

# Flavoured jets and how to define them

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# Flavoured jets are everywhere

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- Flavoured jets as signature
  - Top-quarks
  - Vector+heavy flavour:  $pp \rightarrow W/Z/A + c/b$
  - Higgs  $\rightarrow$  charm, Higgs  $\rightarrow$  bottom
  - New physics searches
  - Studies of QCD dynamics
- Focus of this talk:  $V + \text{heavy-flavour}$  ( $\rightarrow$  but many aspects are generalisable)
  - Benchmark for flavour tagging
  - IR safety/sensitivity



Rely on our capability to  
 $\rightarrow$  **identify** (i.e. tag) flavoured jets  
 $\rightarrow$  **interpret** (i.e. predict) them

# Outline

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- What do we mean by "flavoured jets" and why are there problems?
- Anti-kT "like" flavoured jet algorithms
- Phenomenology → Les Houches study
- Interface to experiment
- Wrap up & outlook

# A look back: Snowmass accord 1990 ...

## Toward a Standardization of Jet Definitions

John E. Huth and Naor Wainer  
*Fermi National Accelerator Laboratory  
P.O. Box 500  
Batavia, Illinois 60510*

Karlheinz Meier  
*Deutsches Elektronen Synchrotron (DESY)  
Hamburg 52, Germany*

Nicholas Hadley  
*University of Maryland  
College Park, Maryland 20742*

F. Aversa and Mario Greco  
*Istituto Nazionale di Fisica Nucleare (INFN)  
Frascati, Italy*

P. Chiappetta and J. Ph. Guillet  
*CTP-CRNS, Luminy  
Marseille, France*

Stephen Ellis  
*University of Washington  
Seattle, Washington 98195*

Zoltan Kunszt  
*Eidg. Technische Hochschule  
Zurich, Switzerland*

Davison Soper  
*University of Oregon  
Eugene, Oregon 97403*

December 1990

A sensible jet definition should be:

- 1) Simple to implement in experimental analysis
- 2) Simple to implement in theoretical calculations
- 3) Defined at any order of perturbation theory
- 4) Yields finite cross section at any order of perturbation theory
- 5) Yields a cross section that is relatively insensitive to hadronization

Purpose: “undo” parton evolution to define the “hard scattering” process

# A look back: Snowmass accord 1990 ...

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For theory:

- Infrared collinear (IRC) safety
- Small sensitivity of 'inclusive observables' to parton-shower & hadronisation

A sensible jet definition should be:

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# Jets at the LHC

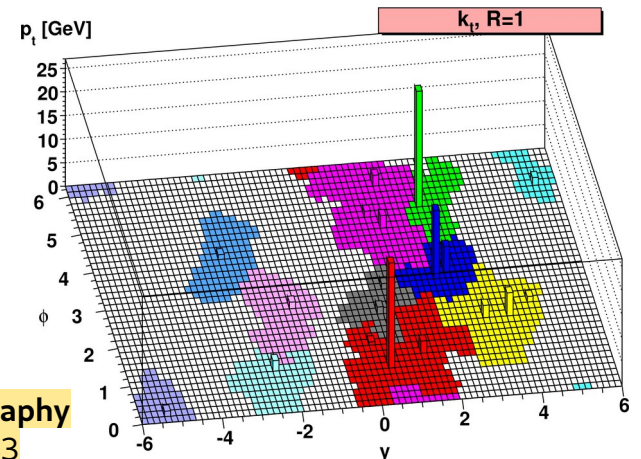
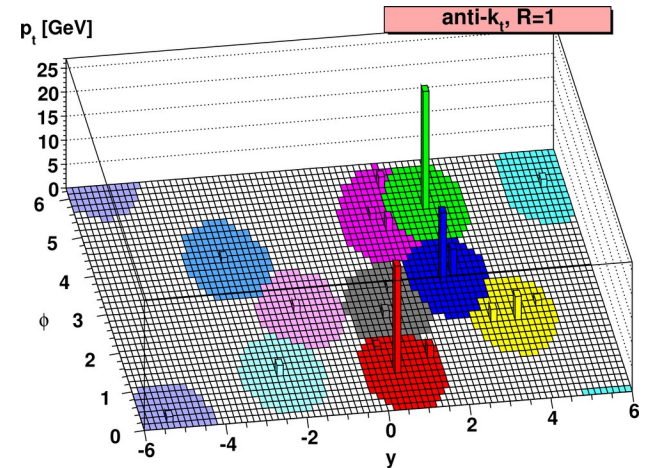
Many proposals of jet algorithms since '90:

- Cone-based algorithms: PxCone, midpoint, seedless, SIScone, ...
- 2-to-1 recombination algorithms: C/A, Jade, kT, anti-kT, ...

The **standard** algorithm for the LHC is the **anti-kT algorithm**:

$$d_{ij} = \min(k_{T,i}^{-2}, k_{T,j}^{-2}) R_{ij}^2 \quad d_i = k_{T,i}^{-2}$$

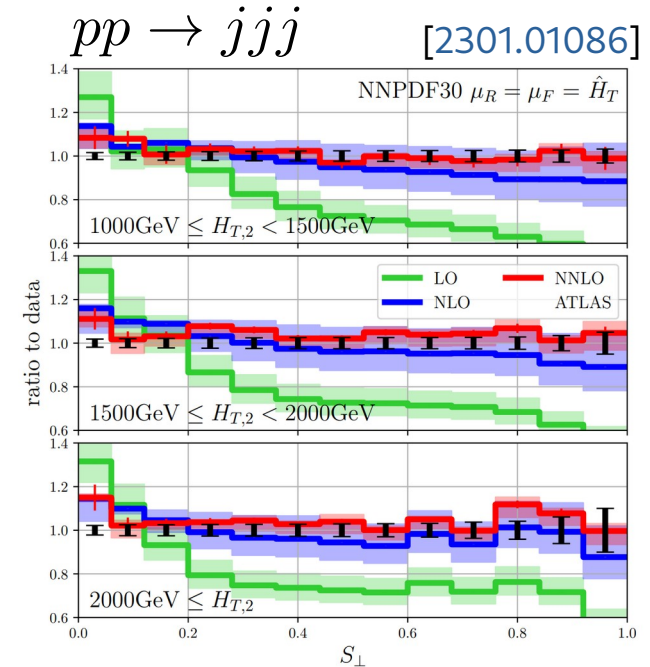
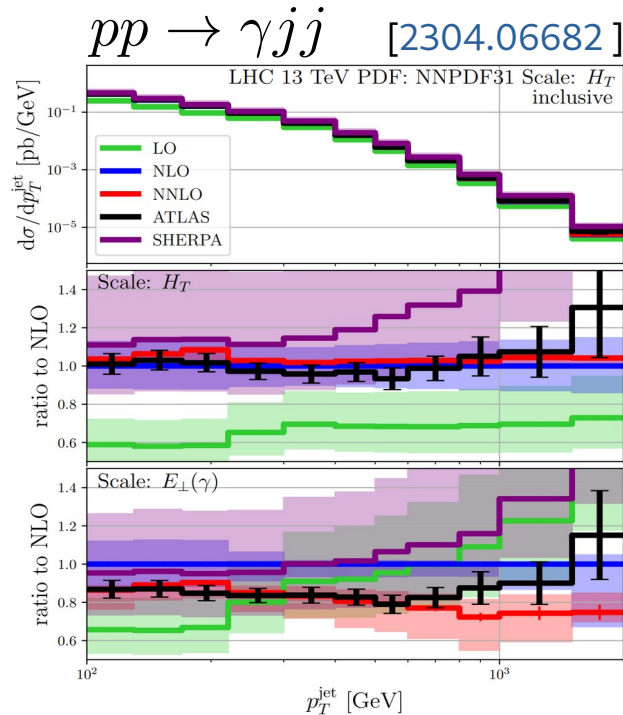
- nice geometric properties
- theoretically okay
- insensitive to soft physics, pile up, etc.



**Towards Jetography**  
Salam 0906.1833

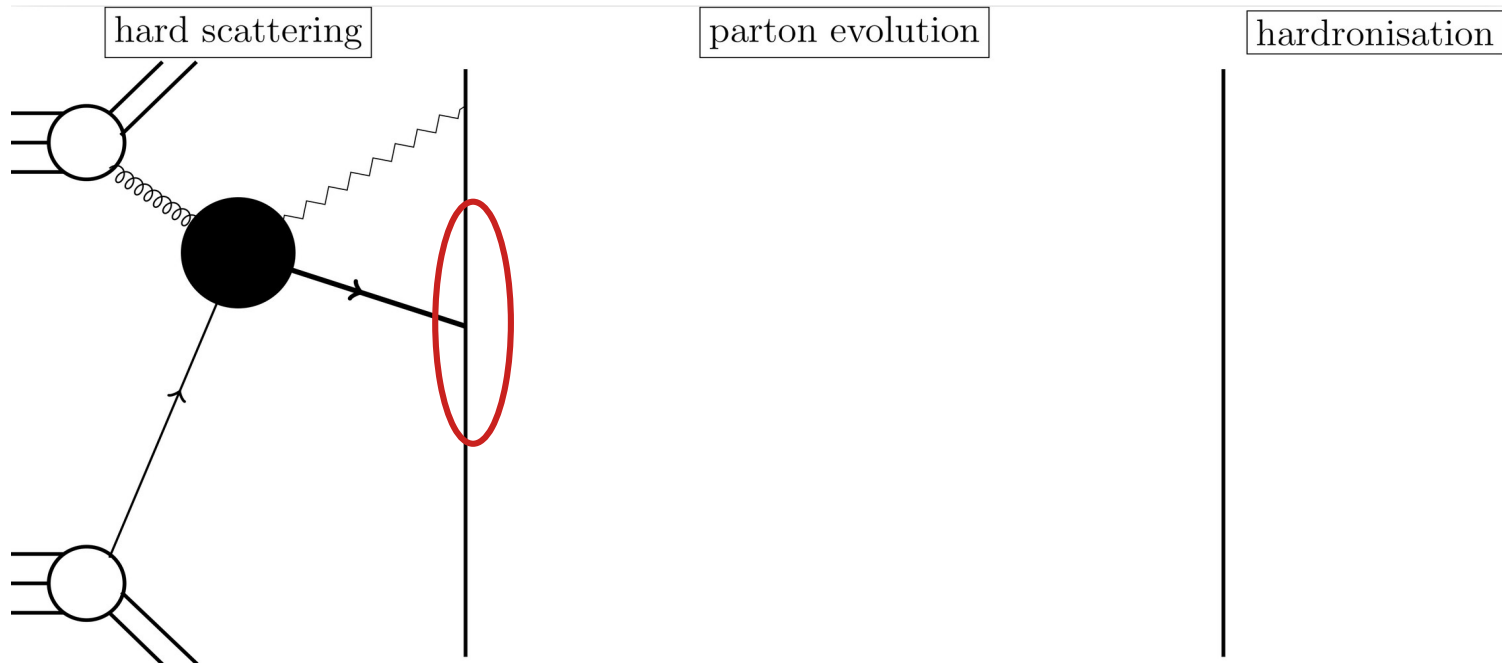
# Precision comparisons of jet cross sections

Following these guidelines means that we can compare theory and experiment even though theorists talk about quarks+gluons and experimentalists about particles



# Flavoured jets

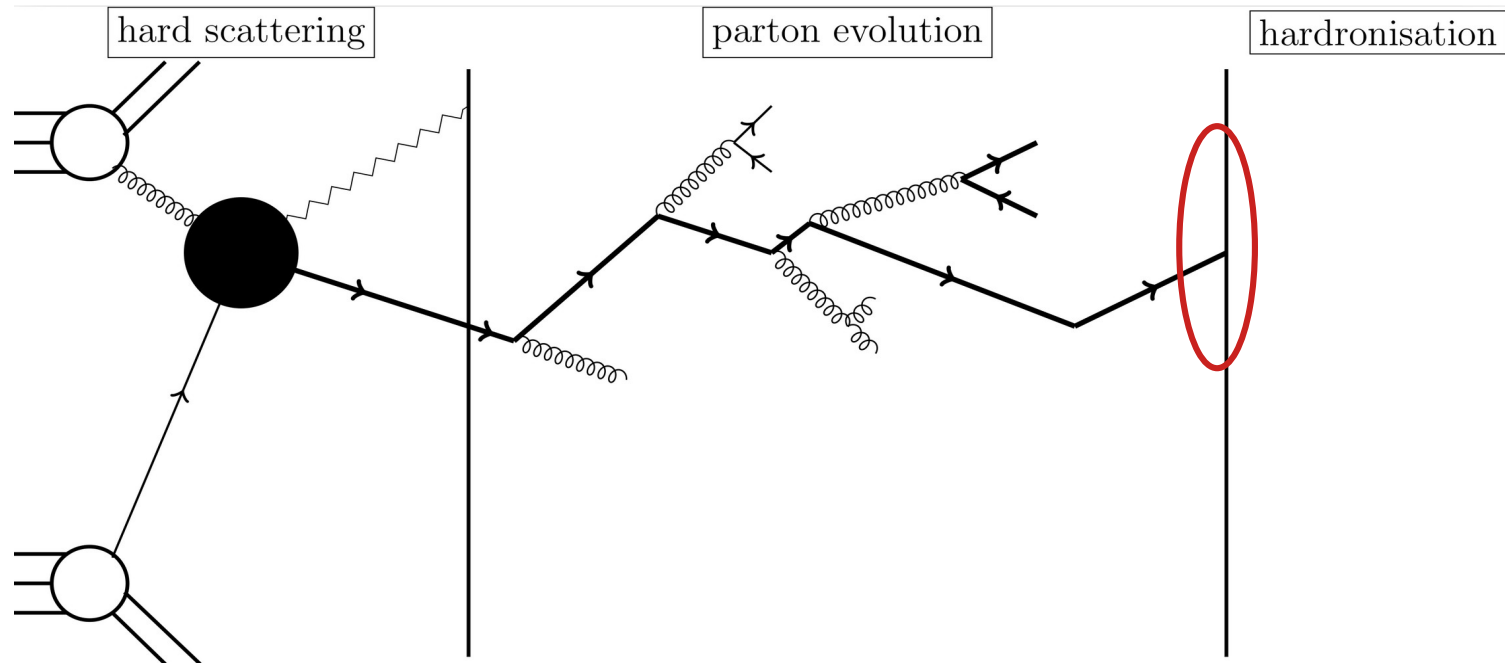
jet that initiated from a "hard scatter" product of specific flavour:  
bottom, charm, "quark/gluon"





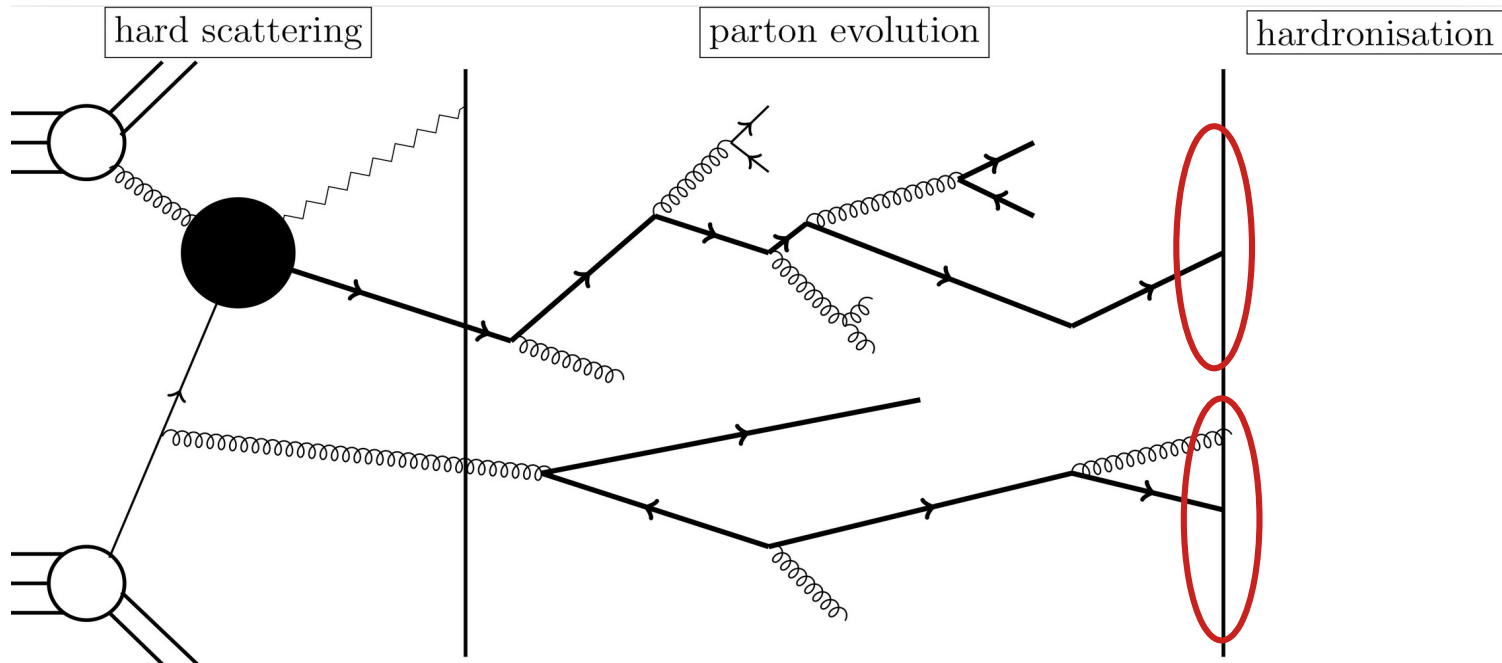
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jet that initiated from a "hard scatter" product of specific flavour:  
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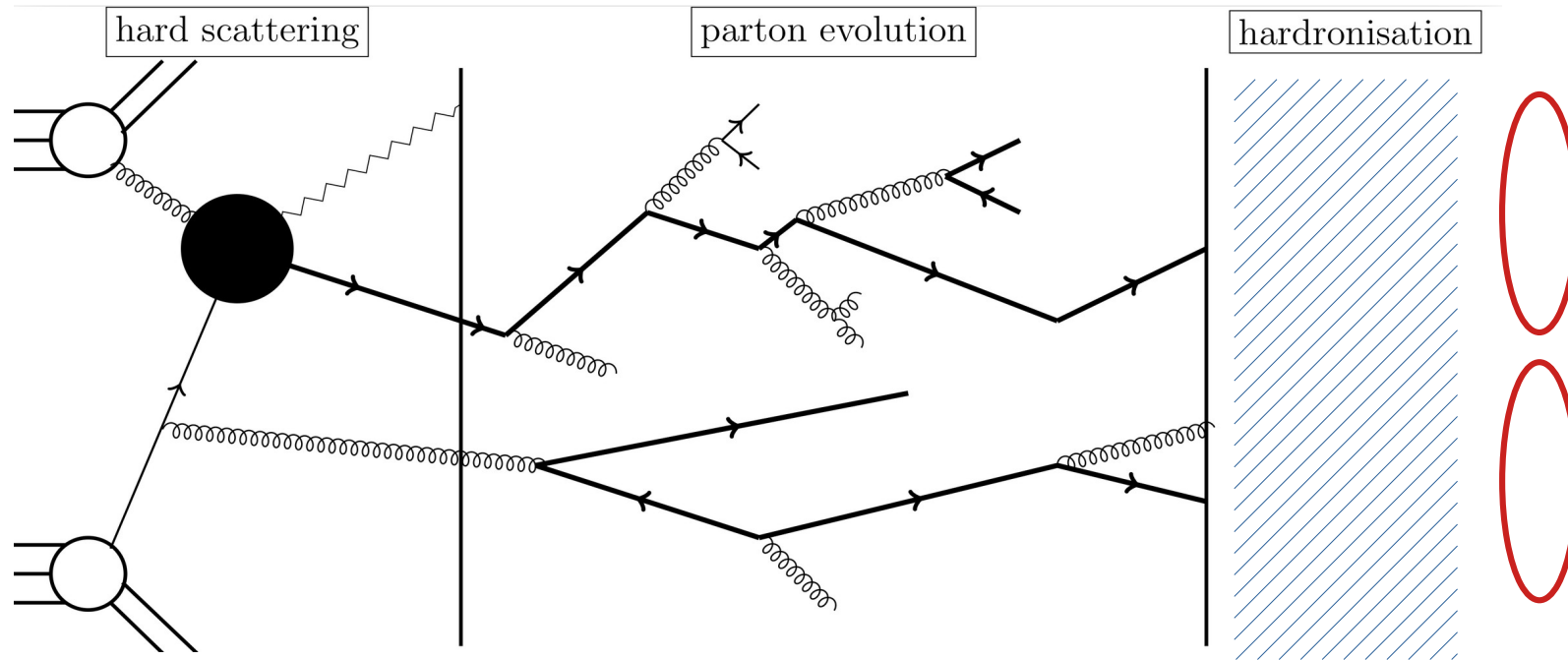
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jet that initiated from a "hard scatter" product of specific flavour:  
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# Flavoured jets

jet that initiated from a "hard scatter" product of specific flavour:  
bottom, charm, "quark/gluon"



# Example for experimental 'truth level' flavour tagging

## Example definition for experimental tagging

A 'truth-level' jet is defined as flavoured if:

1) it contains at least one B hadron

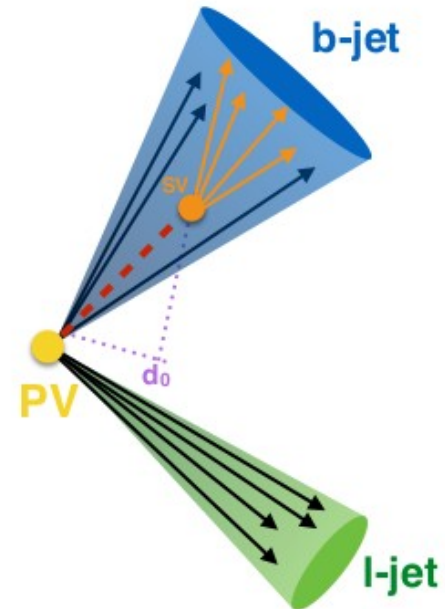
FO: IRC-unsafe because of  $g \rightarrow b \bar{b}$  splitting

2) with  $p_T > p_{T\_cut}$

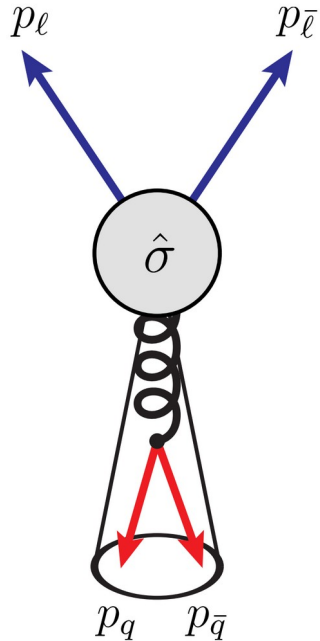
FO: collinear unsafe  $b \rightarrow b g$  splitting  
(okay in fragmentation approach)

3) within  $dR < R$  of jet axis

FO: IRC-unsafe because soft wide angle emission



# Infrared safety issues with flavoured jets I



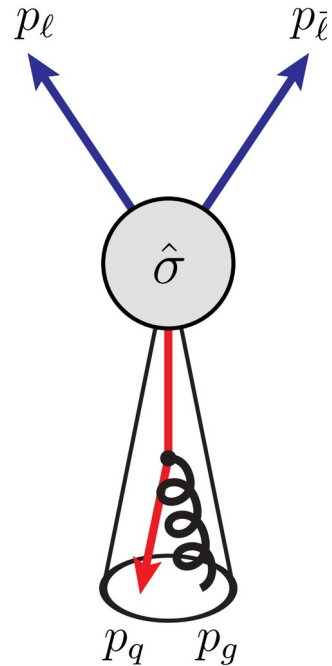
Picture from  
[Gauld et al. 2302.12844]

- IRC unsafe due to  $g \rightarrow \text{quark-anti-quark}$  splitting
- Quarks massless: cross-section not defined
- Quarks massive: logarithmic sensitivity to quark mass
- Can resolved by proper flavour recombination schemes:

<i>jet contents</i>	$b$	$b + \bar{b}$	$b + b$	
<i>scheme</i>				
“any flavour”	$b$	$b$	$b$	simplest experimentally (but collinear unsafe for $m_b \rightarrow 0$ )
net flavour	$b$	$g$	$2b$	theoretically “ideal” definition; but not robust wrt B–Bbar oscillations
flavour modulo 2	$b$	$g$	$g$	theoretically OK; robust wrt B–Bbar oscillations

[Salam]

# Infrared safety issues with flavoured jets II

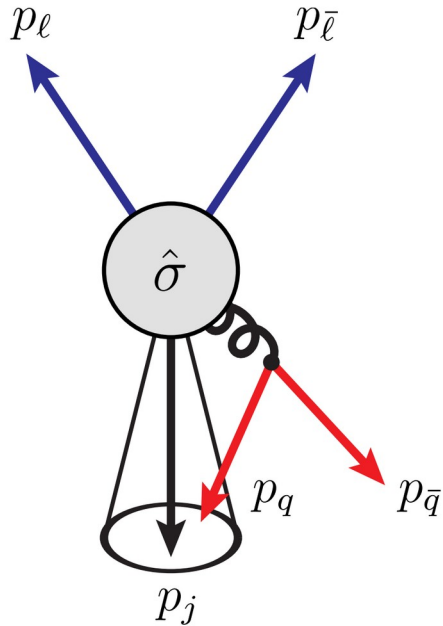


- Collinear unsafe if pT requirement on the quark is present
- Not implementable in pQCD with massless quarks  
→ proper treatment needs fragmentation functions  
→ NNLO QCD example:

**B-hadron production in NNLO QCD: application to LHC ttbar events with leptonic decays,**  
Czakon, Generet, Mitov and Poncelet, 2102.08267

Picture from  
[Gauld et al. 2302.12844]

# Infrared safety issues with flavoured jets III



- Starting at NNLO QCD:  
→ Soft singularity from quark pairs
- Massless quarks → cross section not defined
- Massive quarks → logarithmic IRC sensitivity  $\ln\left(\frac{m}{p_T}\right)$
- **Needs modified jet algorithms!**

Picture from  
[Gauld et al. 2302.12844]

# Solution: Modified jet algorithms

These issues are known since 2006... a solution as well:

Standard kT algorithm:

Pair distance:

$$d_{ij} = \min(k_{T,i}^2, k_{T,j}^2) R_{ij}^2$$

$$R_{ij}^2 = (\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2) / R^2$$

“Beam” distance for determination condition:

$$d_i = k_{T,i}^2$$

Flavour kT algorithm:

**Infrared safe definition of jet flavor,**  
Banfi, Salam, Zanderighi hep-ph/0601139

Pair distance:

$$d_{ij} = R_{ij}^2 \begin{cases} \max(k_{T,i}, k_{T,j})^\alpha \min(k_{T,i}, k_{T,j})^{2-\alpha} & \text{softer of } i, j \text{ is flavoured} \\ \min(k_{T,i}, k_{T,j})^\alpha & \text{else} \end{cases}$$

Beam distance:

$$d_{i,B} = \begin{cases} \max(k_{T,i}, k_{T,B}(y_i))^\alpha \min(k_{T,i}, k_{T,B}(y_i))^{2-\alpha} & i \text{ is flavoured} \\ \min(k_{T,i}, k_{T,B}(y_i))^\alpha & \text{else} \end{cases}$$

$$d_B(\eta) = \sum_i k_{T,i} (\theta(\eta_i - \eta) + \theta(\eta - \eta_i)) e^{\eta_i - \eta}$$

$$d_{\bar{B}}(\eta) = \sum_i k_{T,i} (\theta(\eta - \eta_i) + \theta(\eta_i - \eta)) e^{\eta - \eta_i}$$

→ problem for LHC: this is a kT algorithm → ‘apples to apples’ comparison not possible



# New proposals for flavour-safe anti-kT jets

- Flavour with Soft-drop **Practical Jet Flavour Through NNLO**  
Caletti, Larkoski, Marzani, Reichelt 2205.01109
- Flavour anti-kT **Infrared-safe flavoured anti-kT jets,**  
Czakon, Mitov, Poncelet 2205.11879
- Fragmentation approach **A Fragmentation Approach to Jet Flavor**  
Caletti, Larkoski, Marzani, Reichelt 2205.01117  
**B-hadron production in NNLO QCD: application to LHC ttbar events with leptonic decays,**  
Czakon, Generet, Mitov and Poncelet, 2102.08267
- Flavour dressing → standard anti-kT + flavour assignment  
**QCD-aware partonic jet clustering for truth-jet flavour labelling**  
Buckley, Pollard 1507.00508  
**A dress of flavour to suit any jet**  
Gauld, Huss, Stagnitto 2208.11138
- Interleaved flavour neutralisation  
**Flavoured jets with exact anti-kT kinematics and tests of infrared and collinear safety**  
Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler 2306.07314
- TBC...

# New proposals for flavour-safe anti-kT jets

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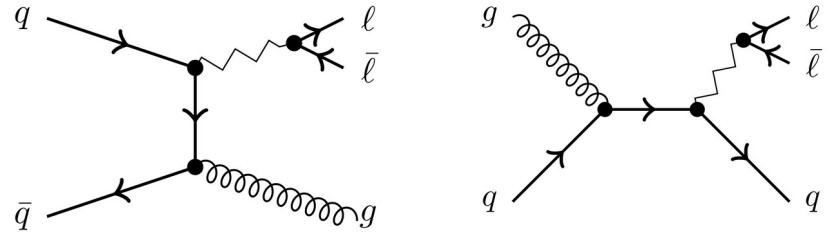
*IRC-safe anti-kT flavoured jet algorithm? Yes, but which one?*

Criteria:

- IRC-safety ← Highly desirable: fulfilled by all candidates (at least through NNLO)
- "truthfully" reconstruct reconstruct the original "hard" object  
→ insensitive to PS+HAD+SOFT ← Desirable: robust theory predictions!
- experimentally implementable ← comment at the end
- numerically efficient ← not yet the focus of the effort  
→ important for experimental implementation
- easy to implement in analysis ← wip towards full release  
→ FastJet-contrib (test implementations: <https://github.com/jetflav/>)
- Jet-substructure?
- ...

# Les Houches “FlavourFest”

$$pp \rightarrow Z + \text{jet}/b - \text{jet}/c - \text{jet}$$

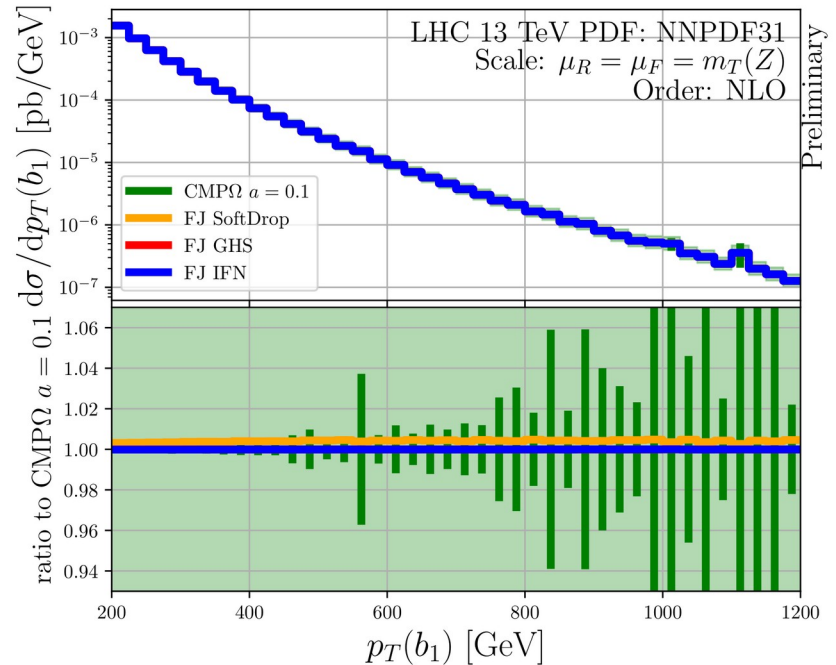
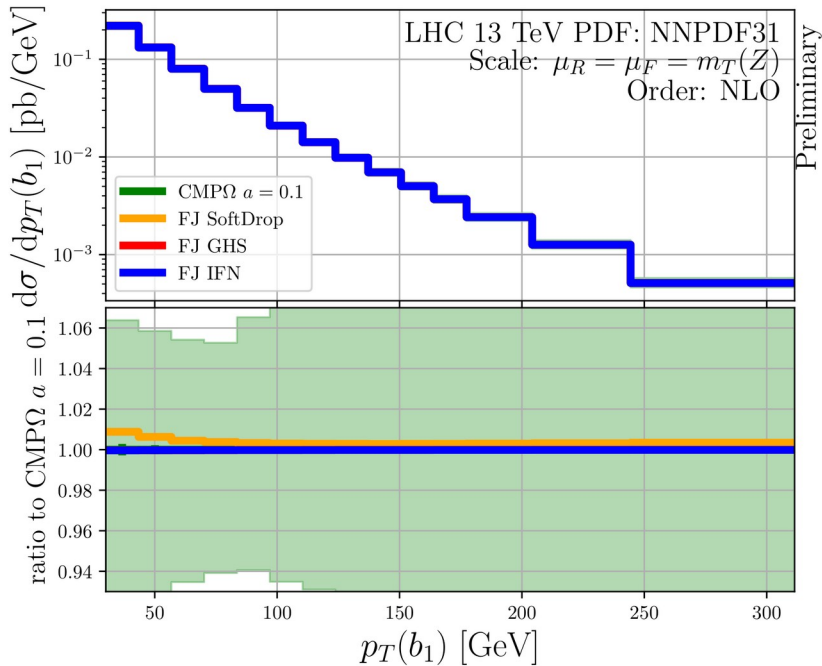


- @13 TeV
- Algorithms: Flavour anti-kT (**CMP**), Flavour dressing (**GHS**), Interleaved flavour neutralisation (**IFN**), Soft-drop (**SDF**)
- pQCD computations: up to NNLO QCD in the  $n_f=5$  scheme  $\rightarrow$  massless b, c quarks
- NLO PS matched calculations:
  - SHERPA (massive quarks, dipole)
  - HERWIG7 (massive quarks, angular) and HERWIG7 (massless quarks, dipole)
- Parton-level and Particle-level

Many people contribute to this:

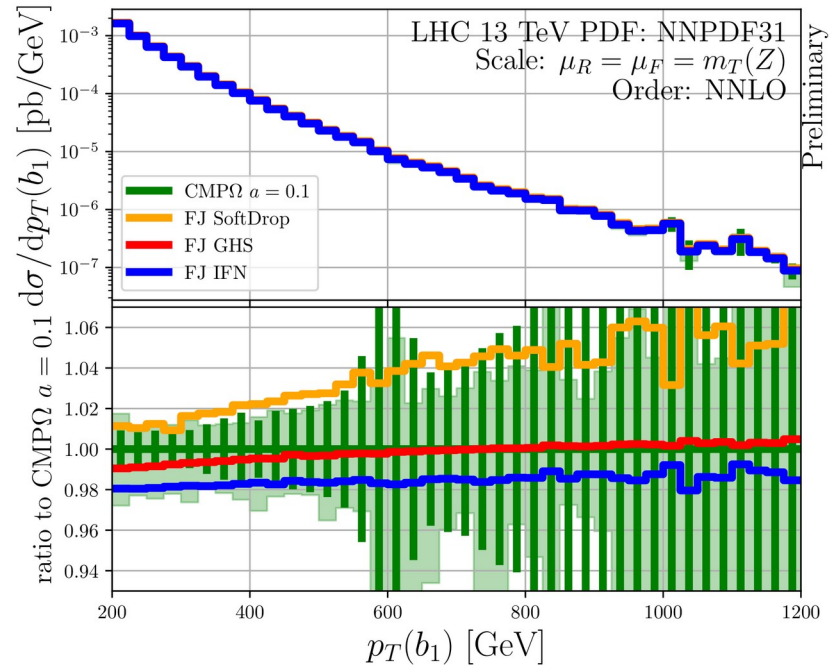
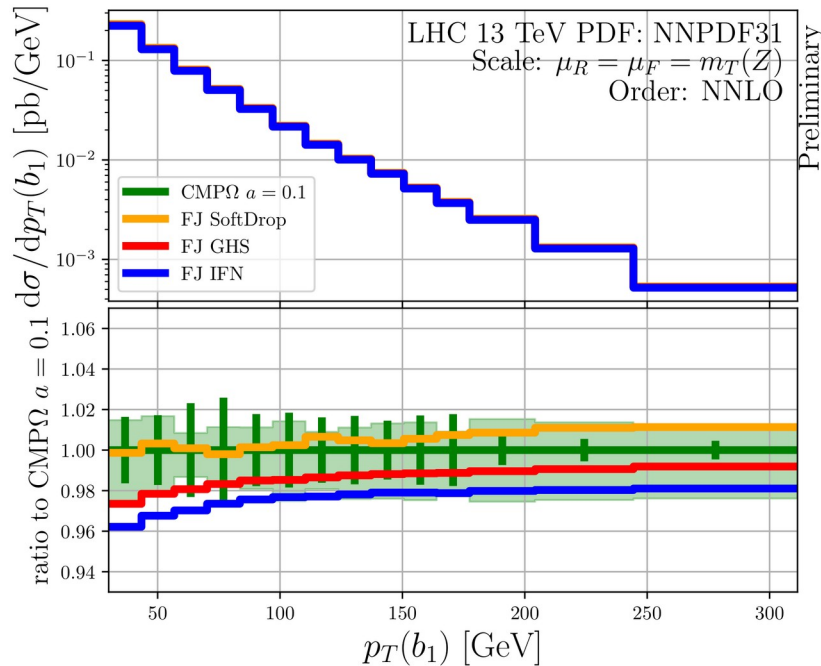
Simone Marzani, Arnd Behring, Daniel Reichelt, James Whitehead, Andrzej Siódmok, Ludovic Scyboz, Gavin Salam, Ezra Lesser, Giovanni Stagnitto, Rene Poncelet, ...

# Fixed-order comparisons NLO QCD



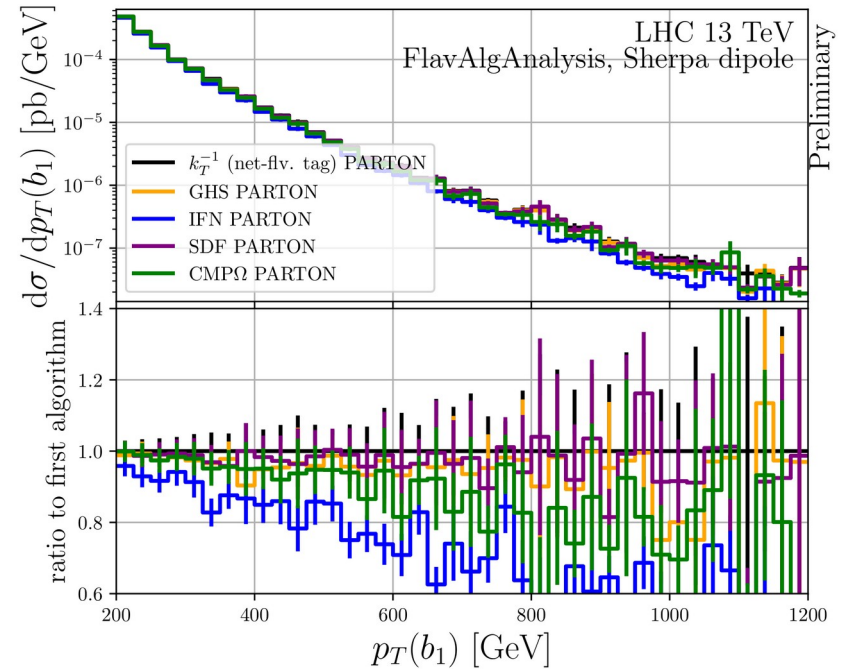
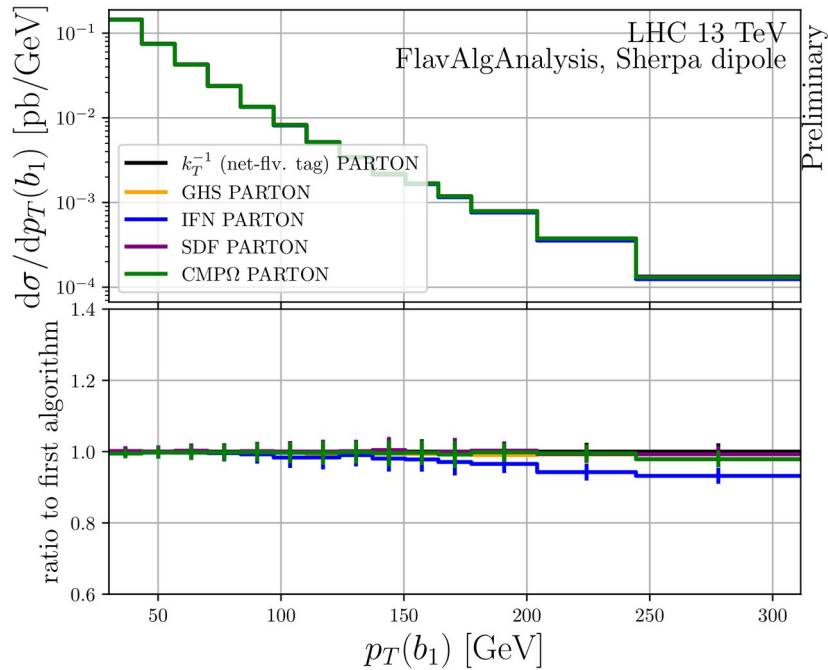
- minimal differences at FO 2-5%
- overall consistent definition of the "hard" object
- other processes tested: WH, W+charm, ttbar+decays

# Fixed-order comparisons NNLO QCD



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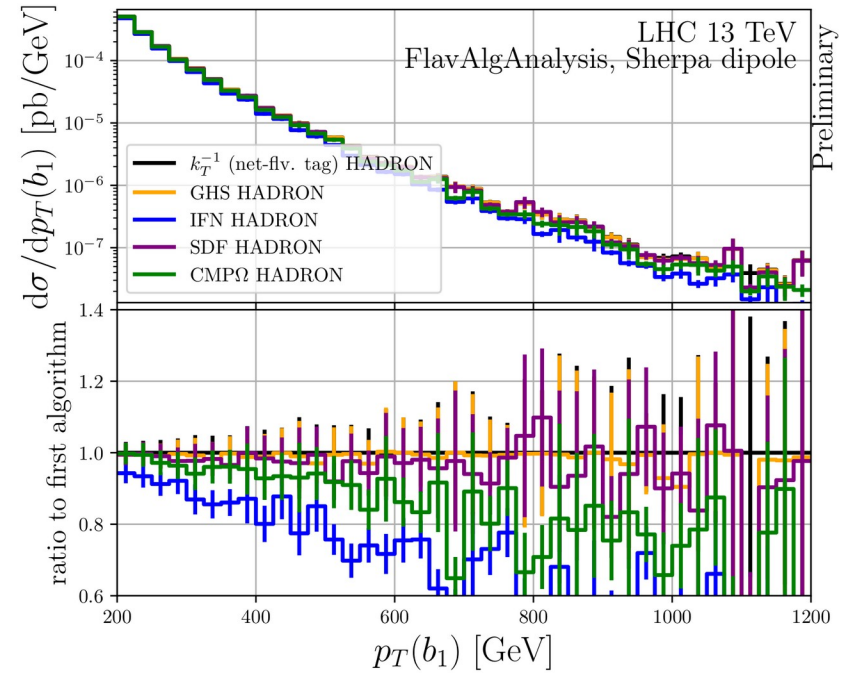
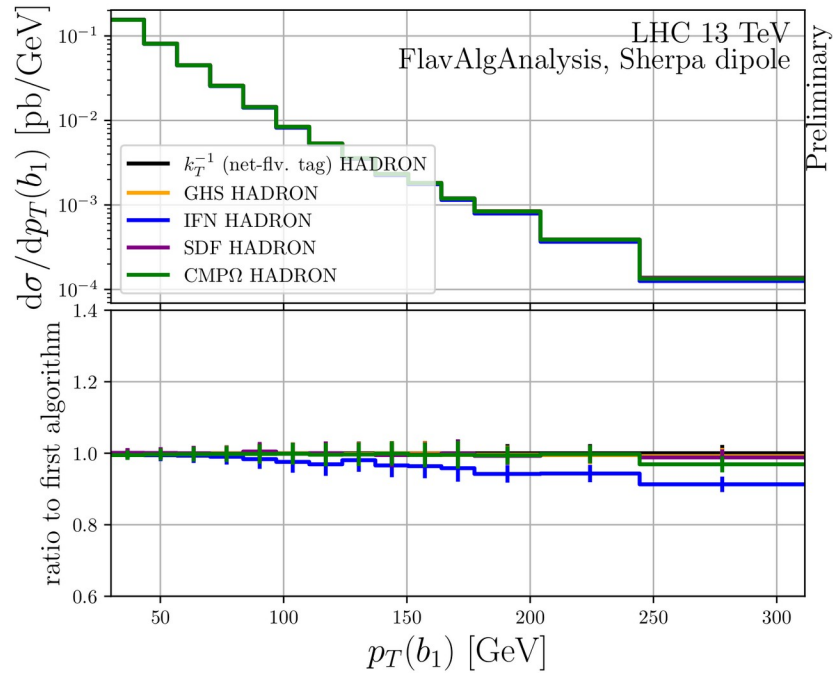
# NLO+PS at parton-level



Larger differences  $\rightarrow$  in particular in high  $p_T$  tail

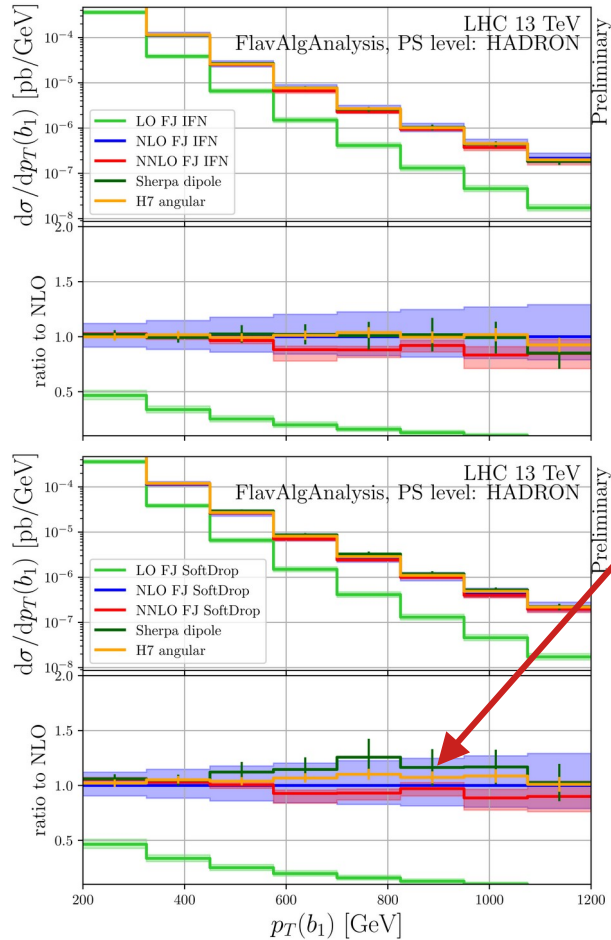
$\rightarrow$  IFN removes flavour more aggressive than  $\text{CMP} > \text{GHS} > \text{SDF}$

# NLO+PS at particle-level

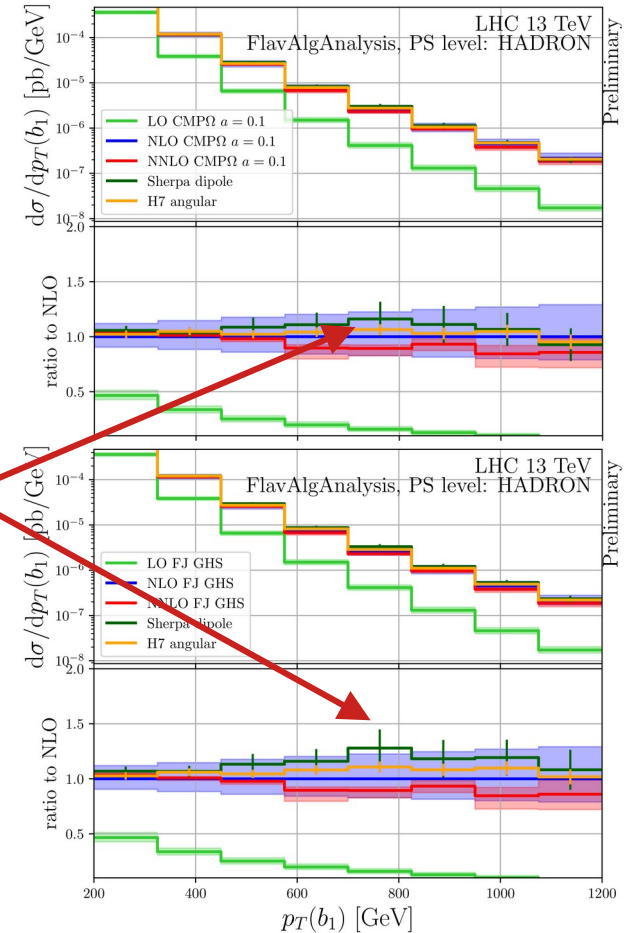


Larger differences  $\rightarrow$  in particular in high  $p_T$  tail  
 $\rightarrow$  observations insensitive to hadronisation effects

# Fixed-order and NLO+PS comparisons: b-jets

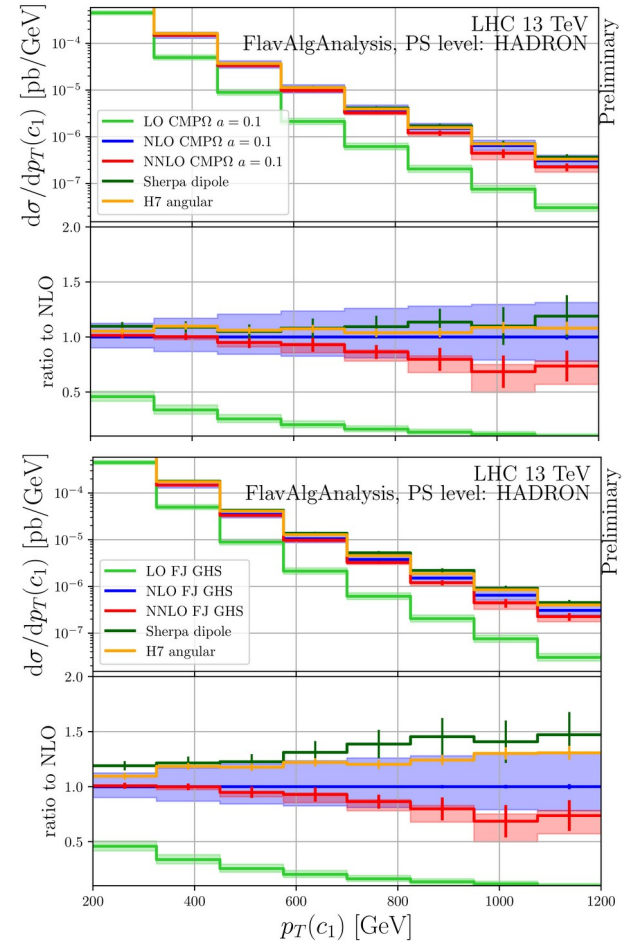
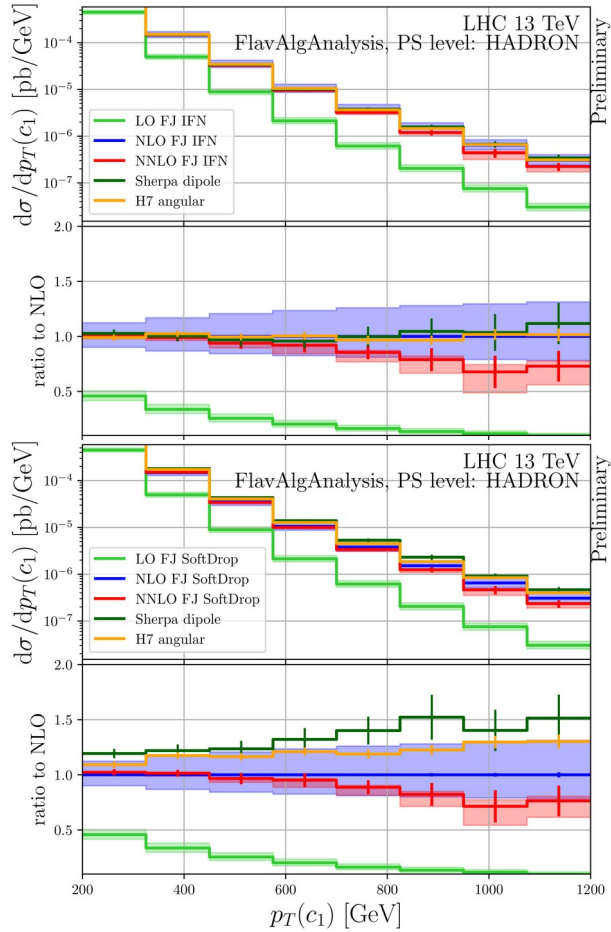


- Overall good agreement
- SDF, CMP & GHS show some shape & PS model dependence
- IFN more stable
- H7 and SHERPA give consistent results





# Fixed-order and NLO+PS comparisons: c-jets

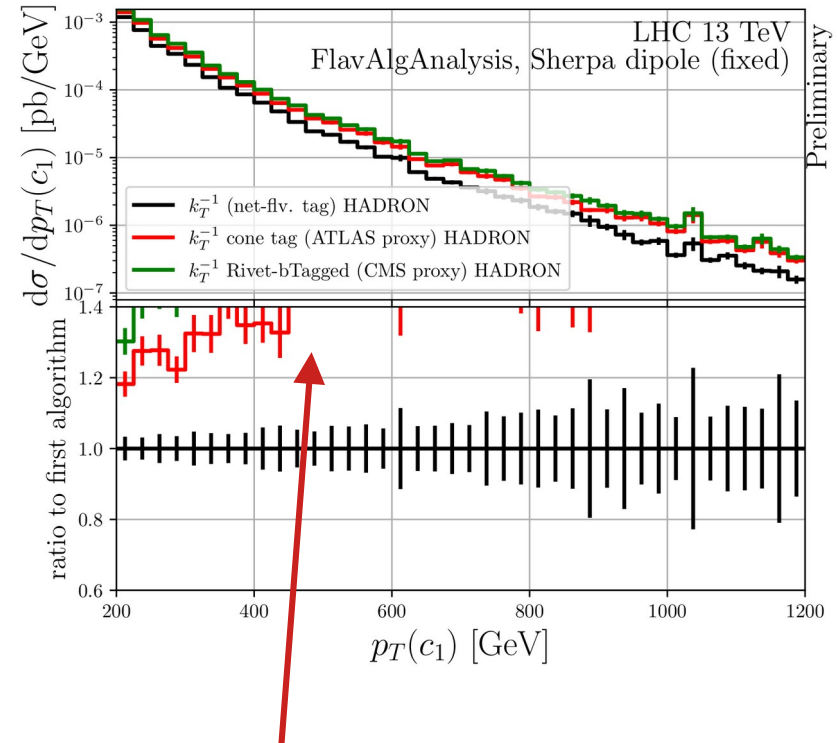
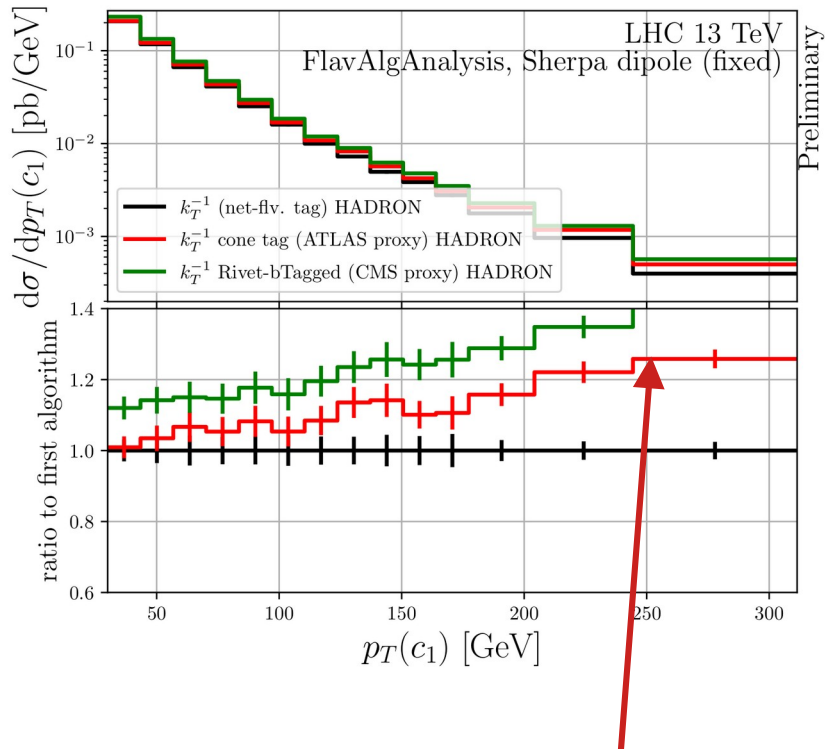


Similar to b-jets  
but differences  
enhanced

→ smaller mass leads to  
larger flavour abundance

→ enhances sensitivity to  
flavour treatment

# Comparison of experimental tagging



Net-flavour tagging makes already a huge difference!  
Driven by  $g \rightarrow cc$  splittings

# Towards experimental implementation

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- The flavoured jet algorithms require detailed flavour information
  - flavour algorithms difficult to implement experimentally
  - Limited by detector-resolution & efficiencies!
- 1) Unfolding (that is done so far):
  - $g \rightarrow b \bar{b}$  splitting if both  $b$ 's hadronise to B-hadrons (this is different to  $b \bar{b} = g$  @ fixed order)
  - Hadronisation/non-perturbative models
  - Unfolding corrections can be sizeable  $O(5-10\%)$  and relies on IR sensitive anti-kT
- 2) Improvement on experimental side:
  - Potential improvements if  $g \rightarrow b \bar{b}$  splittings can be captured experimentally
- 3) Using IRC-safe truth labels in ML – tagger training

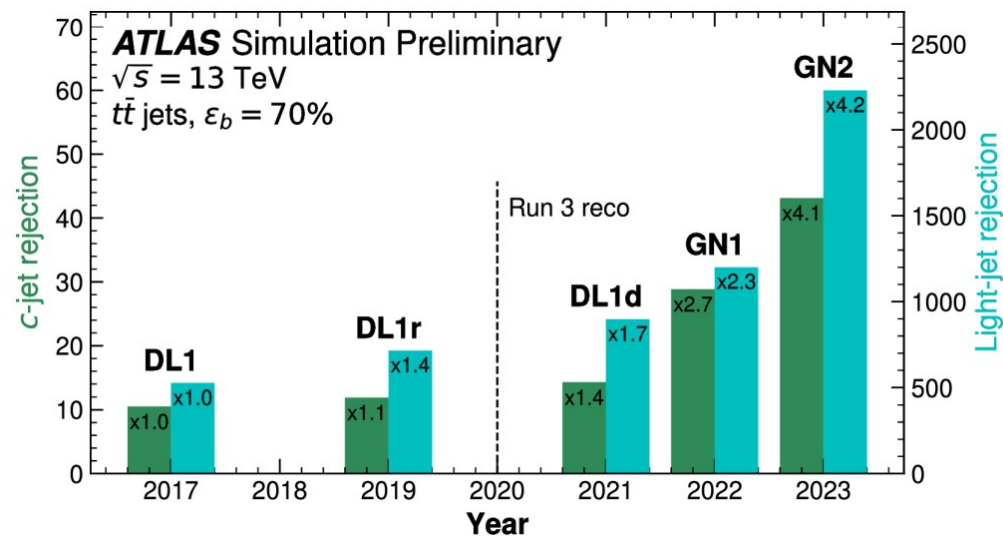
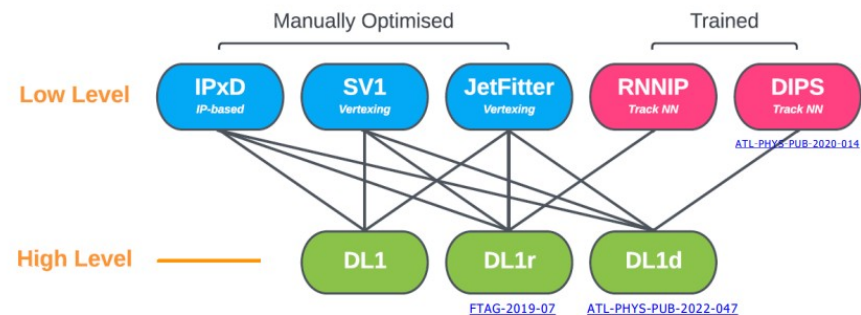
# Experimental b/c-tagging with NN

## Using NN to perform b-tagging

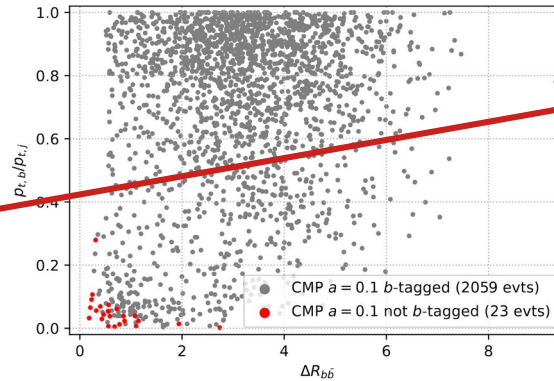
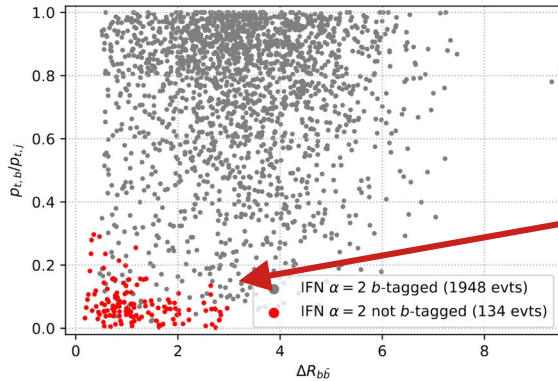
- Many Run II/III analysis use already NN based taggers
- For example ATLAS: DL1
  - uses precomputed low-level infos
- Next generation will directly use hit, track and jet information
  - further performance boost

The truth level information comes (partially or indirectly) from MC simulations

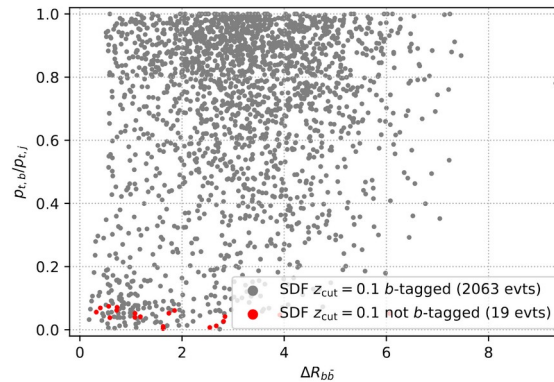
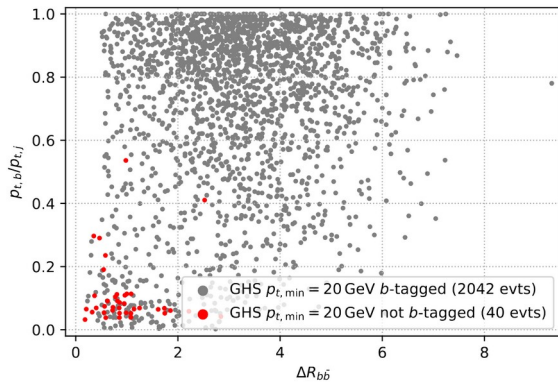
Credit: Arnaud Duperrin (DIS23 talk)



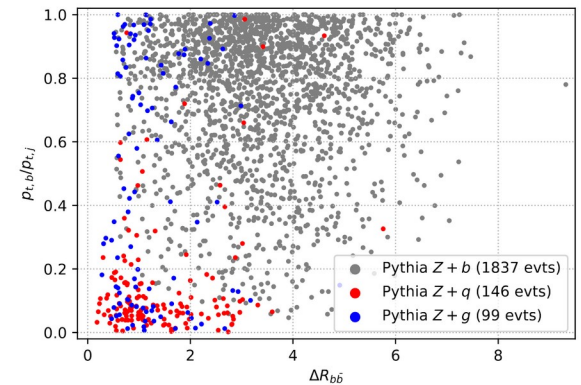
# Truth-level input



Red dots show jets that would be removed w.r.t. to anti-kT



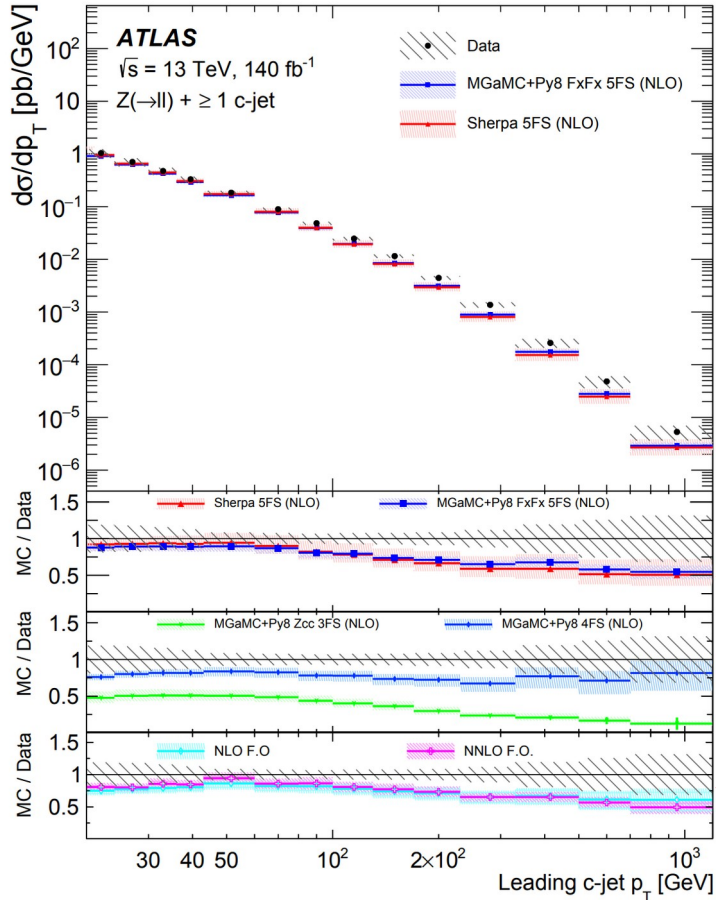
Red dots show 'light' jets



Take away:  
Flavoured algorithms remove  
**wrongly tagged jets!**

[Thanks to Ludovic Scyboz & Gavin Salam]

# Example flavoured jet measurement



Measurements of the production cross-section for a Z boson in association with b- or c-jets in proton-proton collisions at  $\sqrt{s}=13 \text{ TeV}$  with the ATLAS detector, 2403.15093

- Using unfolding to compare to GHS algorithm

Clear mismodelling of high- $p_T$  tails  
→ likely to due  $g \rightarrow b\bar{b}$  splittings

# Take home messages

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- Accurate modelling of (heavy) flavour jets requires improvement on the jet definition  
→ needed for precision phenomenology
- New flavoured jet-algorithms provide IRC safe definitions
- Les Houches Study to study qualitative and quantitative differences between proposals  
→ implementation in fastJet framework
- Experimental implementation still an open questions
  - Unfolding? → Large uncertainties (still uses the IRC unsafe anti-kT jets)
  - Improvement on tagging procedures? Challenging! (maybe  $g \rightarrow b\bar{b}$  tagging?)
  - Truth label for ML-tagger training? Seems a sensible way forward!

Thanks to all the contributors to the Les Houches study!

# Backup

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# LHC precision computations with flavoured jets

## Associated Higgs production + decays in b-quarks:

**Associated production of a Higgs boson decaying into bottom quarks at the LHC in full NNLO QCD**

Ferrera, Somogyi, Tramontano 1705.10304

**NNLO QCD corrections to associated WH production and  $H \rightarrow b\bar{b}$  decay**

Caola, Luisoni, Melnikov, Röntsch 1712.06954

**Associated production of a Higgs boson decaying into bottom quarks and a weak vector boson decaying leptonically at NNLO in QCD**

Gauld, Gehrmann-De Ridder, Glover, Huss, Majer 1907.05836

**Bottom quark mass effects in associated WH production with the  $H \rightarrow b\bar{b}$  decay through NNLO QCD**

Behring, Bizoń, Caola, Melnikov, Röntsch 2003.08321

**VH + jet production in hadron-hadron collisions up to order  $\alpha_s^3$  in perturbative QCD**

Gauld, Gehrmann-De Ridder, Glover, Huss, Majer 2110.12992

## +Partonshower:

**NNLOPS accurate associated HZ production with  $H \rightarrow b\bar{b}$  decay at NLO**

Astill, Bizoń, Re, Zanderighi 1804.08141

**NNLOPS description of the  $H \rightarrow b\bar{b}$  decay with MiNLO**

Bizoń, Re, Zanderighi 1912.09982

**Next-to-next-to-leading order event generation for VH production with  $H \rightarrow b\bar{b}$  decay**

Zanoli, Chiesa, Re, Wiesemann, Zanderighi 2112.04168

# LHC precision computations with flavoured jets

## Vector + flavoured jet(s) production:

**NLO QCD predictions for  $Wb\bar{b}$  production in association with up to three light jets at the LHC**

Anger, Cordero, Ita, Sotnikov 1712.05721

**Predictions for Z-Boson Production in Association with a b-jet at  $O(\alpha_s^3)$**

Gauld, Gehrmann-De Ridder, Glover, Huss, Majer 2005.03016

**NNLO QCD predictions for  $W+c$ -jet production at the LHC,**

Czakon, Mitov, Pellen, Poncelet 2011.01011

**NNLO QCD corrections to  $Wb\bar{b}$  production at the LHC,**

Hartanto, Poncelet, Popescu, Zoia 2205.01687

**A detailed investigation of  $W+c$ -jet at the LHC,**

Czakon, Mitov, Pellen, Poncelet 2212.00467

**Associated production of a W boson and massive bottom quarks at next-to-next-to-leading order in QCD,**

Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini, 2212.04954

**NNLO QCD predictions for Z-boson production in association with a charm jet within the LHCb fiducial region**

Gauld, Gehrmann-De Ridder, Glover, Huss, Rodriguez Garcia, Stagnitto 2302.12844

**Precise QCD predictions for W-boson production in association with a charm jets**

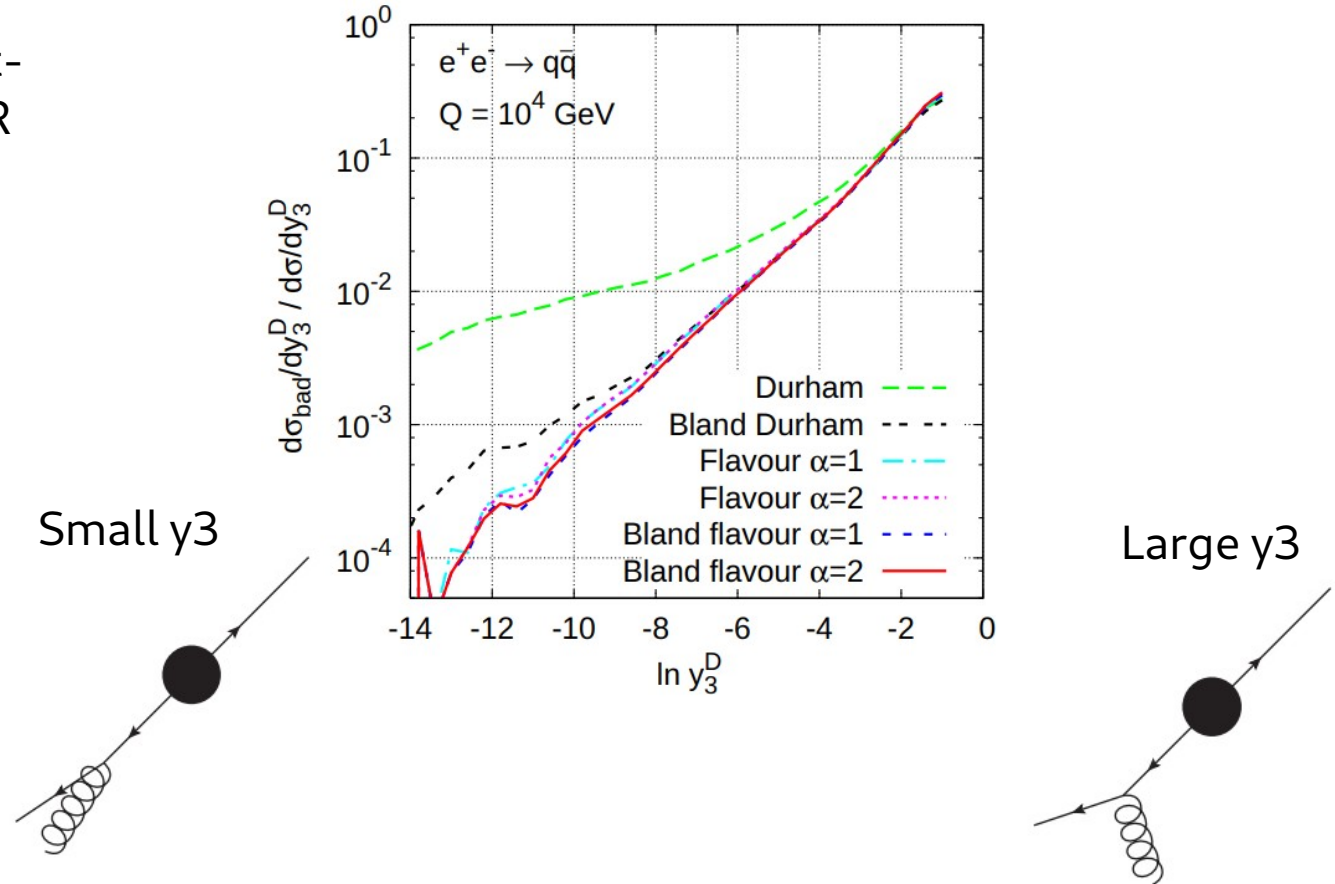
Gehrmann-De Ridder, Gehrmann, Glover, Huss, Garcia, Stagnitto, 2311.14991

# Tests of IR safety

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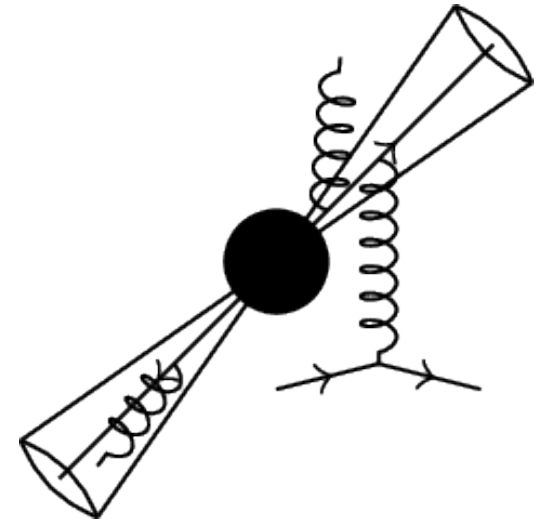
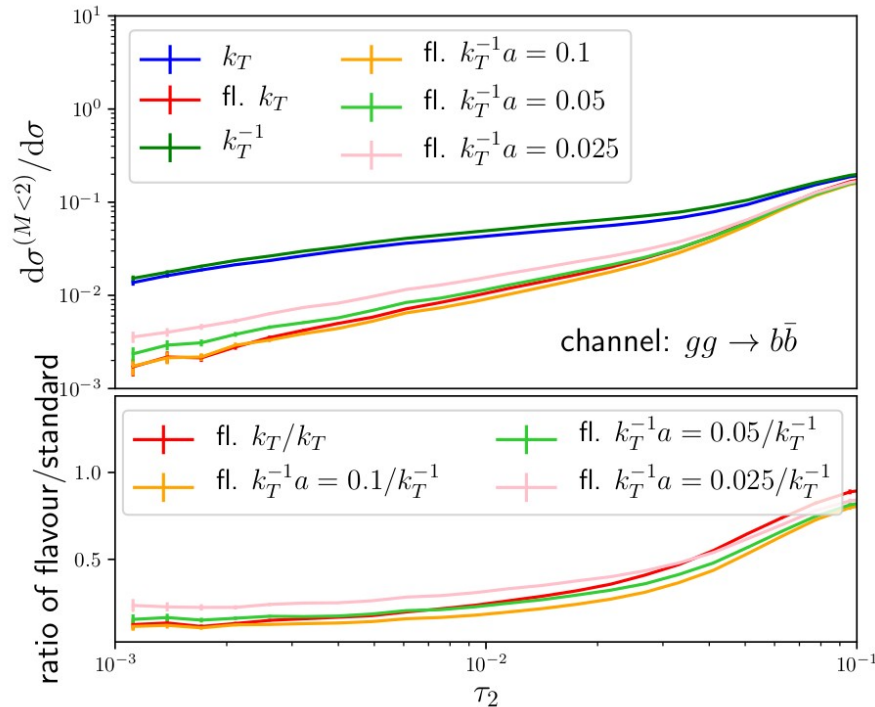
# Tests of IR safety

- Rate of bad-identified jet-flavour as a function of IR sensitive variable
- Parton-shower to model many emissions



# Tests of IR safety with parton showers

- In the di-jet limit the flavour needs to correspond to tree level flavours
- misidentification rate needs to vanish in di-jet back-to-back limit
- IR sensitive observable 2-jettiness



# Tests of IR safety with NNLO FO computations

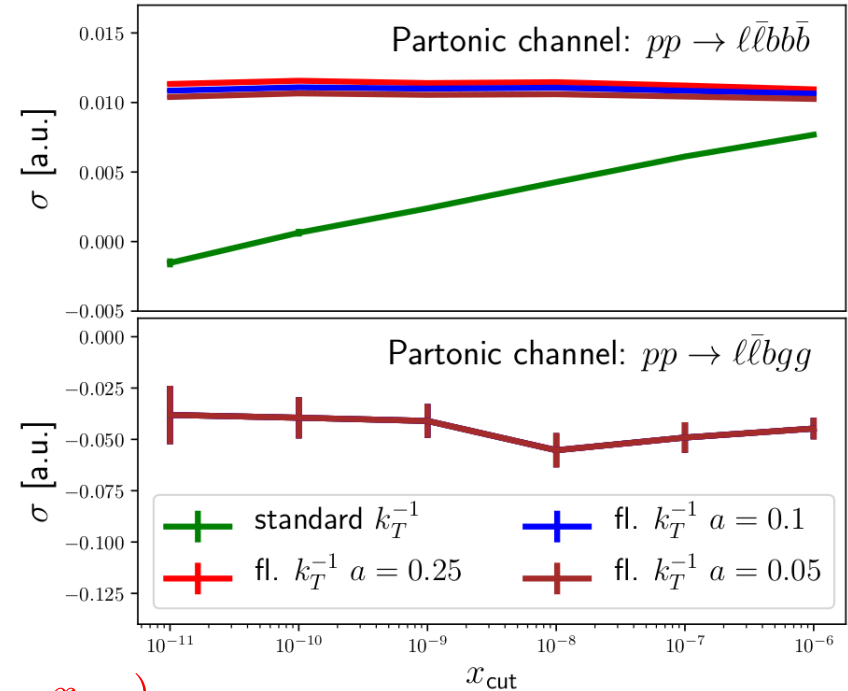
IR sensitivity of jet cross sections on (technical)  
IR regulating parameter  $x$

In the limit  $x_{\text{cut}} \rightarrow 0$ :

IR safe jet flavour  $\rightarrow$  no dependence on  $x_{\text{cut}}$

IR non-safe jet flavour  $\rightarrow$  logarithmic divergent

$$d\sigma \ni d\Phi_{n+2} \left( \text{Diagram 1} \right) F_{n+2} + \dots + d\tilde{\Phi}_{n+2} \mathcal{S}_2 \left( \text{Diagram 2} \right) F_n \theta(x - x_{\text{cut}})$$

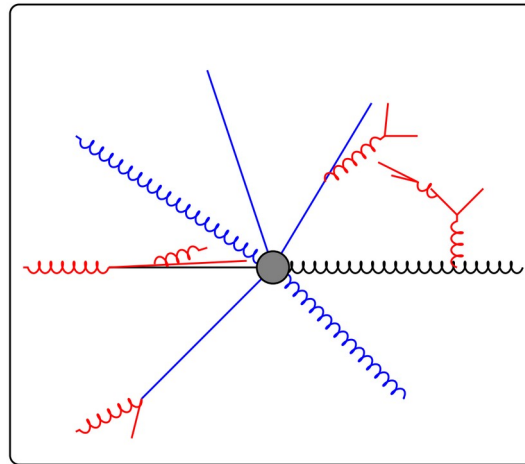


# More tests...

**Flavoured jets with exact anti-kT kinematics and tests of infrared and collinear safety**  
Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler 2306.07314

- IRC safety testing suite:

Credit: Ludo Scyboz



Set of hard jets  
 $\mathcal{J}_{\text{hard}} = \{(p_1, f_1), \dots\}$

$\stackrel{!}{=}$

Set of hard+IRC jets  
 $\mathcal{J}_{\text{hard+IRC}} = \{(\tilde{p}_1, \tilde{f}_1), \dots\}$

# More tests...

Flavoured jets with exact anti-kT kinematics and tests of infrared and collinear safety  
 Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler 2306.07314

order relative to Born		anti- $k_t$	flav- $k_t$ ( $\alpha = 2$ )	CMP	GHS $_{\alpha,\beta}$ (2, 2)	anti- $k_t$ +IFN $_{\alpha}$	C/A+IFN $_{\alpha}$
$\alpha_s$	FHC	✓	✓	✓	✓	✓	✓
	IHC	✓	✓	✓	✓	✓	✓
$\alpha_s^2$	FDS	✗ <b>II B</b>	✓	✓	✓	✓	✓
	IDS	✗ <b>II B</b>	✓	✓	✓	✓	✓
	FHC×IHC	✓	✓	✓	✓	✓	✓
	IHC <sup>2</sup>	✓	✓	✗ <b>C2</b>	✓	✓	✓
	FHC <sup>2</sup>	✓	✓	✓	✗ <b>C4</b>	✓	✓
$\alpha_s^3$	IHC×IDS		~ <b>C1</b>	✗ <b>C3</b>	~ <b>C1</b>	✓	✓
	rest					✓	✓
$\alpha_s^4$	IDS×FDS				✗ <b>C5</b>	✓	✓
	rest					✓	✓
$\alpha_s^5$						✓	✓
$\alpha_s^6$						✓	✓



# Improved distance for CMP/flavour anti-kT

Issue for double collinear limits wrt. to initial states

Many thanks to  
Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler



2306.07314

if  $\gamma_{12} - \gamma_3 < R$  ☹️

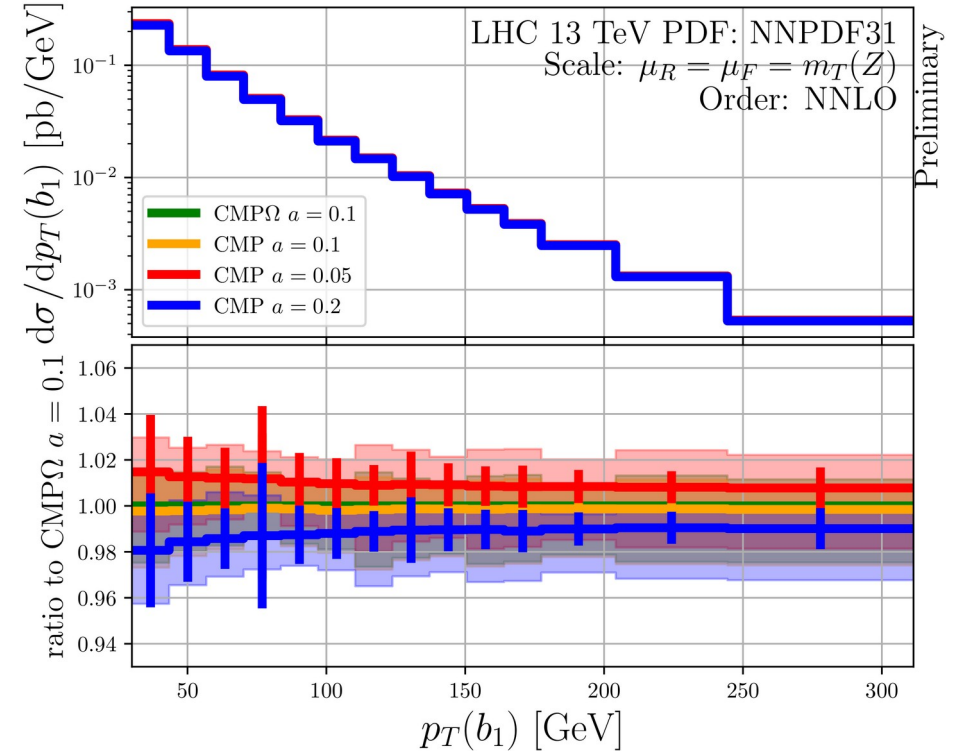
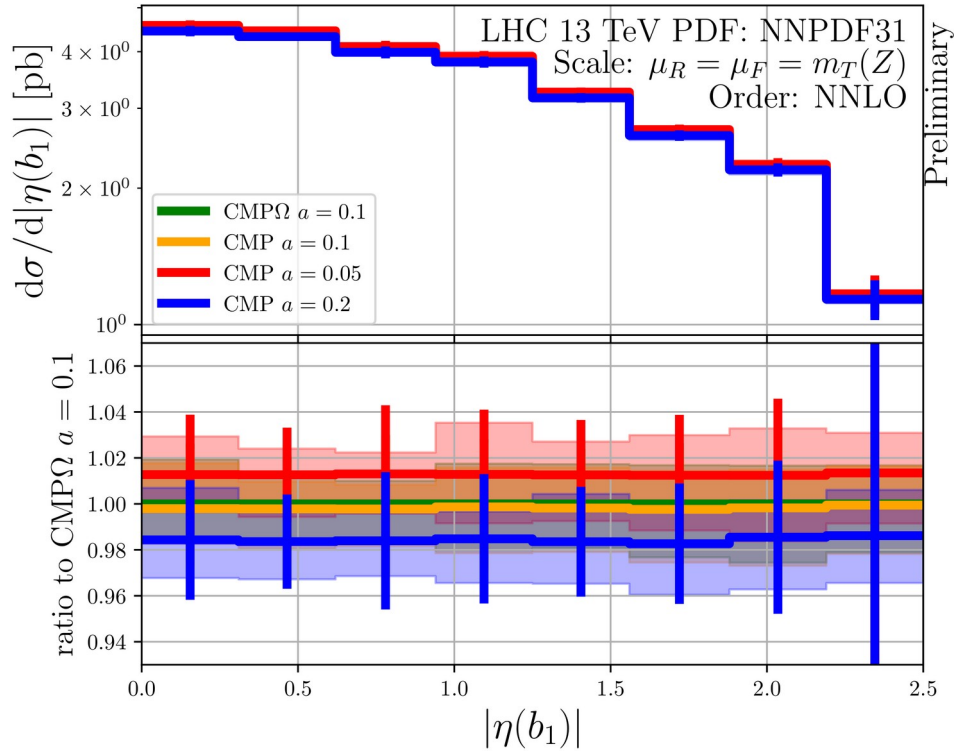
$$S_{ij} \equiv 1 - \theta(1 - \kappa_{ij}) \cos\left(\frac{\pi}{2} \kappa_{ij}\right) \quad \text{with} \quad \kappa_{ij} \equiv \frac{1}{a} \frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,\text{max}}^2}$$

Their proposal:  $S_{ij} \rightarrow \bar{S}_{ij} = S_{ij} \frac{\Omega_{ij}^2}{\Delta R_{ij}^2}$        $\Omega_{ik}^2 \equiv 2 \left[ \frac{1}{\omega^2} (\cosh(\omega \Delta y_{ik}) - 1) - (\cos \Delta \phi_{ik} - 1) \right]$

# Flavour anti-kT: impact of $\Omega_{ij}$

Calculations performed with sector-improved residue subtraction scheme  
1408.2500 & 1907.12911

Les Houches Jet Flavour WG



**Negligible difference between CMPΩ and CMP**

# Benchmark process: Z+b-jet

Well studied up to  $\mathcal{O}(\alpha_s^3)$ :

**Predictions for Z-Boson Production in Association with a b-jet at  $\mathcal{O}(\alpha_s^3)$ ,**  
 Gauld, Gehrmann-De Ridder, Glover, Huss, Majer 2005.03016

- Flavour-kT algorithm
- Unfolding of experimental data (RooUnfold, bin-by-bin unfolding)
- Matching between four- and five-flavour schemes (FONLL)

$$d\sigma^{\text{FONLL}} = d\sigma^{5\text{fs}} + (d\sigma_{m_b}^{4\text{fs}} - d\sigma_{m_b \rightarrow 0}^{4\text{fs}})$$

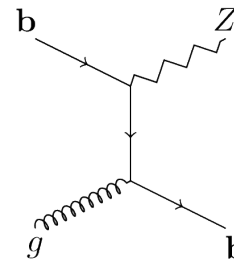
- CMS measurement @ 8 TeV

**Measurements of the associated production of a Z boson and b jets in pp collisions at  $\sqrt{s} = 8 \text{ TeV}$ , CMS 1611.06507**

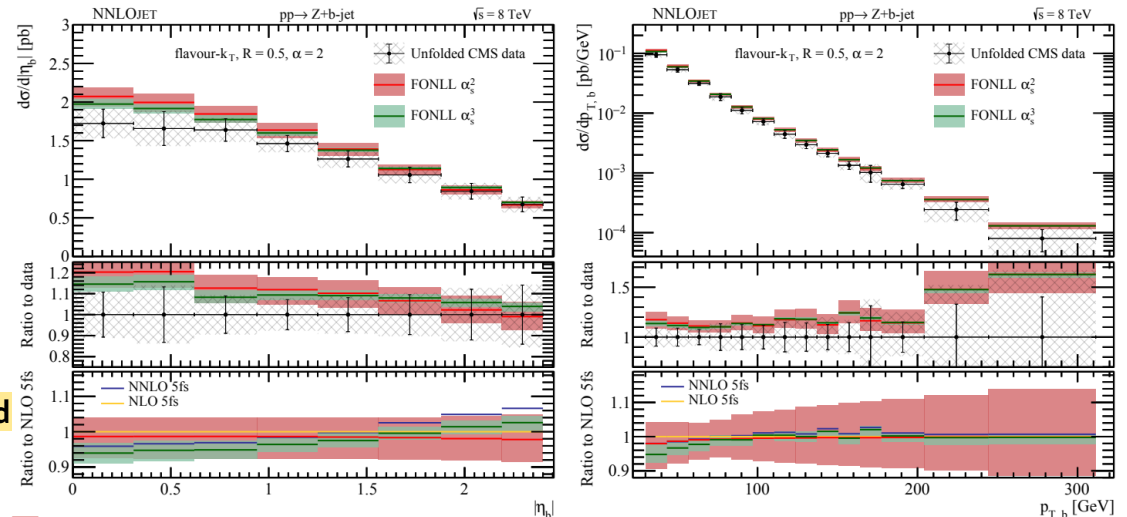
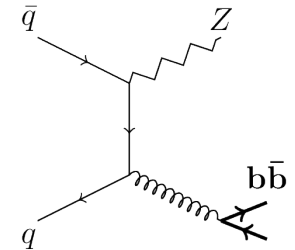
→ Ideal testing ground for flavour anti-kT

$pp \rightarrow Z(\text{ll}) + \text{b-jet}$

5fs:



4fs:



# Bin-by-bin unfolding

Estimation of hadronisation and experimental tagging corrections  
→ NLO + PS (Madraph+Pythia8)

Unfolding factor = NLO+PS (had = Off) / NLO+PS (had = On)

