Precision Calculation of the Small Angle Bhabha Cross Section*

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Abstract

We present the theoretical basis and sample Monte Carlo data for the YFS exponentiated second order (LL) calculation of low angle Bhabha scattering in the LEP/SLC luminosity regime. Tests for below .1% combined physical and technical precision are discussed. We conclude that the current error on the luminosity as calculated using BHLUMI4.00 is $^{+.16\%}_{-.089\%}$, where the upper(lower) error corresponds to the symmetric narrow(asymmetric) ALEPH SICAL-type acceptance.

1. Introduction

The uncertainty on the LEP/SLC luminosity process, low angle bhabha scattering eneters all cross sections measured therein. As Pietrzyk [1] has emphasized, for example, each .1% uncertainty in the respective luminosity cross section corresponds to .0075 uncertainty in the number N_ν of massless neutrino generations. Thus, it is important to keep the theoretical contribution to the luminosity cross section uncertainty at the level of $\sim \frac{1}{2} - \frac{1}{3}$ of the corresponding experimental error so that it does not affect the accuracy of the precision tests of the Standard Model in Z^0 physics in an unacceptable way. In this paper, we present the initial results on our new YFS Monte Carlo BHLUMI4.00 in which we show that we have achieved below .1% precision on the low angle bhabha scattering cross section in the new ALEPH SICAL[1] detector acceptance. In this way, we illustrate that the desired regime of below .1% precision theoretical predictions for this cross section is indeed feasible.

More specifically, in Ref. [2] we showed that our YFS methods as realized in the Monte Carlo event generator BHLUMI2.00 were able to achieve 0.25% precision on the LEP/SLC luminosity process. If we recall that error budget from Table 3 of this latter reference, in view of the result for the Z exchange contribution given in Ref. [3], we see that the dominant error in the budget is that of the of the $\mathcal{O}(\alpha^2)$ bremsstrahlung effects and that these should be our focus in improving on the precision of BHLUMI2.00. In the 1992 time frame of the publication of BHLUMI2.00 in Ref. [4], the typical experimental precision on the respective cross section was $\sim$ 0.5% [5] so that indeed the theoretical precision of 0.25% was acceptable. Recently, the LEP Collaborations have introduced more precise luminometers [1] and two of them, ALEPH and OPAL [6, 7] , have broken the 0.1% barrier on the experimental error, with results of .09% and .07% respectively. Accordingly, it is now important to reduce the theoretical error on the luminosity cross section to the respective below 0.1% regime as well. Here, we show how we achieve this precision in the acceptance of the new ALEPH SICAL luminometer in the LEP energy regime. Similar results for the other LEP Collaborations' new luminometers will appear elsewhere [8].

Our discussion is organized as follows. In the

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next section, we present our analysis of the key issues necessary to improve the precision of BHLUMI2.00 to the below .1% regime. We give explicit Monte Carlo data which shows that we have achieved this precision for the ALEPH SICAL luminometer. Our final section contains our summary remarks.

2. Analysis

Our analysis for achieving the below .1% regime for the theoretical error on $\sigma_{\mathcal{L}}$, the small angle bhabha cross section in the LEP/SLC luminosity regime, proceeds according to our general strategy [2] in which we establish a baseline calculational framework on which we can anchor the new level of precision to known semi-analytical and MC results with known technical and physical precisions. In this section, we present the respective analysis.

The fundamental point is that we need to control both the physical and the technical precision to below .1% precision. To this end, we construct a series of triggers ,$TR_i, i = 0, \cdots, n$, in which we pass smoothely from an academic trigger, TR_0, which is not exactly the ALEPH SICAL acceptance but which allows us to compute the cross section analytically to the trigger TR_n which corresponds to the ALEPH SICAL. For each trigger, we control the corresponding technical and physical errors in BHLUMI4.00 by comparing with the appropriate analytical and/or $OLBIS \oplus LUMLOG$[9] known exact $\mathcal{O}(\alpha) + \mathcal{O}(\alpha^m)_{LL}, m = 2, 3$, results, for example, where LL denotes leading logs and we will refer to the case $m = 2$ as $\mathcal{O}(\alpha^2)_{prag}$. For our academic trigger, we define the cuts as follows: $|t_{min}| < |t| < |t_{max}|$ and $V < V_{max}$, where t is the four-momentum transfer squared transmitted through t-channel photon exchange, and the variable V represents a measure of the total energy carried away by all emitted real photons; we take $V = 1 - \frac{2(p_1p_2)|t|}{(2(p_1p_2)+2(p_1K_p))^2} \frac{2(q_1q_2)|t|}{(2(q_1q_2)+2(q_1K_q))^2}$, where $p_i = 1, 2$ are the four-momenta of incoming and outgoing electron, $q_i = 1, 2$ are four-momenta of incoming and outgoing positron, and K_p and K_q are the total four-momenta of all photons emitted from electron and positron lines respectively. With the above definition of the phase-space window, it is possible to integrate the $\mathcal{O}(\alpha^2)_{prag}$ matrix element keeping all terms within the $\mathcal{O}(\alpha^2)_{prag}$ approximation, where our big logarithm variable is taken as $L = \ln |t|/m_e^2$, for definiteness. In order to establish the technical precision at the 0.03% level, we actually followed the integration to the third order $\mathcal{O}(\alpha^3)_{prag}$ approximation, which includes terms up to NLL at $\mathcal{O}(\alpha^2)$ and up to LL at $\mathcal{O}(\alpha^3)$ in addition to the exact $\mathcal{O}(\alpha)$ cross section. The resulting integrated cross section is recorded in Eq.(2) of Ref. [10]; we do not record it here due to lack of space. Using results such as Figs. 3 and 4 in Ref. [10], respectively, we have checked that the technical precision of BHLUMI4.00 is indeed .03% for the academic trigger(based on comparison with our Eq.(2) in Ref. [10]) and that the missing $\mathcal{O}(\alpha^2L^2)$ correction in BHLUMI2.00 for the original LEP luminosity-type of trigger analyzed in Ref. [2] is below 0.04%. Continuing in this way, if we repeat the check of the old solution $OLDBIS \oplus LUMLOG$ vs BHLUMI as we did in Ref. [2] but for version 4.00 for the new ALEPH SICAL acceptance instead of version 2.00 for the old LEP-type luminometer acceptance, we get the results in Fig. 1.

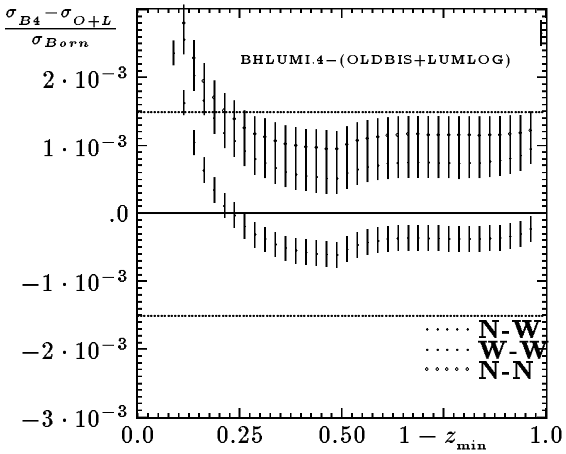

Figure 1. *Trigger and angular range of ALEPH luminosity detector SICAL. MC results from the new BHLUMI version 4.0 (unpublished). Dotted lines mark the usual 0.15% limit.*

Accordingly, already, we can improve significantly over the .25% precision estimate given in Ref. [2] using the results given herein and others derived from the same theoretical apparatus. Specifically, if we recall the error budget given in Table 3 of the latter reference, we may use the results in Ref. [10] together with those in Fig. 1 to conclude that the total error, physical + technical, on the bremsstrahlung contribution to the BHLUMI4.0 differential cross section is now $^{+.15\%}_{-.065\%}$ for the standard calorimetric energy cut between 0.5 and 0.9. Further, from Ref. [3] we have that the error on the QED vacuum polarization contribution is now $\pm.05\%$ and from Ref. [11], we have confirmed our original estimate that the missing pairs contribution gives an error of $\pm.025\%$. Here, we may advocate taking this pairs effect from Refs. [2, 11] with a corresonding reduction in the error associated with it to $\pm.01\%$. In this way, we may now quote the error budget for BHLUMI4.00 in the ALEPH SICAL-type angular regime as shown in Table 1, where the total theoretical error on respective luminosity calculation is now $^{+.16\%}_{-.089\%}$ for the respective $^{narrow}_{asymmetric}$ SICAL acceptance.

BHLUMI 4.00 precision		
Type of contribution	Physical	Technical
{1} Vacuum polarization hadronic	0.05%	
{2} Missing part of $\left(\frac{\alpha}{\pi}\right)^2 L$ brems.	$^{+.15\%}_{-.065\%}$	
{3} Missing part of $\left(\frac{\alpha}{\pi}\right)^3 L^3$ brems.	0.000%	0.008%
{4} Up-down interference	+0.004%	0.001%
{5} e^+e^- pairs LL non-singlet	0.007%	0.002%
{6} e^+e^- pairs LL singlet	$< 0.002\%$	
{7} pairs NLL, quarks, μ	$< 0.007\%$	
{8} Z exchange	0.033%	
{9} γ exchange s-channel	$< 0.0002\%$	
Total	$^{+.16\%}_{-.089\%}$	

Table 1. *The precision on the cross section at $\sqrt{s} = 92.5$ GeV for the SICAL-type acceptance. The precision listed is for BHLUMI4.00, with the understanding that the Z exchange correction is taken from Ref. [5] and pairs effects are taken from Refs. [1,9]. The program calculates, for the SICAL-type trigger, the LL $\mathcal{O}(\alpha^2)$ exponentiated differential cross section (this latter differential cross section , upon integration, leads to Eq.(2) in Ref. [10] for an academic trigger). We list here the theoretical uncertainties of the corrections which are included in BHLUMI 4.00, e.g. {1,9}, which are known from other sources , e.g. {5-8}, but are still left-out and are to be taken there from [1,5,9] or corrections which are left-out , e.g. {2-4}, but are calculated {3-4} or estimated {2} by us. Total theoretical error is indicated, where the upper error corresponds to the symmetric narrow SICAL acceptance and the lower error to the asymmetric SICAL acceptance.*

3. Conclusions

We conclude that the way to below .1% precision LEP/SLC luminosity calculations is open and we already have begun to peer into its beckoning horizon. This is exciting indeed!

Specifically, the main $\mathcal{O}(\alpha^2 L^2)$ part of the missing 2nd order bremsstrahlung correction in BHLUMI2.0) is now realized in BHLUMI4.00. Work on the remaining $\mathcal{O}(\alpha^2 L)$ is in progress[8].(See also the work of Fadin *et al.* [12].) The missing part of the $\mathcal{O}(\alpha^3 L^3)$ correction in BHLUMI4.00 is known and is small in the relevant LEP luminosity acceptance. Our current error on the ALEPH SICAL accepted cross section is now .089% in the relevant asymmetric case. We look forward to further reduction in this theoretical precision tag in the not-too-distant future.

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References

[1] B. Pietrzyk, Proc. of the Tennessee International Symp. on Radiative Corrections 1994, in press; Ed. B.F.L. Ward (World Scientific 1994); and references therein.

[2] S. Jadach, E. Richter-Wąs, B.F.L. Ward and Z. Wąs, Phys. Lett. **B268** (1991) 253.

[3] B. Pietrzyk and W. Beenakker, Phys. Lett. **B304** (1993) 366.

[4] S. Jadach, E. Richter-Wąs, B.F.L. Ward and Z. Wąs, Computer Phys. Comm. **70** (1992) 305.

[5] H. Meinhard, Proc. of the Rochester Conf. on High Energy Phys. 1992, p. 687; Ed. J. Sanford (World Scientific 1992).

[6] M. Gruenwald, These Proceedings 1994; Eds. P.J. Bussey and I.G. Knowles (IOP Publishing).

[7] D. Schaile, These Proceedings 1994; Eds. P.J. Bussey and I.G. Knowles (IOP Publishing).

[8] S. Jadach *et al.*, to appear.

[9] S. Jadach, E. Richter-Wąs, B.F.L. Ward and Z. Wąs, Phys. Lett. **B260** (1991) 438.

[10] S. Jadach *et al.*, Proc. of the Rencontre de Moriond 1994, in press; Ed. J. T. Thanh Van(Frontieres 1994).

[11] S. Jadach, M. Skrzypek and B. F. L. Ward, Phys. Rev. **D47** (1993) 3733.

[12] V. Fadin *et al.*, Proc. of the Tennessee International Symp. on Radiative Corrections 1994, in press; Ed. B.F.L. Ward (World Scientific 1994).