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Analytical O(α) distributions for Bhabha scattering at low angles*

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Abstract

We calculate analytically in QED the O(α) corrected total cross section and selected distributions for low angle Bhabha scattering with semi-realistic cuts at LEP/SLC energies. In the angular range from 0.1^0 to 10^0 the total analytical cross-section agrees with the Monte-Carlo result to within 0.03% and the differential distributions better than 1%. This is the first test of the O(α) calculations (Monte Carlo versus analytical) for low angle Bhabha at this the precision level. Our calculation provides a solid basis, i.e. *baseline calculation*, for future discussion of the O(α^2) corrections, exponentiation and other higher order summation techniques at the overall QED precision tag of 0.2% for this process.

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In LEP/SLC experiments the luminosity and therefore the normalization of all measured cross sections are determined by measuring experimentally and calculating from the theory the pure QED process of e^+e^- scattering at low angles, the so-called Bhabha process. The scattering angles should be small enough such that t-channel photon exchange dominates over the resonant s-channel Z production. The essential part of the procedure is the theoretical calculation of the QED cross section for a given experimental luminosity measurement set-up. The calculation has to include the effects due to QED bremsstrahlung which are quite non-trivial to calculate if the precision requirement is pushed below the 1% limit (for the integrated cross section). The main difficulty is in the calculation of the real hard photon emission contribution for the t-channel photon exchange in the presence of the realistic experimental cut-offs. In such a case the reliable Monte Carlo or analytical calculation is more difficult than for the analogous s-channel annihilation process. There exist a number of $\mathrm{O}(\alpha)$ calculations which are probably sufficient at the 1% precision level [1, 2, 3]. The precision of the experimental luminosity measurement is recently going below that level [4]. There is therefore an urgent need to improve on the precision of the theoretical calculation; this practically means that the QED calculation beyond the $\mathrm{O}(\alpha)$ has to be done. Depending on the magnitude of the $\mathrm{O}(\alpha^2)$ correction this contribution may be either included in the Monte-Carlo or merely serve as a basis for estimation of the systematic QED error.

Although the QED $\mathrm{O}(\alpha)$ calculation[1] gives a unique answer for the total cross section and other measurable distributions[2], on the technical level, any numerical calculation has a limited "technical precision" and it is necessary to prove that the existing $\mathrm{O}(\alpha)$ calculations are actually "technically" correct down to 0.1% level or better[3]. Such proof will be indispensable when the $\mathrm{O}(\alpha^2)$ result becomes available and one will have to inspect the differences $\mathrm{O}(\alpha^2)-\mathrm{O}(\alpha^1)$ and understand them perfectly at the level 0.1% for the total cross section and at approximately the 1% level for the differential distributions. This difference will also play a role as a calculation of the QED *theoretical uncertainty* in the luminosity measurement.

How does one check the technical correctness of the $\mathrm{O}(\alpha)$ calculations? One has to compare two or more *independent* calculations. The comparison of two Monte Carlo programs or two analytical calculations (with possible Gauss integrations) may have common biases. A much more powerful cross check which verifies *technical precision* is the comparison of the Monte Carlo calculation with the analytical one. Documented tests of technical precision below 1% level (for various processes and measurables) based on such a comparison can be found in the recent papers [5, 6, 7, 8, 9]. In this paper we also present such a test. We use a modified/improved version

[1]We refere to an $\mathrm{O}(\alpha)$ calculation as defined in the standard textbooks.

[2]The users of the Monte Carlo event generators often get a false impression that the results of $\mathrm{O}(\alpha)$ calculation are not unique because the numerical results may depend on the infrared cut-off or other "dummy" parameters in the computer program.

[3]The calculation may be technically imprecise due to machine rounding errors, correlated random numbers, apparently unimportant approximation, bugs in the programs, etc.

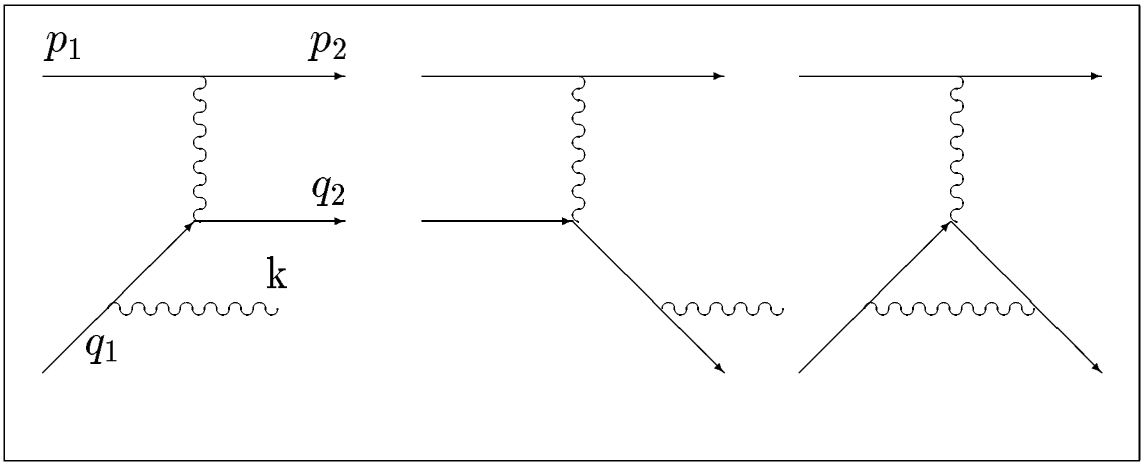

Figure 1: *Noninterference Feynman diagrams, emission from lower line.*

of the existing MC program OLDBAB [2] and the main effort is invested into an analytical calculation. To our knowledge there is no published comparison of this type for the process of low angle Bhabha scattering with semi-realistic cut-offs at the 0.1% precision level. Let us note that the authors of ref. [2] have checked their Monte Carlo [10] against the older non-Monte-Carlo exact $O(\alpha)$ calculations of refs. [11, 12] but they did not specify the resulting estimate of the *technical precision* for their Monte Carlo[4]. Let us also stress that our analytical formulas are extremely compact as compared with the older semi-analytical calculations [12]. The more recent semi-analytical calculation of ref. [13] is aimed at the wide angle Bhabha and a precision of 0.5% in the total cross section is claimed[5]. Let us finally stress that due to the complicated experimental acceptance the Monte Carlo will always be the ultimate tool for the calculation of QED effects in the Bhabha process. Our analytical calculation will be mainly used to test the Monte Carlo calculation, in particular, to test its most important aspect – the overall normalization.

Let us define the framework of our calculation. We drop any contributions which are irrelevant to our main concern - the real hard photon emission. We neglect completely s-channel exchange and the vacuum polarization correction, see fig. 1. In all calculations we shall require both outgoing electrons to be within the trigger limits $\vartheta_{min} < \vartheta_{1,2} < \vartheta_{max}$ (so-called symmetric cuts) where the electron angles $\vartheta_1 \angle(\vec{p_1}, \vec{p_2})$ and $\vartheta_2 \angle(\vec{q_1}, \vec{q_2})$ are defined in the laboratory frame. We often use variable $\xi = (1 - \cos\vartheta)/2$ instead of angle ϑ and we define the angular range by fixing $R_\xi = \xi_{max}/\xi_{min} - 1$. We omit the QED interferences between photon emission from the upper electron line and the lower positron line, which we call *up-down* QED interference, i.e. t-channel boxes and the corresponding real bremsstrahlung interferences. In the case of no cut-offs this type of QED interference is known to be very small, i.e. of $O(m_e^2/t)$, see ref. [14]. In the presence of the experimental

[4]At the time precision below 1% was not of much interest.

[5]The discussion of technical precision in this paper is not very specific.

ϑ_{min}[deg .]	σ	$\sigma_{noninterf}$	$(\sigma-\sigma_{noninterf})/\sigma$	$\Delta\sigma/\sigma$
9	2.14096	2.13908	0.088%	0.018%
3	19.6656	19.6634	0.011%	0.017%
1	179.978	179.975	0.001%	0.015%
1/3	1646.09	1646.09	0.000%	0.013%
1/9	15057.9	15057.9	0.000%	0.011%

Table 1: *Up-down QED interference effects at $s^{1/2}=92GeV$. Photon energy range is $0<k<0.9999$. Electrons are in the (symmetric) angular range $\vartheta_{min}<\vartheta_{1,2}<\vartheta_{max}$ where $\vartheta_{min}=9^0$ and $R_\xi=1$ (i.e., $\vartheta_{max}\simeq 2^{1/2}\vartheta_{min}$). Statistical Monte Carlo error is $\Delta\sigma$. Vacuum polarization and s-channel exchange are excluded. Monte Carlo samples of $2\cdot 10^7$ events were used.*

cuts, for instance for scattering angles ϑ_i of both electrons kept in a definite range, we resort to one of the existing O(α) M.C. to check if it is still true. In the test we have used the M.C. program OLDBAB of ref. [2]. For the purpose of the test we had to modify this program, see later discussion of numerical results for more details. The result of this numerical exercise is shown in Table 1. The interference contribution in question is below the 0.1% level[6] for scattering angles up to 9^0. In our case the inclusion of the interferences would render our final analytical results nontransparent and lengthy. For the same reason our analytical calculation will not be the exact one but we shall systematically drop terms which contribute to the total cross section the *non-infrared* and *non-logarithmic* terms[7] of order $\frac{\alpha}{\pi}\vartheta^2$. They are completely negligible up to quite large angles, at the precision level 0.1%. The approximation simplifies substantially our final analytical results with negligible loss of precision[8]. Needless to say that the standard approximations $s>>m_e^2$ and $s\vartheta^2>>m_e^2$ are also used wherever possible.

Let us concentrate on the following O(α) corrected Bhabha differential cross section with pure photon t-channel exchange and no up-down interferences

$$\sigma \;=\; \frac{1}{2s}\Big[(1+2\beta_t\ln\varepsilon+\Delta_{soft})\int X_0\;\Theta_{trig}\;d\Phi_2(p_1+q_1;p_2,q_2,k)$$

[6]The above result is quite essential for this and and future works on the low angle Bhabha scattering. We also believe that it was not shown quantitatively before this work.

[7]Our calculation is not the leading-log type. We control subleading terms, see later discussion.

[8]With our calculational technique it is feasible to perform the exact analytical O(α) calculation, see further comments below.

$$+\int (X_1 + X_1')\ \Theta_{soft}\Theta_{trig}\ d\Phi_3(p_1+p_2;q_1,q_2,k)\ \Bigg],\qquad (1)$$

where

$$\begin{aligned}
X_0 &= 2e^4\ \frac{s^2+u^2}{t^2}, \quad s=2p_1q_1, \quad t=-2p_1p_2, \quad u=-2p_1q_2,\\
\beta_t &= 2\frac{\alpha}{\pi}\left(\ln\frac{|t|}{m_e^2}-1\right),\\
\Delta_{soft} &= \frac{3}{2}\beta_t + 2\frac{\alpha}{\pi}\left(-\frac{1}{2}-\frac{1}{3}\pi^2+\mathrm{Li}_2\frac{s}{|t|}+\ln\frac{|t|}{s}\ln\frac{t}{u}-\frac{1}{2}\ln^2\frac{|t|}{s}\right),\\
X_1 &= \frac{e^6}{(kp_1)(kp_2)}\left\{\frac{s^2+u_1^2}{|t_1|}\left(1-\frac{2m_e^2}{|t_1|}\frac{kp_1}{kp_2}\right)+\frac{s_1^2+u^2}{|t_1|}\left(1-\frac{2m_e^2}{|t_1|}\frac{kp_2}{kp_1}\right)\right\}, \qquad (2)\\
X_1' &= \frac{e^6}{(kq_1)(kq_2)}\left\{\frac{s^2+u_1^2}{|t|}\left(1-\frac{2m_e^2}{|t|}\frac{kq_1}{kq_2}\right)+\frac{s_1^2+u^2}{|t|}\left(1-\frac{2m_e^2}{|t|}\frac{kq_2}{kq_1}\right)\right\}, \qquad (3)\\
s_1 &= 2p_2q_2, \quad t_1=-2q_1q_2, \quad u_1=-2p_2q_1
\end{aligned}$$

and $\Theta_{soft} = \Theta(k^0 - \varepsilon\sqrt{s}/2)$ defines the energy upper bound on infrared soft photons in the laboratory system $\vec{p_1}+\vec{q_1}=\vec{0}$ (the limit $\varepsilon \to 0$ is always understood), and Θ_{trig} defines the (idealized) experimental trigger

$$\Theta_{trig} = \Theta(\vartheta_1-\vartheta_{min})\Theta(\vartheta_{max}-\vartheta_1)\ \Theta(\vartheta_2-\vartheta_{min})\Theta(\vartheta_{max}-\vartheta_2). \qquad (4)$$

The phase space is $d\Phi_n(P;p_1\dots p_n) = (2\pi)^4\delta^4(P-\sum_{i=1}^{n}p_i)\prod_{i=1}^{n}d^3p_i/(2\pi)^3 2p_i^0$.

Let us describe very briefly the integration technique for the hard photon phase space, see ref. [15] for more details. We parametrize the hard photon phase space and the matrix element with light-cone, Sudakov-type, variables introduced in the LESKOF and BHLUMI Monte Carlo programs [3, 16]: $y = 2kq_2/(|t|+2kq_2)$ and $z = 2kq_1/(|t|+2kq_2)$. For the emission from the lower line, see fig. 1, we obtain

$$\begin{aligned}
d\sigma_1' &= \Theta_{trig}X_1'\ d\Phi_3(p_1+q_1;p_2,q_2,k) = \frac{2\pi\alpha^2}{s}\ \Theta_{trig}Y\ \frac{\alpha}{\pi}\ \frac{sdt}{t^2}\ \frac{d\psi}{2\pi}\ dydz, \qquad (5)\\
Y &= \frac{s_1^2+u^2}{4s^2}\left(\frac{1}{(y+z\Delta)(z+y\Delta)}-\frac{2m_e^2}{|t|}\frac{1}{(z+y\Delta)^2}\right)\\
&+ \frac{s^2+u_1^2}{4s^2}\left(\frac{1}{(y+z\Delta)(z+y\Delta)}-\frac{2m_e^2}{|t|}\frac{1}{(y+z\Delta)^2}\right),
\end{aligned}$$

where $\Delta = m_e^2/|t_1|$ and ψ is the azimuthal angle of the photon around $\vec{p_1}-\vec{p_2}$ in the frame where $p_1^0 - p_2^0 = 0$. We fix the laboratory photon energy $k^0 = \frac{1}{2}k\sqrt{s}$ at the expense of the ψ angle

$$d\sigma_1' = \frac{4\pi\alpha^2}{s}\ \left(\frac{\alpha}{\pi^2}\right)\frac{\Theta(F(y,z))}{\sqrt{F(y,z)}}\ \Theta_{trig}\ Y\ \frac{sdt}{t^2}\ dkdydz, \qquad (6)$$

where $F(y,z) = -(1-k)^2 + 2(1-k)[(1-y)(1-z) + yz(1-|t|/(s(1-y)))] - [(1-y)(1-z) - yz(1-|t|/(s(1-y)))]^2$. The integration domain defined by $F(y,z) > 0$ includes collinear poles at $(y,z) = (k,0)$ and $(0,k)$ corresponding to initial and final state emission, while the infrared point $(0,0)$ is excluded. In order to deal with the trigger function Θ_{trig} we have to control the laboratory electron angles $\vartheta_{1,2}$. They are introduced through the relations

$$\frac{|t|}{s} = \frac{\xi_1(1-y)}{1-y+y\xi_1},\; 1-z = (1-y)\left[\frac{\xi_2}{\xi_1}(1-k) - \xi_2(2-k)\right] + \xi_2(2-k). \quad (7)$$

The resulting triple differential distribution $d\sigma/dkd\xi_1 d\xi_2$ is expressed as a one dimensional integral over a section of a straight line on the (y,z) plain. The integral can be expressed in terms of standard functions (elliptical integrals of the first kind). We calculate it however in an approximate way: we replace the function $F(y,z)$ by the approximate one $\tilde{F}(y,z)$ such that: (i) F and $\tilde{F}$ coincide in the soft limit $k \to 0$ for all (y,z), (ii) for any k they are identical in the neighborhood of the collinear singularities $(k,0)$ and $(0,k)$ and (iii) away from collinear and/or soft points we require $|F-\tilde{F}| \sim \xi_{1,2}$ at most. The above conditions assure exact soft and leading-log (LL) limits. The integrated cross section is correct up to non-logarithmic $\frac{\alpha}{\pi}\vartheta^2$ terms. We realize this approximation as follows

$$\tilde{F}(y,z) = \begin{cases} = -(k-y-z)^2 + 4yz(1-k)[1-\xi_1/(1-k+k\xi_1)], & \text{for } z < y, \\ = -(k-y-z)^2 + 4yz(1-k)(1-\xi_1), & \text{for } z > y, \end{cases} \quad (8)$$

The factors $(s_1^2+u^2)/s^2$ and $(s^2+u_1^2)/s^2$ in Y are also simplified in the same spirit[9]. The integral (6) reduces now to logarithms and the distribution $d\sigma_1'/dkd\xi_1 d\xi_2$ is quite simple [15]. In the following we shall show the double differential distributions $\rho_1(k,\xi_1) = d\sigma_1'/dkd\xi_1$ and $\rho_2(k,\xi_2) = d\sigma_1'/dkd\xi_2$ derived from eq. (5). The distributions $d\sigma/dkd\xi$ corresponding to emission from both lower and upper lines, see eq. (1), is obtained simply by summing them up

$$\frac{d\sigma}{dkd\xi} = \rho_1(k,\xi) + \rho_2(k,\xi), \quad (9)$$

where ξ is now either ξ_1 or ξ_2. Note that the other ξ_i is kept within the trigger angular range, see eq. (5).

The resulting angular distribution of the electron from the no-emission side reads as follows

$$\begin{aligned} \rho_1(k,\xi_1) &= \frac{4\pi\alpha^2}{s}\left(\frac{\alpha}{\pi}\right)\frac{1}{\xi_1^2} \\ &\quad \times\Theta(\xi_1-\xi_{min})\Theta(\xi_{max}-\xi_1)\left[G(k,\tilde{a}_{up},\xi_1) - G(k,\tilde{a}_{low},\xi_1)\right], \quad (10) \\ \tilde{a}_{up} &= \min\left(\frac{(1-k)\xi_{max}}{\xi_1}, 1+\frac{k}{\sqrt{\xi_1}}\right),\; \tilde{a}_{low} = \max\left(\frac{(1-k)\xi_{min}}{\xi_1}, 1-\frac{k}{\sqrt{\xi_1}}\right), \end{aligned}$$

[9]The approximations $m_e^2/(s\xi_i) << 1$ and $\xi_i\alpha/\pi << 1$ are used in many intermediate steps of the calculation, see [15] for details.

where

$$
\begin{aligned}
G(k,\tilde a,\xi_1) &= \chi(\xi_1,\tilde a)\,\frac{1+(1-k)^2}{2k} \\
&\times\Bigg\{\ \Theta(\tilde a-\tilde a_1)\left[\ln\frac{s\xi_1}{m_e^2}-\frac{2(1-k)}{1+(1-k)^2}\right] \\
&\quad +\Theta(\tilde a-\tilde a_1)\ln\frac{1}{(1-\xi_1)(1-k)^2\tilde a_1^2(\tilde a-1)^2} \\
&\quad +\ln_+(\tilde a-\tilde a_1)+\ln_+(\tilde a_1-1) \\
&\quad -\Theta(1-\tilde a-k)\left[\ln\frac{s\xi_1}{m_e^2}-\frac{2(1-k)}{1+(1-k)^2}+\ln\frac{1}{(1-\xi_1)(\tilde a-1)^2}\right] \\
&\quad +\ln_+(\tilde a-1+k)-\ln_+ k\Bigg\},
\end{aligned}
$$

$$
\ln_+(x) = \mathrm{sign}(x)\ln|x|, \tag{11}
$$

$$
\tilde a_1 = \frac{1+\nu(\xi_1)}{1+\nu(\xi_1)-k},\quad \nu(\xi)=\frac{(2-k)\xi}{1-k-(2-k)\xi}. \tag{12}
$$

$$
\chi(\xi,\tilde a) = \begin{cases} \frac{1}{2}[1+(1-\xi)^2] & \text{if } \tilde a<1, \\ \frac{1}{2}\left[1+\left(1-\frac{\xi}{1-k(1-\xi)}\right)^2\right] & \text{otherwise.}\end{cases} \tag{13}
$$

The angular distribution of the electron on the emission side reads

$$
\begin{aligned}
\rho_2(k,\xi_2) &= \frac{4\pi\alpha^2}{s}\left(\frac{\alpha}{\pi}\right)\frac{1}{\xi_2^2}\,\frac{1}{1-k} \\
&\times\Theta(\xi_2-\xi_{min})\Theta(\xi_{max}-\xi_2)\,\Big[H(k,\tilde a_{up},\xi_2)-H(k,\tilde a_{low},\xi_2)\Big], \qquad (14) \\
\tilde a_{up} &= \min\left(\frac{(1-k)\xi_2}{\xi_{min}},1+\frac{k}{\sqrt{\xi_2}}\right),\ \tilde a_{low}=\max\left(\frac{(1-k)\xi_2}{\xi_{max}},1-\frac{k}{\sqrt{\xi_2}}\right),
\end{aligned}
$$

where

$$
\begin{aligned}
H(k,\tilde a,\xi_2) &= \chi^*(\xi_2,\tilde a)\,\frac{1+(1-k)^2}{2k} \\
&\times\Bigg\{\ \frac{\Theta(\tilde a-\tilde a_1^*)}{1-k}\left[\ln\frac{s\xi_2}{m_e^2}-\frac{2(1-k)}{1+(1-k)^2}\right] \\
&\quad +\frac{\Theta(\tilde a-\tilde a_1^*)}{1-k}\ln\frac{1}{(1-\xi_2)\tilde a_1^{*2}(\tilde a_1^*-1)^2}+\Theta(\tilde a-\tilde a_1^*)\ln\frac{(\tilde a_1^*-1)^2}{(\tilde a-1)^2} \\
&\quad +\frac{1}{1-k}\Big[\ln_+(\tilde a-\tilde a_1^*)+\ln_+(\tilde a_1^*-1)\Big] \\
&\quad -(1-k)\Theta(1-\tilde a-k)\left[\ln\frac{s\xi_2}{m_e^2}-\frac{2(1-k)}{1+(1-k)^2}\right] \\
&\quad -(1-k)\Theta(1-\tilde a-k)\ln\frac{1}{(1-\xi_2)k^2}-\Theta(1-\tilde a-k)\ln\frac{k^2}{(\tilde a-1)^2}
\end{aligned}
$$

$$+(1-k)\Big[\ln_+(\tilde{a}-1+k)-\ln_+ k\Big]\Big\}, \tag{15}$$

$$\tilde{a}_1^* = \frac{1-\xi_2 k(2-k)}{1-k}, \tag{16}$$

$$\chi^*(\xi,\tilde{a}) = \begin{cases} \frac{1}{2}[1+(1-\xi)^2] & \text{if } \tilde{a}<1, \\ \frac{1}{2}\Big[1+\Big(1-\frac{\xi(1-k)}{1-\xi k}\Big)^2\Big] & \text{otherwise.} \end{cases} \tag{17}$$

In the following we present selected numerical results illustrating very good agreement of our analytical calculation with the Monte Carlo. In the tests we use the OLDBAB Monte Carlo of ref. [2] because its algorithm allows for fixing the laboratory photon energy and one of the electron angles in the entire MC series. This feature is rather useful for some class of tests. The original version of OLDBAB was not suited for our high precision test – we had to implement an option for calculations with weighted events (to avoid dependence on $k_0=\varepsilon$) and to rewrite completely the QED matrix elements in order to disentangle up-down interference from the rest. Events with negative weights are necessarily admitted in the MC calculation.

In figs. 2 and 3 we compare the angular distributions of electrons from the Monte Carlo with formulas of eqs. (10) and (14) at $k=0.1$. They agree up to statistical errors. The shape of the distribution in fig. 2 can be understood easily in the LL approximation. The sharp kink in the curve is of kinematical origin. It is due to the fact that for events with the photon collinear to the incoming beam we have $\xi_1/\xi_2 \sim (1-k)^2$. As a result, for ξ_1 close to ξ_{max} we may have $\xi_2 > \xi_{max}$ and such an event will not be accepted by our trigger. Since roughly every second photon is in the initial state we get, above certain ξ_1, roughly a factor one-half in $\rho_1(k,\xi_1)$. This can be seen quite clearly in fig. 3 where, for a while, we omit in the plotted $\rho_2(k,\xi_2)$ the factor $\Theta(\xi_2-\xi_{min})\Theta(\xi_{max}-\xi_2)$. The events to be rejected, i.e. with $\xi_2>\xi_{max}$, are included in the plot. For higher k such a rejection would occur for all initial state (in the LL sense) photons and the entire $\rho_1(k,\xi_1)$ would be uniformly lower by factor two, as we shall see in the next figure. In fig. 3 we also include the pure LL analytical result for ρ_2. It is obtained from eq. (14) by means of rejecting all terms except those proportional to $\ln(s\xi_i/m_e^2)$. As we see, the difference between the two analytical results representing the next-to-leading contributions is quite sizeable and it is under control in our analytical calculation. The subleading corrections smear out the sharp LL shape of the distribution. These and remaining results were obtained [10] for $\varepsilon = 10^{-4}$.

The same “kinematical kink” due to initial state bremsstrahlung is visible in the photon energy distribution $\rho_1(k,\xi_1)$ displayed in fig. 4. Here ξ_1 is fixed in the middle of the ξ-range. As already discussed, roughly half of the distribution is lost above some value of k. To see it clearly we also plot the MC distribution without

[10]We have checked that varying ε from 10^{-3} to 10^{-6} changes our results within the statistical error of $2\cdot 10^{-4}$.

ϑ_{min}[deg .]	σ_{MC}	$\sigma_{Anal.}$	$(\sigma_{MC}-\sigma_{Anal.})/\sigma_{MC}$	$\Delta\sigma_{MC}/\sigma_{MC}$
9	2.13908	2.13895	0.006%	0.018%
3	19.6634	19.6659	-0.013%	0.017%
1	179.975	179.999	-0.013%	0.015%
1/3	1646.09	1646.49	-0.024%	0.013%
1/9	15057.9	15059.8	-0.012%	0.011%

Table 2: *Analytical and Monte Carlo total Bhabha cross section at* $s^{1/2} = 92 GeV$. *Photon energy range and angular range for electrons is the same as in table (1). Up-down QED interference, vacuum polarization and s-channel exchange are excluded. Monte Carlo samples of* $2\cdot 10^7$ *events were used.*

angular trigger on ξ_2. As expected it does not have the "kink". Let us note that the position of the "kink" depends on how close is ξ_2 to the limit ξ_{max} (or ξ_1 to ξ_{min}). For ξ_i close to one of the edges the "kink" may occur at arbitrarily low k. For this reason "exponentiation" for low angle Bhabha is more difficult (although feasible) than for $s-$channel exchange, see [15].

In fig. 5 we compare the O(α) QED angular distribution (with matrix element and trigger defined in eq. (1)) for $d\sigma/d\xi$ from analytical and Monte Carlo calculations. They agree very well but in order to see it clearly we plot the corresponding ratio of the two distributions in the fig. 6. As we see, they agree to within 1%, i.e. up to statistical fluctuations[11]. Finally, we also integrate numerically over ξ and obtain the total cross section. The Monte Carlo and analytical results for the total cross section are compared for θ_{min} varying from 0.1^0 to 9^0 in tab. 2. They differ less than 0.03% while the statistical Monte Carlo error is 0.02% and the precision of the analytical result (involving numerical integration over ξ and k) is below 0.01%. Taking some security factor we estimate the *technical precision* of the tested analytical and Monte Carlo calculations to be better than 0.05%. The above numerical results represent a small fraction of the actual tests which were done. Generally, we have tested the parameter region: $0.1^0 < \vartheta_{min} < 9^0$, $0.01 < R_\xi < 50$ for $10^{-6} < \varepsilon < 10^{-3}$.

Let us summarize what is the significance of our results:

- The aim of this study was to calculate the systematic error of the QED O(α) calculation for low angle Bhabha and it was not to give the complete QED prediction for this process.

[11]For such a high statistics and precision it is essential to use REAL*8 variables in the histograming package, not a common feature in the public program libraries.

- Concentrating on the pure $t-$channel exchange we show that we *can* control the $O(\alpha)$ QED calculation at a numerical precision below 0.05% for the integrated cross section with (symmetric) simultaneous cuts on both the electron and positron angles.

- The above result opens the way to many semi-analytical QED calculations for the low angle Bhabha process beyond the $O(\alpha)$. Inclusion of the LL $O(\alpha^2)$ correction and "ad hoc" exponentiation are the two natural candidates.

- The most important application of these semi-analytical calculations will be testing of the Monte Carlo programs for the low angle Bhabha process like for example BHLUMI [3] at the new 0.2% overall precision level. In these tests our $O(\alpha)$ calculation will play a role of the reference *baseline calculation* with well controlled numerical *technical* precision.

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Figure 2: *Comparison of analytical and Monte Carlo results for the electron angular distribution* $\rho_1(k,\xi_1) = d\sigma_1'/dkd\xi_1$ *at fixed photon energy, where* $\xi_1 = (1-\cos\vartheta_1)/2$ *and* ϑ_1 *is the angle of the electron on the no-emission side. Monte Carlo sample of* 10^7 *events was produced at* $s^{1/2} = 92GeV$ *with* $\varepsilon = 10^{-4}$, $\theta_{min} = 9^0$ *and* $R_\xi = 1$. *Analytical results (dots) are from eq. (10). Vertical lines mark the trigger angular range* $\vartheta_{min} < \vartheta_1 < \vartheta_{max}$.

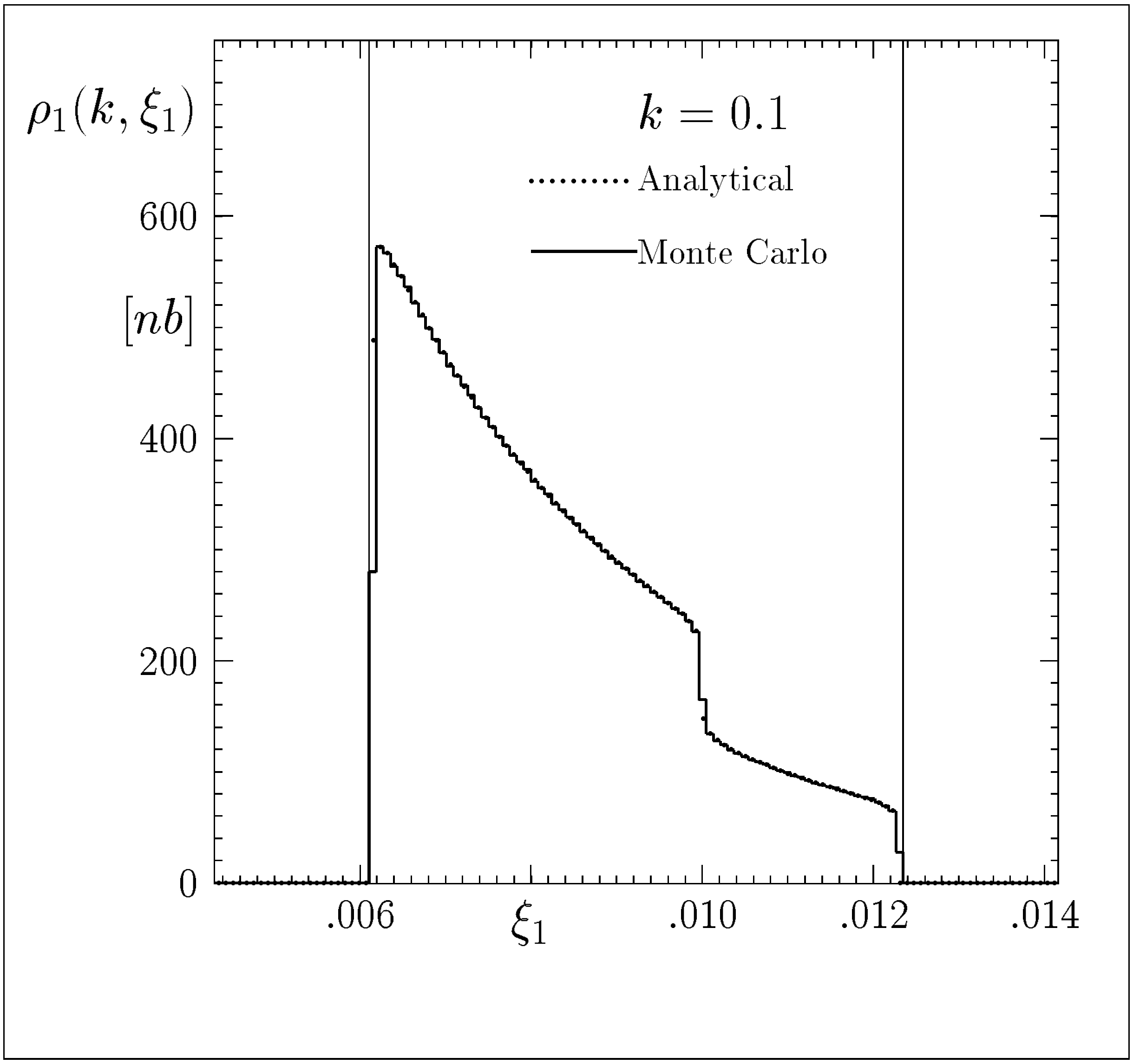

Figure 3: *Comparison of the analytical and Monte Carlo results for the electron angular distribution* $\rho_2(k,\xi_2) = d\sigma'_1/dkd\xi_2$ *at fixed photon energy,* $k = 0.1$*, where* $\xi_2 = (1-\cos\vartheta_2)/2$ *and* ϑ_2 *is the angle of the electron on emission side. A Monte Carlo sample of* 10^7 *events was produced with the same input as in fig. 2. Analytical results (dots) are from eq. (14). The pure leading-logarithmic analytical result is represented by open circles. Vertical lines mark the same angular range as in fig. 2* $\vartheta_{min} < \vartheta_2 < \vartheta_{max}$*. The* ρ_2 *distribution is nonzero outside this range due to temporary (only in this figure) omission of* $\Theta(\vartheta_2-\vartheta_{min})\Theta(\vartheta_{max}-\vartheta_2)$*.*

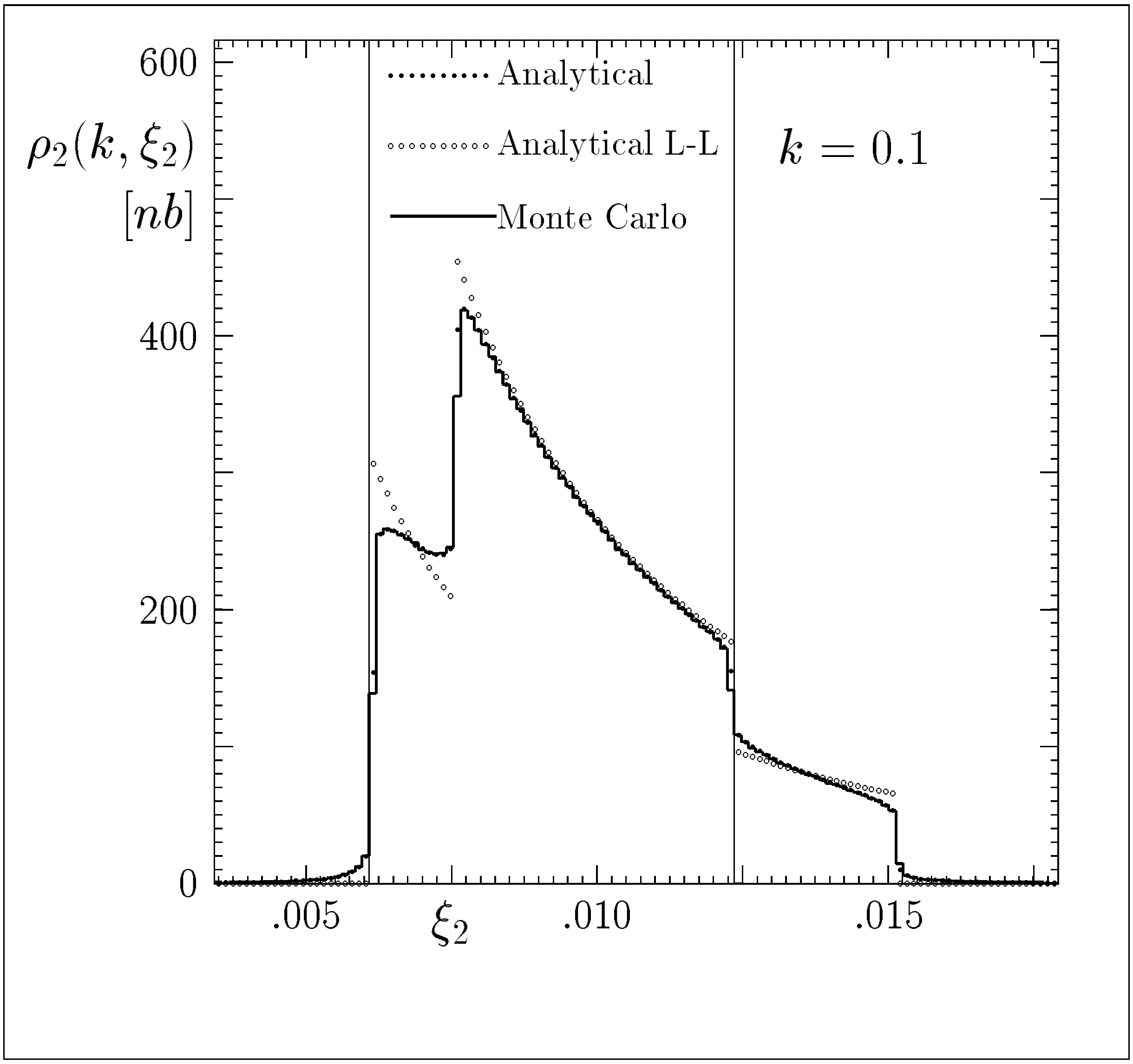

Figure 4: *Comparison of the analytical (dots) and Monte Carlo (histogram) results for photon energy distribution* $k\rho_1(k,\xi_1) = kd\sigma_1'/dkd\xi_1$ *at fixed electron scattering angle of the electron on emission side (i.e. for fixed* ξ_1*). In the Monte Carlo results the angle of the second electron (emission side) remains in the tag region* $\vartheta_{min} < \vartheta_2 < \vartheta_{max}$ *(histogram with thick line) or it is unlimited (thin line). Monte Carlo sample of* 10^7 *events was produced with the same input as in fig. 2. Analytical results (dots) are from eq. (10).*

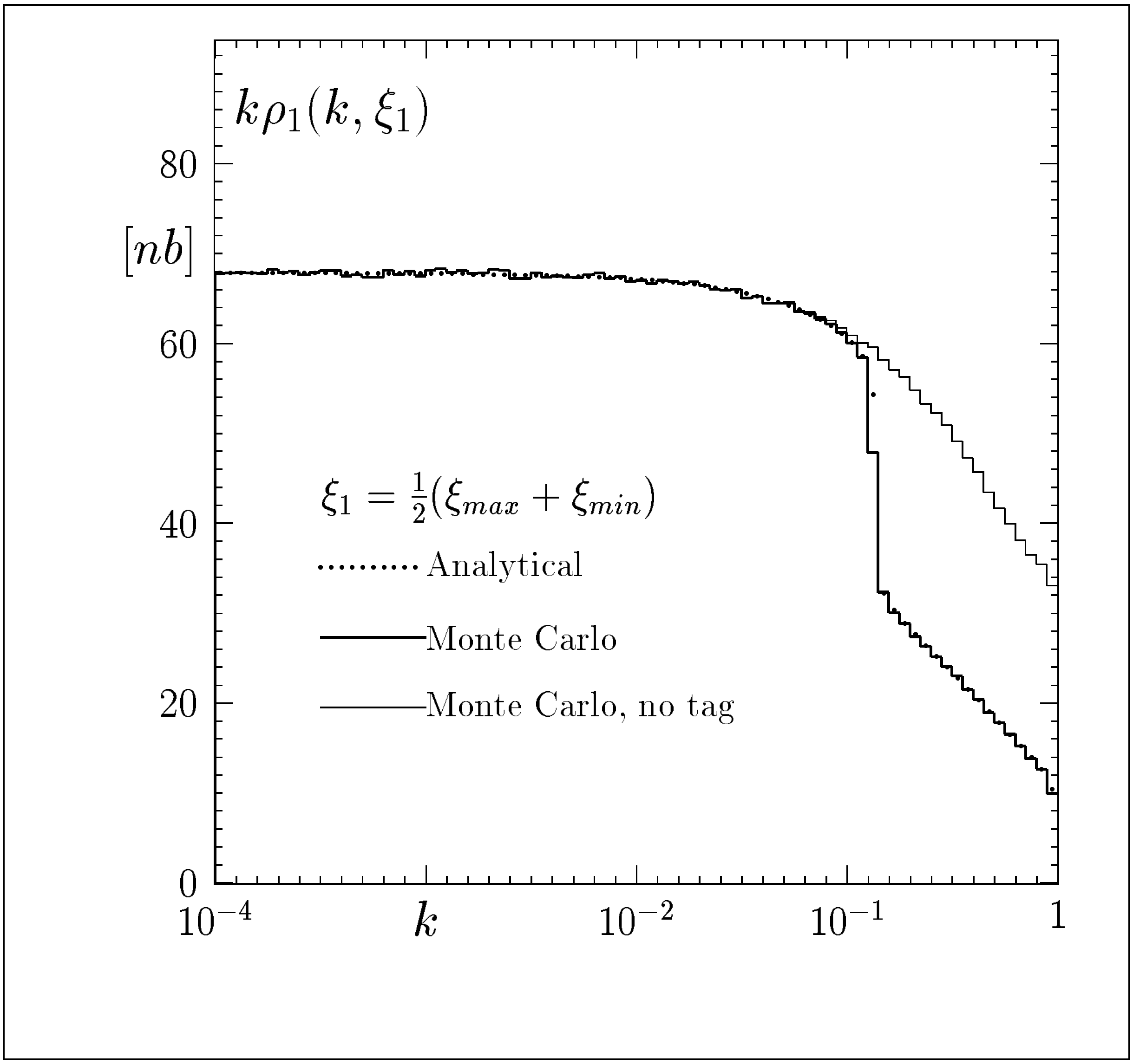

Figure 5: *Comparison of the analytical (dots) and Monte Carlo (histogram) results for the angular distribution $d\sigma/d\xi$ where $\xi = (1-\cos\vartheta)/2$ and ϑ is either ϑ_1 or ϑ_2. The contribution from soft and hard real emission are combined. Born distribution (open circles) is also plotted. Both electrons are tagged $\vartheta_{min} < \vartheta_{1,2} < \vartheta_{max}$ where $\vartheta_{min} = 9^0$ and $R_\xi = 1$. The Monte Carlo sample includes $2 \cdot 10^7$ events. Up-down interference, s-channel exchange and vacuum polarization are excluded.*

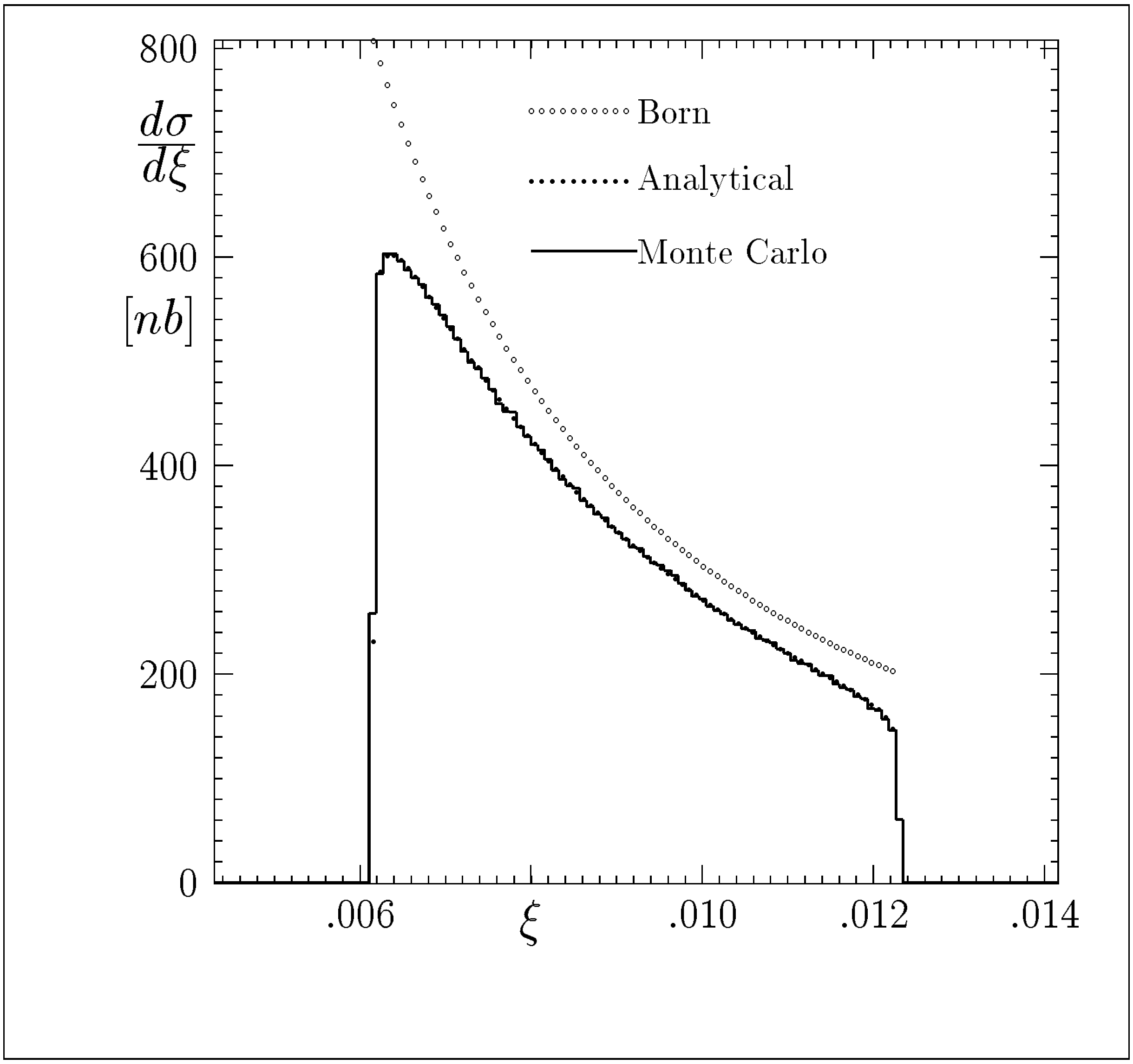

Figure 6: *The ratio of Monte Carlo to the analytical results for $d\sigma/d\xi$ of the fig. 5 are presented as dots. The corresponding ratio for the real emission part, $k > \varepsilon$, is also plotted with the continuous line.*