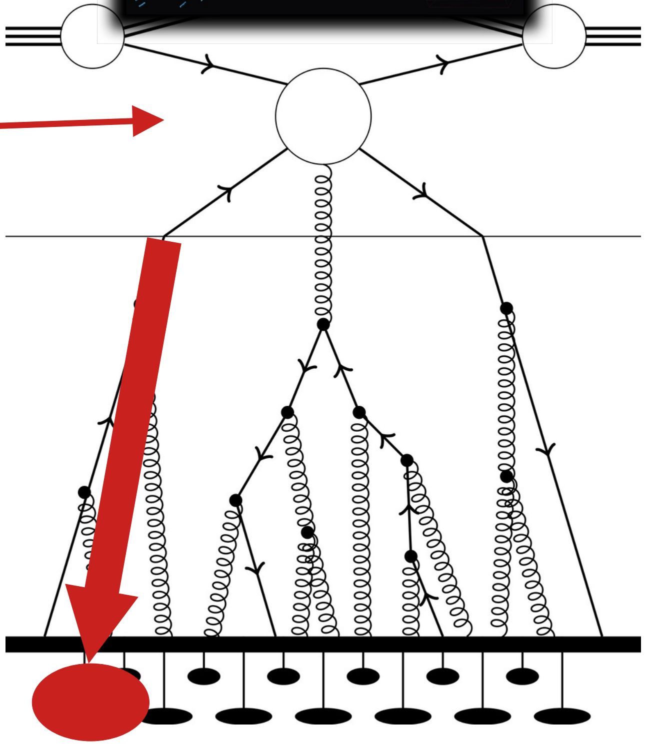




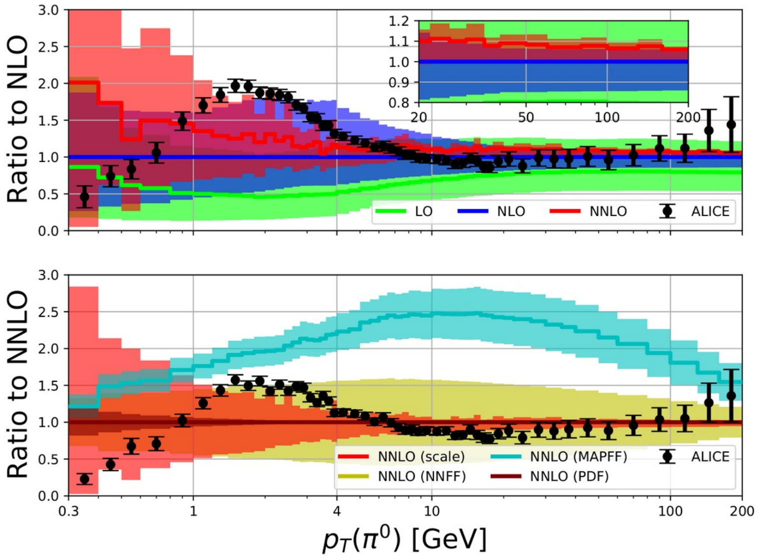
Parton to identified object transition “**Fragmentation**”
→ Resummation of collinear logs through ‘DGLAP’
→ Non perturbative fragmentation functions
Example: B-hadrons in e+e-

$$\frac{d\sigma_B(m_b,z)}{dz} = \sum_i \left\{ \frac{d\sigma_i(\mu_{Fr},z)}{dz} \otimes D_{i\to B}(\mu_{Fr},m_b,z) \right\}(z) + \mathcal{O}(m_b^2)$$

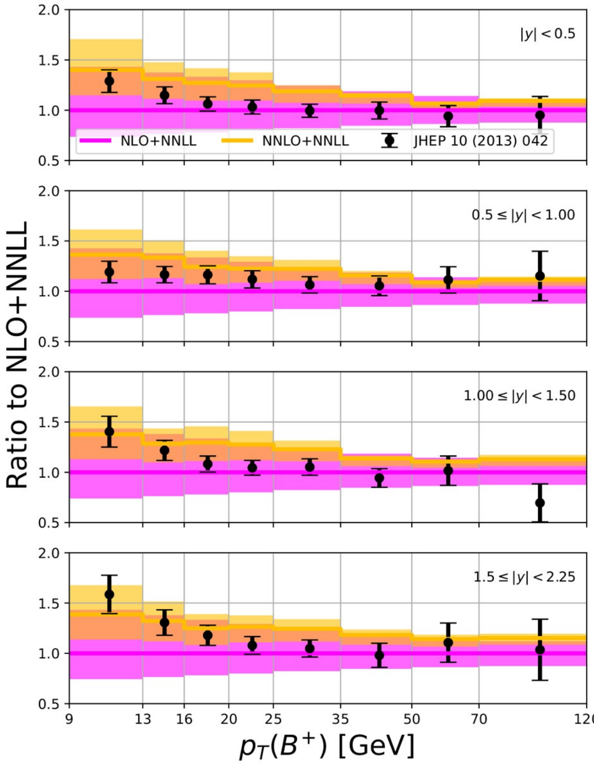
Identified hadrons

Inclusion of fragmentation through NNLO QCD:
[Czakon, Generet, Mitov, Poncelet]
- B-hadrons in top-decays [2210.06078,2102.08267]
- Open-bottom [2411.09684] → accepted in PRL
- Identified hadrons [2503.11489] → accepted in PRL

$$d\sigma_{pp\to h}(p) = \sum_i \int dz\ d\hat{\sigma}_{pp\to i}\left(\frac{p}{z}\right) D_{i\to h}(z)$$



Pion production



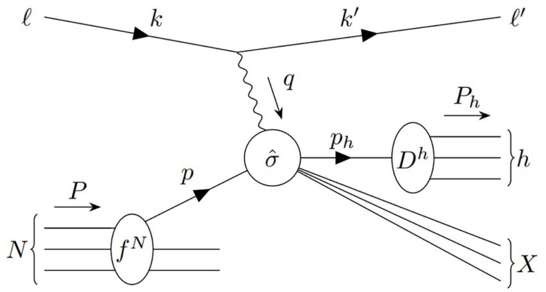
Open-bottom @FONNLL:

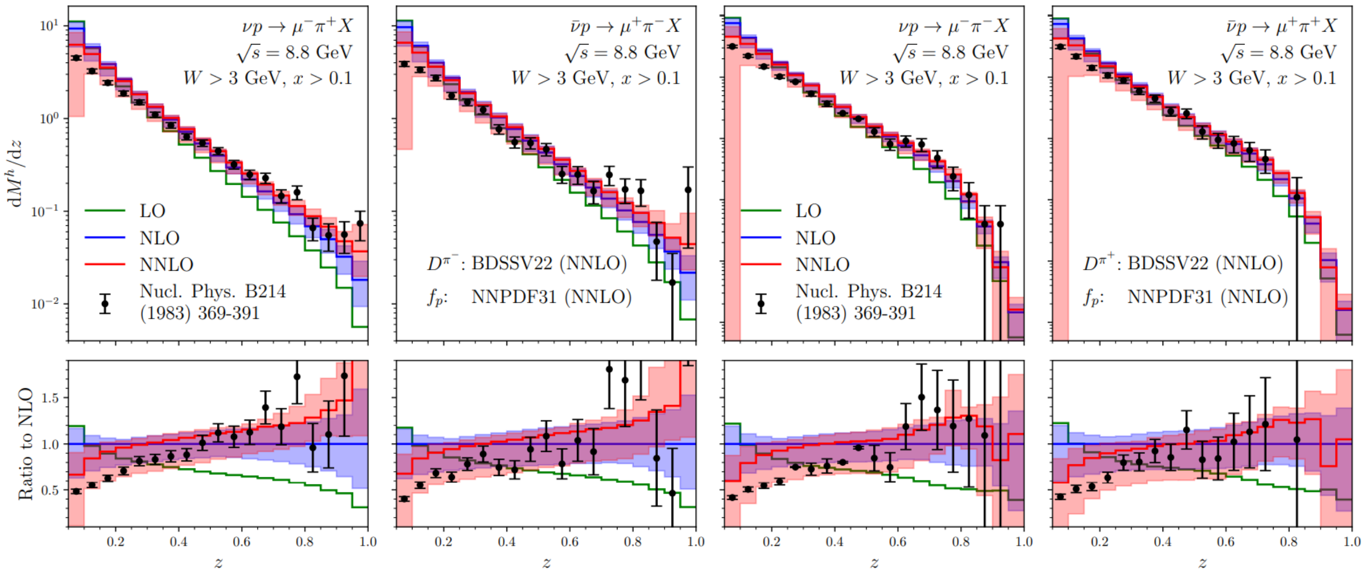
$$\sigma(p_T) = \sigma(m, p_T) + G(p_T)(\sigma(0, p_T) - \sigma(0, p_T)|_{\mathrm{FO}})$$

Semi-inclusive Deep Inelastic Scattering

Series of works on SIDIS through NNLO QCD:
[Bonino, Gehrmann, Loechner, Schoenwald, Stagnitto]

- Polarised initial states [2404.08597]
- Neutrino-Nucleon Scattering [2504.05376]
- CC and NC [2506.19926]





[2504.05376]

Jet substructure

Semi-inclusive jet function [1606.06732, 2410.01902]

$$\frac{d\sigma_{\rm LP}}{dp_T d\eta} = \sum_{i,j,k} \boxed{\int_{x_{i,\min}}^1 \frac{dx_i}{x_i} f_{i/P}(x_i,\mu) \int_{x_{j,\min}}^1 \frac{dx_j}{x_j} f_{j/P}(x_j,\mu) \int_{z_{\min}}^1 \frac{dz}{z} \mathcal{H}_{ij}^k(x_i, x_j, p_T/z, \eta, \mu)}$$
$$\times \boxed{J_k\left(z, \ln\frac{p_T^2 R^2}{z^2\mu^2}, \mu\right)},$$

The same hard function as for identified hadrons!

Modified RGE:

[2402.05170,2410.01902]

$$\frac{d\vec{J}\left(z, \ln\frac{p_T^2R^2}{z^2\mu^2}, \mu\right)}{d\ln\mu^2} = \int_z^1 \frac{dy}{y} \vec{J}\left(\frac{z}{y}, \ln \boxed{\frac{y^2 p_T^2 R^2}{z^2\mu^2}}, \mu\right) \cdot \widehat{P}_T(y)$$

Energy-Energy correlators obey similar factorization!

Small-R jets

Application to small-R jets
[Generet, Lee, Moult, Poncelet, Zhang]
[2503.21866]

‘Triple’ differential measurement by CMS:
Y, pT, R [2005.05159]