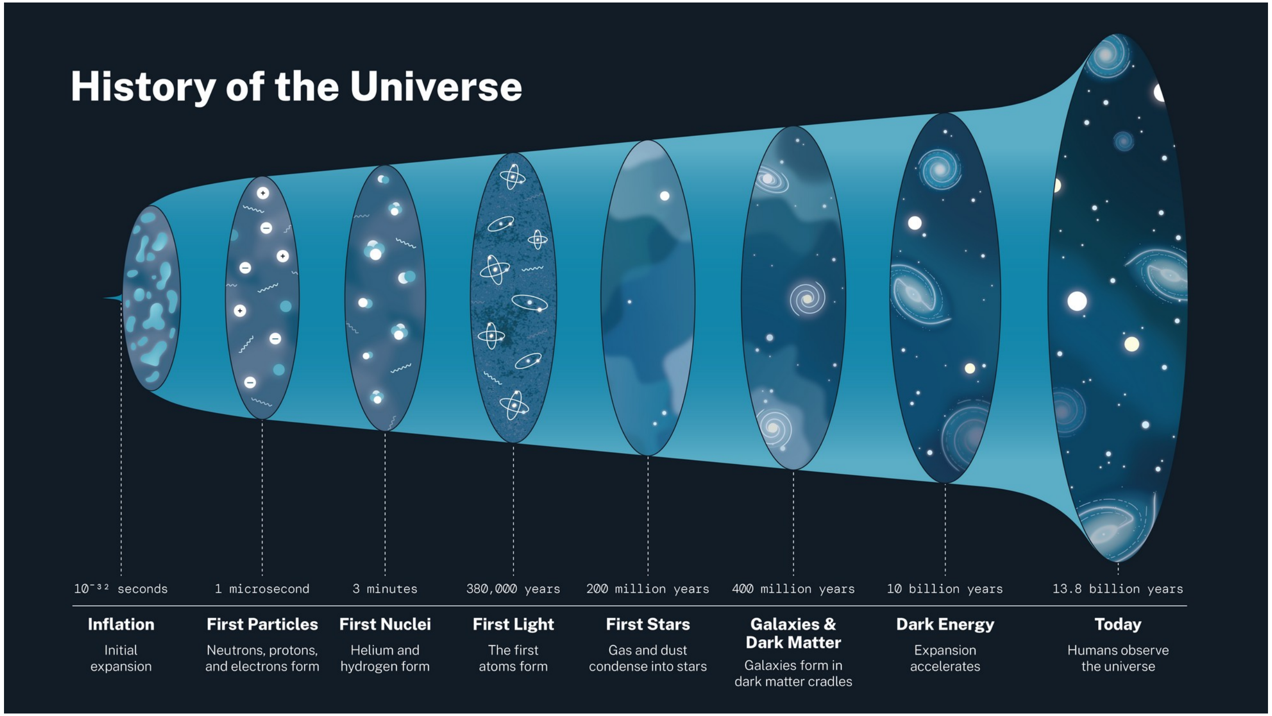
High precision prediction for multi-scale processes at the LHC

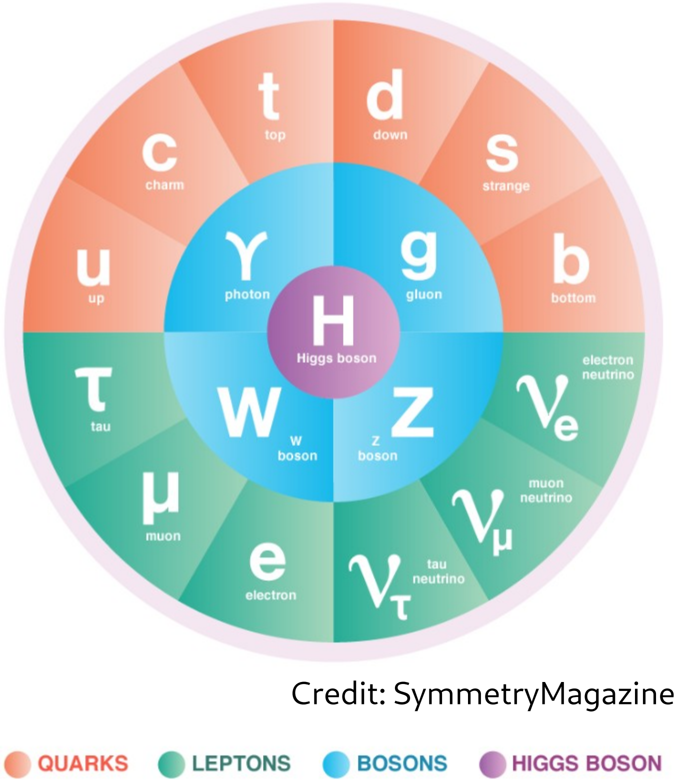
Rene Poncelet

THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

What is the universe made of and where does it come from?



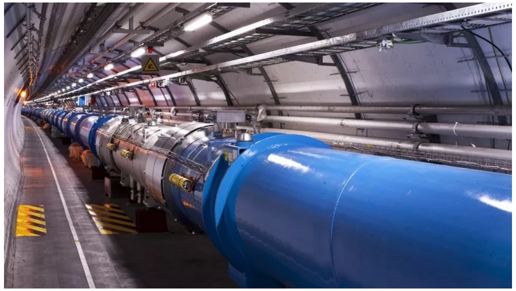
Credit: NASA



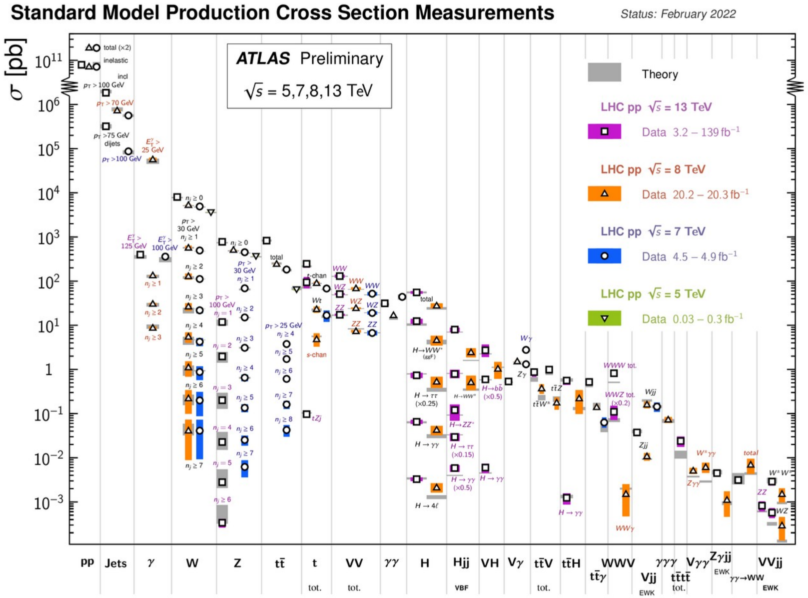
Credit: SymmetryMagazine

What are the fundamental building blocks of matter?

Scattering experiments

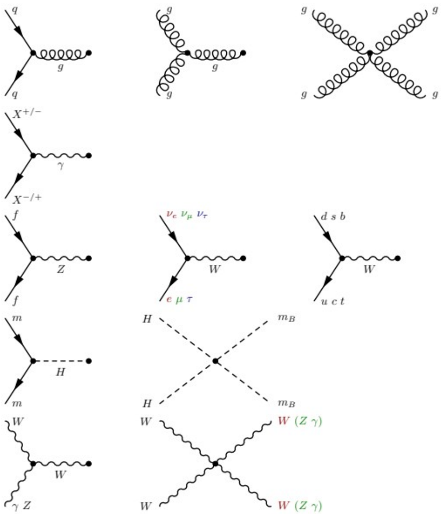


Credit: CERN



Credit: ATLAS

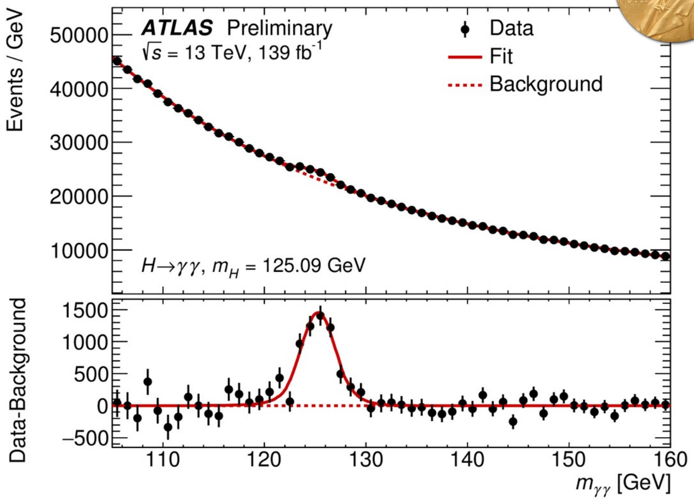
Theory/Model



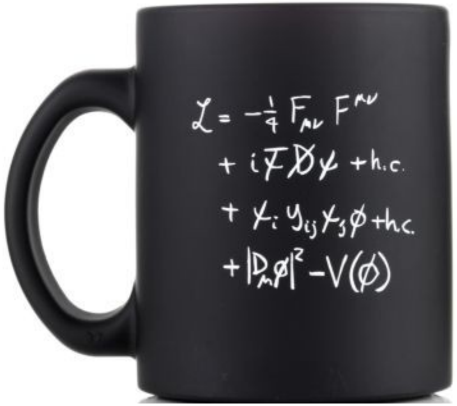
Credit: Jack Lindon, CERN

Standard Model of Particle Physics and beyond

Higgs discovery 2012



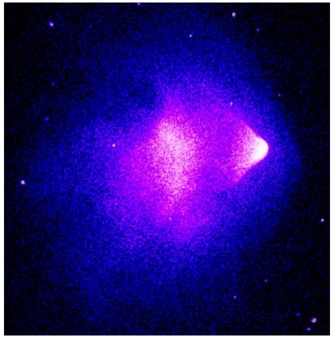
Credit: ATLAS



Credit: CERN

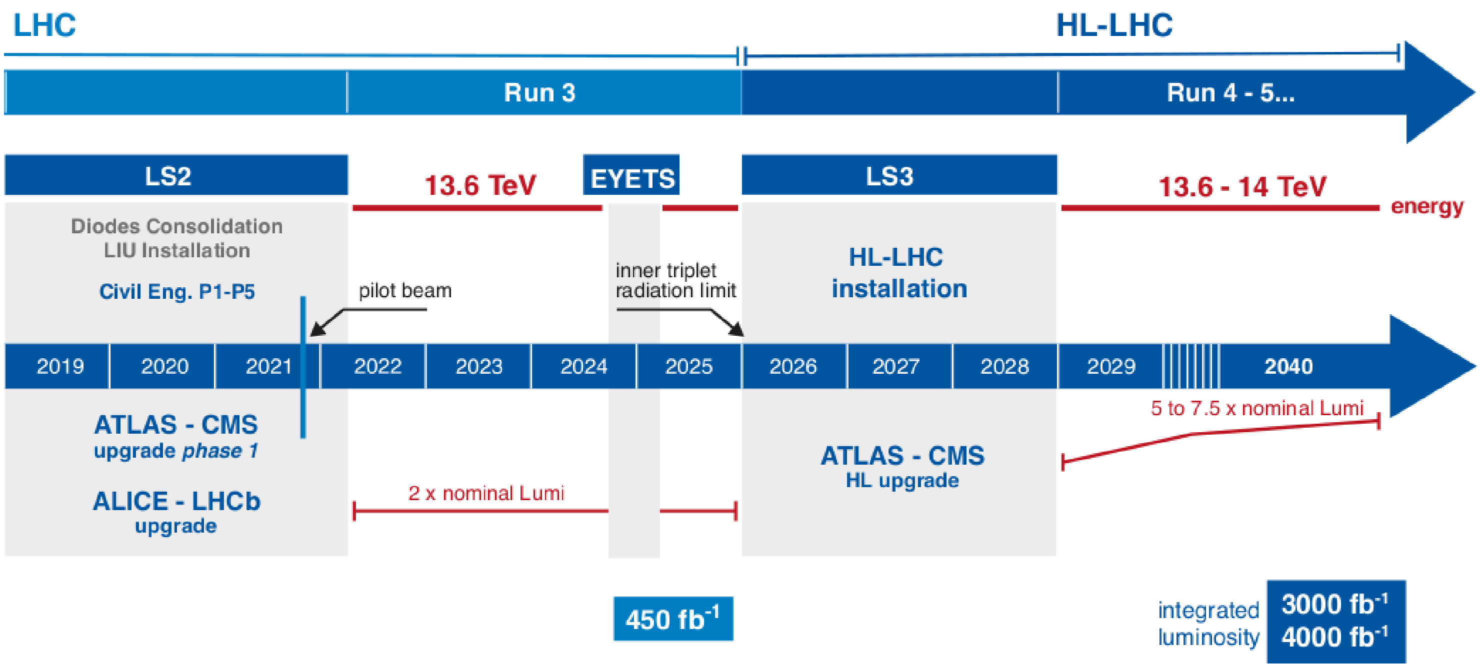
BUT:

- Is the Higgs a fundamental scalar?
- What is dark matter?
- Why is there a matter-anti-matter asymmetry?
- …



Credit: NASA

LHC Precision era and future experiments



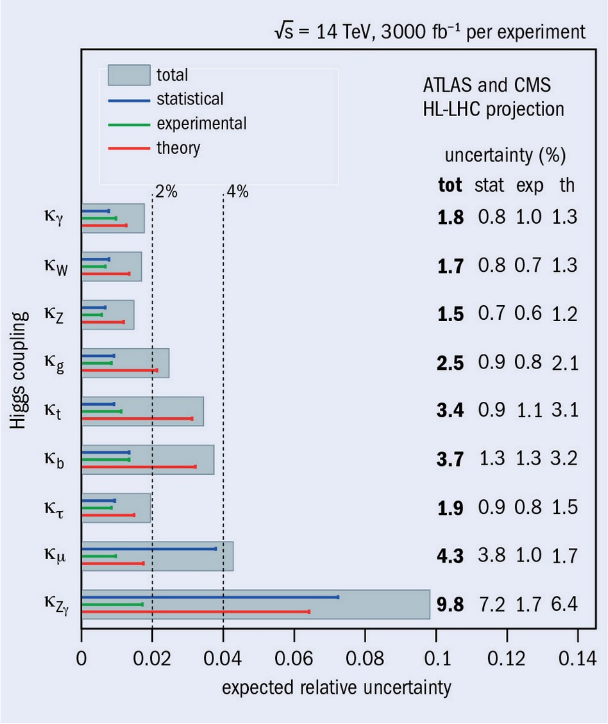
Credit: CERN

Precision phenomenology

- Populate the tails and corners of phase space to direct discover new particles/interactions.
- Constraining missing pieces such as Higgs potential (self-coupling and overall consistency of the Higgs mechanism) and missing Yukawa model
- Precision test of the EW and QCD theory: particle masses, (running of) coupling constants, PDFs, …
- Tests of the quantum nature of our universe at the highest possible energies.

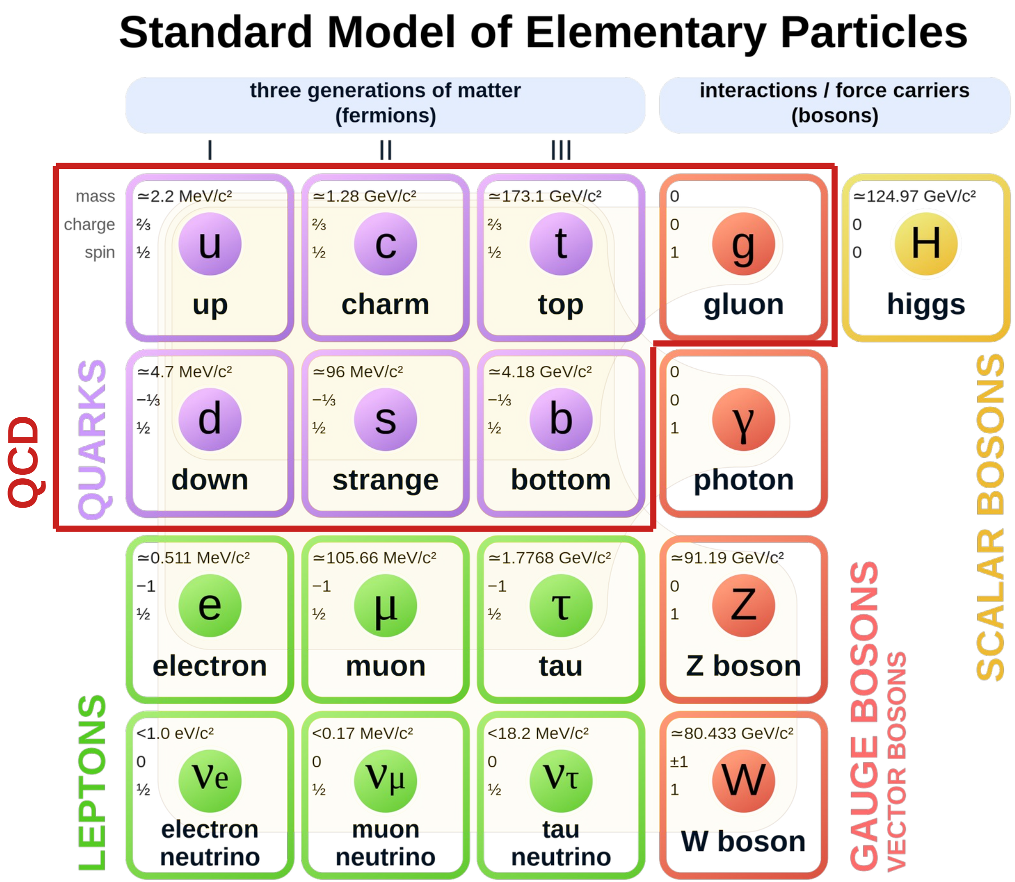
→ All these directions need precision theory input!

Example:
Projected Higgs coupling measurements



[1902.00134]

Precision era of the LHC



- At the LHC QCD is part of any process!
  1) The limiting factor in many analyses is QCD and associated uncertainties.
     → Radiative corrections indispensable
  2) How well we do know QCD? Coupling constant, running, PDFs, …
- Asymptotic freedom: high energy scattering processes allow to probe pQCD directly

$$\mathcal{L}_{\rm QCD} = \bar{q}_i(\gamma^\mu \mathcal{D}_\mu - m_i)q_i - \frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}$$

Precision predictions

Resummation

Fixed order perturbation theory

Parton-showers

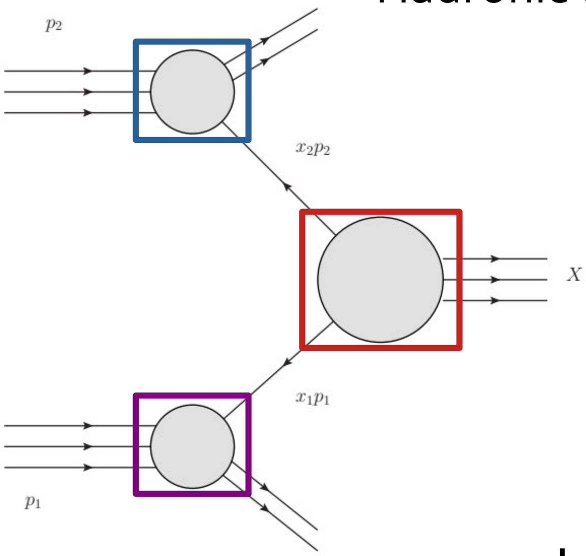
Precision theory predictions

Parametric input:
PDFs, couplings (α_s), ...

Soft physics:
MPI, colour reconnection,
...

Fragmentation/hadronisation

Precision through higher orders



Hadronic cross section in collinear factorisation:

$$\sigma_{h_1h_2\to X} = \sum_{ij}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\phi_{i,h_1}(x_1,\mu_F^2)\phi_{j,h_2}(x_2,\mu_F^2)\hat{\sigma}_{ij\to X}(\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)$$

Parton distribution functions

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \underbrace{\alpha_s^0\hat{\sigma}^{(0)}_{ab\to X}}_{\text{Leading order}} + \underbrace{\alpha_s^1\hat{\sigma}^{(1)}_{ab\to X}}_{\text{Next-to-leading order}} + \underbrace{\alpha_s^2\hat{\sigma}^{(2)}_{ab\to X}}_{\text{Next-to-next-to-leading order}} + \mathcal{O}(\alpha_s^3)$$

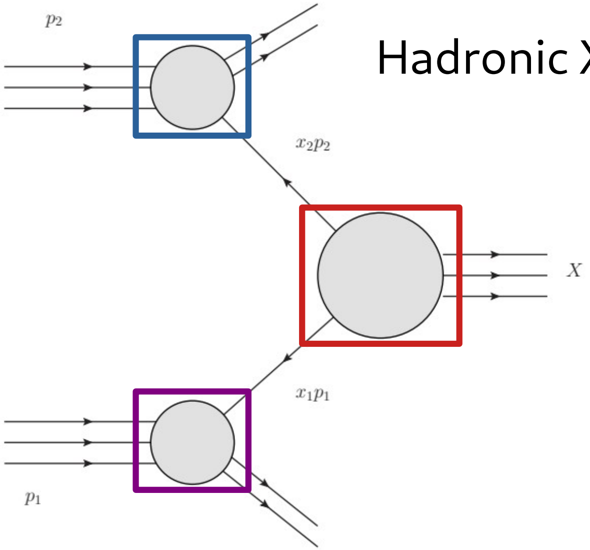
Uncertainty: Order of magnitude O(10%) O(1%)

$\alpha_s(m_Z) \approx 0.118$

Next-to-next-to-leading order QCD needed to match experimental precision!

→ In some cases even next-to-next-to-next-to-leading order!

Hadronic cross section in collinear factorization – NNLO QCD



Hadronic X-section: $\sigma_{h_1 h_2 \to X} = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \phi_{i,h_1}(x_1, \mu_F^2) \phi_{j,h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij \to X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$

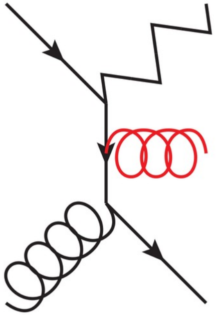
Parton distribution functions

Perturbative expansion of partonic cross section:

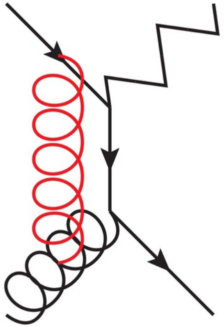
$$\hat{\sigma}_{ab \to X} = \hat{\sigma}^{(0)}_{ab \to X} + \hat{\sigma}^{(1)}_{ab \to X} + \hat{\sigma}^{(2)}_{ab \to X} + \mathcal{O}(\alpha_s^3)$$

The NLO bit: $\hat{\sigma}^{(1)}_{ab} = \hat{\sigma}^{\mathrm{R}}_{ab} + \hat{\sigma}^{\mathrm{V}}_{ab} + \hat{\sigma}^{\mathrm{C}}_{ab}$

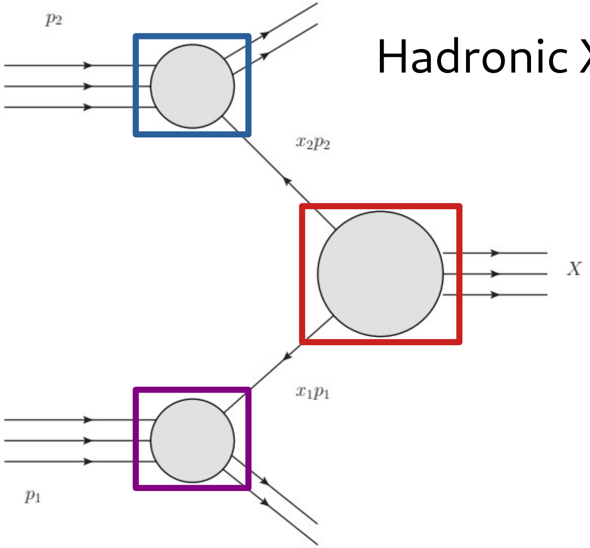
Real radiation



Virtual correction



Hadronic cross section in collinear factorization – NNLO QCD



Hadronic X-section: $\sigma_{h_1 h_2 \to X} = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \phi_{i,h_1}(x_1, \mu_F^2) \phi_{j,h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij \to X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$

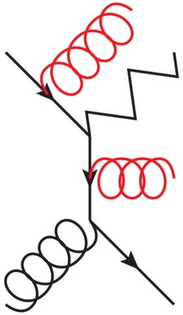
Parton distribution functions

Perturbative expansion of partonic cross section:

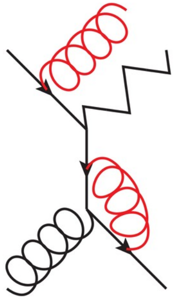
$$\hat{\sigma}_{ab \to X} = \hat{\sigma}^{(0)}_{ab \to X} + \hat{\sigma}^{(1)}_{ab \to X} + \hat{\sigma}^{(2)}_{ab \to X} + \mathcal{O}(\alpha_s^3)$$

The NNLO bit: $\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$

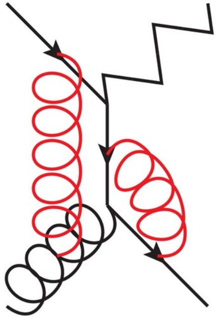
Double real radiation



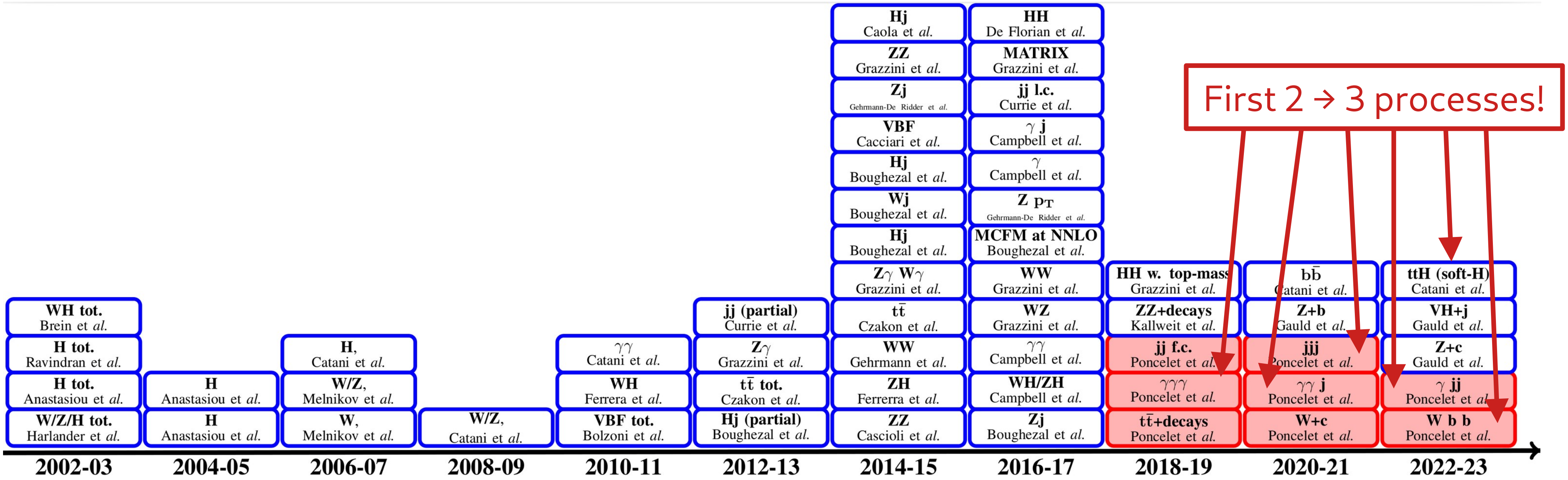
Real/Virtual correction



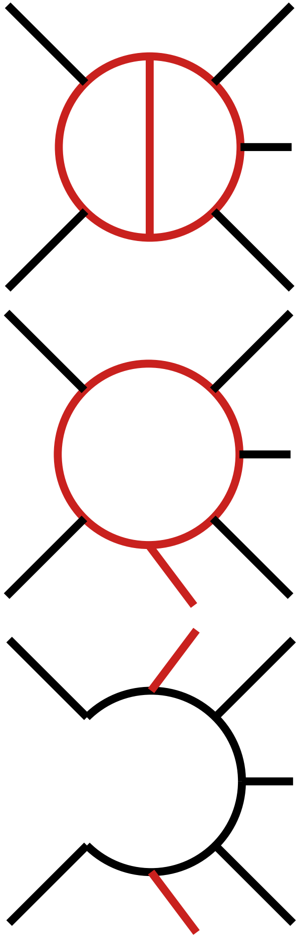
Double virtual corrections



The NNLO QCD revolution



NNLO QCD for 2→3 processes - inputs



Two-loop amplitudes

- (Non-) planar 5 point massless external states [Chawdry'19'20'21,Abreu'20'21'23,Agarwal'21'23,Badger'21'23]
  → triggered by efficient MI representation [Chicherin'20]
- 5 point with one external mass [Abreu'20,Syrrakos'20,Canko'20,Badger'21'22,Chicherin'22]

One-loop amplitudes → OpenLoops [Buccioni'19]

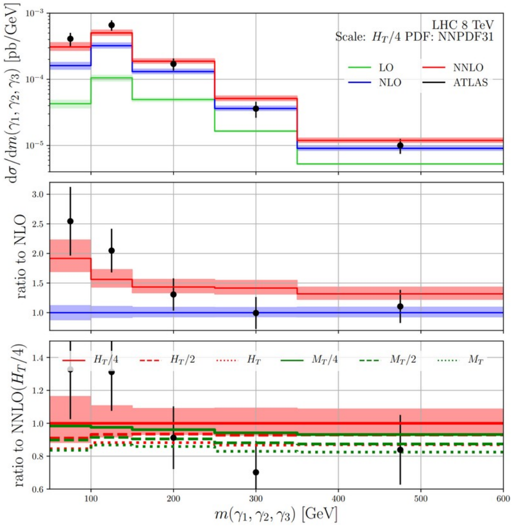
- Many legs and IR stable (soft and collinear limits)

Double-real Born amplitudes → AvHlib[Bury'15]

- IR finite cross-sections → NNLO subtraction schemes
  qT-slicing [Catani'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projetction [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17], Local Analytic [Magnea'18], **Sector-improved residue subtraction** [Czakon'10-'14,'19]

NNLO QCD cross sections for massless 2 → 3 processes

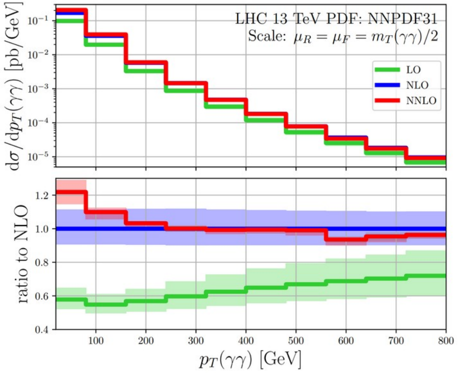
$pp \to \gamma\gamma\gamma$



Chawdhry, Czakon, Mitov, RP [1911.00479]

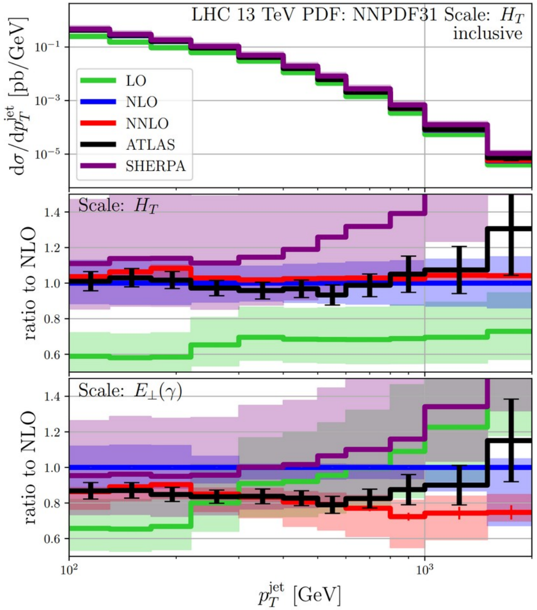
Kallweit, Sotnikov, Wiesemann [2010.04681]

$pp \to \gamma\gamma j$



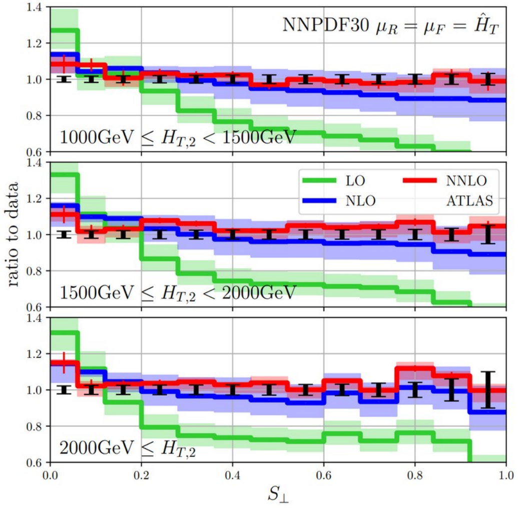
Chawdhry, Czakon, Mitov, RP [2103.04319]

$pp \to \gamma jj$



Badger, Czakon, Hartanto, Moodie, Peraro, RP, Zoia [2304.06682]

$pp \to jjj$



Czakon, Mitov, RP [2106.05331]
+ Alvarez, Cantero, Llorente [2301.01086]

Multi-jet observables

Test of pQCD and extraction of strong coupling constant

NLO theory unc. > experimental unc.

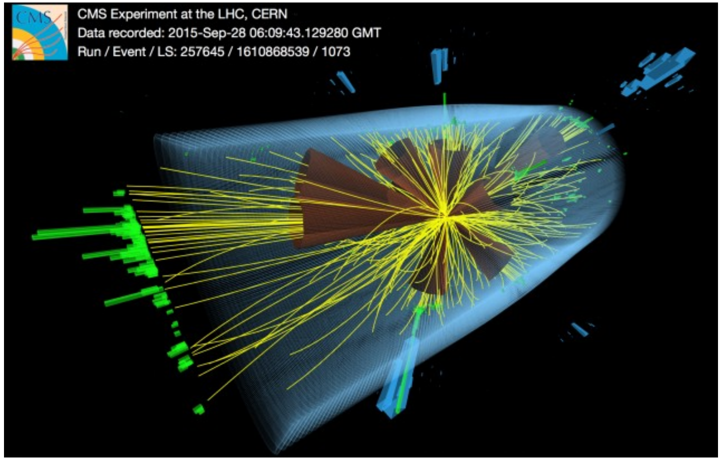
- NNLO QCD needed for precise theory-data comparisons
  → Restricted to two-jet data [Currie'17+later][Czakon'19]
- New NNLO QCD three-jet → access to more observables
  - Jet ratios

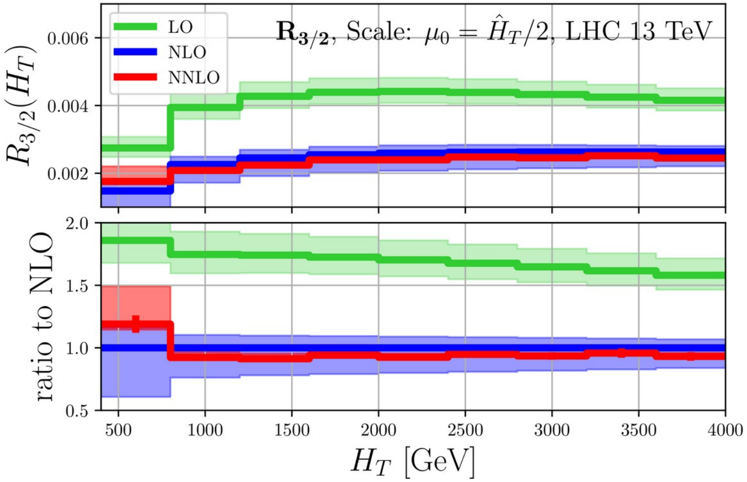
    Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC
    Czakon, Mitov, **Poncelet** [2106.05331]

$$R^i(\mu_R,\mu_F,\mathrm{PDF},\alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R,\mu_F,\mathrm{PDF},\alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R,\mu_F,\mathrm{PDF},\alpha_{S,0})}$$

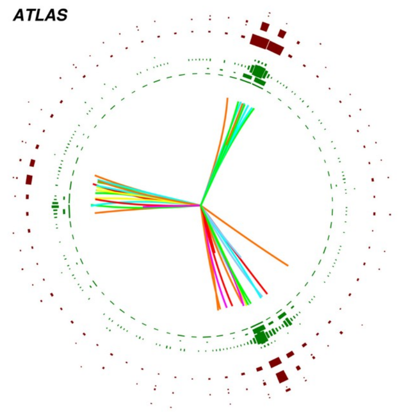
  - Event shapes

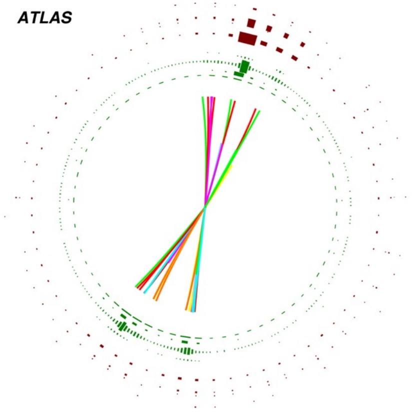
    NNLO QCD corrections to event shapes at the LHC
    Alvarez, Cantero, Czakon, Llorente, Mitov, **Poncelet** [2301.01086]





Encoding QCD dynamics in event shapes





Using (global) event information to separate different regimes of QCD event evolution:

- **Thrust & Thrust-Minor**

$$T_\perp = \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}\,, \quad \text{and} \quad T_m = \frac{\sum_i |\vec{p}_{T,i} \times \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}\,.$$

- **Energy-energy correlators**

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$

Separation of energy scales: $H_{T,2} = p_{T,1} + p_{T,2}$

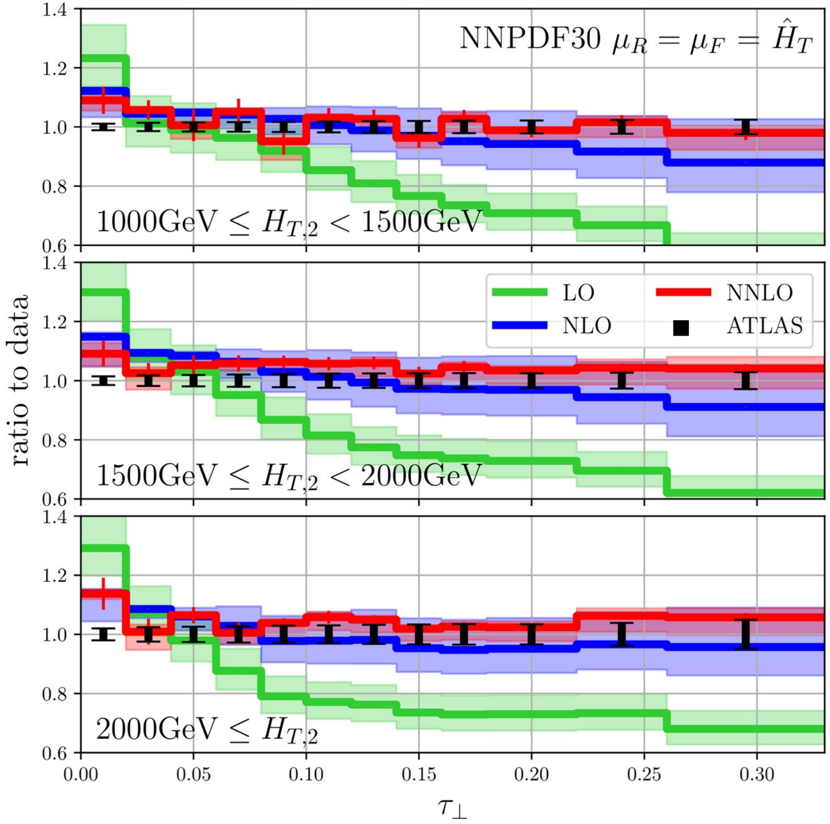
Ratio to 2-jet:

$$R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$$

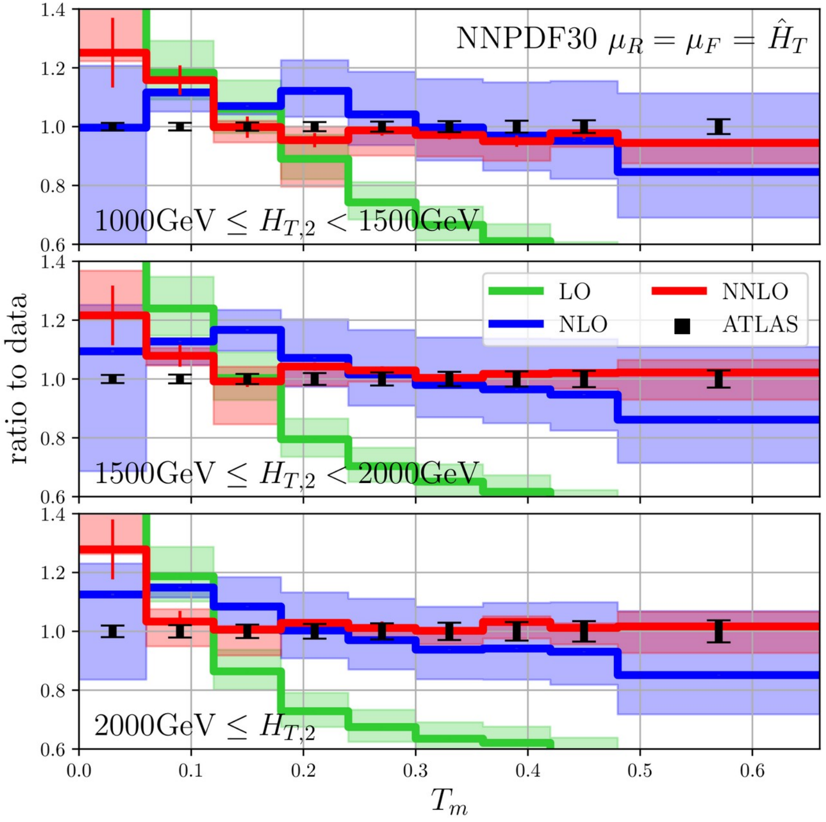
Here: use jets as input → experimentally advantageous (better calibrated, smaller non-pert.)

Transverse Thrust @ NNLO QCD

NNLO QCD corrections to event shapes at the LHC
Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet [2301.01086]



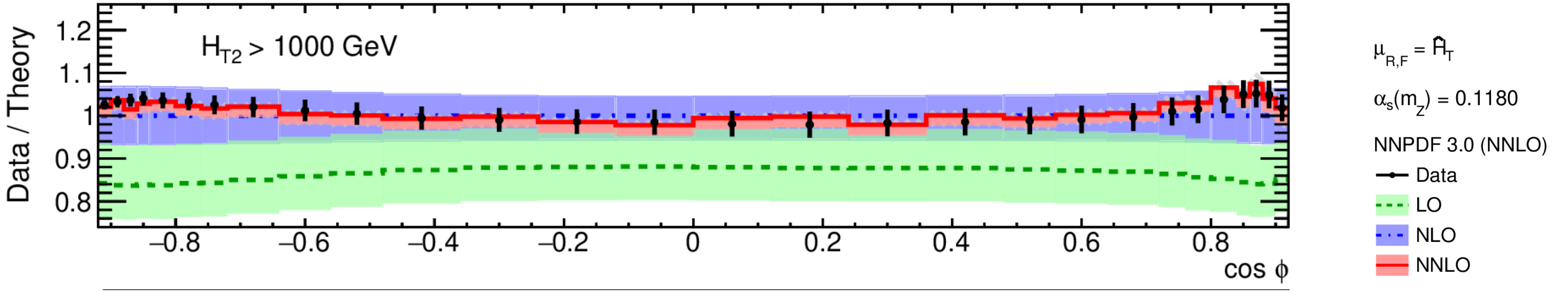
ATLAS [2007.12600]



The transverse energy-energy correlator

$$\frac{1}{\sigma_2}\frac{d\sigma}{d\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{d\sigma\; x_{\perp,i}x_{\perp,j}}{dx_{\perp,i}dx_{\perp,j}d\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})dx_{\perp,i}dx_{\perp,j}d\cos\Delta\phi_{ij}\,,$$

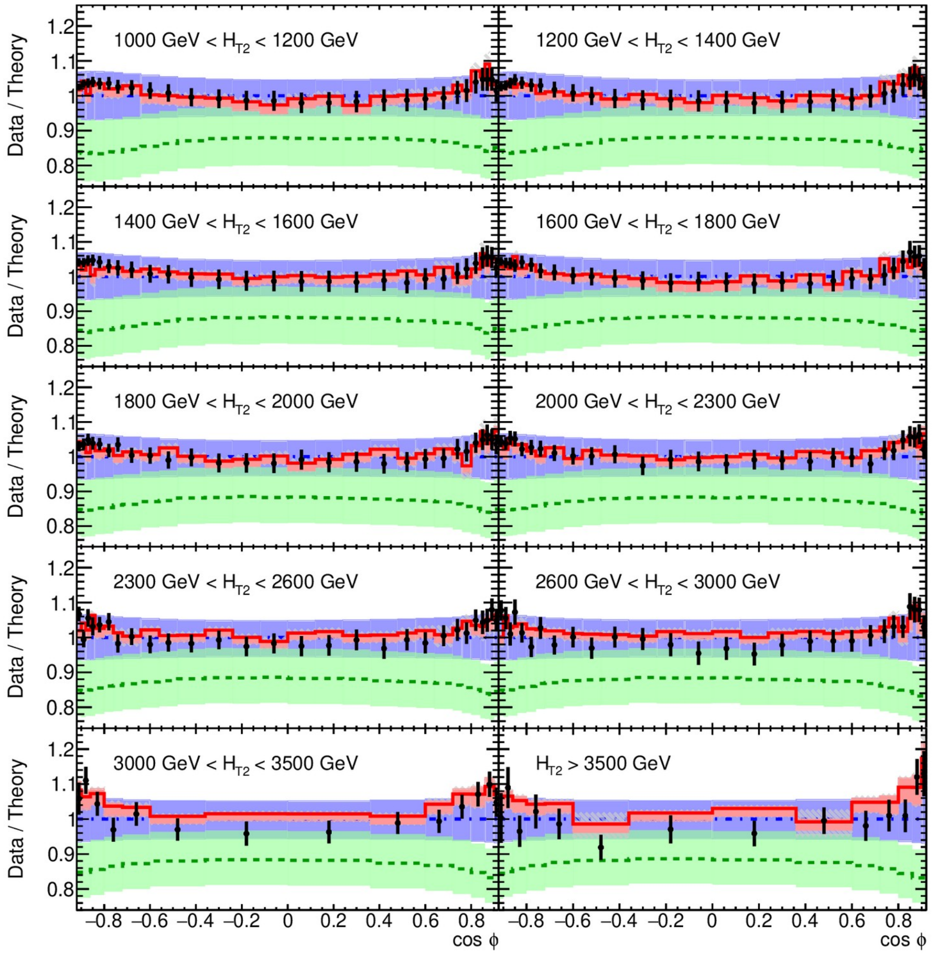
- Insensitive to soft radiation through energy weighting $x_{T,i} = E_{T,i}/\sum_j E_{T,j}$
- Event topology separation:
  - Central plateau contain isotropic events
  - To the right: self-correlations, collinear and in-plane splitting
  - To the left: back-to-back



[ATLAS 2301.09351]

Double differential TEEC

[ATLAS 2301.09351]



ATLAS

Particle-level TEEC

$\sqrt{s}$ = 13 TeV; 139 fb^{-1}

anti-k_t R = 0.4

p_T > 60 GeV

$|\eta|$ < 2.4

$\mu_{R,F} = \hat{H}_T$

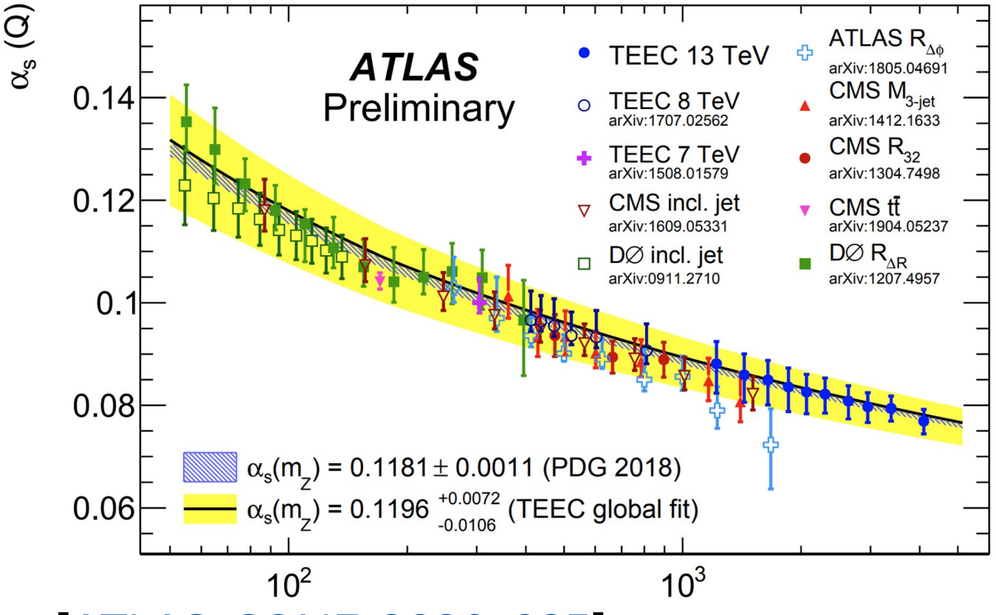
$\alpha_s(m_Z)$ = 0.1180

NNPDF 3.0 (NNLO)

- Data
- LO
- NLO
- NNLO

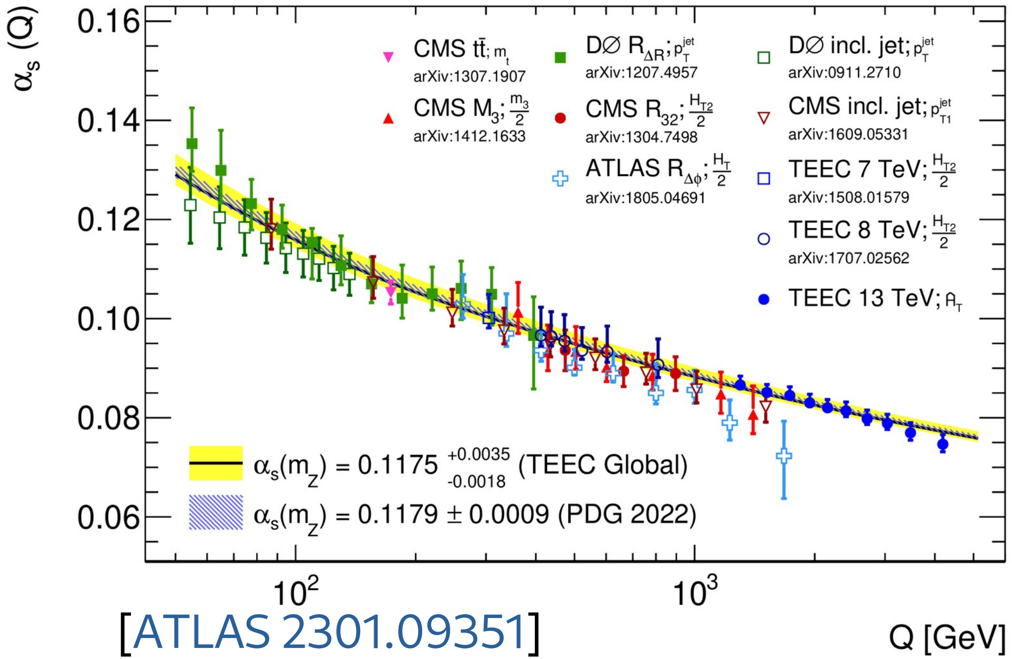
Running of α_S

NLO QCD



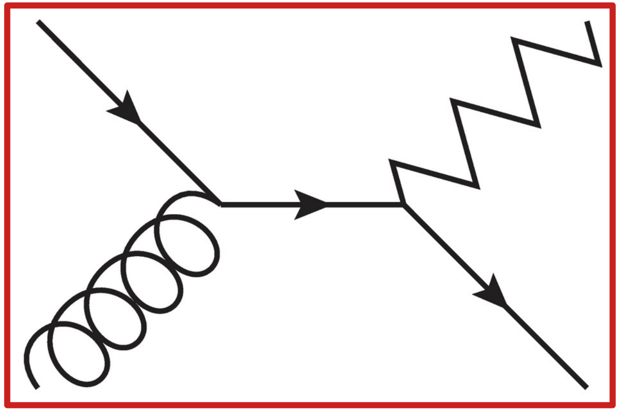
[ATLAS-CONF-2020-025]

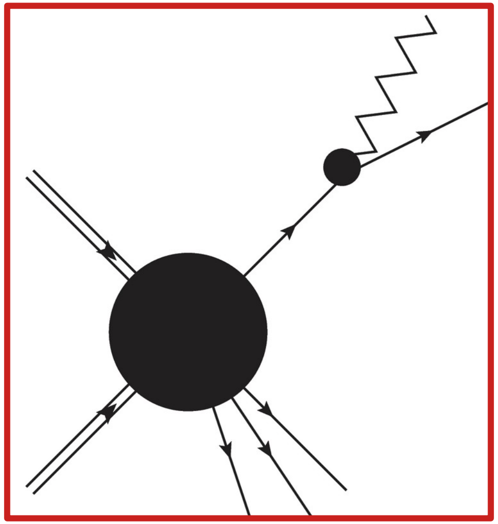
NNLO QCD



[ATLAS 2301.09351]

Prompt photon production

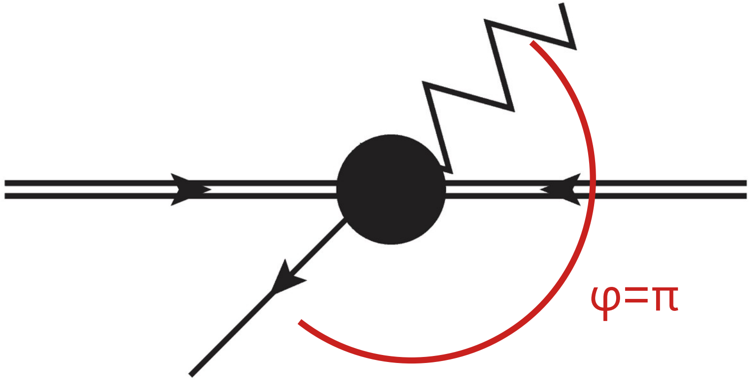


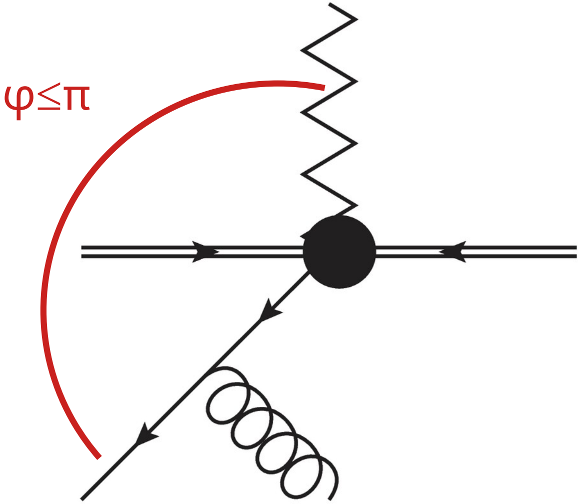


Direct production
- Test of perturbative QCD
- Gluon PDF sensitivity
- Estimates for BSM backgrounds

Fragmentation
- Depends on non-perturbative fragmentation functions
- Separation from “direct” not unique

Why photon plus a jet pair?





- Non-back-to-back Born configurations
  → access to angular correlations between the photon and jets
- Access to different kinematic regimes through distinguishable photon
  → enhance direct, high- or low-z fragmentation
- Background process for BSM: $pp \to \gamma + Y(\to jj)$

Photon plus jet pair

Measurement of isolated-photon plus two-jet production in pp collisions at sqrt(s) = 13 TeV with the ATLAS detector [1912.09866]

Requirements on photon	$E_T^\gamma > 150$ GeV, $\lvert\eta^\gamma\rvert < 2.37$ (excluding $1.37 < \lvert\eta^\gamma\rvert < 1.56$) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 4.8$ GeV (reconstruction level) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 10$ GeV (particle level)		
Requirements on jets	at least two jets using anti-k_t algorithm with $R = 0.4$ $p_T^{\text{jet}} > 100$ GeV, $\lvert y^{\text{jet}}\rvert < 2.5$, $\Delta R^{\gamma-\text{jet}} > 0.8$		
Phase space	**total**	**fragmentation enriched** $E_T^\gamma < p_T^{\text{jet2}}$	**direct enriched** $E_T^\gamma > p_T^{\text{jet1}}$
Number of events	755 270	111 666	386 846

Modelled with hybrid isolation

$$E_\perp(r) \leq E_{\perp\max}(r) = 0.1\, E_\perp(\gamma)\left(\frac{1-\cos(r)}{1-\cos(R_{\max})}\right)^2 \quad \text{for} \quad r \leq R_{\max} = 0.1$$

+

$$E_\perp(r) \leq E_{\perp\max} = 0.0042\, E_\perp(\gamma) + 10 \text{ GeV} \quad \text{for} \quad r \leq R_{\max} = 0.4$$

No fragmentation contribution
→ Purely pQCD through NNLO
→ focus on “inclusive” and “direct” PS

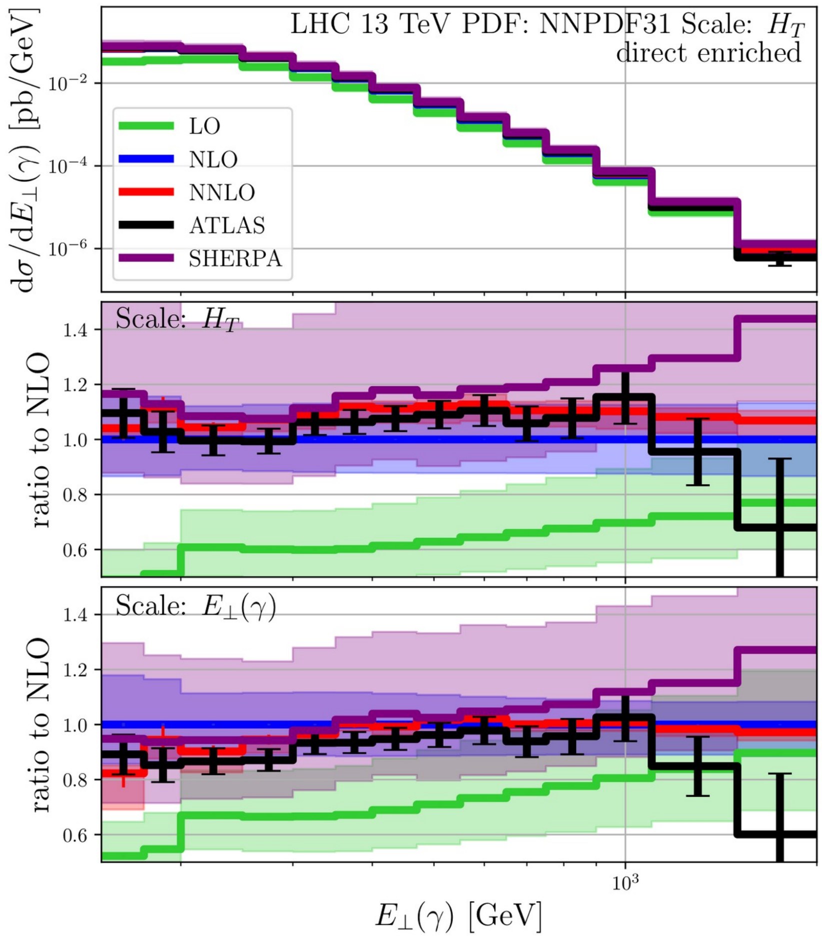
Theory - data comparisons

NNLO QCD

- Describes data well
- Improvements on the shape
- Small corrections
- Small remaining scale dependence

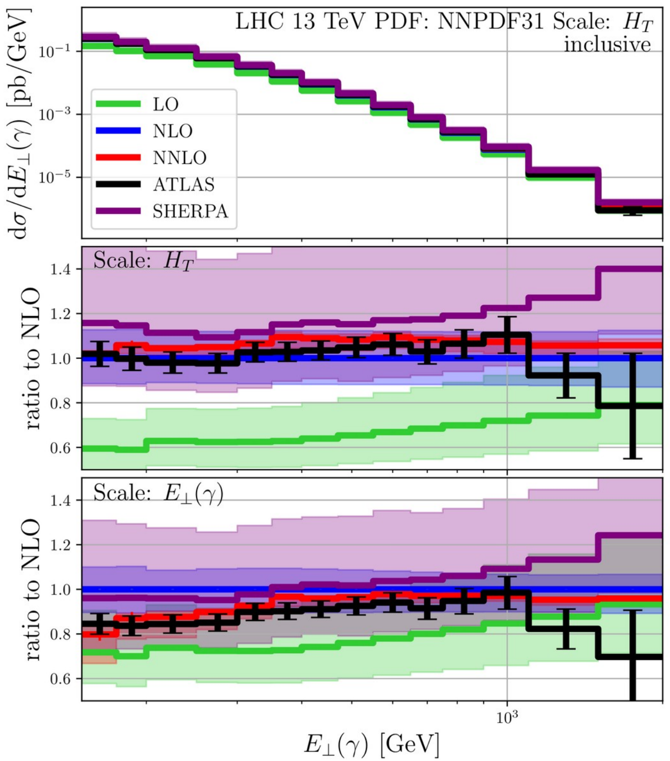
Comment on the SHERPA predictions

- Large NLO scale uncertainties
- The shape is not well described
- → Newer version fix this issue

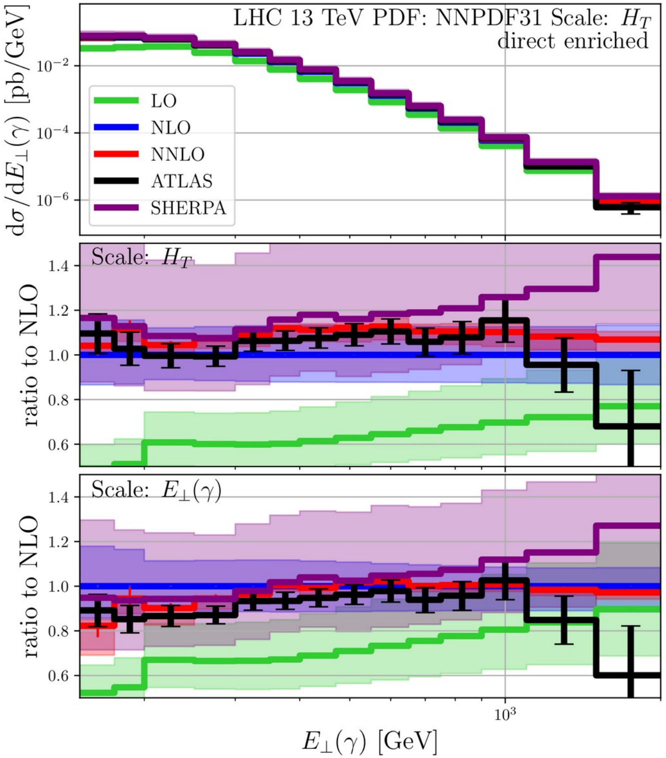


Inclusive vs. direct vs. fragmentation

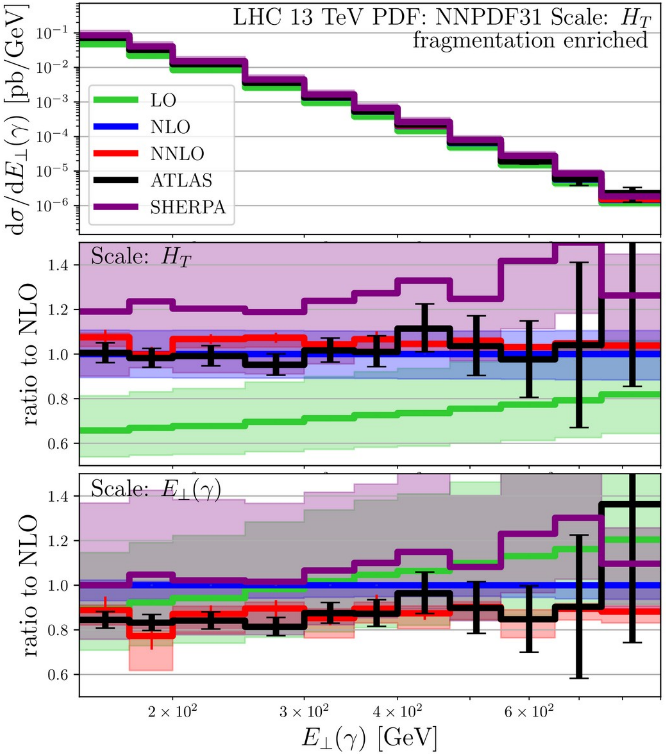
Inclusive



Direct-enriched



Fragmentation

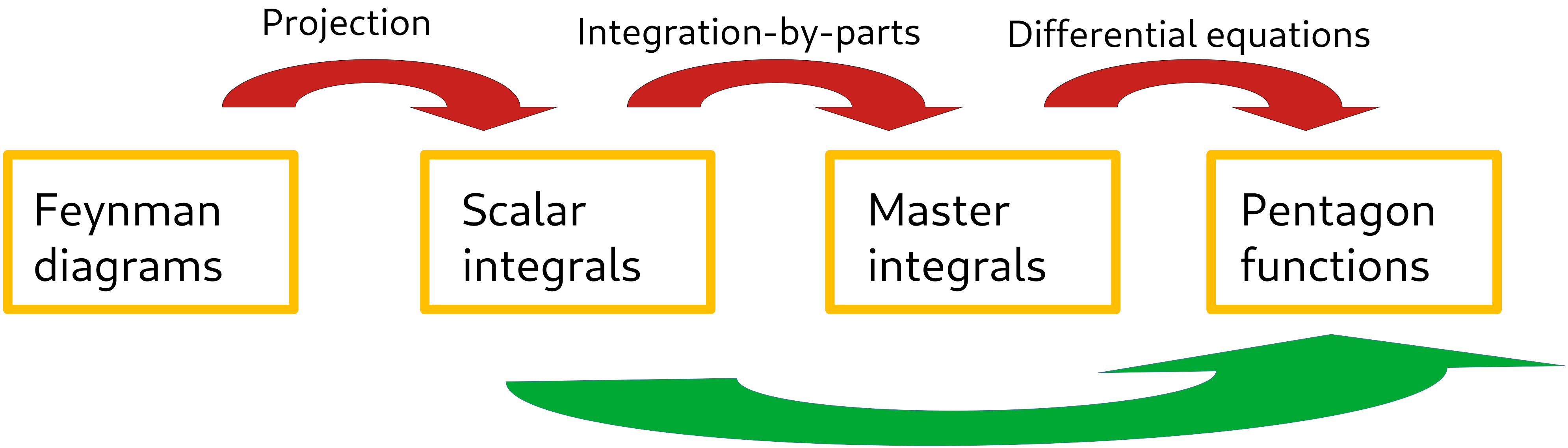


Transverse photon energy

Five-point amplitude with one mass

Overview

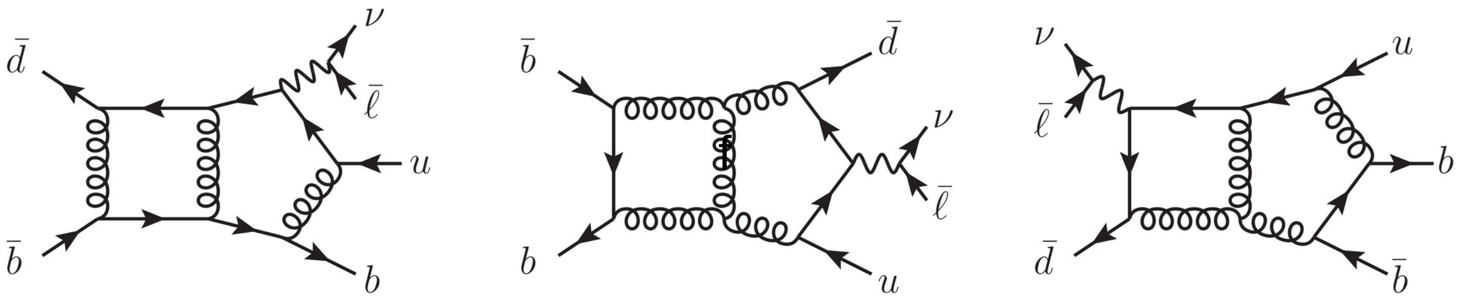
"Old school" approach:



Automated framework using finite fields to avoid expression swell based on FiniteFlow [Peraro'19]

Projection to scalar integrals

Generate diagrams (contributing to leading-colour) with QGRAF



Factorizing decay: $A_6^{(L)} = A_5^{(L)\mu} D_\mu P$ $\qquad$ $M_6^{2(L)} = \sum_{\text{spin}} A_6^{(0)^*} A_6^{(L)} = M_5^{(L)\mu\nu} D_{\mu\nu} |P|^2$

Projection on scalar functions (FORM+Mathematica):
→ anti-commuting γ_5 + Larin prescription

$$M_5^{(L)\mu\nu} = \sum_{i=1}^{16} a_i^{(L)} v_i^{\mu\nu}$$

$$a_i^{(L)} = a_i^{(L),\text{even}} + \text{tr}_5 a_i^{(L),\text{odd}} \qquad a_i^{(L),p} = \sum_i c_{j,i}(\{p\},\epsilon)\mathcal{I}(\{p\},\epsilon)$$

Integration-By-Parts reduction

$$a_i^{(L),p} = \sum_i c_{j,i}(\{p\},\epsilon)\mathcal{I}(\{p\},\epsilon)$$

→ prohibitively large number of integrals

$$\mathcal{I}_i(\{p\},\epsilon) \equiv \mathcal{I}(\vec{n_i},\{p\},\epsilon) = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d}\frac{\mathrm{d}^d k_2}{(2\pi)^d}\prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\},\{k\})$$

Integration-By-Parts identities connect different integrals → system of equations
→ only a small number of independent “master” integrals

$$0 = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d}\frac{\mathrm{d}^d k_2}{(2\pi)^d} l_\mu \frac{\partial}{\partial l^\mu}\prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\},\{k\}) \quad \text{with} \quad l \in \{p\}\cap\{k\}$$

LiteRed (+ Finite Fields)

$$a_i^{(L),p} = \sum_i d_{j,i}(\{p\},\epsilon)\mathrm{MI}(\{p\},\epsilon)$$

Master integrals & finite remainder

Differential Equations: $\mathrm{d}\vec{\mathrm{MI}} = \mathrm{d}A(\{p\},\epsilon)\vec{\mathrm{MI}}$

Canonical basis: $\mathrm{d}\vec{\mathrm{MI}} = \epsilon \mathrm{d}\tilde{A}(\{p\})\vec{\mathrm{MI}}$

[Remiddi, 97]
[Gehrmann, Remiddi, 99]
[Henn, 13]

Simple iterative solution

$$\mathrm{MI}_i = \sum_w \epsilon^w \tilde{\mathrm{MI}}_i^w \quad \text{with} \quad \tilde{\mathrm{MI}}_i^w = \sum_j c_{i,j} m_j$$

Chen-iterated integrals
"Pentagon"-functions

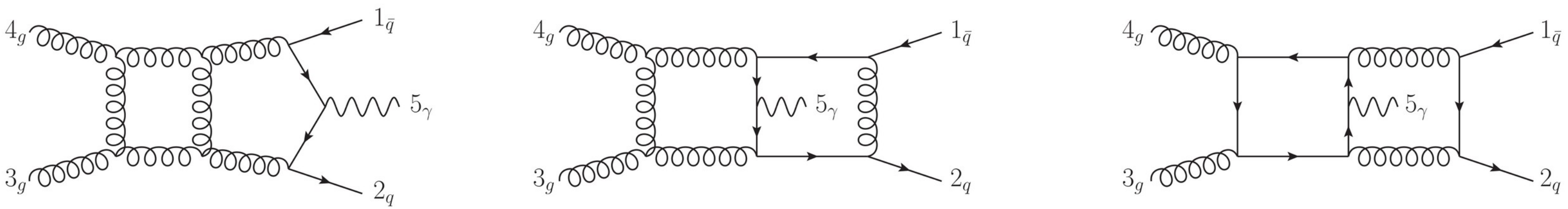
[Chicherin, Sotnikov, 20]
[Chicherin, Sotnikov, Zoia, 21]

Putting everything together (and removing of IR poles):

$$f_i^{(L),p} = a_i^{(L),p} - \text{poles} \qquad f_i^{(L),p} = \sum_j c_{i,j}(\{p\}) m_j + \mathcal{O}(\epsilon)$$

Reconstruction of Amplitudes

[Badger'23]



$A^{(2),N_c^2}_{34;q}$, $A^{(2),N_c}_{\delta;q}$ — $A^{(2),1}_{34;q}$, $A^{(2),N_c}_{\delta;q}$, $A^{(2),1/N_c}_{\delta;q}$ — $A^{(2),N_c}_{34;l}$, $A^{(2),1/N_c}_{34;l}$, $A^{(2),1/N_c^2}_{\delta;l}$

New optimizations

- Syzygy's to simplify IBPs
- Exploitation of Q-linear relations
- Denominator Ansaetze
- On-the-fly partial fractioning

amplitude	helicity	original	stage 1	stage 2	stage 3	stage 4
$A^{(2),1}_{34;q}$	$-++-+$	94/91	74/71	74/0	22/18	22/0
$A^{(2),1}_{34;q}$	$-+-++$	93/89	90/86	90/0	24/14	18/0
$A^{(2),1/N_c^2}_{34;q}$	$-++-+$	90/88	73/71	73/0	23/18	22/0
$A^{(2),1/N_c^2}_{34;q}$	$-+-++$	90/86	86/82	86/0	24/14	19/0
$A^{(2),1/N_c}_{34;l}$	$-+-++$	89/82	74/67	73/0	27/14	20/0
$A^{(2),1/N_c}_{34;l}$	$-++-+$	85/81	61/58	60/0	27/18	20/0
$A^{(2),N_c^2}_{34;q}$	$-+-++$	58/55	54/51	53/0	20/16	20/0

Massive reduction of complexity

Wbb @ NNLO QCD

[Hartanto, Poncelet, Popescu, Zoia '22]

- LHC @ 8 TeV in 5 FS, NNPDF31, scale: $H_T = E_T(l\nu) + pT(b1) + pT(b2)$
- Phasespace definition to model **[CMS, 1608.07561]:** pT(l) ≥ 30 GeV |y(l)| < 2.1 pT(j) ≥ 25 GeV, |y(j)| < 2.4
- Inclusive (at least 2 b-jets) and exclusive (exactly 2 b-jets, no other jets) jet phase spaces (defined by the flavour-kT jet algorithm [Banfi'06])

- Inclusive :
  ~ +20% corrections
  ~ 7% scale dependence
- Exclusive:
  ~ + 6% corrections
  ~ 2.5% scale dependence (7-pt)
  Compare decorrelated model: [Steward'12]
  ~ 11% scale dependence

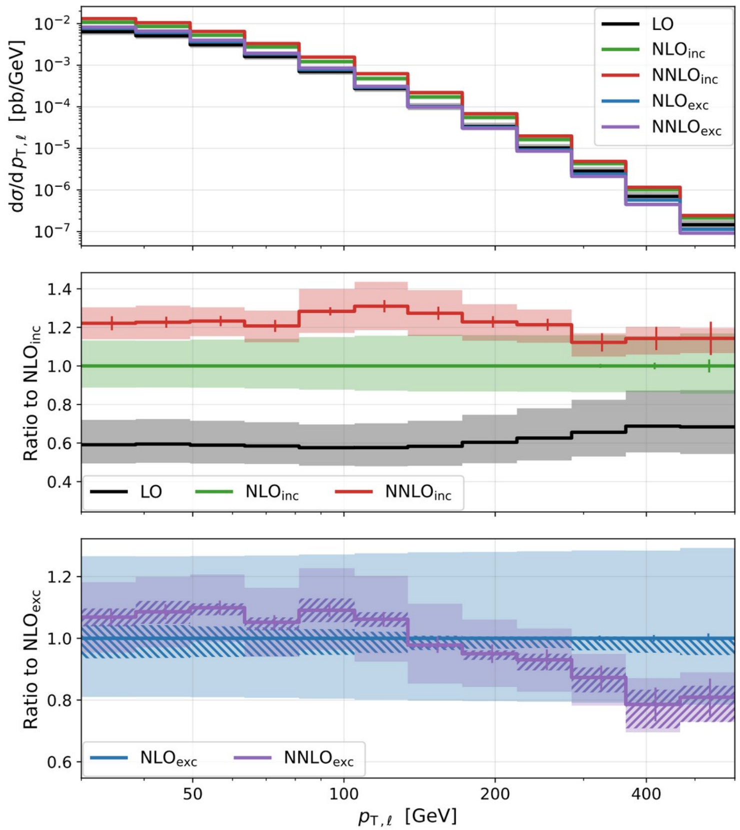
	inclusive [fb]	$\mathcal{K}_{\rm inc}$	exclusive [fb]	$\mathcal{K}_{\rm exc}$
$\sigma_{\rm LO}$	$213.2(1)^{+21.4\%}_{-16.1\%}$	-	$213.2(1)^{+21.4\%}_{-16.1\%}$	-
$\sigma_{\rm NLO}$	$362.0(6)^{+13.7\%}_{-11.4\%}$	1.7	$249.8(4)^{+3.9(+27)\%}_{-6.0(-19)\%}$	1.17
$\sigma_{\rm NNLO}$	$445(5)^{+6.7\%}_{-7.0\%}$	1.23	$267(3)^{+1.8(+11)\%}_{-2.5(-11)\%}$	1.067

$$\sigma_{Wb\bar{b},\rm excl.} = \sigma_{Wb\bar{b},\rm incl.} - \sigma_{Wb\bar{b}j,\rm incl.}$$

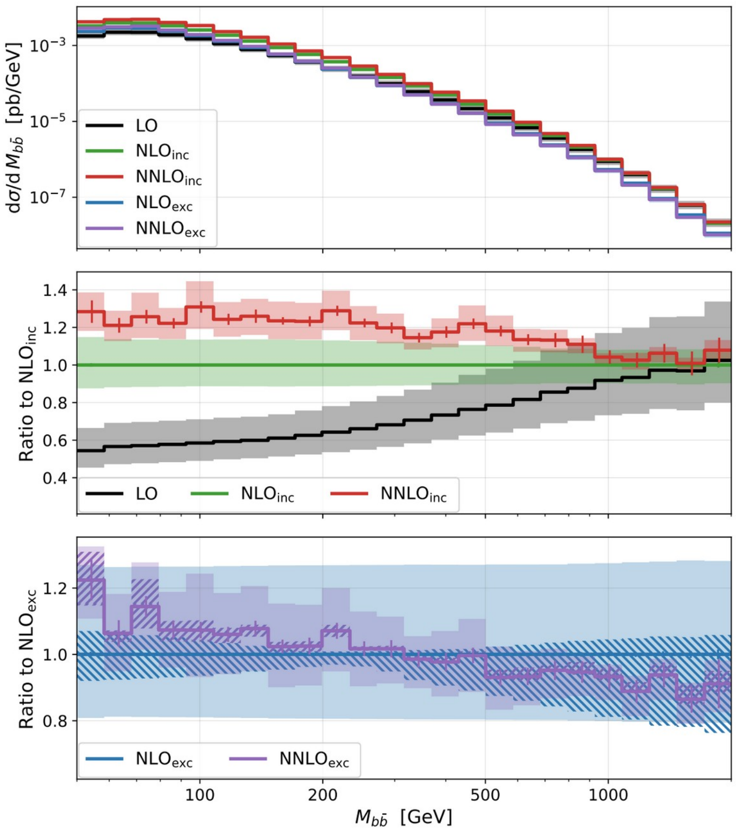
$$\Delta\sigma_{Wb\bar{b},excl.} = \sqrt{(\Delta\sigma_{Wb\bar{b},incl.})^2 + (\Delta\sigma_{Wb\bar{b}j,incl.})^2}$$

Differential cross sections

Transverse momentum of lepton



Invariant mass b-jet pair



Take home messages

- Precision at collider needs precise predictions
  → Higher order QCD corrections are crucial to compare to current and future LHC data

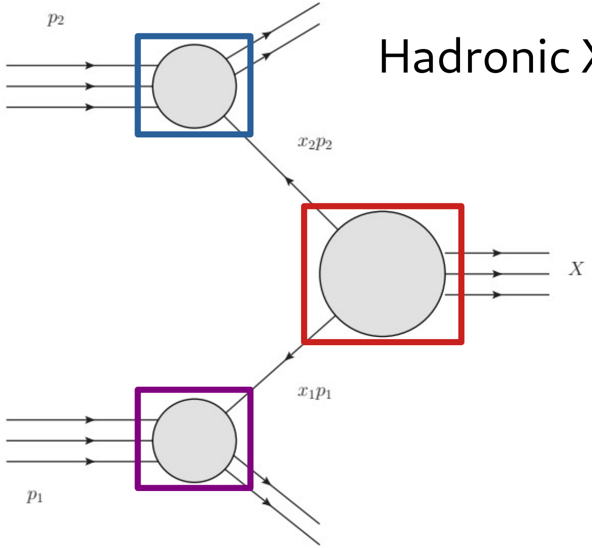
- Very good description of data using perturbative NNLO QCD
  → Significantly improved theory uncertainty estimates
  → First phenomenological applications: extraction of the strong coupling constant

- Completion of massless 2 → 3 processes at hadron colliders through NNLO QCD
  $pp \to \gamma\gamma\gamma \qquad pp \to \gamma\gamma j \qquad pp \to \gamma j j \qquad pp \to j j j$

- Most important bottlenecks:
  → Monte Carlo integration of real radiation contributions → improved methods needed!
  → Two-loop amplitudes
  (including external/internal masses are the current frontier)

Backup

Hadronic cross section



Hadronic X-section: $\sigma_{h_1h_2\to X} = \sum_{ij}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\phi_{i,h_1}(x_1,\mu_F^2)\phi_{j/h_2}(x_2,\mu_F^2)\hat{\sigma}_{ij\to X}(\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)$

Parton distribution functions

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

The NNLO bit: $\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$

Double real radiation

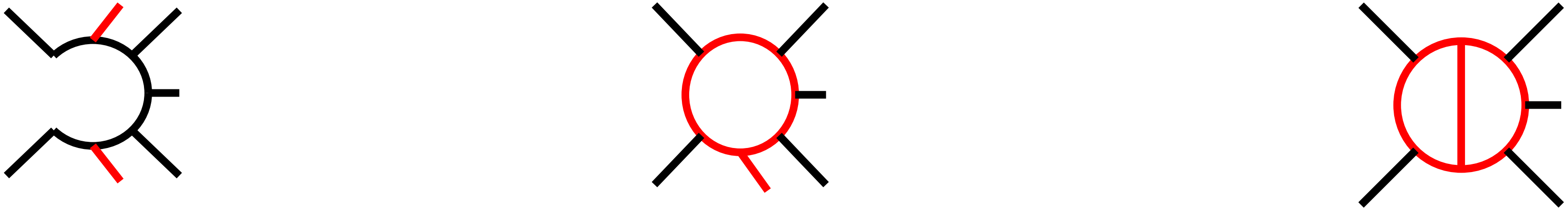
$$\hat{\sigma}^{\mathrm{RR}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\left\langle \mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2}$$

Real/Virtual correction

$$\hat{\sigma}^{\mathrm{RV}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re}\left\langle \mathcal{M}^{(0)}_{n+1}\middle|\mathcal{M}^{(1)}_{n+1}\right\rangle \mathrm{F}_{n+1}$$

Double virtual corrections

$$\hat{\sigma}^{\mathrm{VV}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n}\left(2\mathrm{Re}\left\langle \mathcal{M}^{(0)}_{n}\middle|\mathcal{M}^{(2)}_{n}\right\rangle + \left\langle \mathcal{M}^{(1)}_{n}\middle|\mathcal{M}^{(1)}_{n}\right\rangle\right) \mathrm{F}_{n}$$



Partonic cross section beyond LO

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

Contributions with different multiplicities and # convolutions:

$$\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$$

$$\hat{\sigma}^{\mathrm{RR}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}^{(0)}_{n+2} \middle| \mathcal{M}^{(0)}_{n+2} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}^{\mathrm{RV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(1)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\mathrm{VV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n} \middle| \mathcal{M}^{(2)}_{n} \right\rangle + \left\langle \mathcal{M}^{(1)}_{n} \middle| \mathcal{M}^{(1)}_{n} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}^{\mathrm{C1}}_{ab} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\mathrm{C2}}_{ab} = (\text{double convolution})\, \mathrm{F}_{n}$$

Each term separately IR divergent. But sum is:

→ finite

→ regularization scheme independent

Considering CDR ($d = 4 - 2\epsilon$):

→ Laurent expansion:

$$\hat{\sigma}^{C}_{ab} = \sum_{i=-4}^{0} c_i \epsilon^i + \mathcal{O}(\epsilon)$$

Sector decomposition I

Considering working in CDR:

→ Virtuals are usually done in this regularization

→ Real radiation:

→ Very difficult integrals, analytical impractical (except very simple cases)!

→ Numerics not possible, integrals are divergent: ε-poles!

How to extract these poles? → Sector decomposition!

Divide and conquer the phase space:

$$1 = \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right]$$

$$\hat{\sigma}_{ab}^{\rm RR} = \frac{1}{2\hat{s}} \int {\rm d}\Phi_{n+2} \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle {\rm F}_{n+2}$$

Sector decomposition II

Divide and conquer the phase space:

→ Each $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$ has simpler divergences.

appearing as $1/s_{ijk}$ $1/s_{ik}/s_{jl}$

Soft and collinear (w.r.t parton k,l) of partons i and j

→ Parametrization w.r.t. reference parton:

$$\hat{\eta}_i = \frac{1}{2}(1-\cos\theta_{ir}) \in [0,1] \qquad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$$

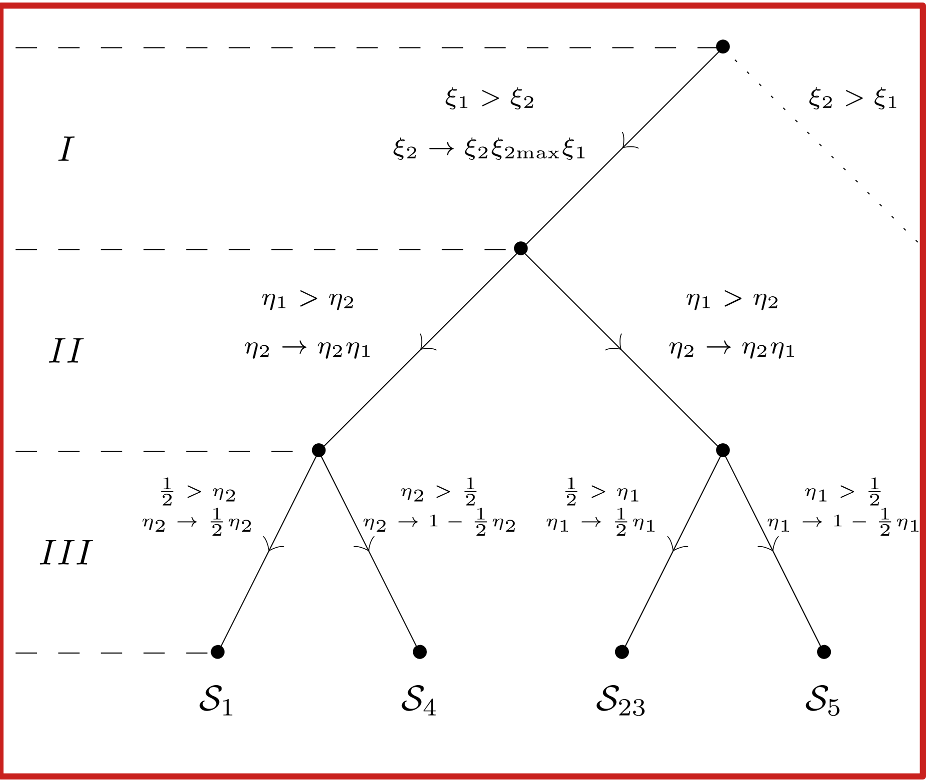
→ Subdivide to factorize divergences

$$s_{u_1u_2k} = (p_k+u_1+u_2)^2 \sim \hat{\eta}_1 u_1^0 + \hat{\eta}_2 u_2^0 + \hat{\eta}_3 u_1^0 u_2^0$$

→ double soft factorization:

$$\theta(u_1^0-u_2^0)+\theta(u_2^0-u_1^0)$$

→ triple collinear factorization



[Czakon'10,Caola'17]

Sector decomposition III

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\,\mathcal{S}_{kl,m}\left\langle \mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2} = \sum_{\text{sub-sec.}}\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i\,\underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}}\,\mathrm{d}\tilde{\mu}(\{x_i\})\underbrace{\prod x_i^{a_i+1}\left\langle \mathcal{M}_{n+2}\middle|\mathcal{M}_{n+2}\right\rangle}_{\text{regular}}\mathrm{F}_{n+2}$$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 \mathrm{d}x\,\left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x)-f(0)}{x^{1+b\epsilon}}$$

Finite NNLO cross section

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_{n}$$

sector decomposition and master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\left(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}\right) \quad \left(\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}\right) \quad \left(\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}\right) \quad \left(\sigma_{SU}^{C1}, \sigma_{DU}^{C1}\right) \quad \left(\sigma_{DU}^{C2}, \sigma_{FR}^{C2}\right)$$

re-arrangement of terms → 4-dim. formulation [Czakon'14, Czakon'19]

$$\left(\sigma_F^{RR}\right) \quad \left(\sigma_F^{RV}\right) \quad \left(\sigma_F^{VV}\right) \quad \left(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}\right) \quad \left(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2}\right) \quad \left(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}\right)$$

separately finite: ε poles cancel

C++ framework

- Formulation allows efficient algorithmic implementation
- **High degree of automation**:
  - Partonic processes (taking into account all symmetries)
  - Sectors and subtraction terms
  - Interfaces to Matrix-element providers:
    AvH, OpenLoops, Recola, NJET, HardCoded

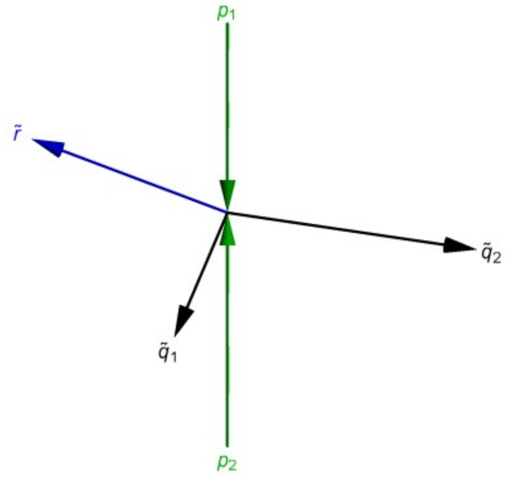
    → Only two-loop matrix elements required
- **Broad range of applications** through additional facilities:
  - Narrow-Width & Double-Pole Approximation
  - Fragmentation
  - Polarised intermediate massive bosons
  - (Partial) Unweighting → Event generation for **HighTEA**
  - Interfaces: FastNLO, FastJet

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed : u_i $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,,\ \ \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

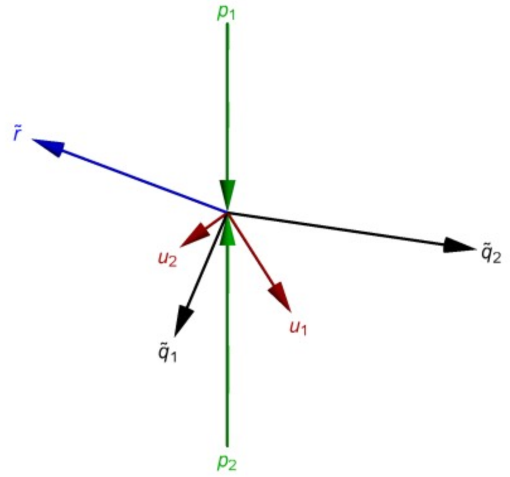
- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed : u_i $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,,\;\; \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

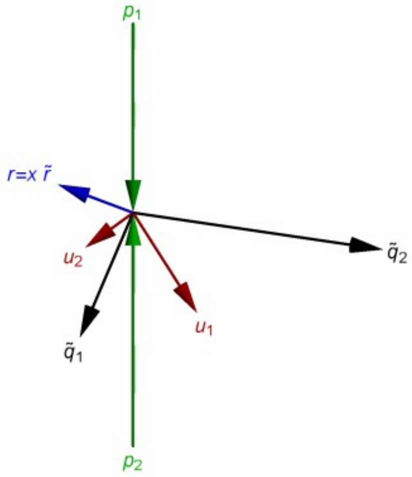
- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed : u_i $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,,\ \ \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

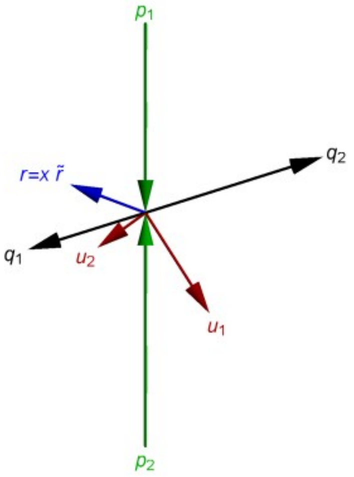
- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



Requirements:

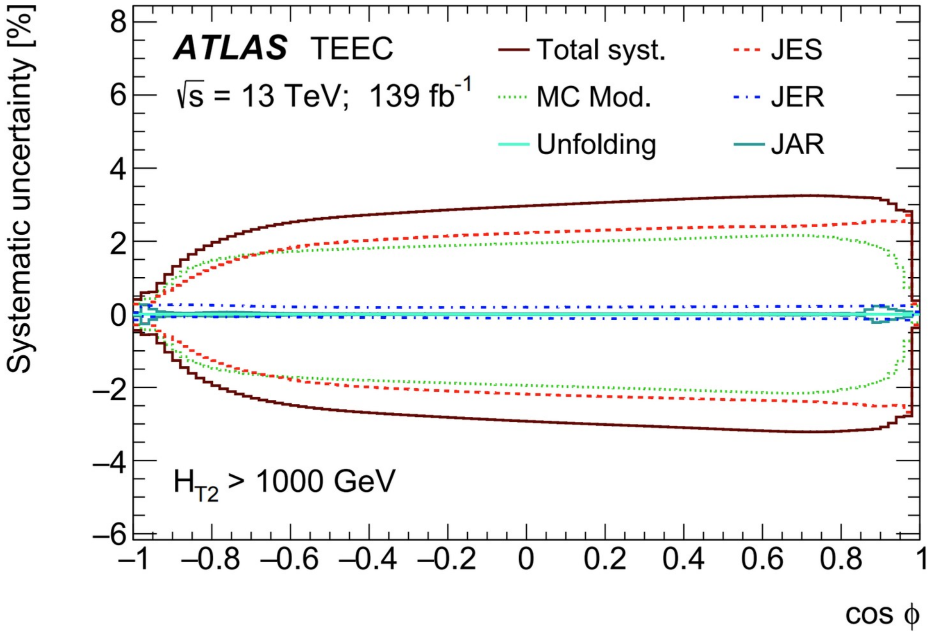
- Keep direction of reference r fixed
- Invertible for fixed : u_i $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,,\;\; \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

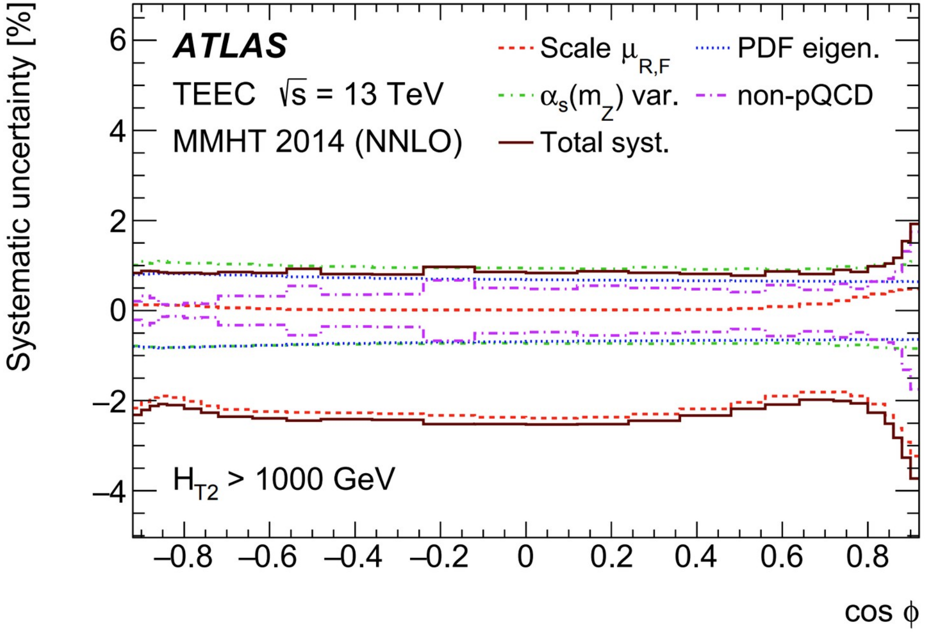
- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

Systematic Uncertainties TEEC

Experimental uncertainties



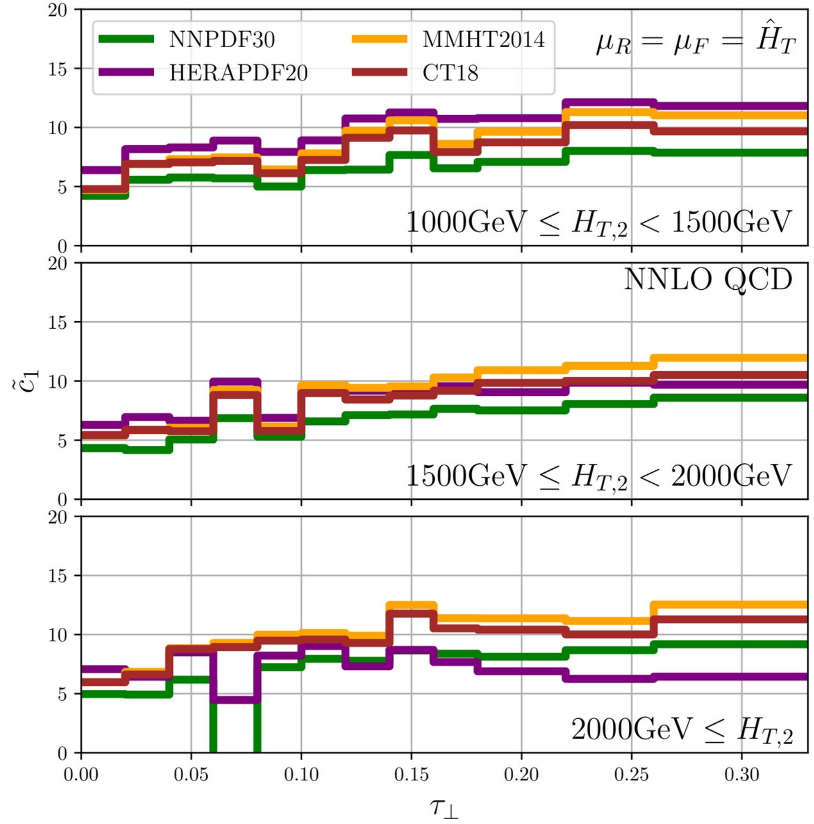
Theory uncertainties



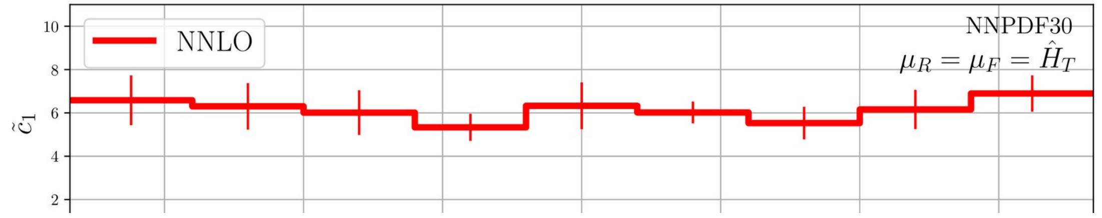
Scale dependence is the dominating uncertainty → NNLO QCD required to match exp.

Strong coupling dependence

Thrust



TEEC



$$R^{\mathrm{NNLO,fit}}(\mu, \alpha_{S,0}) = c_0 + c_1(\alpha_{S,0} - 0.118) + c_2(\alpha_{S,0} - 0.118)^2 + c_3(\alpha_{S,0} - 0.118)^3$$

mostly linear dependence

Visualisation of α_S dependence

$$\tilde{c}_1 = \frac{c_1}{R^{\mathrm{NNLO}}(\alpha_{S,0} = 0.118)}$$

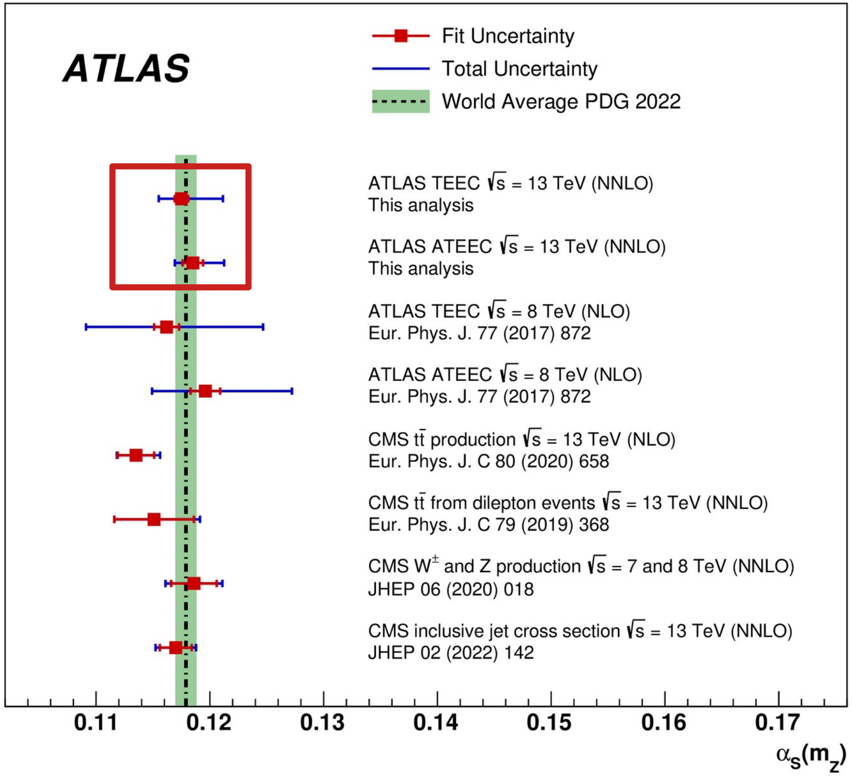
For comparison:
scale dependence (dominant theory uncertainty)

- TEEC ($H_{T,2} > 1$ TeV) : ~2%
- Thrust : ~3-5 %

O(1%) sensitivity

α_S from TEEC @ NNLO by ATLAS

[ATLAS 2301.09351]



- NNLO QCD extraction from multi-jets → will contribute to PDG for the first time
- Significant improvement to 8 TeV → driven by NNLO QCD corrections
- Individual precision large but comparable to top or jets-data.
- However: extraction at high energy scales

Using the running of α_S to probe NP

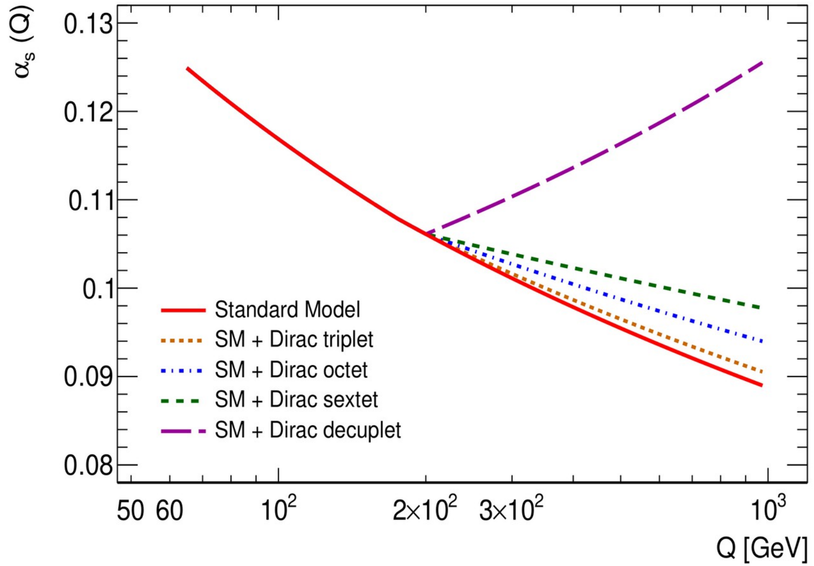
[Llorente, Nachman 1807.00894]

Indirect constraints to NP through modified running:

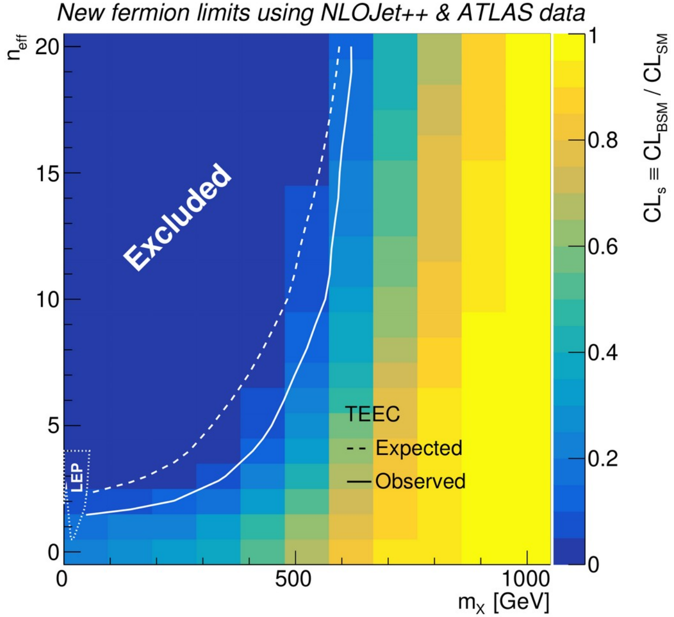
$$\beta_0 = \frac{1}{4\pi}\left(11 - \frac{2}{3}n_f - \frac{4}{3}n_X T_X\right)$$

$$\beta_1 = \frac{1}{(4\pi)^2}\left[102 - \frac{38}{3}n_f - 20 n_X T_X\left(1 + \frac{C_X}{5}\right)\right]$$

$$\alpha_s(Q) = \frac{1}{\beta_0 \log z}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\log(\log z)}{\log z}\right]; \quad z = \frac{Q^2}{\Lambda^2_{\rm QCD}}$$

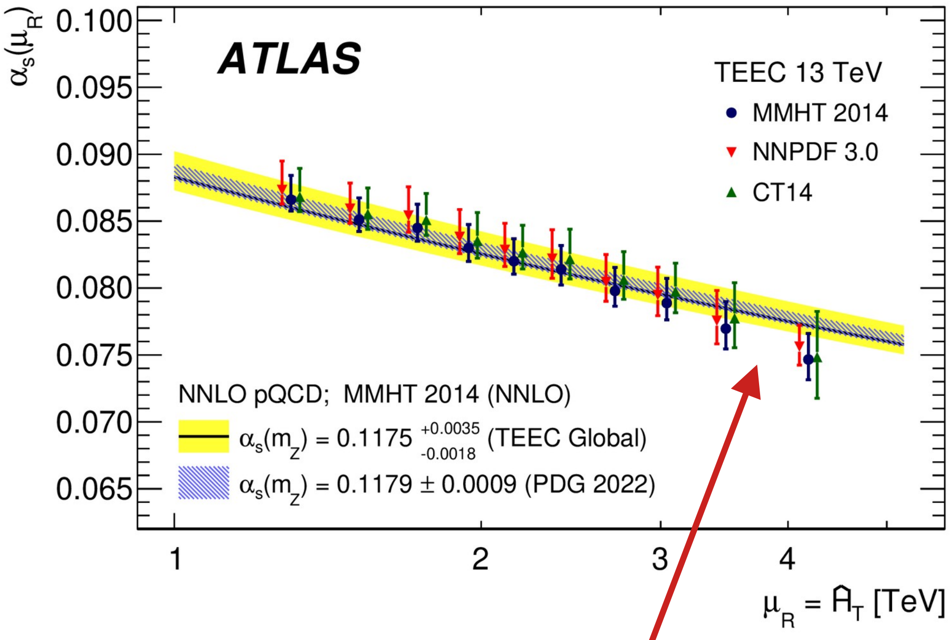


ATLAS TEEC @ 7 TeV data



Update with TEEC@13 TeV
→ much improved bounds

… or 'new' SM dynamics



Systematic slope
→ New physics?

Possible SM explanations

- Residual PDF effects → high x,Q^2 ?
- EW corrections?
- Maybe effect from LC approximation in two-loop ME?

$$\mathcal{R}^{(2)}(\mu_R^2) = 2\,\mathrm{Re}\left[\mathcal{M}^{\dagger(0)}\mathcal{F}^{(2)}\right](\mu_R^2) + \left|\mathcal{F}^{(1)}\right|^2(\mu_R^2)$$

$$\equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^{4} c_i \ln^i\left(\frac{\mu_R^2}{s_{12}}\right)$$

$$\mathcal{R}^{(2)}(s_{12}) \approx \mathcal{R}^{(2)l.c.}(s_{12})$$

- Experimental systematics?
- Resummation?

Either case interesting!

Photon isolation

Hard cone

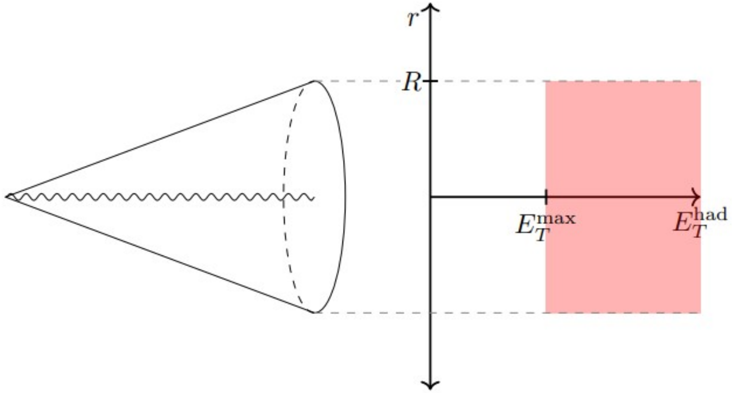
- Experimental hard cone:

$$E_\perp(r) \leq E_{\perp\max} = 0.0042\, E_\perp(\gamma) + 10 \text{ GeV} \quad \text{for} \quad r \leq R_{\max} = 0.4$$

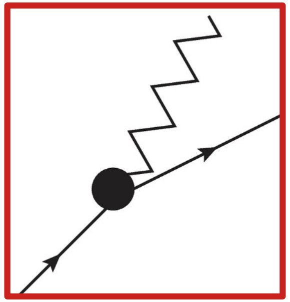
- Theory perspective:

Not collinear safe in perturbative QCD
due to q → qγ splittings

→ Non-vanishing fragmentation contribution
(NNLO QCD with frag. [2201.06982][2205.01516])



Credit: Marius Hoefer (talk@SM@LHC22)



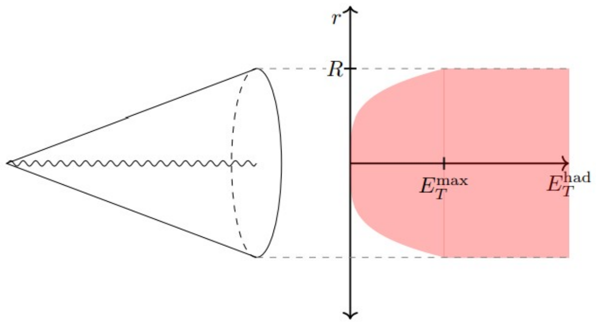
Photon isolation

Smooth cone

- by Frixione [hep-ph/9801442]

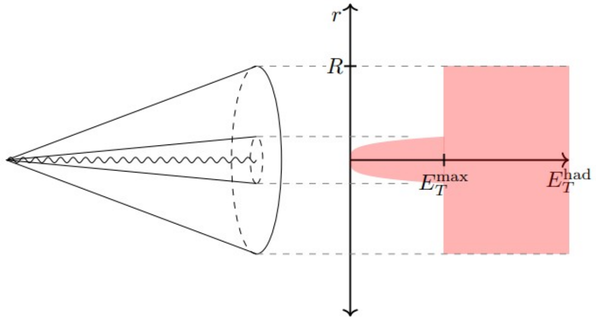
$$E_\perp(r) \leq E_{\perp\max}(r) = 0.1\, E_\perp(\gamma)\left(\frac{1-\cos(r)}{1-\cos(R_{\max})}\right)^2 \quad \text{for} \quad r \leq R_{\max} = 0.1$$

→ Theoretically convenient
→ Removes fragmentation contribution
→ Experimentally limited by detector resolution



Hybrid cone

- [1611.07226][2205.01516]
  - Combines smooth & hard cone
  - Fair approx. to hard cone [2205.01516]

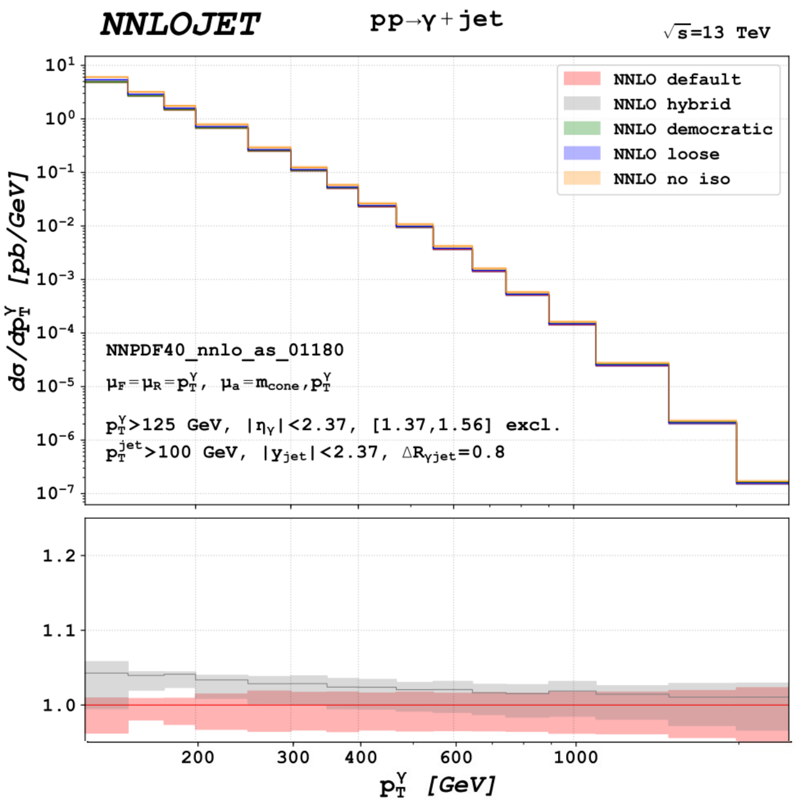


Credit: Marius Hoefer (talk@SM@LHC22)

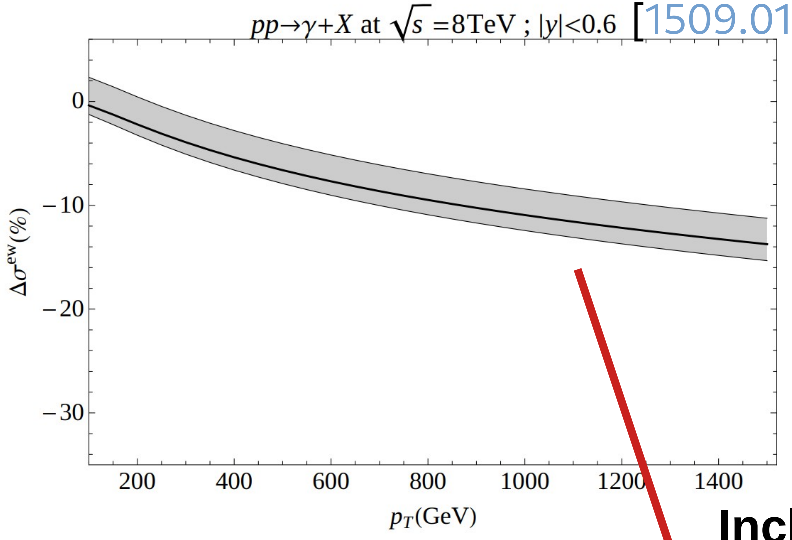
Fragmentation contribution

[2205.01516]

- ATLAS photon requirements (same as for $pp \to \gamma + 2j$)
- Comparison between:
  - “default” NNLO with fragmentation
  - “hybrid” NNLO with hybrid isolation
- Fragmentation contr.
  - ~5% at small $E_T(\gamma)$
  - ~<1% at high $E_T(\gamma)$



Missing effects



[1509.01961]

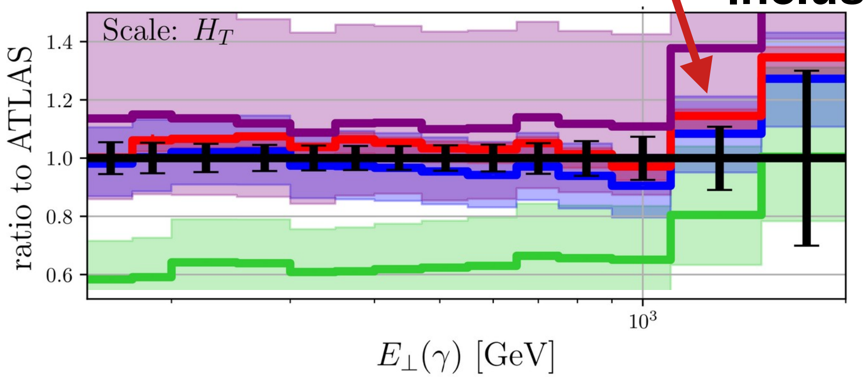
Electro-weak corrections

- EW Sudakov logs at high $E_\perp(\gamma)$
- ~O(-10%) above 1 TeV
- Further improvement of theory/data

Fragmentation

- More relevant at small $E_\perp(\gamma)$
- For $pp \to \gamma + X:\ \ \sigma(\text{hybrid}) > \sigma(\text{frag.})$
- Inclusion might cure slightly high normalisation

Inclusive



Missing effects

Electro-weak corrections

- EW Sudakov logs at high $E_\perp(\gamma)$
- ~O(-10%) above 1 TeV
- Further improvement of theory/data

Fragmentation

- More relevant at small $E_\perp(\gamma)$
- For $pp \to \gamma + X: \;\; \sigma(\text{hybrid}) > \sigma(\text{frag.})$
- Inclusion might cure slightly high normalisation

[2205.01516]