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PHOTOS–A universal Monte Carlo for QED radiative corrections: version 2.0

Elisabetta Barberio
PPE DIVISION
CERN, 1211 Geneva 23, Switzerland
and
Fachbereich Physik, Universität Siegen, 5900 Siegen , Fed. Rep. Germany.
and
Zbigniew Wąs
TH DIVISION
CERN, 1211 Geneva 23, Switzerland
and
Institute of Nuclear Physics, Kraków, ul. Kawiory 26a, Poland

ABSTRACT

This paper documents an update of the PHOTOS algorithm, by E. Barberio, B. van Eijk and Z. Wąs, for the Monte Carlo simulation of QED photon radiative corrections in decays. The algorithm is implemented as an independent package written in FORTRAN 77. The program is universal, i.e. it allows for easy interface with "any" program generating decays of "any" particle. The program can be used to estimate the size of the QED bremsstrahlung in the leading-logarithmic (collinear) approximation, but it provides final states with their full topology, including kinematical effects due to massive particles. The proper soft photon behaviour is also reproduced. The algorithm is designed in such a way that the inclusion (if necessary) of the exact, process-dependent, matrix element subgenerator is *possible* and straightforward. This update of PHOTOS provides new options to improve the performances of the program in some specific cases: the interference for two-body particle-antiparticle decays, the double bremsstrahlung, and refinements important for the decays into massive non-relativistic decay products. The comparison of the program with the exact matrix-element calculation for the process $Z \to \mu^+\mu^-(\gamma)$, $W \to e\nu(\gamma)$, $\tau \to e\nu(\gamma)$, $B^\pm \to D^0 e^\pm \nu(\gamma)$ and $gg \to t\bar{t}(\gamma), t\bar{t}(\gamma\gamma)$ is also discussed.

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NEW VERSION SUMMARY

Title of the new version of the program: PHOTOS, version: 2.0
Authors of the original program: E. Barberio, B. van Eijk, Z. Wąs.
Reference to the original program: Comput. Phys. Commun. **66** (1991) 115.
Computer: IBM 3090, VAX; *Installation:* CERN
Operating system (on which the new version has been tested): VM/CMS, VMS, ULTRIX, UNIX
Programing language used: FORTRAN 77
No. of bits in a word: 32
Peripherals used: Line printer.
No. of cards in combined program and test deck: 2495
Keywords: radiative corrections, heavy leptons, resonances, particle decays, Monte Carlo simulation, quantum electrodynamics, leading-log approximations.
Nature of physical problem: Most of the decay modes of particles or resonances are accompanied by the QED real photon corrections. The presence of (a) hard photon(s) may affect the shape of the measured distributions, detection efficiencies and/or experimental selection criteria.
Method of solution: The Monte Carlo simulation is best suited for studying the combined process of production and decay of unstable particles, including QED corrections and detector efficiencies.
Restrictions on the complexity of the problem: The QED $O(\alpha)$ and $O(\alpha^2)$ corrections are implemented in the leading-logarithm approximation with the proper soft photon behaviour. Emission of non-collinear hard photons is simulated, but the validity of distribution may be restricted.
Typical running time: 500 events/sec (VAX station 4000-90) for the $Z \to \mu^+\mu^-$ decay mode.

1 Introduction

The aim of this paper is to present an update of the Monte Carlo utility program PHOTOS [1], which implements an algorithm for QED bremsstrahlung. QED (bremsstrahlung) corrections are well known (see eg. [2, 3]) to affect measurements of decay properties. Hard photons may affect selection criteria and, indirectly, measured branching ratios or be background for some processes. Knowledge of the kinematics of such bremsstrahlung in decay configurations may also be important in studies for the development of dedicated detectors, e.g. for the LHC or SSC [4]. The complete answer on the size of the bremsstrahlung correction can be obtained from a full Monte Carlo study, which takes into account all classes of QED bremsstrahlung, that is bremsstrahlung in production, bremsstrahlung in decays and also detector properties.

Monte Carlo programs including radiative corrections in both production and decay are not always available. Attempts to introduce bremsstrahlung corrections in decays, will in general require the construction of programs including specific hadron-structure-dependent matrix elements for each decay channel of any particle or resonance. Furthermore, in the case of the cascade decays it may be necessary to control the QED bremsstrahlung from different sources simultaneously. All these complications may lead to the situation where a complete matrix element treatment is very difficult or even impossible. This treatment may not be necessary if the required experimental precision is not very high.

As in the previous version, PHOTOS can be combined with any Monte Carlo program generating decays. The only requirement is that the particle codes and the output common block format as defined by the Particle Data Group [5, 6] have to be provided by the host program.

The outline of the paper is the following. In section 2 we recall the basic elements of QED allowing for the construction of a universal algorithm for the generation of bremsstrahlung in decays. In section 3 we describe the matrix element and phase-space parametrization of the decay. In section 4 we give a functional description of the algorithm used in our program for single photon radiation. Section 5 presents how this algorithm can be used for the double-bremsstrahlung generation. In section 6 we describe how bremsstrahlung is generated in the case of cascade decays. Section 7 is devoted to the discussion of the quality of approximations used in the algorithm. To this end comparisons with the results obtained from simulations with exact matrix element programs are presented. Section 8 concludes the paper. In Appendix A instructions on how to run the program is given. Appendix B presents the algorithm of the cascade decay scanning.

To ease reading, we have included in this paper parts of ref. [1], e.g. the description of the single photon algorithm. Necessary corrections of misprints and other updates of the text were of course introduced.

2 Structure of QED corrections

Let us recapitulate here the properties of QED corrections that are essential to the construction of our algorithm, and fortunately independent of a particular process.

The first property is that of the factorization of bremsstrahlung kernels[1]. If the photon is soft [9] or collinear to the charged particle [10], then the differential cross section for such regions of phase-space can be written as Born-level differential cross sections multiplied by the bremsstrahlung kernel.

The $O(\alpha)$ real photon corrections alone lead to infrared singularities. These infrared singularities are cancelled order by order by virtual corrections, which are infinite as well, but of opposite sign [9]. From the Lee-Nauenberg-Kinoshita theorem [11], we know that if a summation of virtual and bremsstrahlung corrections is performed, large mass logarithms[2] cancel for the inclusive quantities. Instead of calculating virtual corrections analytically, we can thus reconstruct them numerically, up to the leading logs, from the real photon corrections.

Finally, let us recall that higher order leading logarithmic corrections can be obtained by convolution of the first-order result. This leads to the concept of structure functions and evolution equations [12, 13].

3 Phase space parametrization and matrix element

In the following we will discuss the decay of particle P, $P(p) \to ch(q_1)Y_2(q_2)Y_3(q_3)...(\gamma)$. Here ch is the charged decay product "emitting" a bremsstrahlung photon γ. The Y_i denote all other decay products. Let us define the variables used for the phase-space parametrization:

- k is the photon energy in the P rest frame;
- θ_1 , ϕ_1 are the angles defining the Y direction in the P rest frame; Y is equal to Y_2 for the two-body decay modes and denotes the sum of all Y_i otherwise;
- θ_2, ϕ_2 are the angles defining the direction of the photon momentum with respect to ch in the rest frame of the $(ch, \sum_i Y_i)$ system (thus excluding the photon);
- for a two-body decay, this completes the parametrization of the phase space; in the case of a multi-body final state, parametrization of the phase space is completed with the introduction of M_Y, the invariant mass of the $\sum_i Y_i$ system, and of any set of variables defining the sub-phase space ω_Y of Y_i in the $\sum_i Y_i$ rest frame.

[1]This factorization, or rather semi-factorization, extends in some cases also to exact $O(\alpha)$ calculations. This was exploited in the algorithm of MUSTRAAL [7] for the muon pair production and later in Monte Carlo for Z and W decays into a pair of quarks and a gluon [8].

[2]We exclude vacuum polarization effects from our consideration. The bulk of these effects can usually be incorporated into the overall normalization of the branching ratio.

Let us start with the parametrization of the phase space for the Born cross section $P \to chY$. Using the Bjorken convention [14]:

$$d\sigma^{Born}(P \to chY) = |M_B|^2 \frac{1}{m_P 16(2\pi)^2} \frac{\lambda^{1/2}\Big(m_P^2, m_{ch}^2, M_Y^2\Big)}{m_P^2} d\cos\theta_1 d\phi_1, \tag{1}$$

where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. For two-particle final states, M_Y is the mass of $Y = Y_2$. For a multi-body final state, the phase space given by formula (1) is completed by the factor $dM_Y^2 d\omega_Y$, where $d\omega_Y$ is the "decay phase space" of the system Y and dM_Y^2 is the integral over its mass. According to the spinor normalization adopted in [14] for each final-state fermion, a factor $2m_f$ should be included in the phase-space expression. If there are any identical particles, the respective statistical factor should also be included.

The differential cross-section for the decay $P \to chY\gamma$ takes the following form:

$$\begin{aligned} d\sigma(P \to chY\gamma) &= |M|^2 \frac{1}{m_P 32(2\pi)^5} \frac{\lambda^{1/2}\Big(m_P^2(1-\frac{2k}{m_P}), m_{ch}^2, M_Y^2\Big)}{m_P^2(1-\frac{2k}{m_P})} \\ & \quad kdkd\cos\theta_1 d\phi_1 d\cos\theta_2 d\phi_2, \end{aligned} \tag{2}$$

with the photon energy constraint: $0 < k < k_{max} = \frac{m_P}{2} - \frac{(m_{ch}+M_Y)^2}{2m_P}$. Note also that for the multi-body final states the same extension of the phase space has to be performed as in the Born case.

We can approximate expression (2) by factorizing the matrix element squared $\frac{1}{2(2\pi)^3}|M|^2$ into the Born one $|M_B|^2$ and a bremsstrahlung factor $f(k, c_2, \phi_2)$, obtaining the formula for the *raw* cross section:

$$\begin{aligned} d\sigma^R\Big(P \to chY(\gamma)\Big) &= |M_B(\cos\theta_1,\phi_1)|^2 \frac{1}{m_P 16(2\pi)^2} f(k,\cos\theta_2,\phi_2) \\ & \quad \frac{\lambda^{1/2}\Big(m_P^2, m_{ch}^2, M_Y^2\Big)}{m_P^2} kdkd\cos\theta_1 d\phi_1 d\cos\theta_2 d\phi_2, \end{aligned} \tag{3}$$

which can be rewritten as

$$d\sigma^R(P \to ch\ Y(\gamma)) = d\sigma^B(P \to ch\ Y) f(k,\cos\theta_2,\phi_2) kdkd\cos\theta_2 d\phi_2. \tag{4}$$

This factorization is valid in the leading-log approximation as well as in the soft photon limit. The factor $f(k,\cos\theta_2,\phi_2)$ is a function given to O(α) in leading-logs. Neglecting for a while the soft photon limit and concentrating on the case of ch of spin $\frac{1}{2}$, we derive including also virtual corrections and Born cross section:

$$f(k,\cos\theta_2,\phi_2) \;=\; \frac{1}{4\pi m_P^2}\times$$
$$\left\{\delta(\frac{k}{m_P})\frac{m_P}{k}(1-\frac{\alpha}{\pi}N) \;+\; \Theta(k-\epsilon)\frac{\alpha}{\pi}(1+(1-\frac{k}{k_{max}})^2)\frac{m_P^2}{2k^2}\frac{2}{1-\cos\theta_2\sqrt{1-\frac{4m_{ch}^2m_P^2}{\left(m_P^2+m_{ch}^2\right)^2}}}\right\}, \tag{5}$$

with

$$N \;=\; \frac{1}{\sqrt{1-\frac{4m_{ch}^2m_P^2}{\left(m_P^2+m_{ch}^2\right)^2}}}\ln\left(\left(1+\sqrt{1-\frac{4m_{ch}^2m_P^2}{(m_P^2+m_{ch}^2)^2}}\right)^2\frac{(m_P^2+m_{ch}^2)^2}{4m_{ch}^2m_P^2}\right)$$
$$\left(\ln\frac{k_{max}}{\epsilon}-\frac{3}{4}+\frac{\epsilon}{k_{max}}-\frac{\epsilon^2}{4k_{max}^2}\right), \tag{6}$$

and ϵ denotes the soft photon cut-off. If the exact matrix element for a certain decay channel is known, we can correct for possible deviations due to the leading logarithm approximation used in eq. (3). The ratio between the exact expression (2) and the approximation (3) is given in the form of a weight:

$$W = \frac{|M|^2}{2(2\pi)^3|M_B|^2f(k,c_2,\phi_2)}J, \tag{7}$$

where the Jacobian factor (J) reads:

$$J = \frac{\lambda^{1/2}\left(1,\frac{m_{ch}^2}{m_P^2(1-\frac{2k}{m_P})},\frac{M_Y^2}{m_P^2(1-\frac{2k}{m_P})}\right)}{\lambda^{1/2}(1,\frac{m_{ch}^2}{m_P^2},\frac{M_Y^2}{m_P^2})}. \tag{8}$$

When $k=0$, $|M|^2$ should be understood as the sum of Born, virtual and, integrated up to energy ϵ, real photon contributions. The function $f(k,\cos\theta_2,\phi_2)$ was chosen such that $W\to 1$ in regions of phase space contributing to leading-logs (for ch of $\frac{1}{2}$ spin).

Let us now return to the discussion of formula (4). This expression forms the basis of the algorithm presented in this paper. For the generation of events, one can simply use the Born level algorithm and then supply it with the photon variables (k, θ_2, ϕ_2) generated according to the distributions (4)-(6). The leading-log content of these formulae is that the bremsstrahlung constitutes a process of fragmentation of the charged P decay product into itself and a photon, according to the Altarelli-Parisi-Lipatov splitting function. Note, however, that we control the approximate distributions we use. The generation of the photon variables depends only on the electromagnetic charge and spin of ch and does not depend on the properties of the particular process under consideration.

One has to be careful when applying this algorithm. It is assumed that the Born matrix element M_B is a slowly varying function of the phase space. Special care has to be taken in the case of the cascade decays. For example one should apply the algorithm presented here twice, for the $\tau^+ \to \rho^+\nu$ and $\rho^+ \to \pi^+\pi^0\nu$, rather than in one step to the $\tau^+ \to \pi^+\pi^0\nu$, because of the strong variation of the matrix element in the presence of the ρ resonance in the $\tau^+ \to \pi^+\pi^0\nu$ three-body phase space.

4 Single photon algorithm

In this section we will present an algorithm, which we use for simulation of single photon bremsstrahlung in the decay of particle P. As an input it uses the complete kinematic configuration of the P particle decay. Clearly, the decay of P may be obtained with the help of any package that reconstructs complete kinematics.

It is organized as follows:

(1) We calculate two angles, $\cos\theta_1$, ϕ_1 (defining the ch direction), and the masses of ch and spectator system $\sum_i Y_i$ from the four-momenta of the P decay products defined in the P rest frame.

(2) According to the distribution (5) we generate a bremsstrahlung photon energy k and if it is non-zero also the direction: $\cos\theta_2$, ϕ_2.

(3) If the generated photon energy is zero, we leave the original event unmodified and go to point (9) of the algorithm.

(4) The modified momentum q of the charged particle ch in the rest frame of the $(ch, \sum_i Y_i)$ system is obtained through the relation

$$q = \frac{\lambda^{1/2}\Big(m_P^2(1-\frac{2k}{m_P}), m_{ch}^2, M_Y^2\Big)}{2m_P\sqrt{1-\frac{2k}{m_P}}}. \tag{9}$$

(5) We rotate all four-momenta of P decay products to obtain the momentum of ch anti-parallel to the third axis of the reference frame.

(6) We boost the charged particle (and its subsequent decay products) along the third reference axis, reducing its momentum to $-q$. A second boost is performed, in which the resultant momentum of the Y system is reduced to q.

(7) We construct the photon 4-momentum from variables k, $\cos\theta_2$, ϕ_2. The complete event is constructed in terms of four-momenta in the rest frame of the decay particles, excluding the photon.

(8) Next, a series of Lorentz transformations is performed, which are the same for all P decay products. First, they are rotated in order to get the photon along the third

axis of the reference frame. Next, all particles are boosted to the P rest frame. The boost is followed by a rotation to get the resultant momentum of Y along the third axis [3]. A final rotation restores the original direction (as defined at the Born event) of Y.

(9) An iterative procedure is applied in the case where the charged multiplicity is greater than 1, that is more than one particle is able to radiate a photon. Since this algorithm treats only $O(\alpha)$ corrections, the *simultaneous* emission of photons from several charged decay products is *excluded*. Thus the iteration over charged particles is ended as soon as the first photon emission occurs or when no charged particles are left. In the latter case, PHOTOS leaves the event untouched. To compensate for the reduced probability of the program to reach further iterations of the algorithm, we accordingly increase the probability of the hard bremsstrahlung. We divide the raw hard photon probability (see formula (6)) by the probability of not generating a photon in all previous iterations.

(10) If the exact matrix element for a certain decay channel is known, we may correct for the possible deviations of our algorithm from the exact results, with the help of the weight (7). If an event is rejected, PHOTOS leaves the event untouched. Optionally, if the calculation of the weight, as is the case with the universal weight presented later in this section, does not require all information about the event, this rejection can be done just after point (8) of our algorithm.

A universal weight is defined in our program; it will be used instead of weight defined in formula (7). It is always smaller than 1, and we perform the rejection according to it after point (8) of the above algorithm. With the help of this rejection we correct the distribution in the soft photon region and for the differences between Altarelli-Parisi kernels for possible different spins, s, of ch.

This weight reads:

$$W = W_1 W_2 W_3, \tag{10}$$

where

$$\begin{aligned} W_1 &= \frac{1-\cos\theta_2\ \beta_0}{1-\cos\theta_2\ \beta_1}, \\ W_2 &= \left(1 - \frac{1-\beta_1^2}{G_s\left(1-\cos^2\theta_2\beta_1^2\right)}\right)\left(\frac{1}{2} + \frac{\cos\theta_2}{2}\beta_1\right) \end{aligned} \tag{11}$$

and for decay products of different spin, we use

$$W_3 = \begin{cases} \dfrac{1-\frac{k}{k_{max}}}{1-\frac{k}{k_{max}}+\frac{1}{2}\left(\frac{k}{k_{max}}\right)^2} & \text{for} \quad s=0 \\ 1 & \text{for} \quad s=\frac{1}{2} \\ \dfrac{1-\frac{k}{k_{max}}+\frac{1}{2}\left(\frac{k}{k_{max}}\right)^2-\frac{1}{2}\left(\frac{k}{k_{max}}\right)^3}{1-\frac{k}{k_{max}}+\frac{1}{2}\left(\frac{k}{k_{max}}\right)^2} & \text{for} \quad s>\frac{1}{2} \end{cases} \tag{12}$$

[3] In addition to the transformation sequence described above, we make two rotations to assure that e.g. the azimuth of the Y constituents, with respect to the Y-axis, is preserved after the introduction of the bremsstrahlung corrections.

and

$$G_s = \begin{cases} 1 & \text{for} \quad s = 0 \\ \frac{1}{2}\Big(1 - \frac{k}{k_{max}} + \frac{1}{1-\frac{k}{k_{max}}}\Big) & \text{for} \quad s = \frac{1}{2} \\ 1 & \text{for} \quad s > \frac{1}{2} \end{cases} \tag{13}$$

where we define

$$\beta_1 = \sqrt{1 - \frac{4m_{ch}^2(1 - \frac{2k}{m_P})}{m_P^2\Big(1 - \frac{2k}{m_P} + \frac{m_{ch}^2 - M_Y^2}{m_P^2}\Big)^2}} \tag{14}$$

and

$$\beta_0 = \sqrt{1 - \frac{4{m_{ch}}^2 m_P^2}{(m_P^2 + {m_{ch}}^2)^2}}. \tag{15}$$

Let us now briefly explain the components of the weight W. The first two components, W_1 and W_2, correct our original distribution in the soft photon region. Effectively, in this region the matrix element used in the generation is

$$M = M_B Q_1 \Big(\frac{q_1 \cdot \varepsilon}{q_1 \cdot k} - \frac{p \cdot \varepsilon}{p \cdot k}\Big), \tag{16}$$

where Q_1 denotes the charge of the charged decay product ch, the photon polarization is denoted by ε and an appropriate summation over ε is assumed in the calculation of the cross sections. This formula is valid also in the case of a non-relativistic ch. When the algorithm is used for multi-charged final states, an additional weight (calculated from the four-momenta defined in the P rest-frame) has to be introduced in order to ensure a proper infrared behaviour (this weight is not necessarily smaller than 1):

$$W_{multi} = \frac{\sum_\varepsilon \Big| Q_1 \frac{q_1 \cdot \varepsilon}{q_1 \cdot k} + Q_2 \frac{q_2 \cdot \varepsilon}{q_2 \cdot k} + \ldots \Big|^2}{\sum_\varepsilon Q_1^2 \Big| \frac{q_1 \cdot \varepsilon}{q_1 \cdot k} \Big|^2 + Q_2^2 \Big| \frac{q_2 \cdot \varepsilon}{q_2 \cdot k} \Big|^2 + \ldots}, \tag{17}$$

where Q_1, Q_2,..., denote the charges of the decay products of P [4].

This weight (17) is not included in the program, except in the case of the two-body particle-antiparticle decay. In this case W_{multi}, calculated from the internal variables of the program, is[5]:

$$W_{multi} = \frac{2}{1 + \beta_1^2 \cos^2\theta_2}. \tag{18}$$

The weight W_3 accounts for the difference between the Altarelli-Parisi kernel for a spin $\frac{1}{2}$ particle and the kernels for the other spins.

Further improvements, in general, can be performed only for the separate modes and they require inclusion of the corresponding $O(\alpha)$ matrix elements. In the Monte Carlo it can be realized with the help of rejections (point (10) of our algorithm).

[4] In the calculation of this weight, the use of `REAL*8` variables in kinematics may be important if a ratio of the mother particle mass to the mass of the charged decay product is large.

[5] This weight is activated by the `INTERF =.TRUE.` flag, see Appendix A.

5 Double-bremsstrahlung generation

The master formula for the cross section with the bremsstrahlung corrections in the leading-log approximations reads:

$$d\sigma_{LL}(P \to chY_2Y_3...\gamma\gamma...) = dxD(x,\beta_{ch})d\sigma^B(P \to chY_2Y_3...), \tag{19}$$

where x denotes the fraction of the original energy that is carried by the decay product ch after emission of the photon(s), $\beta_{ch} = \frac{\alpha}{\pi}\ln\frac{m_P^2}{4m_{ch}^2}$. The result of the QED perturbative leading-log calculation is located in the so-called structure functions $D(x,\beta_{ch})$, which in QED, contrary to the QCD case, can be calculated with an arbitrary precision (see refs. [15, 16, 17] for more discussions). The infinite order expression for non-singlet $D(x,\beta_{ch})$ reads

$$D(x,\beta_{ch}) = \delta(1-x) + \beta_{ch}P(x) + \frac{1}{2!}\beta_{ch}^2\{P\otimes P\}(x) + \frac{1}{3!}\beta_{ch}^3\{P\otimes P\otimes P\}(x) + \ldots, \tag{20}$$

where $P(x) = \delta(1-x)(\ln\varepsilon + 3/4) + \Theta(1-x-\varepsilon)\frac{1}{2}(1+x^2)/(1-x)$ and $\{P\otimes P\}(x) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(x - x_1x_2)P(x_1)P(x_2)$. The regulator ε has to be small but positive.

It is rather straightforward to realize that, after integrating over angles and neglecting non-leading terms, our bremsstrahlung factor (5) gives:

$$\bar{f}(x) = \int d\cos\theta_2 d\phi_2 f(k_{max}(1-x), \cos\theta_2, \phi_2)k_{max}^2(1-x) = \delta(1-x) + \beta_{ch}P(x). \tag{21}$$

Up to O(α^2), we can thus write a structure function as

$$D(x,\beta_{ch})|_{O(\frac{\alpha}{\pi})^2} = \frac{1}{2}\delta(1-x) + \frac{1}{2}\{\bar{f}\otimes\bar{f}\}(x). \tag{22}$$

This is the basis of our double bremsstrahlung algorithm. We invoke a single photon algorithm twice, but in 50% of the cases only. If a photon is generated in a first try, the kinematics of the event $chY\gamma$ is reconstructed. A second iteration of the algorithm treats the $chY\gamma$ event as an initial configuration. In this way, the convolution $\otimes$ is performed. A discussion of the quality of this approach will be presented elsewhere [18]. Let us note that similarly to the case of first order, part of the non-leading terms will be included as well. It is rather straightforward to extend the algorithm even further, up to O(α^n) leading-log, that is up to n real photons. In this case, numerical instabilities may become a problem.

For the double-bremsstrahlung generation, the infrared cut-off parameter must be treated with particular care. The emission of the second photon changes the kinematics of the first one. Therefore, the infrared cut-off should be sufficiently smaller than the desired photon energy range. It is different from the single-bremsstrahlung case, where if a photon is boosted from the P frame to the laboratory frame, the boost changes the photon energy, but the ratio of the energy of the photon to that of the particle P (after boost) never exceeds the ratio of the photon energy (before boost) to half of the P mass. In the case of the very hard second photon emission, this condition on the first photon energy may not always be fulfilled.

6 Extension to the cascade case

Algorithms presented in the previous two sections can be executed only if the decaying particle is in its rest-frame. This is not usually the case. In the typical application, an event (stored in the `HEPEVT` common block) has the form of a complicated cascade tree of particles decaying (mothers) into their products (daughters), which are often mothers of secondary decays. This cascade tree has to be scanned, every elementary decay has to be copied to the working buffer, and later boosted to its rest-frame. If the photon generation is successful, the event stored in the `HEPEVT` common block has to be modified accordingly.

The technical description of this algorithm is given in Appendix B.

7 Systematic error of the algorithm

There are two main classes of systematic uncertainties defining respectively the technical and physical precision of the results obtained with the help of the Monte Carlo.

The technical precision of the program can be limited by approximations used in the implementation of the algorithm in the FORTRAN code. A good example of the case of single photon generators is the well known bias due to the neglect of kinematic effects of photons of energy lower than a minimal value ϵ. The precision can also be limited by numerical effects due to the simple rounding errors or random number generators. Finally, there may be program bugs in the code. The usual method of checking that these kinds of problems are under control is to calculate certain distributions with the help of the Monte Carlo and compare them with the other, obtained from exactly the same physical assumptions, but using different methods, preferably analytical ones. For this purpose, extensive comparisons of PHOTOS with the MUSTRAAL, exact $O(\alpha)$ Monte Carlo for $e^+e^- \to \mu^+\mu^-(\gamma)$ process, were performed in ref. [1]. To ensure an identical physical input of the two programs, slight modifications of the weight (10) were introduced in PHOTOS. In MUSTRAAL, initial state bremmsstrahlung and QED interference were switched off. An excellent agreement at the level of sample of 10^6 events was found for differential distributions. In this way, we could convince ourself that the program generates results of our approximations with a precision not worse than 0.1%. Comparison of the results obtained from PHOTOS with the semi-analytical *leading-log* formulae for electron spectra in $\tau^\pm \to e^\pm\nu\bar{\nu}(\gamma)$ and $B^\pm \to D^0\nu e^\pm(\gamma)$ decays represents another important test of technical precision of PHOTOS [19].

In general, we do not address the question of the physical precison of the approximations used in our generator. Design of the program guarantees the correctness of the leading-log corrections and distributions in the soft photon region only. In many cases, this would be very bad. In fact, not better than complete neglect of QED corrections. On the basis of the design principles one can estimate that the physical uncertainty of our results for the decay of particle P into ch and Y_i is not smaller than $\frac{\alpha}{\pi}$ or $(\frac{\alpha}{\pi}\log\frac{m_P^2}{m_{ch}^2})^3$, whichever is bigger. This has to be compared with the generic size of the single photon effect, which is $\frac{\alpha}{\pi}\log\frac{m_P^2}{m_{ch}^2}$.

In the following we will address the question: How well can our algorithm reproduce complete $O(\alpha)$ or $O(\alpha^2)$ results? For this purpose we will calculate the size of the missing non-leading terms for some distributions and processes. We will concentrate mainly on the photon energy spectrum in the decaying particle rest-frame and on the angular distribution of the photon.

1. $Z \to \mu^+\mu^-(\gamma)$

   This case was studied carefully in the first paper on PHOTOS [1], where comparisons with the MUSTRAAL Monte Carlo [7] were presented. The photon energy spectrum was found to agree with the one obtained from the exact matrix element better than up to 5% in the photon energy range from zero to about 70% of the maximal energy allowed kinematically. Also, the test on the transverse photon energy leads to a similar level of agreement. Distribution in an angle between photon and μ^+ was found to be indistinguishable from the one obtained from the exact matrix element calculation. All these tests were performed on the 10^6 $Z \to \mu^+\mu^-(\gamma)$ sample. Such good agreement is found only if the interference weight (formula (18)) is included. Otherwise the agreement is much poorer.

2. $W^\pm \to e^\pm\nu(\gamma)$,

   For this channel, comparisons with the generator of ref. [20] were performed in ref. [21]. Results were very similar to the ones presented in the previous point: even discrepancies at the higher end of the photon energy spectrum have a similar shape. Interference weight was obviously not necessary.

3. $\tau^\pm \to e^\pm\nu\bar{\nu}(\gamma)$ and $B^\pm \to D^0\nu e^\pm(\gamma)$

   In ref. [19] the electron energy spectrum was studied. In the case of τ decay, the agreement with the analytical result of ref. [22] was excellent. No statistically meaningful difference was found for the sample of 5×10^6 events. In the case of $B^\pm$ decay, the agreement was poorer, the difference with the formula of ref. [23] was typically of the order of 10% of the QED effect.

4. $gg \to t\bar{t}(\gamma,\gamma\gamma)$

   PHOTOS has been used to simulate the single and double photon bremsstrahlung in the $gg \to t\bar{t}$ process. These results were compared [4] with the exact matrix element calculation. Agreement up to 10% was found. It was obtained for different distributions and complicated cuts. Note that in this case the leading logarithm was usually comparable to 1, thus non-leading at all.

The introduction of our universal weight (10) improved the agreement significantly in the above cases. This, in general, is not guaranteed, even if for the examples recalled above, the program works remarkably better than expectations. It is however always justified to claim that, for the particular new decay channel or distribution, discrepancies will be larger. The only answer in this case is to perform a comparison analogous to one of the presented above.

One should keep in mind that the hard photon emission, in general, may change spin states of the decay products. This affects distributions of the particles in the further part of the cascade. PHOTOS does not have means to correct for this effect, and spin correlations of the host program are preserved. This is justified in the soft photon as well as in the leading-log limits.

8 Conclusion

This paper presents PHOTOS, the Monte Carlo generator for QED corrections in decays. It has the following properties:

(1) It can be combined with any Monte Carlo for the simulation of decays, provided that the host program stores the output in a Standard Common Block as defined by the Particle Data Group.

(2) All phase space is covered by the generation.

(3) Leading-log (collinear) and infrared limits are properly reproduced. An infrared limit is properly simulated also in the case of non-relativistic charged decay products. In some cases, the interference contribution is however not included, see discussion in section 4.

(4) The program can be combined either with simple or with cascade decays.

The main improvements with respect to the previous version include:

1. Double bremsstrahlung option.

2. QED interference correction, but in the case of two-body particle-antiparticle decay channels only.

3. Improvements to ensure the proper soft photon limit in the case of non-ultrarelativistic charged decay products.

4. Changes in program structure, allowing for easy introduction of other modifications to the host program, such as real e^+e^- pair emission.

5. New tests on the program.

6. Option for bremsstrahlung in non-decay $gg(q\bar{q}) \to t\bar{t}$ branch.

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APPENDIX A

Here the basic instruction on how to run the program is given.

A.1 Initialization

Before any event is generated, the package has to be initialized by a call on routine `PHOINI`. Some of the parameters will be set by call on routine `PHOCIN`, and the printout will be produced by the routine `PHOINF`. Default parameter values are given in brackets [].

1. `PHLUN` *Output device number* [6]
2. `ALPHA` *QED coupling constant* [0.00729735039]
3. `XPHCUT` *infrared cut-off parameter* [0.01]
4. `ISEED(1), ISEED(2)` *Seeds for Marsaglia and Zaman random number generator*
5. `INTERF` *Interference weight (for two-body symmetric channels only) switch* [`.TRUE.`] `FINT` *maximum interference weight. Should be* `[2.0]` *if* `INTERF=.TRUE.` *and* `1.0` *otherwise.*
6. `ISEC` *Double bremsstrahlung switch* [`.TRUE.`]
7. `IFTOP` *Switch to activate program for* $gg(q\bar{q}) \to t\bar{t}$ process.[`TRUE.`]
8. `QEDRAD(I)` *radiation flag for particle* `I`. `[.TRUE.]`

These parameters can be later reinitialized before any call on routine `PHOTOS`.

A.2 Algorithm invocation

The execution of PHOTOS is invoked by the call: `CALL PHOTOS(IPPAR)`, where `IPPAR` is the pointer to the particle at position `ABS(IPPAR)` in the common `/HEPEVT/`. If `IPPAR` is negative, PHOTOS acts on `ABS(IPPAR)` decay only. If it is positive, the program acts also on the complete chain of daughters starting from the `IPPAR` position.

Since PHOTOS may be interfaced with any event generator that provides output through `/HEPEVT/`, a situation may occur in which the event generator already includes QED corrections. In order to avoid double counting, a Boolean array `QEDRAD` with the same length as `/HEPEVT/` has been introduced, with which particles may be flagged that are (not) allowed to radiate in PHOTOS. By default, after calling `PHOCIN`, all status flags are set `.TRUE.`, thus allowing each (charged) particle to radiate. Programs that apply exact QED matrix elements like KORALB [24], KORALZ [25] and TAUOLA [26], make use of this facility. Interfacing PHOTOS with other programs including QED corrections requires that the particles are properly (dis-)allowed to radiate.

The input/output common block (the standard HEP common block) is listed here (for a detailed description, see refs. [5, 6]):

```
C--   HEPEVT                        Standard high-energy physics (HEP)
C--                                 common definitions
      INTEGER NMXHEP
      PARAMETER (NMXHEP=2000)
      INTEGER IDHEP,ISTHEP,JDAHEP,JMOHEP,NEVHEP,NHEP
      REAL  PHEP,VHEP
      COMMON/HEPEVT/NEVHEP,NHEP,ISTHEP(NMXHEP),IDHEP(NMXHEP),
     1JMOHEP(2,NMXHEP),JDAHEP(2,NMXHEP),PHEP(5,NMXHEP),VHEP(4,NMXHEP)
```

The subroutine PHOTOS performs a scan over the decay cascade, it invokes the action of the rest of the program. Let us recall here the most important routines, their action, and the main (but not all) input-output variables.

A.2.1 Subroutine PHTYPE

It is used to invoke the action of the program on a particular elementary decay branch at the position `IP` in the `/HEPEVT/` common block. The input parameters are the pointer to the starting particle of the decay branch and generation flags defined in initialization, the output is the common `/HEPEVT/`.

A.2.2 Subroutine PHOIN

The purpose of this routine is to copy the selected decay branch from the common `/HEPEVT/` into the new common `/PHOEVT/` and if necessary to transform it into its c.m. system. The input includes the pointer to the particle that starts the decay branch `IP`. The output is the `/PHOEVT/` common and a flag `BOOST`, whether a boost to c.m. was or was not performed.

```
C--   PHOEVT                        Contains the PDG branch
      INTEGER NMXPHO
      PARAMETER (NMXPHO=2000)
      INTEGER IDPHO,ISTPHO,JDAPHO,JMOPHO,NEVPHO,NPHO
      REAL PPHO,VPHO
      COMMON/PHOEVT/NEVPHO,NPHO,ISTPHO(NMXPHO),IDPHO(NMXPHO),
     1JMOPHO(2,NMXPHO),JDAPHO(2,NMXPHO),PPHO(5,NMXPHO),VPHO(4,NMXPHO)
```

In this common, the meaning of the variables is the same as the corresponding ones in the `HEPEVT` common block.

A.2.3 Subroutine PHOOUT

This routine copies the selected decay branch back to the `/HEPEVT/` from the `/PHOEVT/` common block. The input is the pointer `IP` to the particle, which starts the decay branch and a flag BOOST to show if the boost to c.m. was performed. The output is the common /HEPEVT/, with the new particles added at the end.

A.2.4 Subroutine PHOCHK

This checks whether a branch IP copied into the common block /PHOEVT/ fulfils preconditions for PHOTOS operation. A radiation/non-radiation flag CHKIF(I) for each daughter of IP is defined. Corresponding QEDRAD(J) flag is used in CHKIF(I) definition.

A.2.5 Subroutine PHOBOS

The purpose of this routine is to correct the kinematics of all particles in the cascade sub-tree if the momentum of the sub-tree parent was modified by PHOTOS. The routine performs the same operation also on the subsequent daughters. The input parameters are: the pointer of the particle that starts the chain to be boosted, the old 4-momentum of the parent (before PHOTOS action), its new 4-momentum, and the pointers to its first and last daughters. The output is the modified common block /HEPEVT/.

A.2.6 Subroutine PHOMAK

The single bremsstrahlung radiative corrections are generated in the decay of the IPPAR particle in the common block /HEPEVT/. This routine invokes an algorithm, as explained in section 4. By a call on PHOIN, the selected decay branch is copied into the common block /PHOEVT/ first. If a photon is generated, the kinematics of the event is constructed. In the interference case (INTERF=.TRUE.), the weight of the event is adjusted accordingly. In every case before copying back the decay branch into the /HEPEVT/, a rejection with weight (10) is performed. The input parameters are the pointer IPPAR to the decaying particle in the /HEPEVT/ common and the common block itself. The output is the common /HEPEVT/, if (a) photon(s) is (are) added.

A.2.7 Subroutine PHOTWO

Combines two mothers into one in the case of $gg(q\bar{q}) \rightarrow t\bar{t}$. It acts on the /PHOEVT/ common block.

A.2.8 Further Subroutines

Other routines of PHOTOS are not presented here. They are either less important for the understanding of the program operation or self-explanatory, like PHINT calculating interference weight.

APPENDIX B

In the following, we will give a technical description of the program operation invoked by a call on subroutine PHOTOS(IPPAR). The following set of instructions will be executed:

1. If IPPAR is positive, perform a scan over the whole cascade sub-tree, starting at position IPPAR, to find all the branching points ISTACK(KK). If IPPAR is negative only the ISTACK(0)= | IPPAR | branching point will be registered.

2. Copy the ISTACK(KK) elementary cascade branch, i.e. the mother of this position and daughters from the HEPEVT common block into the PHOTOS local common block PHOEVT.

3. Boost the corresponding cascade branch into the rest-frame of the mother.

4. Check whether the coresponding reaction branch is a proper decay. Except for special cases, mother and all dauhters should be particles, or resonances. For instance, they should not be strings, quarks, gluons or other objects used to decribe hadronic interactions. Then, QED radiation cannot be treated independently from those phenomena. Also daughters of the mother should have their first mother pointers directed back. Sometimes it is not the case. It may also occur that the host program was already generating hard bremsstrahlung in the particular decay channel: PHOTOS has to be desactivated in this case. For this purpose QEDRAD flags are used.

   An exception is invoked if IFTOP flag is set to .TRUE.. In this case, for the processes $gg(q\bar{q}) \to t\bar{t}$, the single mother of the 4-momentum being a sum of gg $(q\bar{q})$ 4-momenta is formed and the photon radiation algorithm is activated.

5. If all conditions are met, apply the generation algorithm described in sections 4 and 5. It is rather easy to see that any other generation algorithm can be called at this moment[6].

6. If new particles are added: boost back the decay branch to the laboratory system and write back daughters from PHOEVT common block into HEPEVT common block. Add newly generated daughters at the end of the HEPEVT common block.

7. If necessary, adjust the momenta of the particles in cascade branches starting from daughters whose momenta may have been modified by the action described in the previous steps.

8. Repeat all steps, starting from the second point for the next ISTACK(KK+1) branching point until exhausting all of them.

[6]We have checked it with the algorithm for the e^+e^- real pair emission [27]. In principle, any exact matrix element generator for the particular decay branch can be interfaced in this way, if only the final state of this generator includes original daughters as a subset of its own decay products. In this case only an overall branching ratio would have to be adjusted in the host program.

9. Reorder whole HEPEVT. Move new daughters to the proper places.

It should be noted that one has to be very careful while using this algorithm. The HEPEVT common block is filled by the different host generators not in a unique way. A typical problem occurs if, for some internal reasons of the host generators the daughter pointers of a mother point to the daughters whose mother pointers do not point to the original mother. A difficulty may arise also if there are many-mother branchings in the cascade, due for instance to simulation of the hadronic interaction in the decay of heavy-flavour resonances. However, so far the case did not arise.

TEST PROGRAM

A small test job has been prepared for PHOTOS. It consists of a main program and a routine that provides a dump of the event, before and after PHOTOS operation. For generation of primary event JETSET [28] (in fact its later update, see below) was used.

Our test program was not modified and is as in ref. [1]. Note, however, that the test output may change with the future updates or different random number initialization in JETSET.

```
********************************************************************************
*                                                                              *
*                           =========================                          *
*                             PHOTOS, Version:  2. 0                           *
*                             Released at:  16/10/93                           *
*                           =========================                          *
*                                                                              *
*                  PHOTOS QED Corrections in Particle Decays                   *
*                                                                              *
*         Monte Carlo Program - by E. Barberio, B. van Eijk and Z. Was         *
*         From version 2.0 on - by E.B. and Z.W.                               *
*                                                                              *
********************************************************************************
*                                                                              *
*                  Internal input parameters:                                  *
*                                                                              *
*                  INTERF= T  ISEC= T  IFTOP= T                                *
*                  ALPHA_QED=  .00730   XPHCUT=  .01000                        *
*                                                                              *
*                  option with interference is active                          *
*                  option with double photons is active                        *
*                  emision in t tbar production is active                      *
*                                                                              *
********************************************************************************
                   The Lund Monte Carlo - JETSET version 7.3
                   **  Last date of change:  20 May 1992  **
0===============================================================================
                             Event No.:         1
0      Particle Parameters
0 Nr   Type   Parent(s)  Daughter(s)     Px        Py        Pz         E    Inv. M.
    1     4      0          3 -    3    37.24     21.82     14.34     45.50     1.35
    2    -4      0          3 -    3   -37.24    -21.82    -14.34     45.50     1.35
    3    94   1 -    2      4 -    5      .00       .00       .00     91.00    91.00
    4     4      3          6 -    7    37.00     21.69     14.25     45.29     3.06
    5    -4      3          8 -    9   -37.00    -21.69    -14.25     45.71     6.85
    6     4      4         16 -   16    36.10     21.05     13.41     43.91     1.35
    7    21      4         17 -   17      .90       .63       .85      1.39      .00
```

8	-4	5	10 - 11	-36.51	-21.93	-14.42	45.14	3.95
9	21	5	18 - 18	-.48	.24	.16	.57	.00
10	-4	8	21 - 21	-15.36	-9.46	-4.99	18.77	1.35
11	21	8	12 - 13	-21.15	-12.47	-9.42	26.37	1.94
12	21	11	14 - 15	-16.24	-9.85	-6.67	20.16	1.10
13	21	11	19 - 19	-4.92	-2.62	-2.76	6.21	.00
14	2	12	22 - 22	-5.30	-3.79	-2.47	6.97	.01
15	-2	12	20 - 20	-10.94	-6.07	-4.20	13.19	.01
16	4	6	25 - 25	36.10	21.06	13.41	43.91	1.35
17	21	7	25 - 25	.90	.63	.85	1.39	.00
18	21	9	25 - 25	-.48	.24	.16	.57	.00
19	21	13	25 - 25	-4.92	-2.62	-2.76	6.21	.00
20	-2	15	25 - 25	-10.94	-6.07	-4.20	13.19	.01
21	-4	10	23 - 23	-15.36	-9.46	-4.99	18.77	1.35
22	2	14	23 - 23	-5.30	-3.79	-2.47	6.97	.01
23	91	21 - 22	24 - 24	-20.66	-13.25	-7.46	25.73	2.05
24	-423	23	39 - 40	-20.66	-13.25	-7.46	25.73	2.01
25	92	16 - 20	26 - 38	20.66	13.25	7.46	65.27	60.02
26	413	25	41 - 42	29.48	17.21	11.54	36.09	2.01
27	-213	25	43 - 44	3.58	1.70	.92	4.10	.48
28	2212	25	Stable	2.12	2.08	1.28	3.37	.94
29	-2112	25	Stable	.00	-.22	-.37	1.03	.94
30	-211	25	Stable	.38	.22	-.01	.46	.14
31	211	25	Stable	-.08	.58	.96	1.13	.14
32	223	25	45 - 46	-.81	-1.02	-1.37	2.05	.78
33	111	25	47 - 48	-2.52	-1.15	-.91	2.92	.14
34	-213	25	49 - 50	-2.53	-1.27	-.50	3.00	.88
35	113	25	51 - 52	-3.50	-1.69	-1.48	4.23	.74
36	211	25	Stable	-2.89	-1.66	-1.03	3.49	.14
37	111	25	53 - 54	-1.39	-.66	-.95	1.81	.14
38	-211	25	Stable	-1.19	-.86	-.61	1.60	.14
39	-421	24	55 - 56	-19.43	-12.50	-7.01	24.22	1.86
40	111	24	57 - 58	-1.23	-.75	-.45	1.51	.14
41	411	26	59 - 61	25.90	15.11	10.05	31.68	1.87
42	22	26	Stable	3.58	2.10	1.49	4.41	.00
43	-211	27	Stable	1.57	.58	.52	1.76	.14
44	111	27	62 - 63	2.01	1.12	.40	2.34	.14
45	-211	32	Stable	-.60	-1.09	-1.04	1.62	.14
46	211	32	Stable	-.21	.06	-.34	.43	.14
47	22	33	Stable	-.95	-.48	-.30	1.11	.00
48	22	33	Stable	-1.57	-.67	-.61	1.81	.00
49	-211	34	Stable	-1.79	-.45	-.24	1.86	.14
50	111	34	64 - 65	-.74	-.82	-.26	1.14	.14
51	211	35	Stable	-2.02	-1.32	-1.03	2.63	.14
52	-211	35	Stable	-1.48	-.37	-.45	1.60	.14
53	22	37	Stable	-.51	-.28	-.43	.73	.00
54	22	37	Stable	-.87	-.38	-.52	1.08	.00
55	323	39	66 - 67	-15.57	-10.39	-5.40	19.50	.95
56	-321	39	Stable	-3.86	-2.11	-1.62	4.72	.49
57	22	40	Stable	-.73	-.47	-.34	.93	.00
58	22	40	Stable	-.49	-.28	-.11	.58	.00
59	-13	41	Stable	3.51	1.78	1.20	4.12	.11
60	14	41	Stable	13.92	7.60	5.69	16.85	.00
61	-311	41	68 - 68	8.46	5.73	3.17	10.71	.50
62	22	44	Stable	1.58	.93	.35	1.87	.00
63	22	44	Stable	.43	.19	.05	.47	.00
64	22	50	Stable	-.30	-.27	-.14	.43	.00
65	22	50	Stable	-.44	-.55	-.12	.71	.00
66	311	55	69 - 69	-9.44	-6.68	-3.41	12.07	.50
67	211	55	Stable	-6.13	-3.71	-1.99	7.44	.14
68	310	61	70 - 71	8.46	5.73	3.17	10.71	.50
69	130	66	Stable	-9.44	-6.68	-3.41	12.07	.50
70	211	68	Stable	2.46	1.69	.74	3.08	.14
71	-211	68	Stable	6.00	4.04	2.43	7.63	.14
0			Vector Sum:	.00	.00	.00	91.00	91.00

```
0==============================================================================
                          === after PHOTOS: ===
0==============================================================================
                             Event No.:        1
0      Particle Parameters
0 Nr   Type   Parent(s)  Daughter(s)     Px        Py        Pz        E     Inv. M.
    1     4       0         3 -    3    37.24     21.82     14.34     45.50     1.35
    2    -4       0         3 -    3   -37.24    -21.82    -14.34     45.50     1.35
    3    94    1 -    2     4 -    5      .00       .00       .00     91.00    91.00
    4     4       3         6 -    7    37.00     21.69     14.25     45.29     3.06
    5    -4       3         8 -    9   -37.00    -21.69    -14.25     45.71     6.85
    6     4       4        16 -   16    36.10     21.05     13.41     43.91     1.35
    7    21       4        17 -   17      .90       .63       .85      1.39      .00
    8    -4       5        10 -   11   -36.51    -21.93    -14.42     45.14     3.95
    9    21       5        18 -   18     -.48       .24       .16       .57      .00
   10    -4       8        21 -   21   -15.36     -9.46     -4.99     18.77     1.35
   11    21       8        12 -   13   -21.15    -12.47     -9.42     26.37     1.94
   12    21      11        14 -   15   -16.24     -9.85     -6.67     20.16     1.10
   13    21      11        19 -   19    -4.92     -2.62     -2.76      6.21      .00
   14     2      12        22 -   22    -5.30     -3.79     -2.47      6.97      .01
   15    -2      12        20 -   20   -10.94     -6.07     -4.20     13.19      .01
   16     4       6        25 -   25    36.10     21.06     13.41     43.91     1.35
   17    21       7        25 -   25      .90       .63       .85      1.39      .00
   18    21       9        25 -   25     -.48       .24       .16       .57      .00
   19    21      13        25 -   25    -4.92     -2.62     -2.76      6.21      .00
   20    -2      15        25 -   25   -10.94     -6.07     -4.20     13.19      .01
   21    -4      10        23 -   23   -15.36     -9.46     -4.99     18.77     1.35
   22     2      14        23 -   23    -5.30     -3.79     -2.47      6.97      .01
   23    91   21 -   22    24 -   24   -20.66    -13.25     -7.46     25.73     2.05
   24  -423      23        39 -   40   -20.66    -13.25     -7.46     25.73     2.01
   25    92   16 -   20    26 -   38    20.66     13.25      7.46     65.27    60.02
   26   413      25        41 -   42    29.48     17.21     11.54     36.09     2.01
   27  -213      25        43 -   44     3.58      1.70       .92      4.10      .48
   28  2212      25         Stable       2.12      2.08      1.28      3.37      .94
   29 -2112      25         Stable        .00      -.22      -.37      1.03      .94
   30  -211      25         Stable        .38       .22      -.01       .46      .14
   31   211      25         Stable       -.08       .58       .96      1.13      .14
   32   223      25        45 -   47     -.81     -1.02     -1.37      2.05      .78
   33   111      25        48 -   49    -2.52     -1.15      -.91      2.92      .14
   34  -213      25        50 -   51    -2.53     -1.27      -.50      3.00      .88
   35   113      25        52 -   53    -3.50     -1.69     -1.48      4.23      .74
   36   211      25         Stable      -2.89     -1.66     -1.03      3.49      .14
   37   111      25        54 -   55    -1.39      -.66      -.95      1.81      .14
   38  -211      25         Stable      -1.19      -.86      -.61      1.60      .14
   39  -421      24        56 -   57   -19.43    -12.50     -7.01     24.22     1.86
   40   111      24        58 -   59    -1.23      -.75      -.45      1.51      .14
   41   411      26        60 -   62    25.90     15.11     10.05     31.68     1.87
   42    22      26         Stable       3.58      2.10      1.49      4.41      .00
   43  -211      27         Stable       1.57       .58       .52      1.76      .14
   44   111      27        63 -   64     2.01      1.12       .40      2.34      .14
   45  -211      32         Stable       -.59     -1.08     -1.03      1.62      .14
   46   211      32         Stable       -.21       .05      -.33       .42      .14
   47    22      32         Stable        .00       .01      -.01       .01      .00
   48    22      33         Stable       -.95      -.48      -.30      1.11      .00
   49    22      33         Stable      -1.57      -.67      -.61      1.81      .00
   50  -211      34         Stable      -1.79      -.45      -.24      1.86      .14
   51   111      34        65 -   66     -.74      -.82      -.26      1.14      .14
   52   211      35         Stable      -2.02     -1.32     -1.03      2.63      .14
   53  -211      35         Stable      -1.48      -.37      -.45      1.60      .14
   54    22      37         Stable       -.51      -.28      -.43       .73      .00
   55    22      37         Stable       -.87      -.38      -.52      1.08      .00
   56   323      39        67 -   68   -15.57    -10.39     -5.40     19.50      .95
   57  -321      39         Stable      -3.86     -2.11     -1.62      4.72      .49
   58    22      40         Stable       -.73      -.47      -.34       .93      .00
   59    22      40         Stable       -.49      -.28      -.11       .58      .00
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   60    -13     41         Stable       3.51      1.78      1.20      4.12       .11
   61     14     41         Stable      13.92      7.60      5.69     16.85       .00
   62   -311     41       69 -   69      8.46      5.73      3.17     10.71       .50
   63     22     44         Stable       1.58       .93       .35      1.87       .00
   64     22     44         Stable        .43       .19       .05       .47       .00
   65     22     51         Stable       -.30      -.27      -.14       .43       .00
   66     22     51         Stable       -.44      -.55      -.12       .71       .00
   67    311     56       70 -   70     -9.44     -6.68     -3.41     12.07       .50
   68    211     56         Stable      -6.13     -3.71     -1.99      7.44       .14
   69    310     62       71 -   72      8.46      5.73      3.17     10.71       .50
   70    130     67         Stable      -9.44     -6.68     -3.41     12.07       .50
   71    211     69         Stable       2.46      1.69       .74      3.08       .14
   72   -211     69         Stable       6.00      4.04      2.43      7.63       .14
0                       Vector Sum:       .00       .00       .00     91.00     91.00
 *********************************************************************************
 **** PHOTOS Test Run has successfully ended                                  ****
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