

Precision QCD phenomenology for multi-scale processes at the Large-Hadron-Collider

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IFJ PAN seminar 25th April 2024

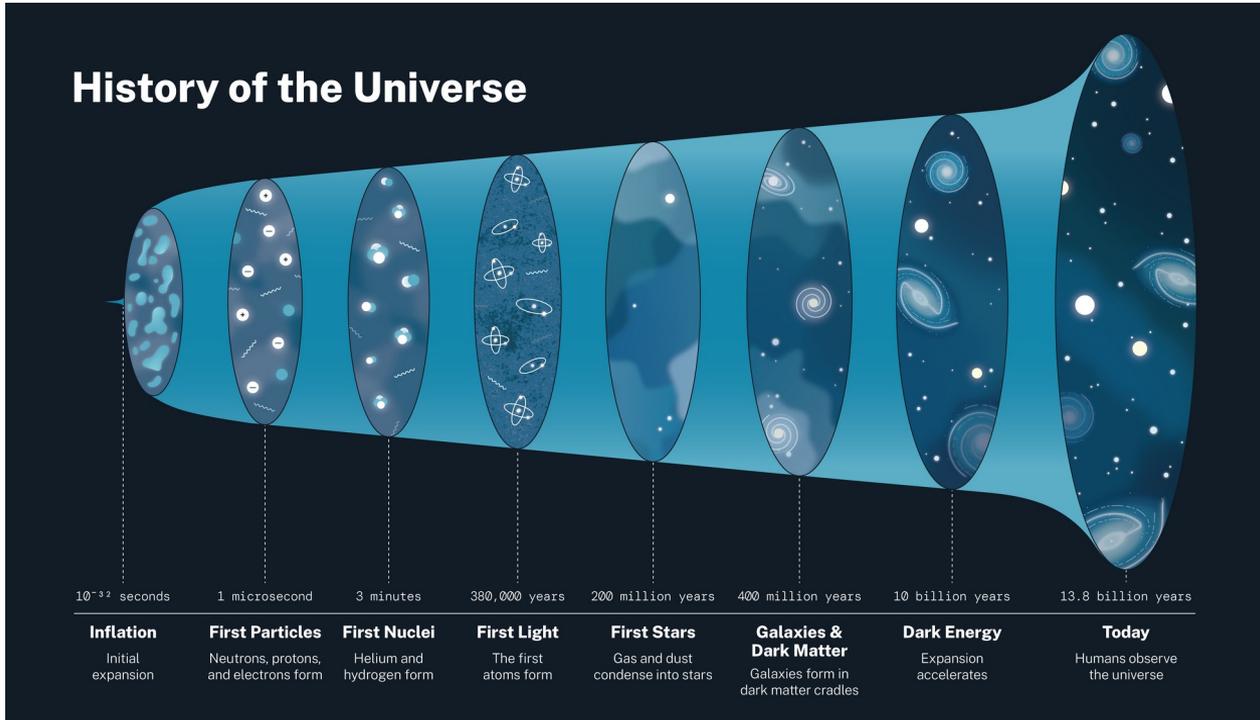


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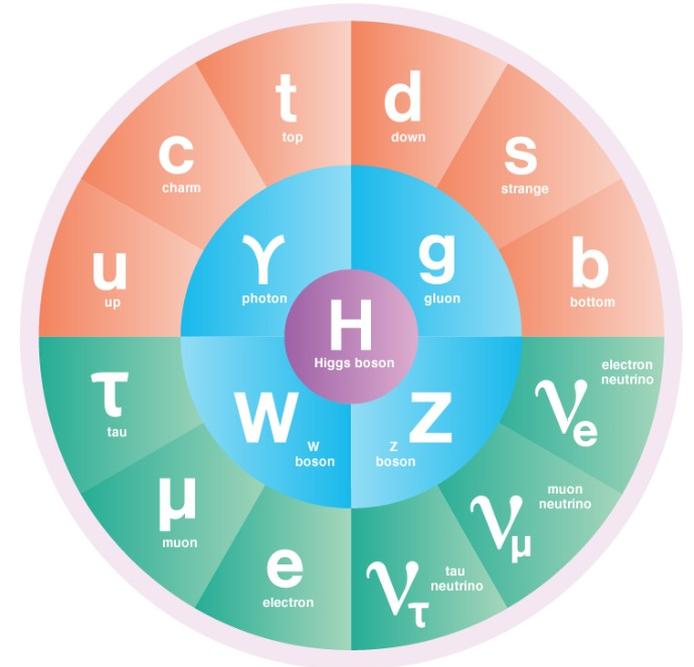
Outline

- Precision phenomenology at the Large Hadron Collider
- Theory predictions with higher-order corrections
- Phenomenology for $2 \rightarrow 3$ processes
- Summary and Outlook

What is the universe made of and where does it come from?



[Credit: NASA]

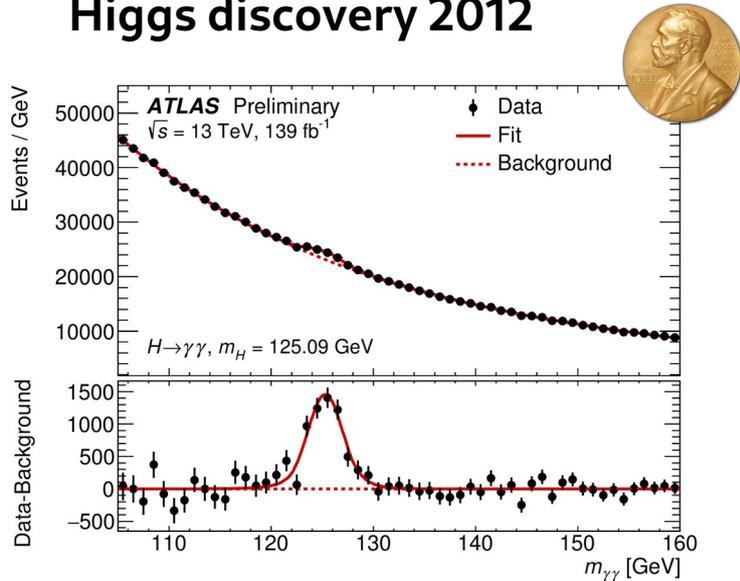


[Credit: SymmetryMagazine]

● QUARKS ● LEPTONS ● BOSONS ● HIGGS BOSON

Standard Model of Particle Physics and beyond

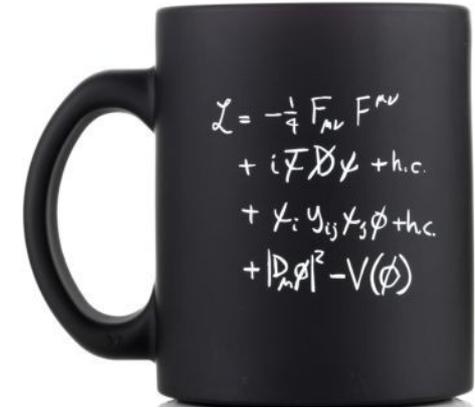
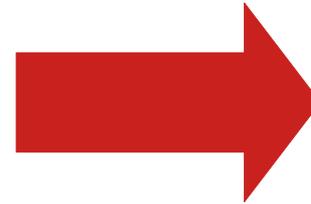
Higgs discovery 2012



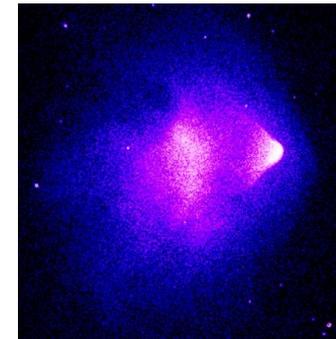
[Credit: ATLAS]

BUT:

- Is the Higgs a fundamental scalar?
- What is dark matter?
- Why is there a matter-anti-matter asymmetry?
- ...

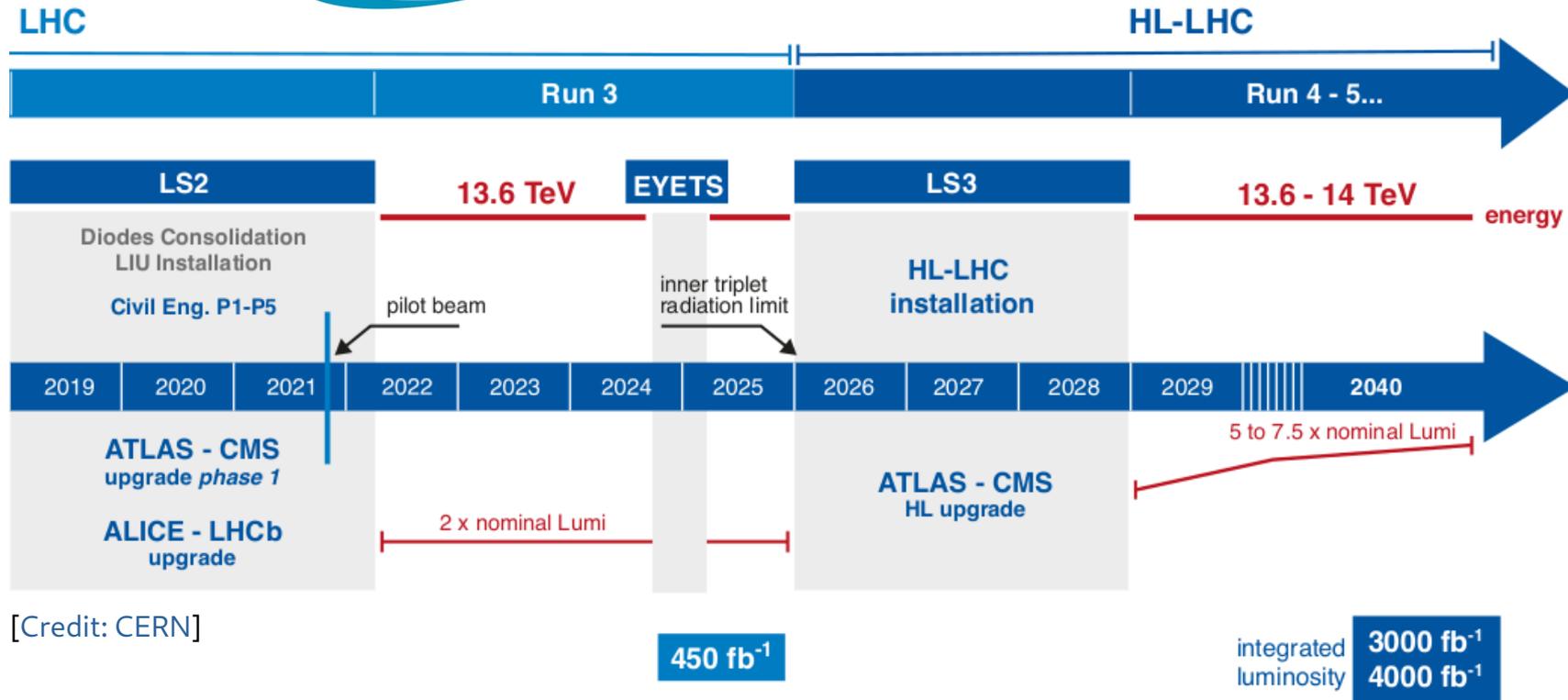


[Credit: CERN]



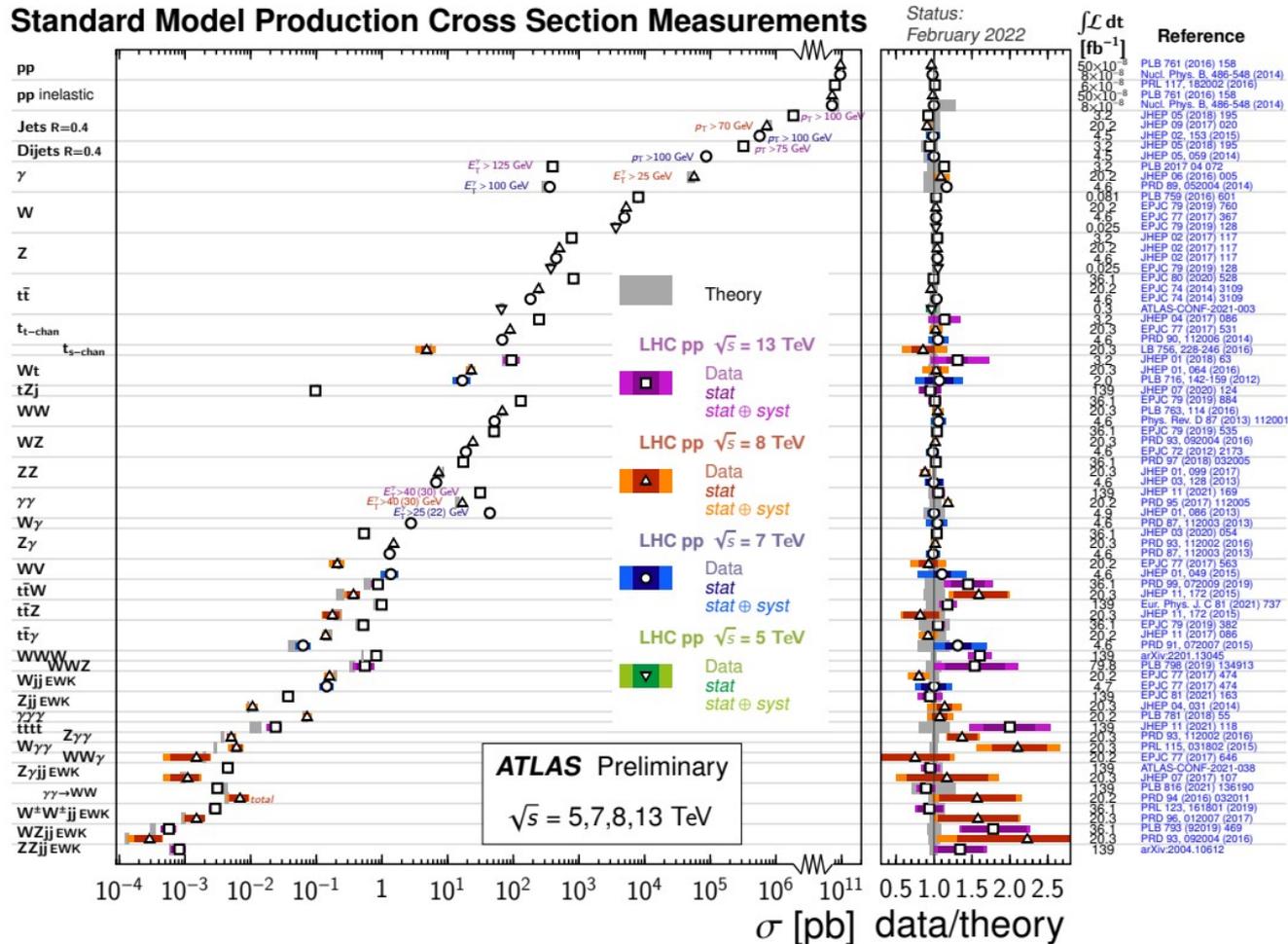
[Credit: NASA]

LHC Precision era and future experiments



[Credit: CERN]

SM measurements at the LHC



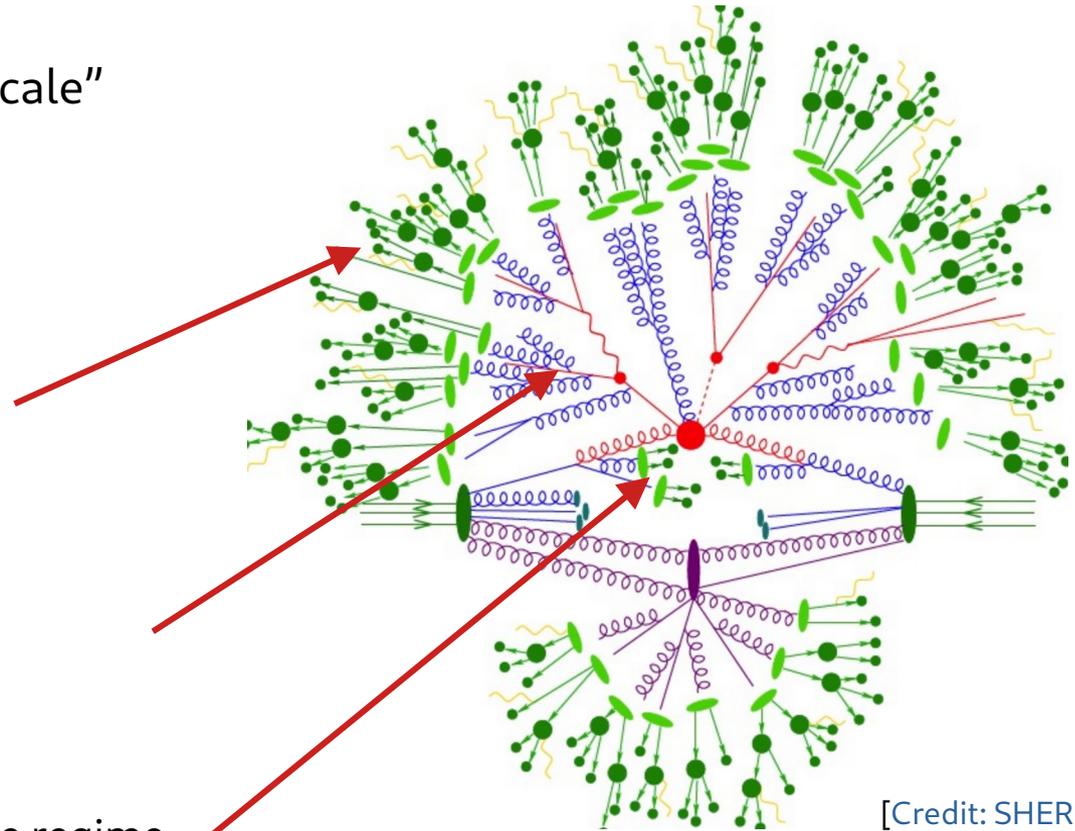
Theory picture of hadron collision events

Factorization

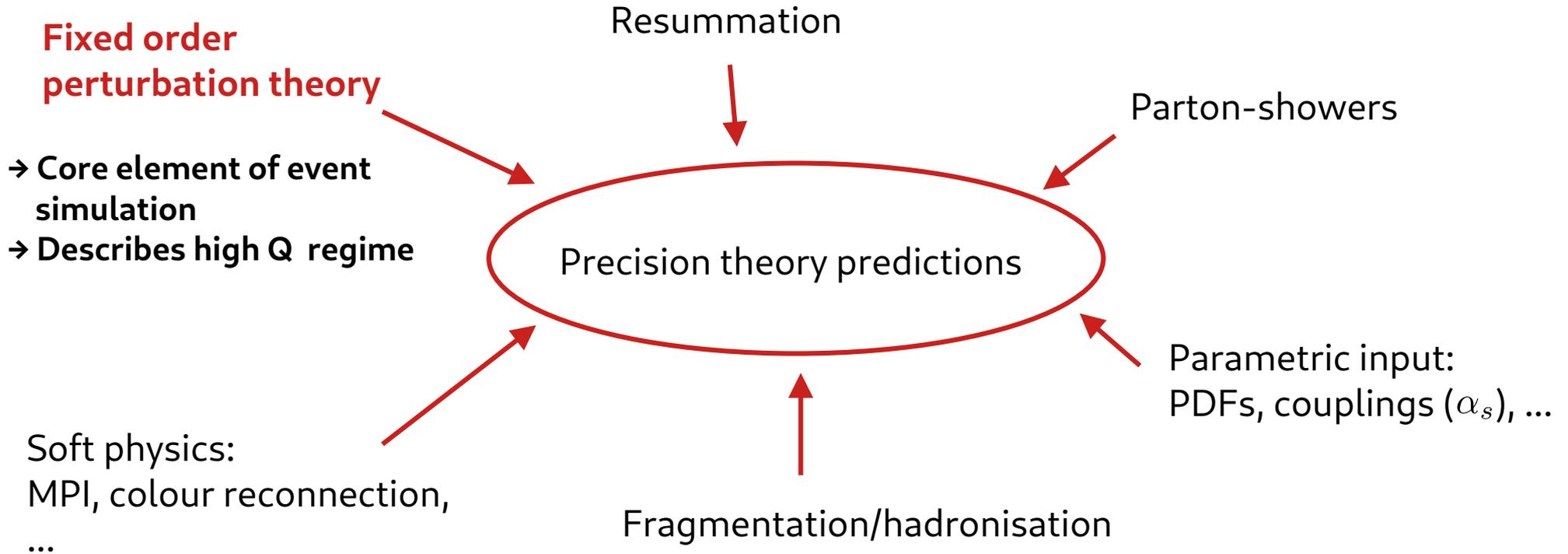
"What you see depends on the energy scale"

In Quantum Chromodynamics (QCD):

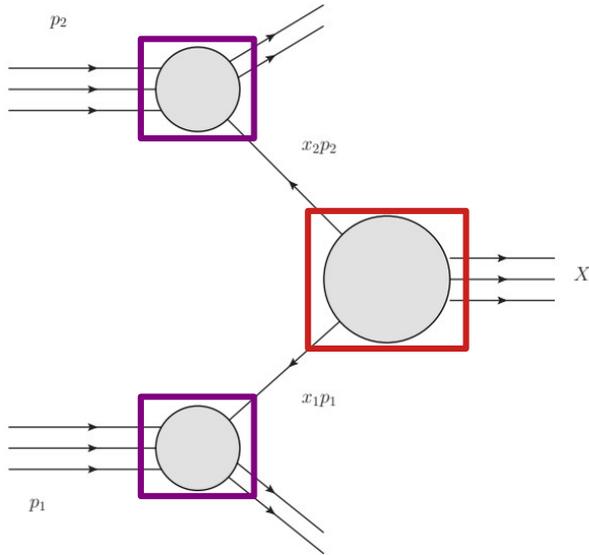
- $Q \sim \Lambda_{\text{QCD}}$
- Strong coupling
 - Realm of confined states
 - non-perturbative physics
- $Q \gtrsim \Lambda_{\text{QCD}}$
- Transition region
 - Parton-shower
 - Resummation
 - DGLAP / PDF evolution
- $Q \gg \Lambda_{\text{QCD}}$
- Small coupling \rightarrow perturbative regime
 - Scattering of individual partons



Precision predictions



Perturbative QCD



Hadronic cross section in collinear factorization:

$$\sigma_{h_1 h_2 \rightarrow X} = \sum_{ij} \int_0^1 \int_0^1 dx_1 dx_2 \underbrace{\phi_{i,h_1}(x_1, \mu_F^2)}_{\text{PDF}} \underbrace{\phi_{j/h_2}(x_2, \mu_F^2)}_{\text{PDF}} \underbrace{\hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)}_{\text{Hard Cross Section}}$$

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \hat{\sigma}_{ab \rightarrow X}^{(1)} + \hat{\sigma}_{ab \rightarrow X}^{(2)} + \mathcal{O}(\alpha_s^3)$$

Typical uncertainties from scale variations: $\delta\text{LO } \mathcal{O}(\sim 100\%)$, $\delta\text{NLO } \mathcal{O}(\sim 10\%)$, $\delta\text{NNLO } (\sim 1\%)$

(estimate for corrections from missing higher orders based on renormalisation scale invariance $\frac{d\sigma_{h_1 h_2 \rightarrow X}}{d\mu} = 0$)

Example: Production of three isolated photons

$$pp \rightarrow \gamma\gamma\gamma$$

Theory to data comparison

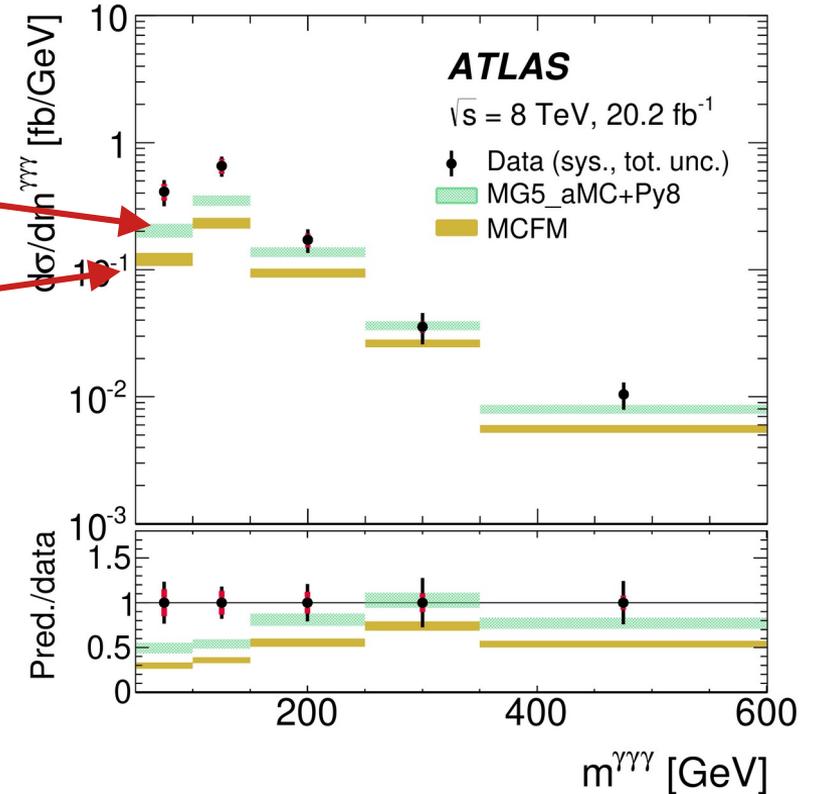
NLO QCD
+ Parton-shower simulation

Fixed-order NLO QCD

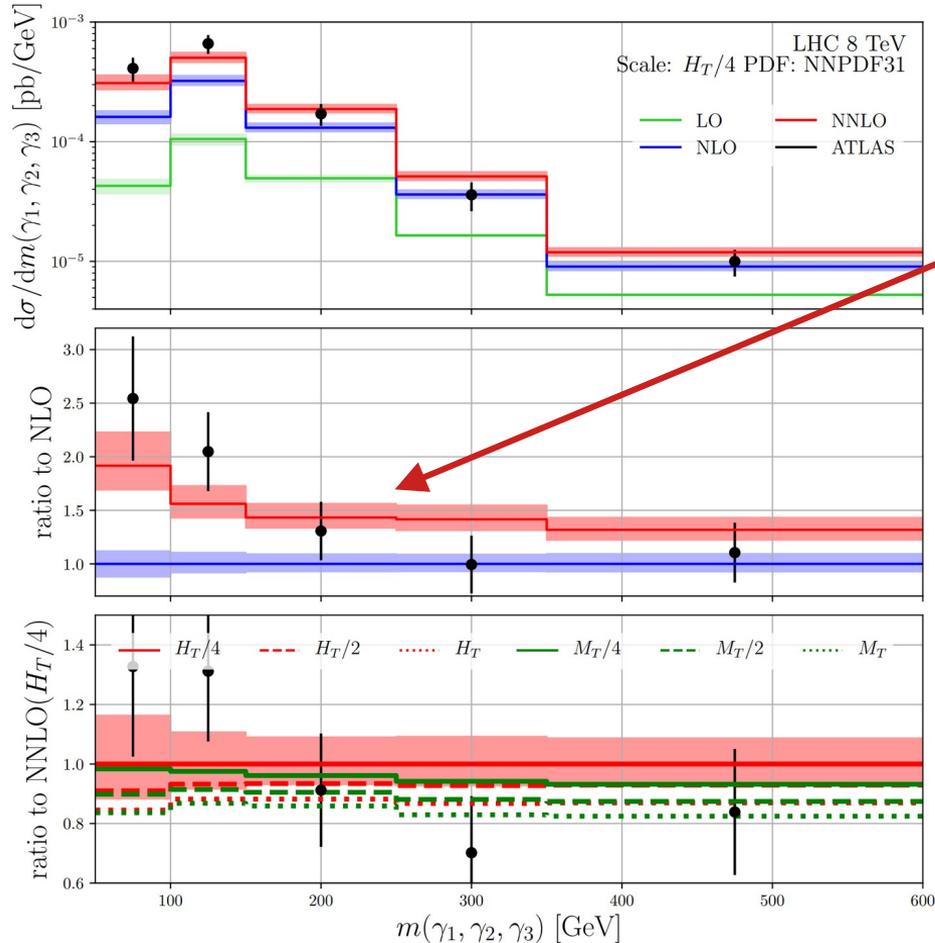
**Both fail to describe data
(normalization and shape)**

**Why?
→ NNLO QCD effects!**

Measurement of the production cross section of three isolated photons in pp collisions at $\sqrt{s} = 8$ TeV using the ATLAS detector, ATLAS [[1712.07291](#)]



NNLO QCD in three photon production



NNLO QCD corrections to three-photon production at the LHC, Chawdhry, Czakon, Mitov, Poncelet [JHEP 02 (2020) 057]

Corrections to **normalization and shape**

→ (Much) improved description of data

Without NNLO QCD corrections the data

- is not interpretable
→ loss of information

or

- is misleading
→ looks like “New Physics” = data - SM

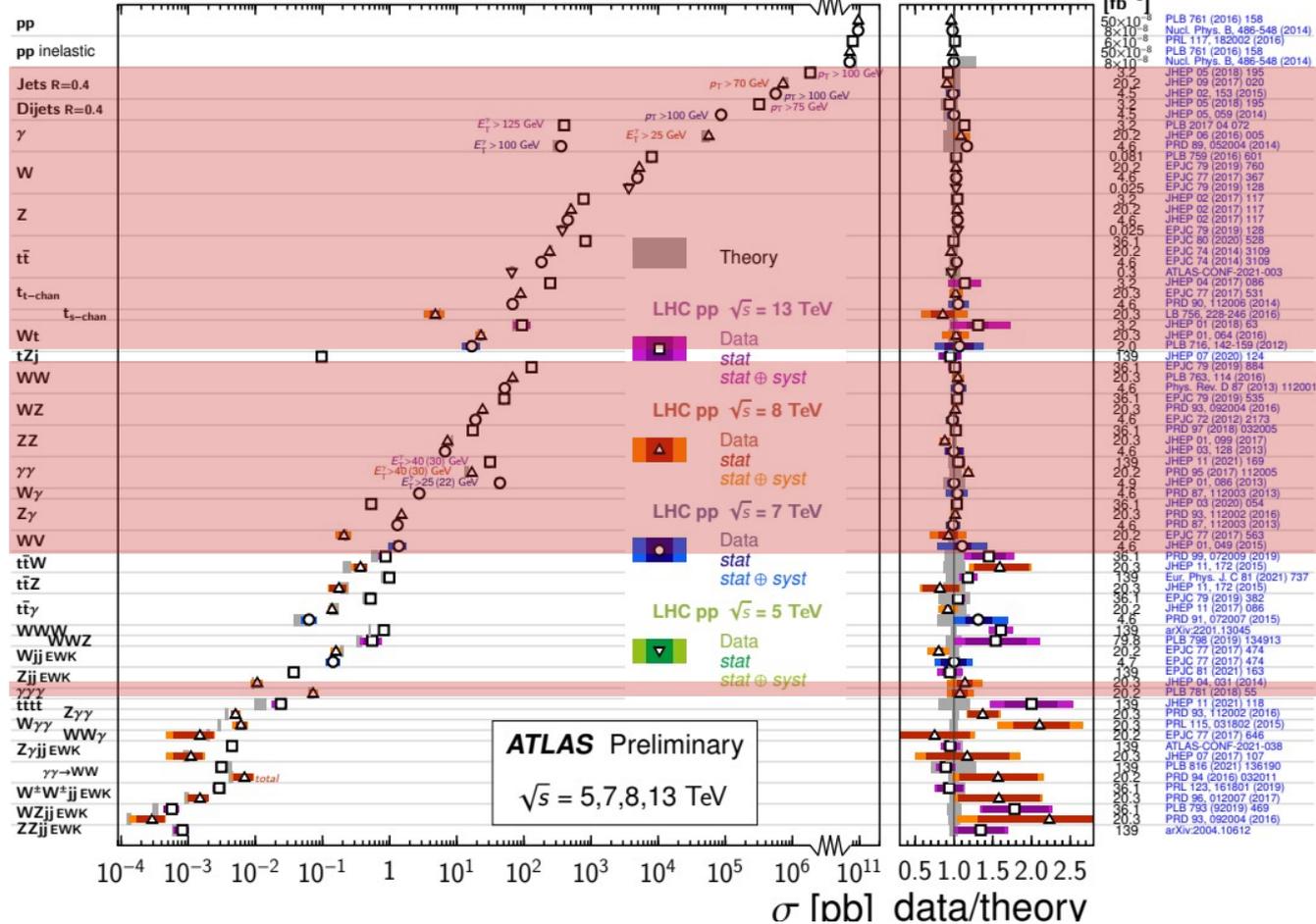
NNLO QCD coverage

Standard Model Production Cross Section Measurements

Status:
February 2022

$\int \mathcal{L} dt$
[fb⁻¹]

Reference



Theory predictions with higher-order corrections

Next-to-leading order case

$$\hat{\sigma}_{ab}^{(1)} = \hat{\sigma}_{ab}^{\text{R}} + \hat{\sigma}_{ab}^{\text{V}} + \hat{\sigma}_{ab}^{\text{C}}$$

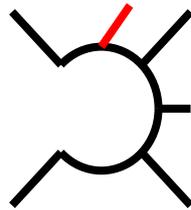


Each term separately infrared (IR) divergent:

KLN theorem

sum is finite for sufficiently inclusive observables and regularization scheme independent

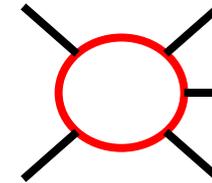
Real corrections:



$$\hat{\sigma}_{ab}^{\text{R}} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1}$$

Phase space integration over unresolved configurations

Virtual corrections:



$$\hat{\sigma}_{ab}^{\text{V}} = \frac{1}{2\hat{s}} \int d\Phi_n 2\text{Re} \langle \mathcal{M}_n^{(0)} | \mathcal{M}_n^{(1)} \rangle F_n$$

Integration over loop-momentum
(UV divergences cured by renormalization)

IR singularities in real radiation

$$\hat{\sigma}_{ab}^{\text{R}} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1}$$



$$\sim \int_0 dE d\theta \frac{1}{E(1 - \cos \theta)} f(E, \cos(\theta))$$

Finite function

Divergent

Regularization in Conventional Dimensional Regularization (CDR) $d = 4 - 2\epsilon$

$$\rightarrow \int_0 dE d\theta \frac{1}{E^{1-2\epsilon}(1 - \cos \theta)^{1-\epsilon}} f(E, \cos(\theta)) \sim \frac{1}{\epsilon^2} + \dots$$

Cancellation against similar divergences in

$$\hat{\sigma}_{ab}^{\text{V}} = \frac{1}{2\hat{s}} \int d\Phi_n 2\text{Re} \langle \mathcal{M}_n^{(0)} | \mathcal{M}_n^{(1)} \rangle F_n$$

How to extract these poles? Slicing and Subtraction

Central idea: Divergences arise from infrared (IR, soft/collinear) limits → Factorization!

Slicing

$$\hat{\sigma}_{ab}^R = \frac{1}{2\hat{s}} \int_{\delta(\Phi) \geq \delta_c} d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1} + \frac{1}{2\hat{s}} \int_{\delta(\Phi) < \delta_c} d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1}$$

$$\approx \frac{1}{2\hat{s}} \int_{\delta(\Phi) \geq \delta_c} d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1} + \frac{1}{2\hat{s}} \int d\Phi_n \tilde{M}(\delta_c) F_n + \mathcal{O}(\delta_c)$$

... + $\hat{\sigma}_{ab}^V$ = finite

Subtraction

$$\hat{\sigma}_{ab}^R = \frac{1}{2\hat{s}} \int \left(d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1} - d\tilde{\Phi}_{n+1} \mathcal{S}F_n \right) + \frac{1}{2\hat{s}} \int d\tilde{\Phi}_{n+1} \mathcal{S}F_n$$

$$\frac{1}{2\hat{s}} \int d\tilde{\Phi}_{n+1} \mathcal{S}F_n = \frac{1}{2\hat{s}} \int \underline{d\Phi_n d\Phi_1} \mathcal{S}F_n$$

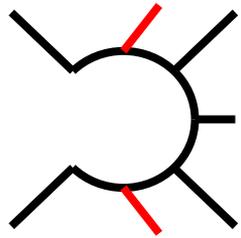
Phase space factorization
→ momentum mappings

Most popular
NLO QCD schemes:
CS [[hep-ph/9605323](https://arxiv.org/abs/hep-ph/9605323)],
FKS [[hep-ph/9512328](https://arxiv.org/abs/hep-ph/9512328)]

→ **Basis of modern event simulation**

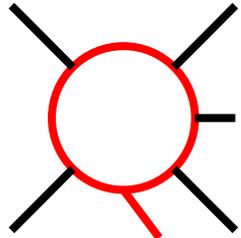
Partonic cross section beyond NLO

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{VV} + \hat{\sigma}_{ab}^{RV} + \hat{\sigma}_{ab}^{RR} + \hat{\sigma}_{ab}^{C2} + \hat{\sigma}_{ab}^{C1}$$



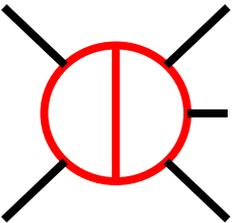
Real-Real

$$\hat{\sigma}_{ab}^{RR} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2}$$



Real-Virtual

$$\hat{\sigma}_{ab}^{RV} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(1)} \rangle F_{n+1}$$



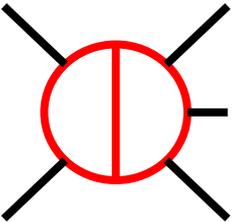
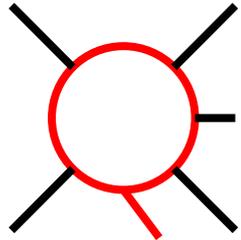
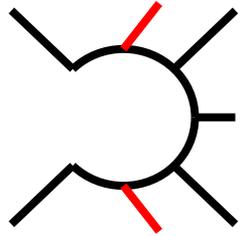
Virtual-Virtual

$$\hat{\sigma}_{ab}^{VV} = \frac{1}{2\hat{s}} \int d\Phi_n \left(2\text{Re} \langle \mathcal{M}_n^{(0)} | \mathcal{M}_n^{(2)} \rangle + \langle \mathcal{M}_n^{(1)} | \mathcal{M}_n^{(1)} \rangle \right) F_n$$

$$\hat{\sigma}_{ab}^{C2} = (\text{double convolution}) F_n \quad \hat{\sigma}_{ab}^{C1} = (\text{single convolution}) F_{n+1}$$

Partonic cross section beyond NLO

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{VV} + \hat{\sigma}_{ab}^{RV} + \hat{\sigma}_{ab}^{RR} + \hat{\sigma}_{ab}^{C2} + \hat{\sigma}_{ab}^{C1}$$



Real-Real

$$\hat{\sigma}_{ab}^{RR} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2}$$

Technically substantially more complicated!

Main bottlenecks:

- Real - real \rightarrow overlapping singularities
Many possible limits \rightarrow good organization principle needed
- Real - virtual \rightarrow stable matrix elements
- Virtual - virtual \rightarrow complicated case-by-case analytic treatment

Real-Virtual

Virtual-Virtual

$$\hat{\sigma}_{ab}^{C2} = (\text{double convolution}) F_n \quad \hat{\sigma}_{ab}^{C1} = (\text{single convolution}) F_{n+1}$$

Slicing and Subtraction

Slicing

- Conceptually simple
- Recycling of lower computations
- Non-local cancellations/power-corrections
→ computationally expensive

Subtraction

- Conceptually more difficult
- Local subtraction → efficient
- Better numerical stability
- Choices:
 - Momentum mapping
 - Subtraction terms
 - Numerics vs. analytic

NNLO QCD schemes

qT-slicing [Catani'07],
N-jettiness slicing [Gaunt'15/Boughezal'15]

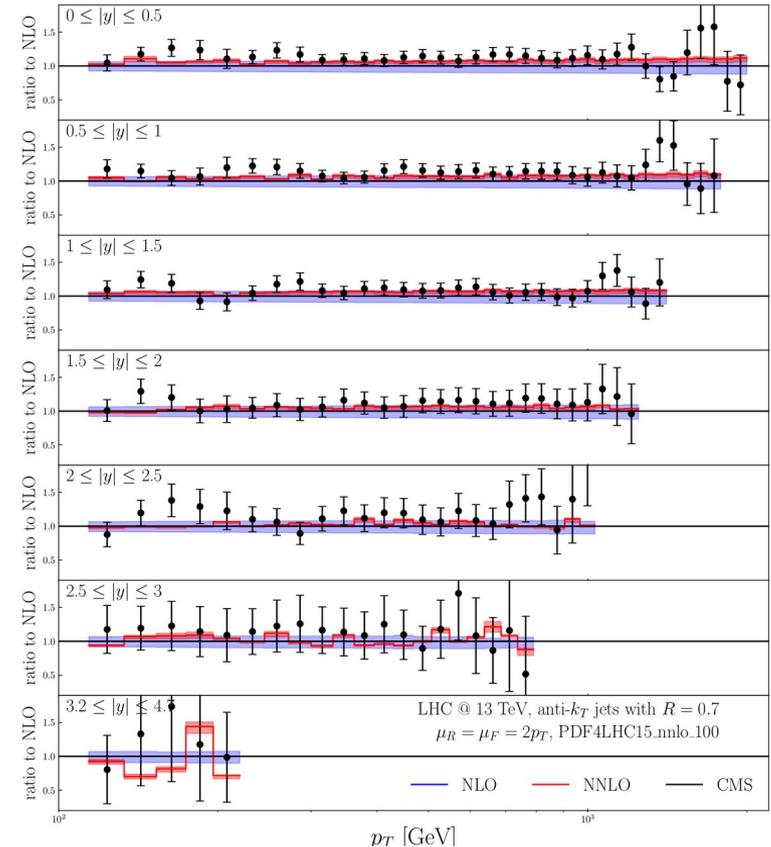
Antenna [Gehrmann'05-'08],
Colorful [DelDuca'05-'15],
Sector-improved residue subtraction [Czakon'10-'14'19]
Projection [Cacciari'15],
Nested collinear [Caola'17],
Geometric [Herzog'18],
Unsubtraction [Aguilera-Verdugo'19],
...

Minimal sector-improved residue subtraction

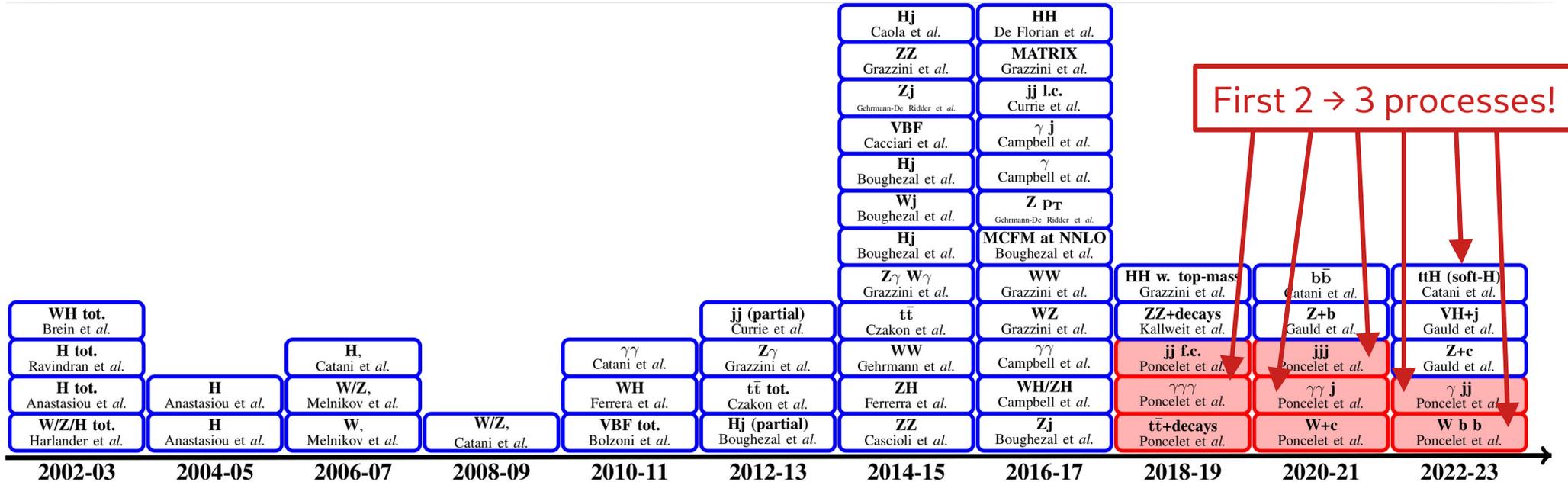
Single-jet inclusive rates with exact color at $\mathcal{O}(\alpha_s^4)$
Czakon, Hameren, Mitov, Poncelet, *JHEP* 10 (2019), 262

Refined formulation of the
sector-improved residue subtraction

- New phase space parametrisation
→ minimization of subtraction kinematics
→ improved computational efficiency/stability
- Improved sector decomposition
- New 4 – dimensional formulation
- First application: inclusive jet production
→ demonstrates that the **scheme is complete**
→ no approximations



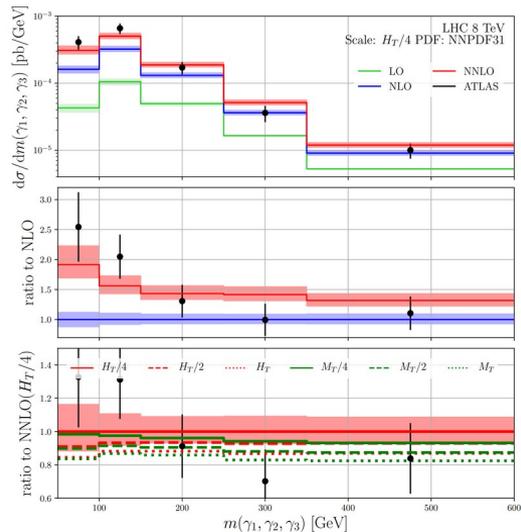
The NNLO QCD revolution



First 2 \rightarrow 3 processes!

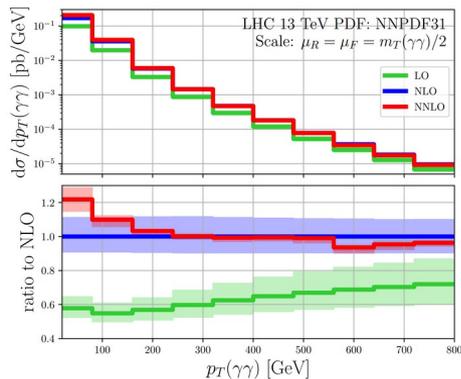
NNLO QCD for massless $2 \rightarrow 3$ processes

$$pp \rightarrow \gamma\gamma\gamma$$



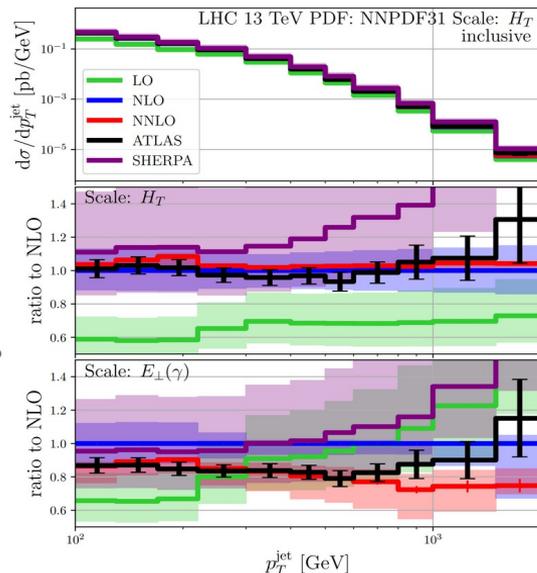
Chawdhry, Czakon, Mitov,
Poncelet [1911.00479]
 Kallweit, Sotnikov,
 Wiesemann [2010.04681]

$$pp \rightarrow \gamma\gamma j$$



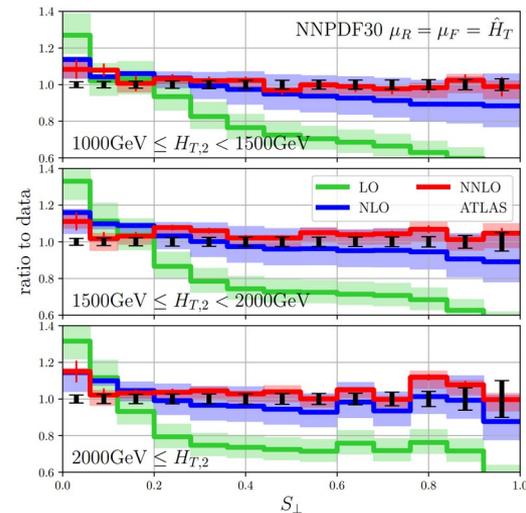
Chawdhry, Czakon, Mitov,
Poncelet [2103.04319]

$$pp \rightarrow \gamma jj$$



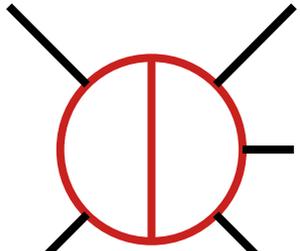
Badger, Czakon, Hartanto,
 Moodie, Peraro, **Poncelet**,
 Zoia [2304.06682]

$$pp \rightarrow jjj$$



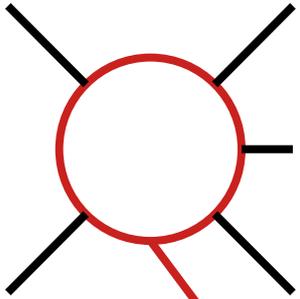
Czakon, Mitov, **Poncelet**
 [2106.05331]
 + Alvarez, Cantero, Llorente
 [2301.01086]

NNLO QCD for 2→3 processes - inputs



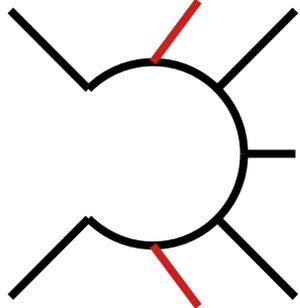
Two-loop amplitudes

- (Non-) planar 5 point massless [[Chawdry'19'20'21](#),[Abreu'20'21'23](#),[Agarwal'21](#),[Badger'21'23](#)]
→ triggered by efficient MI representation [[Chicherin'20](#)]



One-loop amplitudes → OpenLoops [[Buccioni'19](#)]

- Many legs and IR stable (soft and collinear limits)



Double-real Born amplitudes → AvHlib [[Bury'15](#)]

- IR finite cross-sections → NNLO subtraction schemes
qT-slicing [[Catani'07](#)], N-jettiness slicing [[Gaunt'15/Boughezal'15](#)], Antenna [[Gehrmann'05-'08](#)],
Colorful [[DelDuca'05-'15](#)], Projection [[Cacciari'15](#)], Geometric [[Herzog'18](#)],
Unsubtraction [[Aguilera-Verdugo'19](#)], Nested collinear [[Caola'17](#)],
Local Analytic [[Magnea'18](#)], **Sector-improved residue subtraction** [[Czakon'10-'14,'19](#)]

Phenomenology for 2 → 3 processes

Multi-jet observables

Test of pQCD and extraction of strong coupling constant

NLO theory unc. (MHO) > experimental unc.

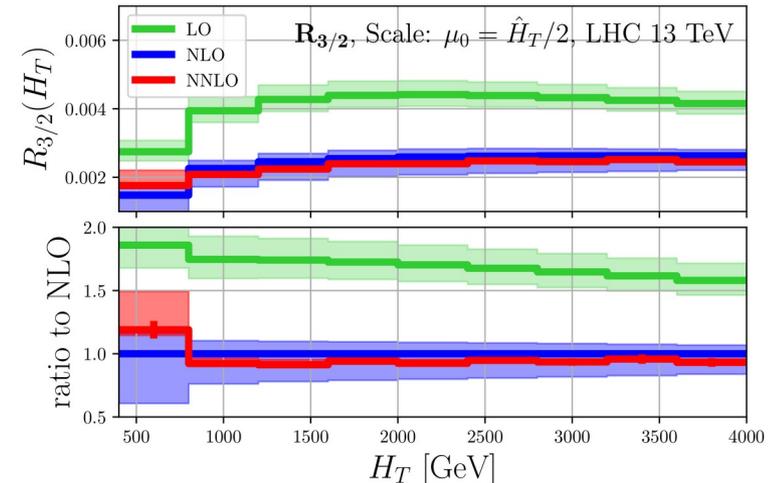
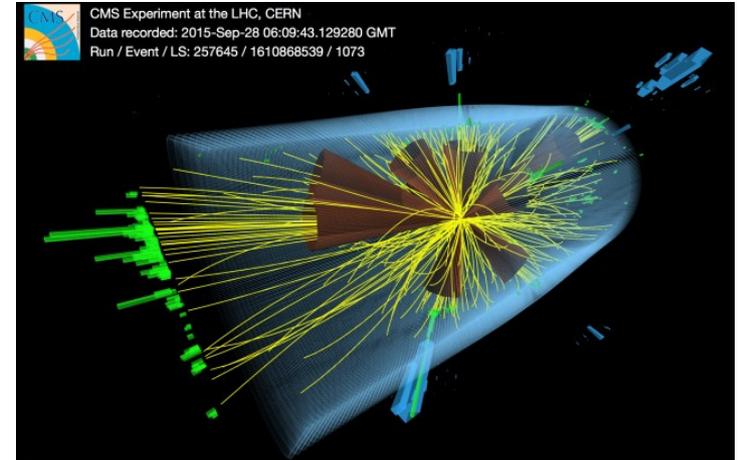
- **NNLO QCD needed for precise theory-data** comparisons
→ Restricted to two-jet data [Currie'17+later][Czakon'19]
- **New NNLO QCD three-jet** → access to more observables
- Jet ratios

Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC
Czakon, Mitov, Poncelet *Phys.Rev.Lett.* 127 (2021) 15, 152001

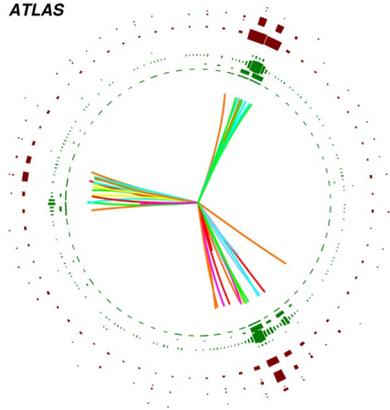
$$R^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0}) = \frac{d\sigma_3^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}{d\sigma_2^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}$$

- Event shapes

NNLO QCD corrections to event shapes at the LHC
Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet *JHEP* 03 (2023) 129



Encoding QCD dynamics in event shapes



Using (global) event information to separate different regimes of QCD event evolution:

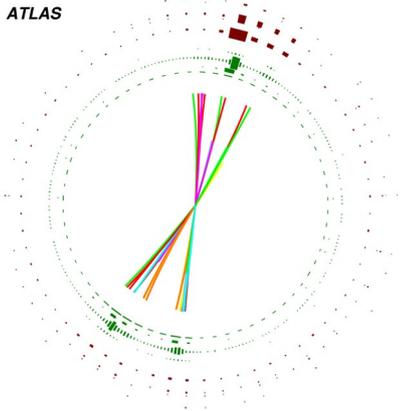
Energy-energy correlators

$$\frac{1}{\sigma_2} \frac{d\sigma}{d \cos \Delta\phi} = \frac{1}{\sigma_2} \sum_{ij} \int \frac{d\sigma x_{\perp,i} x_{\perp,j}}{dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij}} \delta(\cos \Delta\phi - \cos \Delta\phi_{ij}) dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij},$$

Separation of energy scales: $H_{T,2} = p_{T,1} + p_{T,2}$

Ratio to 2-jet: $R^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0}) = \frac{d\sigma_3^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}{d\sigma_2^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}$

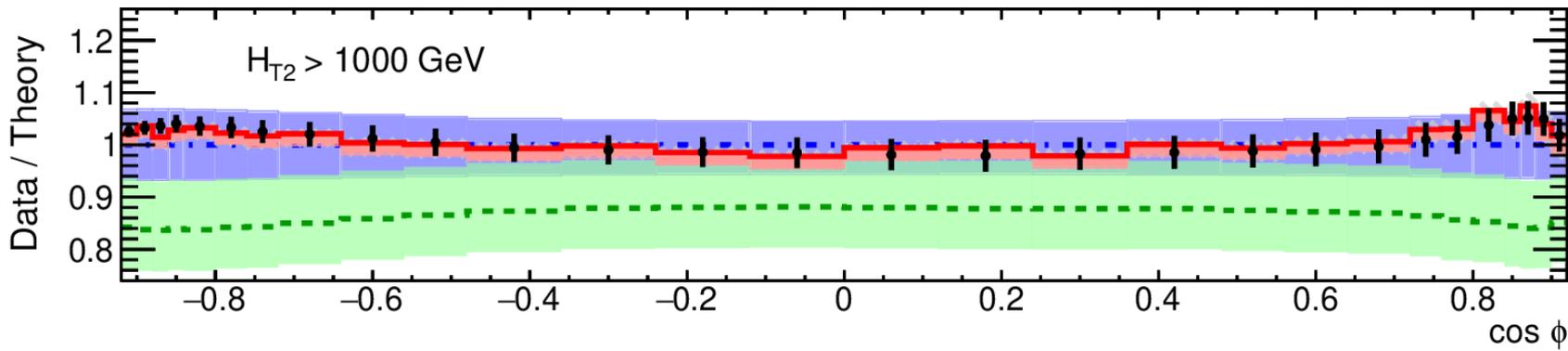
Here: **jets as input** → experimentally advantageous
(better calibrated, smaller non-pert.)



The transverse energy-energy correlator

$$\frac{1}{\sigma_2} \frac{d\sigma}{d \cos \Delta\phi} = \frac{1}{\sigma_2} \sum_{ij} \int \frac{d\sigma x_{\perp,i} x_{\perp,j}}{dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij}} \delta(\cos \Delta\phi - \cos \Delta\phi_{ij}) dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij},$$

- Insensitive to soft radiation through energy weighting $x_{T,i} = E_{T,i} / \sum_j E_{T,j}$
- Event topology separation:
 - Central plateau contain isotropic events
 - To the right: self-correlations, collinear and in-plane splitting
 - To the left: back-to-back



[ATLAS 2301.09351]

ATLAS

Particle-level TEEC

$\sqrt{s} = 13 \text{ TeV}; 139 \text{ fb}^{-1}$

anti- k_t $R = 0.4$

$p_T > 60 \text{ GeV}$

$|\eta| < 2.4$

$\mu_{R,F} = \hat{P}_T$

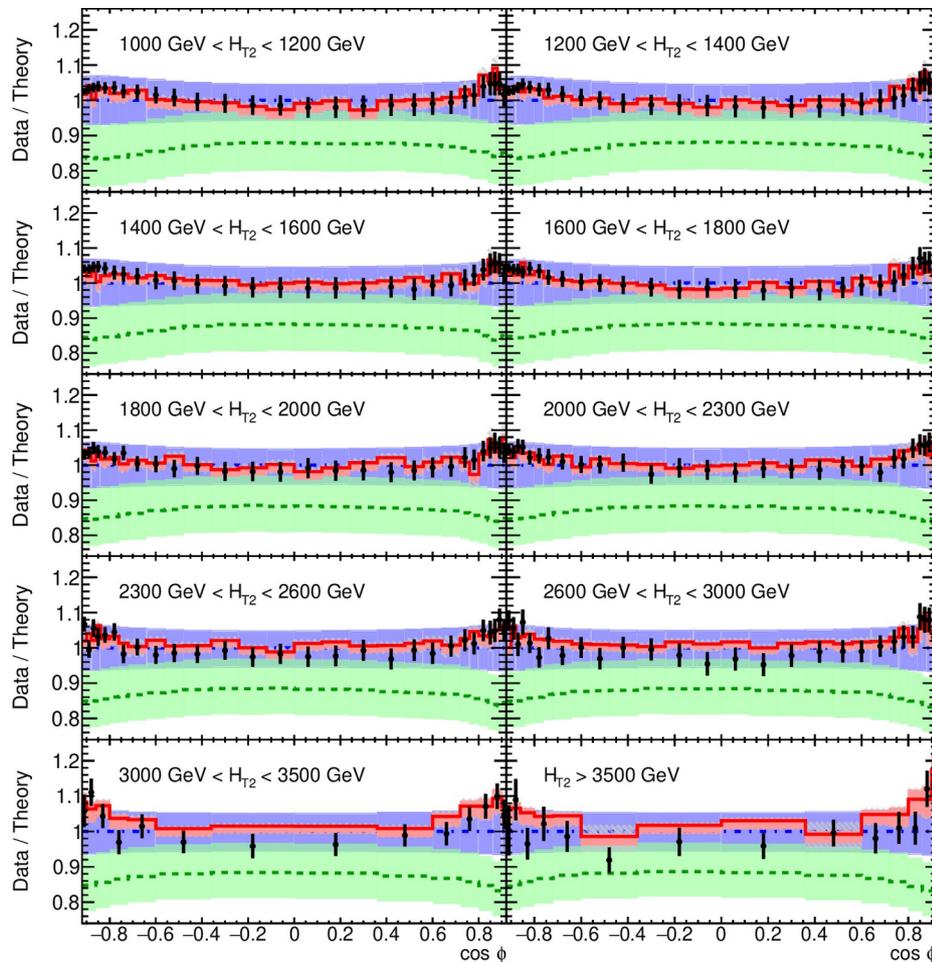
$\alpha_s(m_Z) = 0.1180$

NNPDF 3.0 (NNLO)

—•— Data
 - - - LO
 - · - NLO
 - - - NNLO

Double differential TEEC

[ATLAS 2301.09351]



ATLAS

Particle-level TEEC

$\sqrt{s} = 13 \text{ TeV}; 139 \text{ fb}^{-1}$

anti- k_t $R = 0.4$

$p_T > 60 \text{ GeV}$

$|\eta| < 2.4$

$\mu_{R,F} = \hat{p}_T$

$\alpha_s(m_Z) = 0.1180$

NNPDF 3.0 (NNLO)

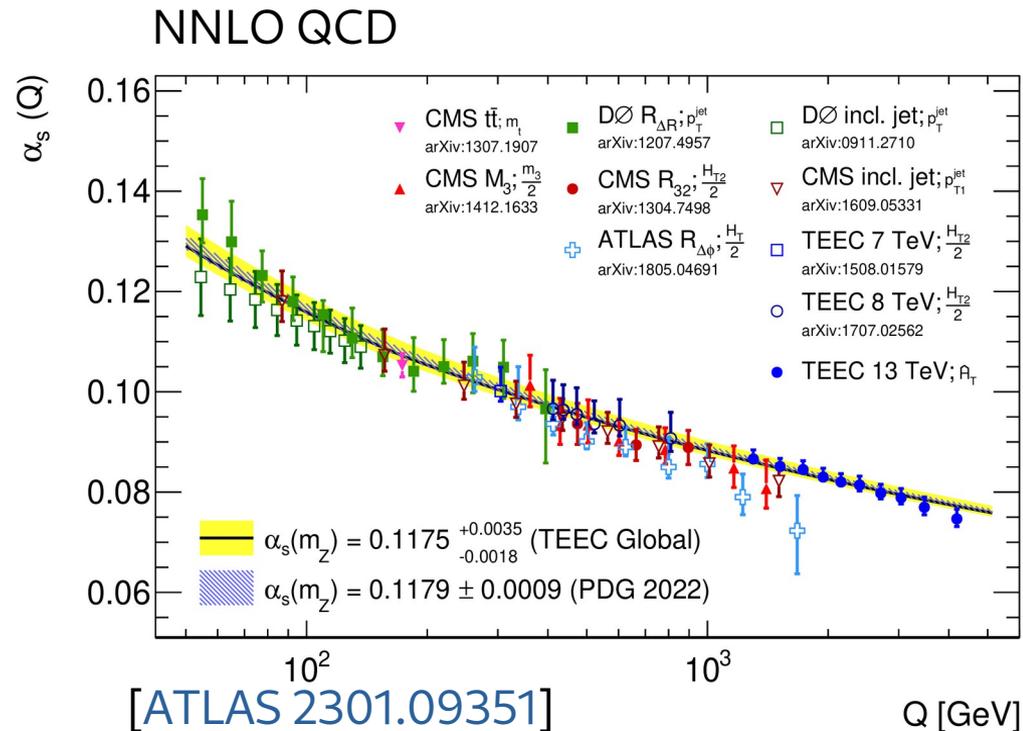
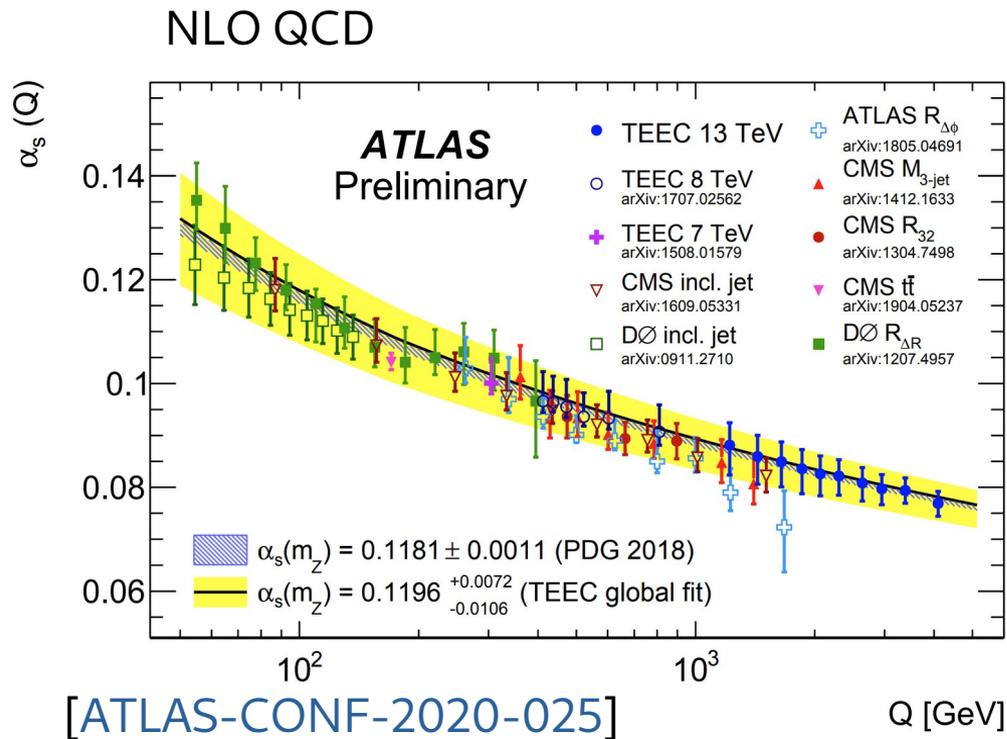
— Data

--- LO

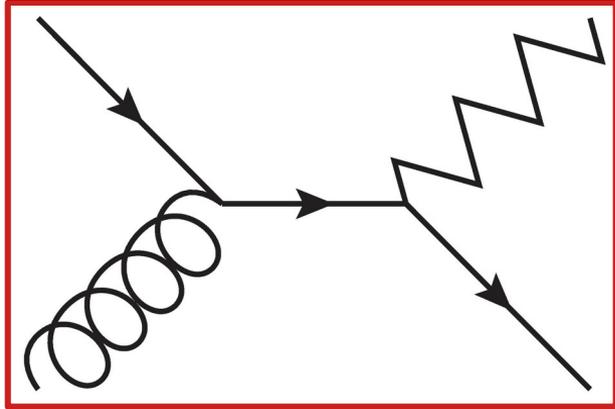
--- NLO

--- NNLO

Running of α_s

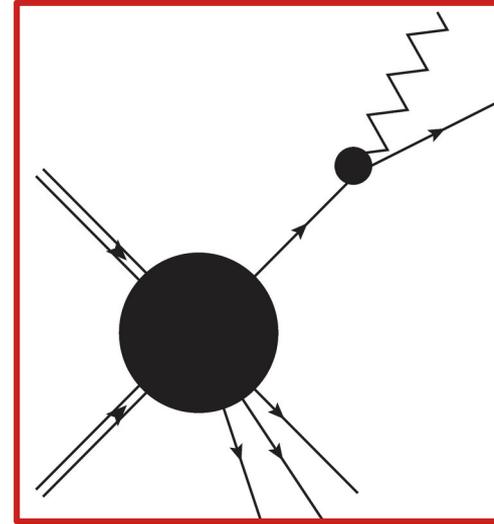


Prompt photon production



Direct production

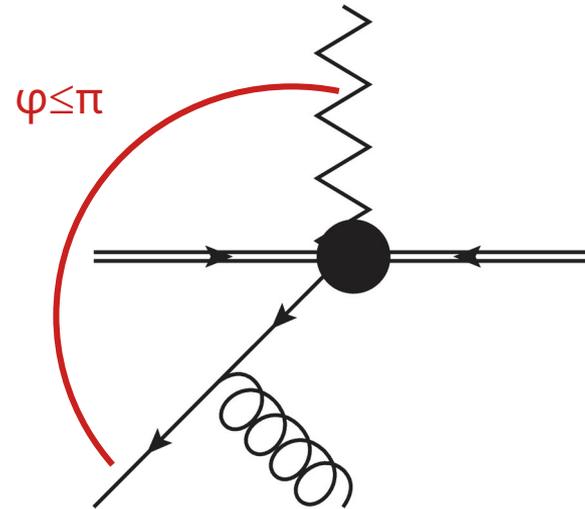
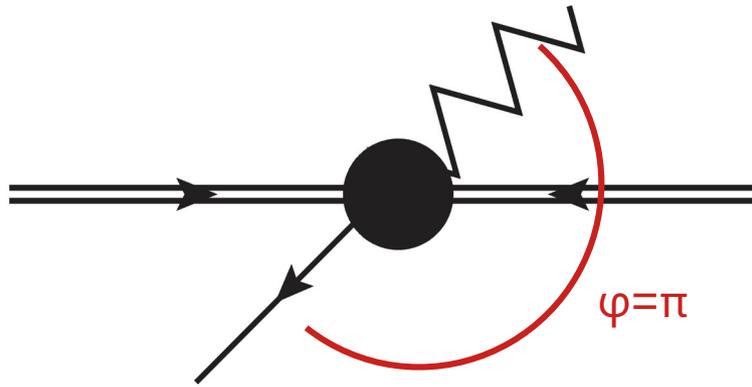
- Test of perturbative QCD
- Gluon PDF sensitivity
- Estimates for BSM backgrounds



Fragmentation

- Depends on non-perturbative fragmentation functions
- Separation from “direct” not unique

Why photon plus a jet pair?



- Non-back-to-back Born configurations
→ access to angular correlations between the photon and jets
- Access to different kinematic regimes through distinguishable photon
→ enhance direct, high- or low- z fragmentation
- Background process for BSM: $pp \rightarrow \gamma + Y (\rightarrow jj)$

Photon plus jet pair

Measurement of isolated-photon plus two-jet production in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector [[1912.09866](#)]

Requirements on photon	$E_T^\gamma > 150$ GeV, $ \eta^\gamma < 2.37$ (excluding $1.37 < \eta^\gamma < 1.56$) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 4.8$ GeV (reconstruction level) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 10$ GeV (particle level)		
Requirements on jets	at least two jets using anti- k_r algorithm with $R = 0.4$ $p_T^{\text{jet}} > 100$ GeV, $ y^{\text{jet}} < 2.5$, $\Delta R^{\gamma\text{-jet}} > 0.8$		
Phase space	total	fragmentation enriched	direct enriched
		$E_T^\gamma < p_T^{\text{jet}2}$	$E_T^\gamma > p_T^{\text{jet}1}$
Number of events	755 270	111 666	386 846

Modelled with hybrid isolation

$$E_\perp(r) \leq E_{\perp\text{max}}(r) = 0.1 E_\perp(\gamma) \left(\frac{1 - \cos(r)}{1 - \cos(R_{\text{max}})} \right)^2 \quad \text{for } r \leq R_{\text{max}} = 0.1$$

+

$$E_\perp(r) \leq E_{\perp\text{max}} = 0.0042 E_\perp(\gamma) + 10 \text{ GeV} \quad \text{for } r \leq R_{\text{max}} = 0.4$$



No fragmentation contribution
 → Purely pQCD through NNLO
 → focus on “inclusive” and “direct” PS

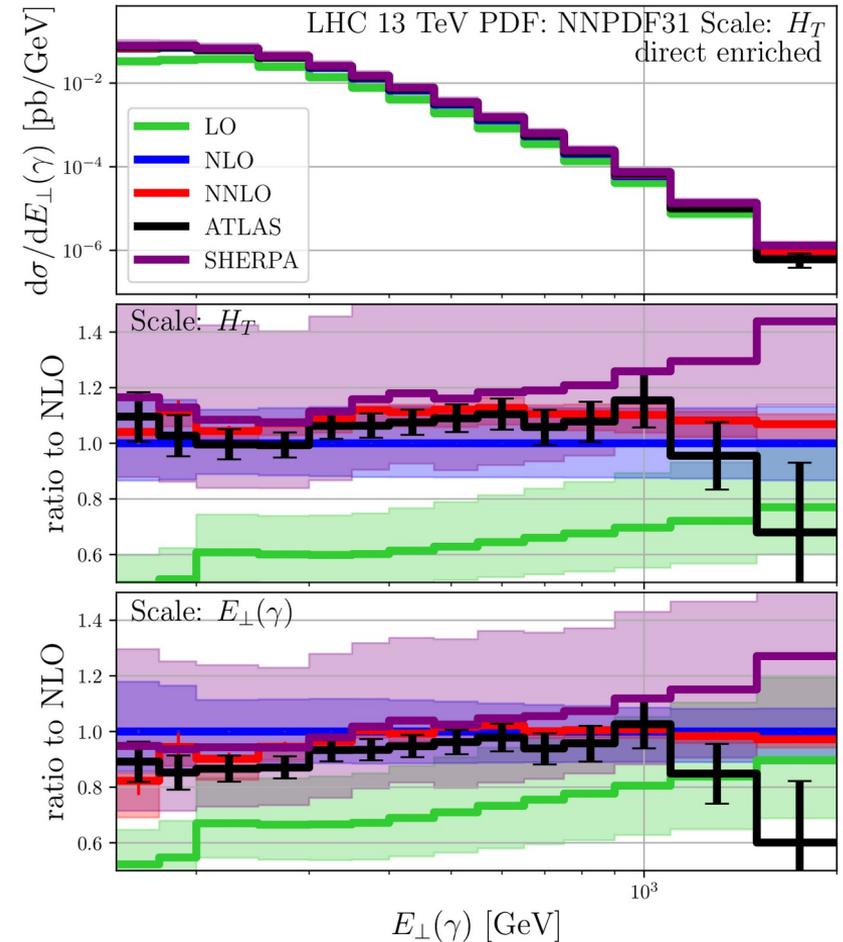
Theory - data comparisons

NNLO QCD

- Describes data well
- Improvements on the shape
- Small corrections
- Small remaining scale dependence

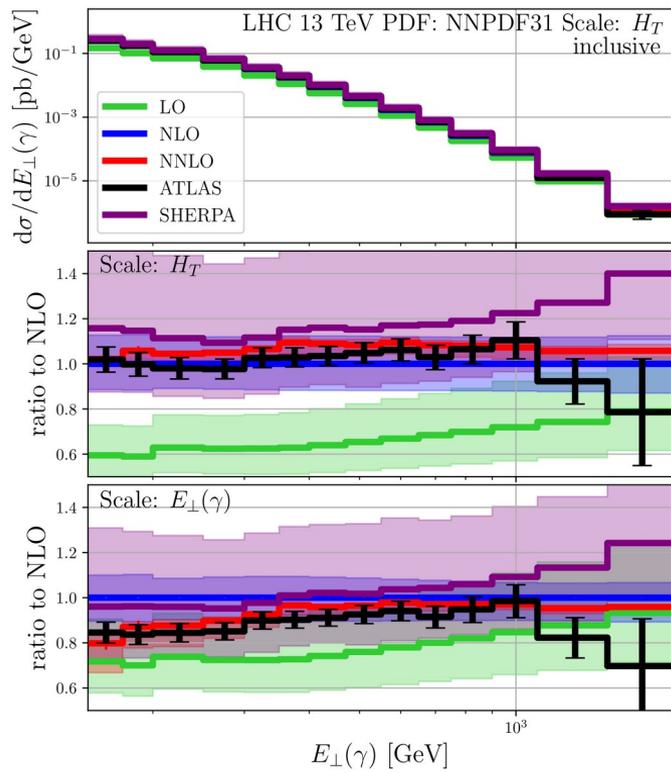
Comment on the SHERPA predictions

- Large NLO scale uncertainties
- The shape is not well described
- Maybe an artefact of multi-jet merging?

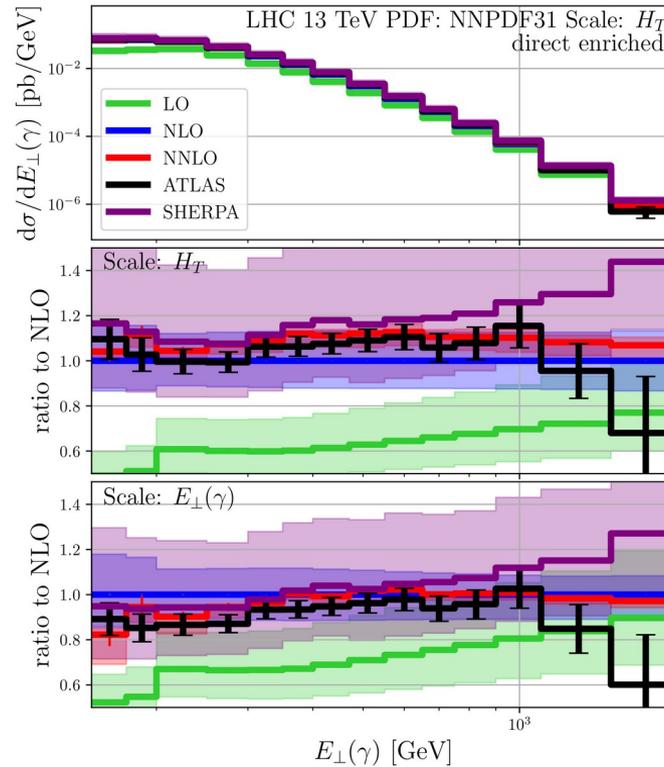


Inclusive vs. direct vs. fragmentation

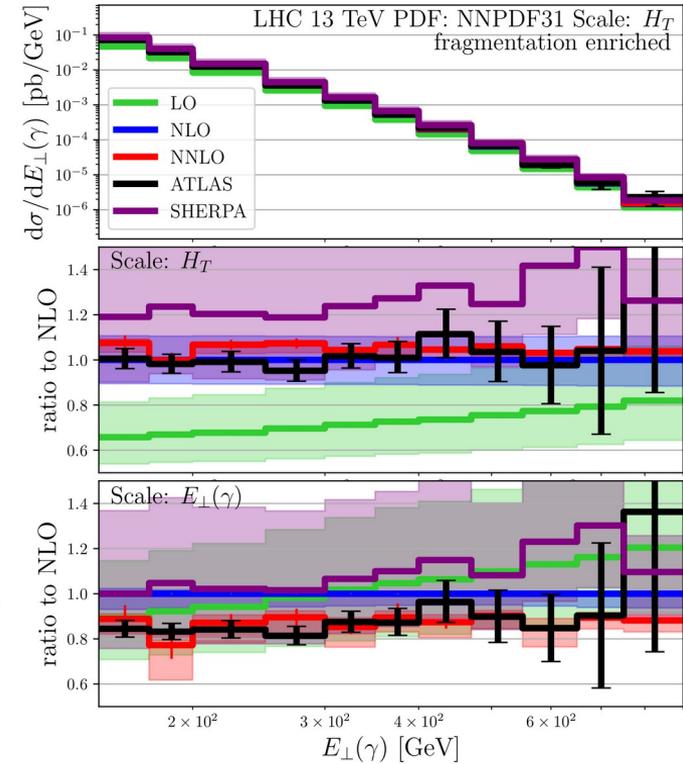
Inclusive



Direct-enriched



Fragmentation



Transverse photon energy

Scale choice

Full tree kinematics

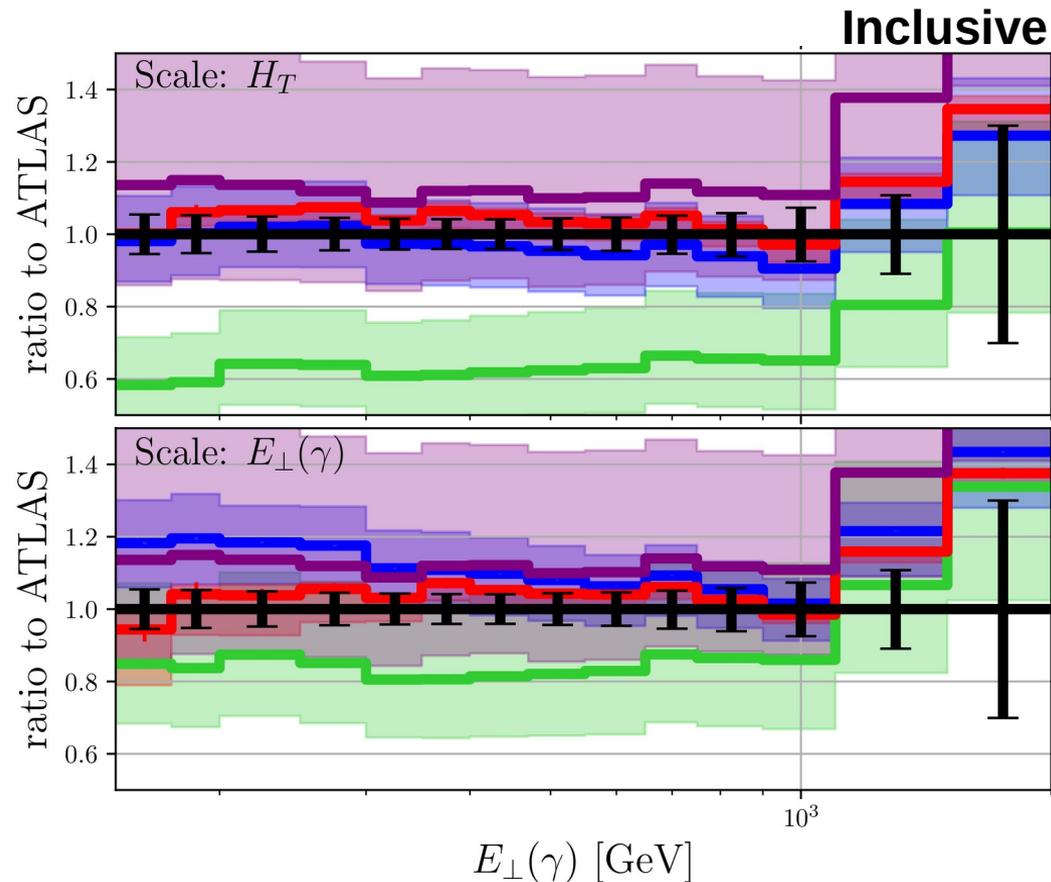
$$\mu_R = \mu_F = H_T = E_\perp(\gamma) + p_T(j_1) + p_T(j_2)$$
$$\mu_R = \mu_F = E_\perp(\gamma),$$

Only photon

Perturbative convergence

NNLO result similar **but** $E_\perp(\gamma)$

- Larger (negative) NNLO corrections
- Larger scale dependence (for jet obs.)



Scale choice

Full tree kinematics

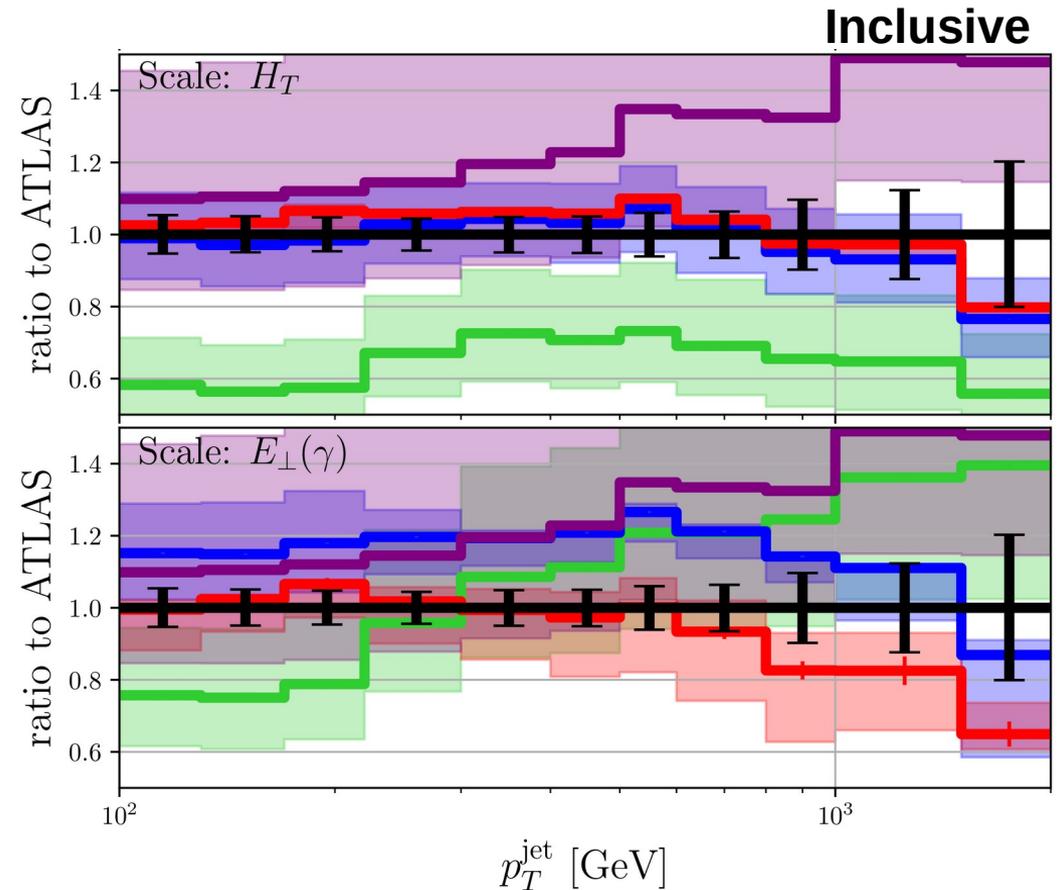
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Only photon

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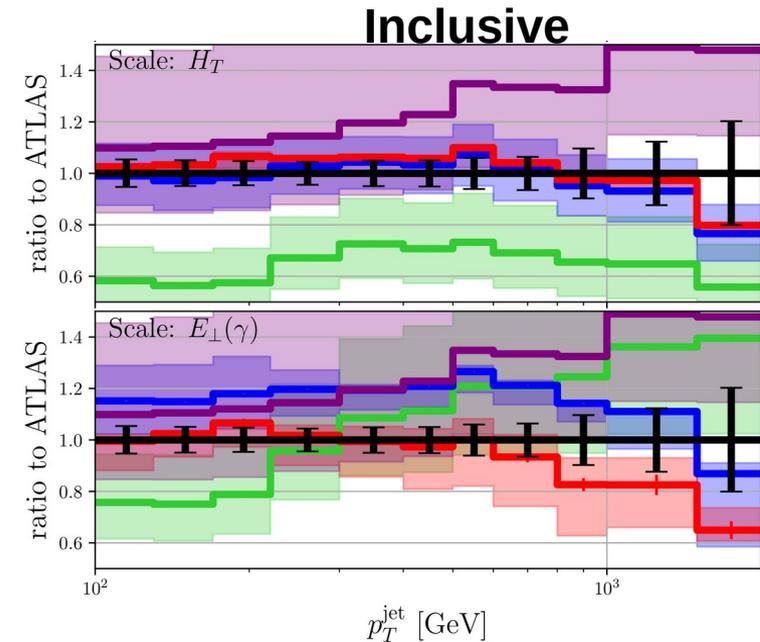
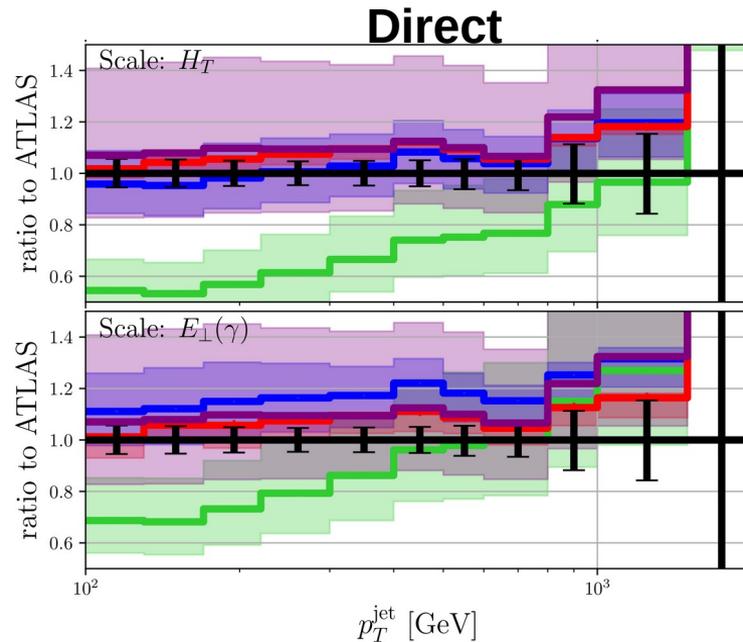


Scale choice

→ $E_{\perp}(\gamma)$ does not capture relevant scales for $pp \rightarrow \gamma + 2j$

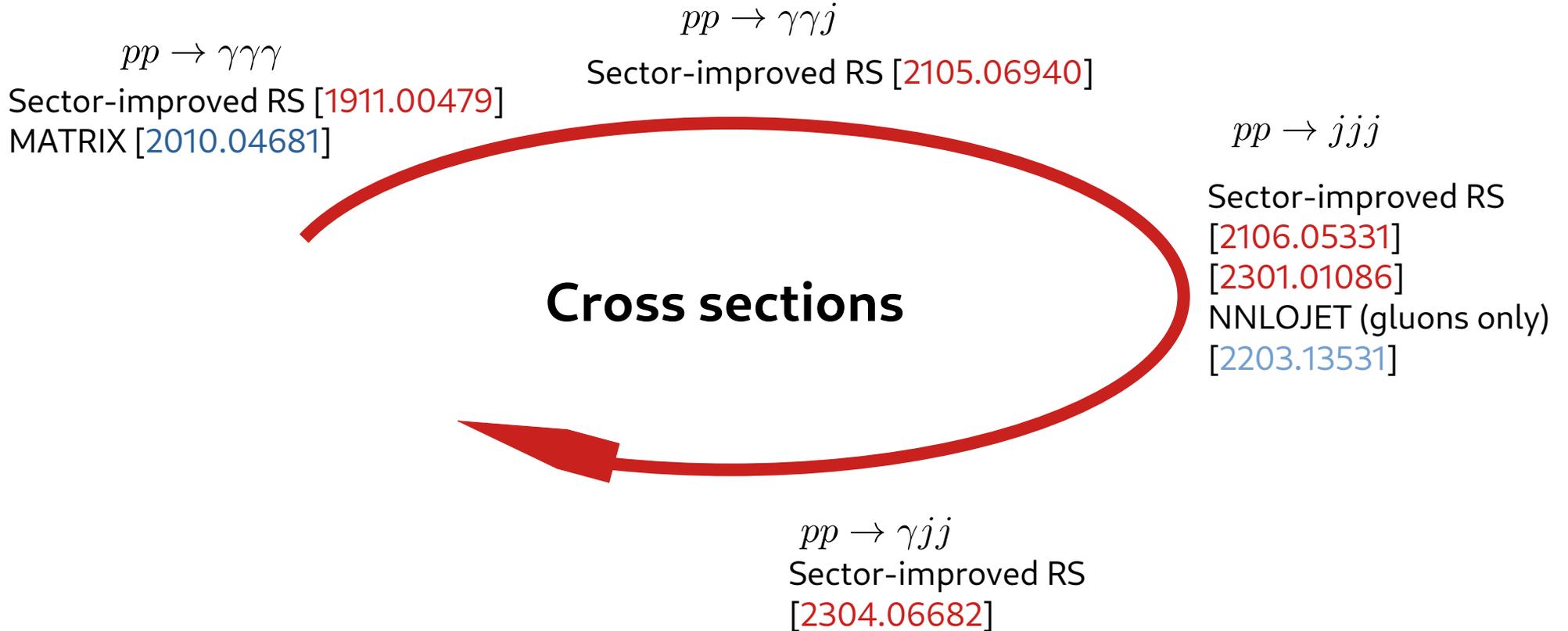
- Better for “direct” enriched phase space $p_T(\gamma) > p_T(j_1)$
→ $E_{\perp}(\gamma)$ closer to $H_T = p_T(\gamma) + p_T(j_1) + p_T(j_2)$

**NNLO QCD needed
for this conclusion**

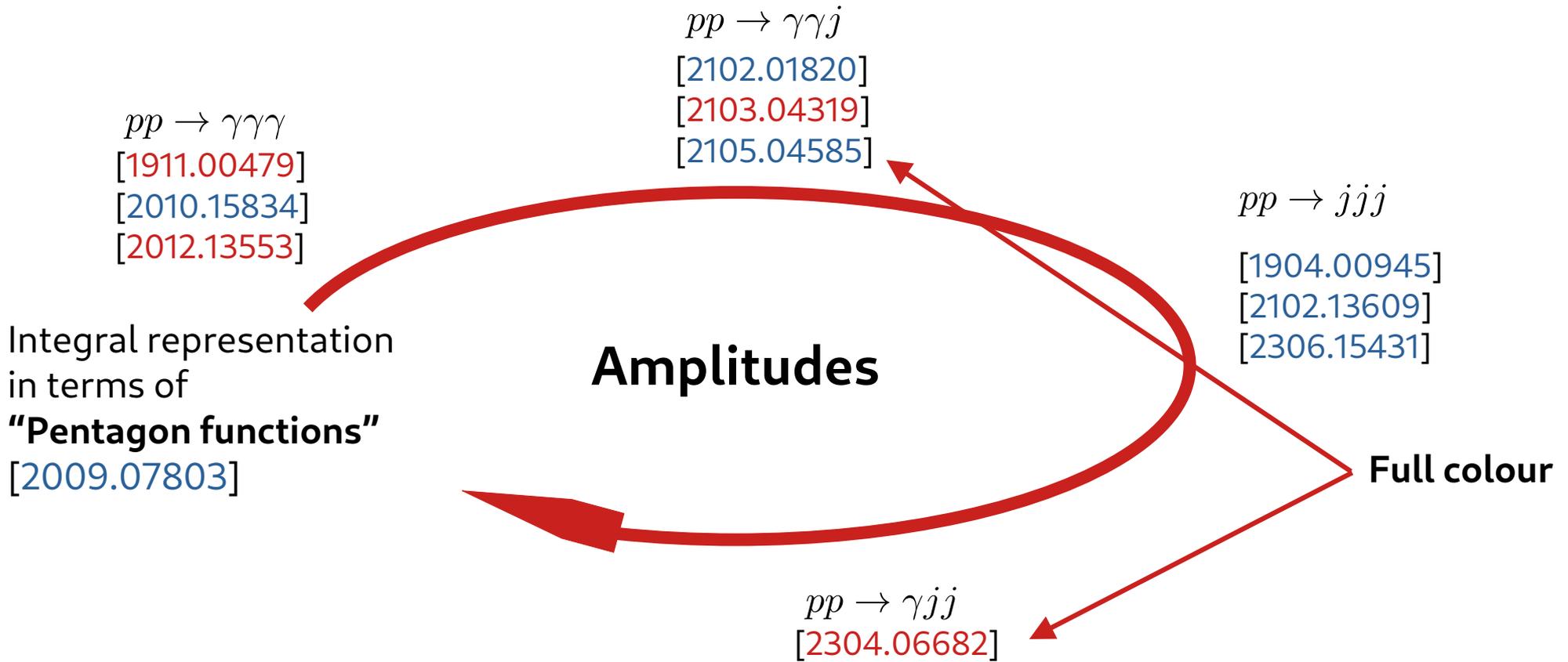


Summary & Outlook

Overview 2 → 3 massless cross sections



Overview 2 → 3 massless amplitudes



→ $pp \rightarrow \gamma jj$ **first computation with full colour two-loop** matrix elements

Take home messages

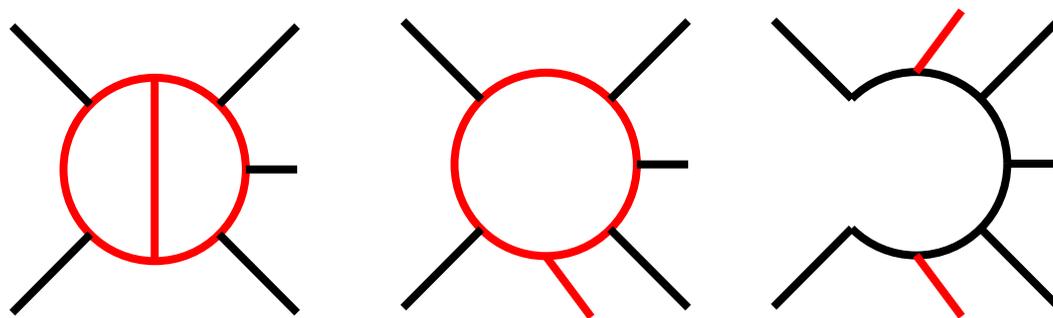
- Precision phenomenology is stable of LHC physics
 - but requires higher-order corrections!
 - NNLO QCD or even higher orders are needed to keep up with experimental precision
- Completion of **massless 2→3** processes at hadron colliders through NNLO QCD

$$pp \rightarrow \gamma\gamma\gamma \quad pp \rightarrow \gamma\gamma j \quad pp \rightarrow \gamma jj \quad pp \rightarrow jjj$$

- Most important bottlenecks from theory side:
 - Real radiation contributions
(subtraction, Monte Carlo methods, efficiency, automation,...)
 - Two-loop amplitudes
(including external/internal masses are the current frontier)

Backup

Sector-improved residue subtraction



Sector decomposition I

Considering working in CDR:

→ Virtuals are usually done in this regularization: $\hat{\sigma}_{ab}^{VV} = \sum_{i=-4}^0 c_i \epsilon^i + \mathcal{O}(\epsilon)$

→ Can we write the real radiation as such expansion?

→ Difficult integrals, analytical impractical (except very simple observables)!

→ Numerics not possible, integrals are divergent → ϵ -poles!



How to extract these poles? → Sector decomposition!

Divide and conquer the phase space:

$$1 = \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \longrightarrow \hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle_{\text{F}_{n+2}}$$

Sector decomposition II

Divide and conquer the phase space

- Each $\mathcal{S}_{i,k}$ (NLO), $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$ (NNLO) has simpler divergences:

- Soft limits of partons i and j
- Collinear w.r.t partons k (and l) of partons i and j

$$\mathcal{S}_{i,k} = \frac{1}{D_1 d_{i,k}} \quad D_1 = \sum_{ik} \frac{1}{d_{i,k}} \quad d_{i,k} = \frac{E_i}{\sqrt{s}} (1 - \cos \theta_{ik})$$

- Parametrization w.r.t. reference parton makes divergences explicit

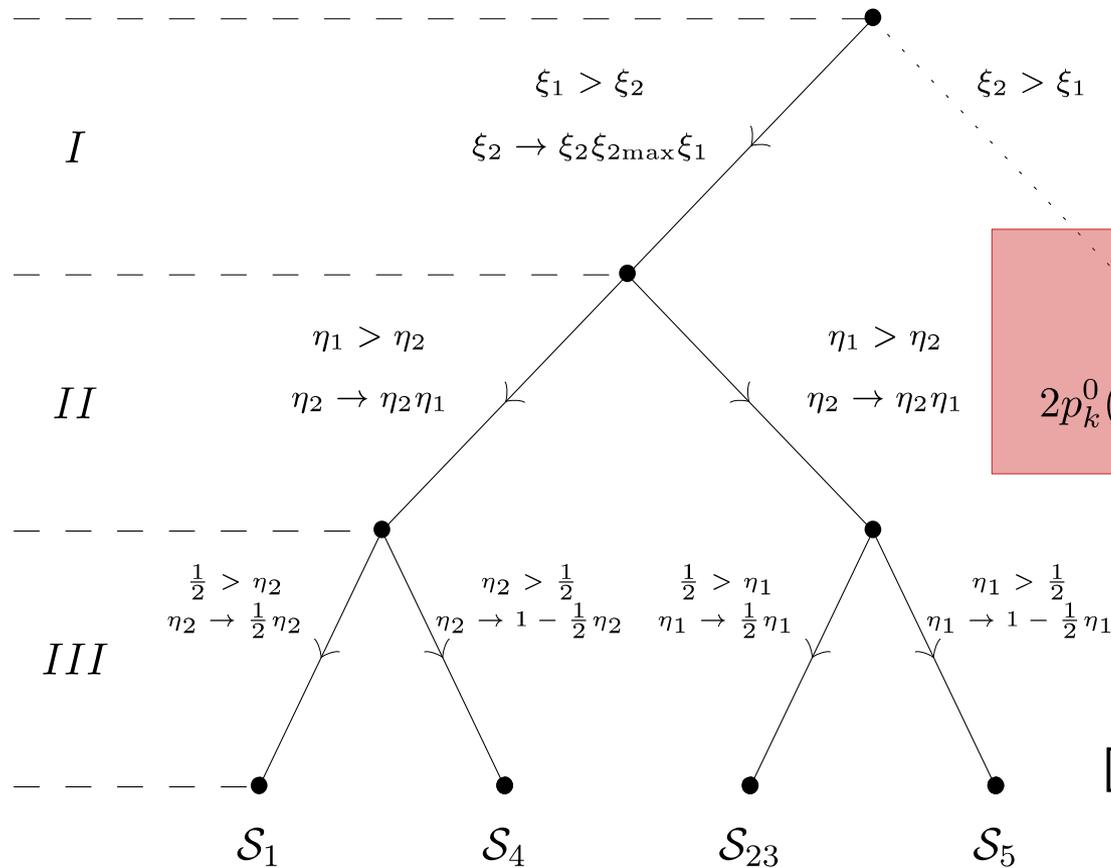
$$\hat{\eta}_i = \frac{1}{2}(1 - \cos \theta_{ik}) \in [0, 1] \quad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0, 1]$$

- Example: Splitting function

$$\sim \frac{1}{s_{ik}} P(z) \quad s_{ik} = (p_i + p_k)^2 = 2p_k^0 u_{\max}^0 \xi_i \eta_i \quad \sim \frac{1}{\eta_i \xi_i}$$

Sector decomposition II – triple collinear factorization

Three particle invariant does not factorize trivially:



Double soft factorization:

$$\theta(u_i^0 - u_j^0) + \theta(u_i^0 - u_j^0)$$

$$(p_k + u_i + u_j)^2 = 2p_k^0 (\xi_i \eta_i u_{\max}^{0,i} + \xi_j \eta_j u_{\max}^{0,j} + \xi_i \xi_j \frac{u_{\max}^{0,i} u_{\max}^{0,j}}{p_k^0} \angle(u_i, u_j))$$

[Czakov'10, Caola'17]

Sector decomposition III

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}} \int d\Phi_{n+2} \mathcal{S}_{kl,m} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2} = \sum_{\text{sub-sec.}} \int d\Phi_n \prod dx_i \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}} d\tilde{\mu}(\{x_i\}) \underbrace{\prod x_i^{a_i+1} \langle \mathcal{M}_{n+2} | \mathcal{M}_{n+2} \rangle}_{\text{regular}} F_{n+2}$$

$x_i \in \{\eta_1, \xi_1, \eta_2, \xi_2\}$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{[x^{-1-b\epsilon}]_+}_{\text{reg. + sub.}} \quad \int_0^1 dx [x^{-1-b\epsilon}]_+ f(x) = \int_0^1 \frac{f(x) - f(0)}{x^{1+b\epsilon}}$$

Finite NNLO cross section

$$\hat{\sigma}_{ab}^{RR} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2}$$

$$\hat{\sigma}_{ab}^{C1} = (\text{single convolution}) F_{n+1}$$

$$\hat{\sigma}_{ab}^{RV} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(1)} \rangle F_{n+1}$$

$$\hat{\sigma}_{ab}^{C2} = (\text{double convolution}) F_n$$

$$\hat{\sigma}_{ab}^{VV} = \frac{1}{2\hat{s}} \int d\Phi_n \left(2\text{Re} \langle \mathcal{M}_n^{(0)} | \mathcal{M}_n^{(2)} \rangle + \langle \mathcal{M}_n^{(1)} | \mathcal{M}_n^{(1)} \rangle \right) F_n$$



sector decomposition and master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{[x^{-1-b\epsilon}]_+}_{\text{reg. + sub.}}$$

$$(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}) \quad (\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}) \quad (\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}) \quad (\sigma_{SU}^{C1}, \sigma_{DU}^{C1}) \quad (\sigma_{DU}^{C2}, \sigma_{FR}^{C2})$$



re-arrangement of terms \rightarrow 4-dim. formulation [Czakon'14, Czakon'19]

$$\underline{(\sigma_F^{RR})} \quad \underline{(\sigma_F^{RV})} \quad \underline{(\sigma_F^{VV})} \quad \underline{(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1})} \quad \underline{(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2})} \quad \underline{(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2})}$$

separately finite: ϵ poles cancel

Improved phase space generation

Phase space cut and differential observable introduce

mis-binning: mismatch between kinematics in subtraction terms

→ leads to increased variance of the integrand

→ slow Monte Carlo convergence

New phase space parametrization [Czakon'19]:

Minimization of # of different subtraction kinematics in each sector

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

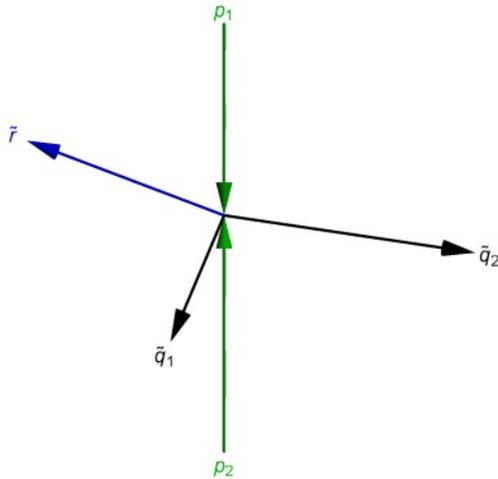
Mapping from $n+2$ to n particle phase space: $\{P, r_j, u_k\} \rightarrow \{\tilde{P}, \tilde{r}_j\}$

Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i : $\{\tilde{P}, \tilde{r}_j, u_k\} \rightarrow \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2, \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration



Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

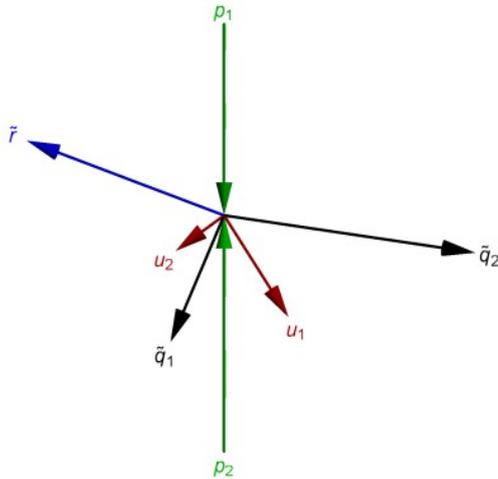
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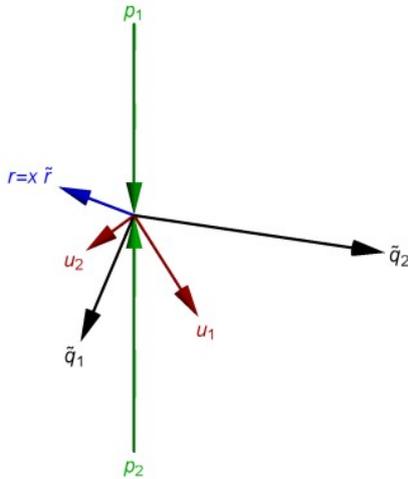
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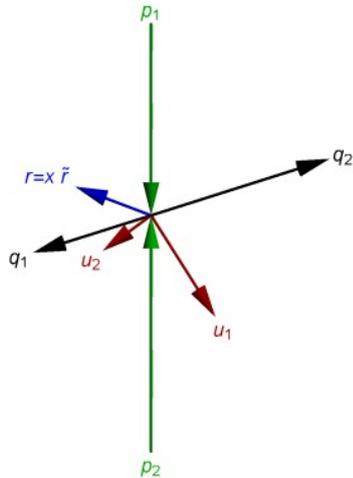
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Main steps:

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- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration



t'HV corrections

Observables: Implemented by infrared safe measurement function (MF) F_m

Infrared property in STRIPPER context:

- $\{x_i\} \rightarrow 0 \leftrightarrow$ single unresolved limit
 $\Rightarrow F_{n+2} \rightarrow F_{n+1}$
- $\{x_i\} \rightarrow 0 \leftrightarrow$ double unresolved limit
 $\Rightarrow F_{n+2} \rightarrow F_n$
 $\Rightarrow F_{n+1} \rightarrow F_n$

Tool for new formulation in the 't Hooft Veltman scheme:

Parameterized MF F_{n+1}^α

- $F_n^\alpha \equiv 0$ for $\alpha \neq 0$
(NLO MF)
- 'arbitrary' F_n^0
(NNLO MF)
- $\alpha \neq 0 \Rightarrow$ DU = 0 and SU separately finite

Example: $F_{n+1}^\alpha = F_{n+1} \Theta_\alpha(\{\alpha_i\})$
with $\Theta_\alpha = 0$ if some $\alpha_i < \alpha$

t'HV corrections

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \quad \text{where} \quad \sigma_{SU}^c = \int d\Phi_{n+1} (I_{n+1}^c F_{n+1} + I_n^c F_n)$$

NLO measurement function ($\alpha \neq 0$):

$$\int d\Phi_{n+1} (I_{n+1}^{RR} + I_{n+1}^{RV} + I_{n+1}^{C1}) F_{n+1}^\alpha = \text{finite in 4 dim.}$$

All divergences cancel in d -dimensions:

$$\sum_c \int d\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1}^\alpha \equiv \sum_c \mathcal{I}^c = 0$$

t'HV corrections

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \underbrace{-\mathcal{I}^{RR} - \mathcal{I}^{RV} - \mathcal{I}^{C1}}_{=0}$$

$$\begin{aligned} \sigma_{SU}^c - \mathcal{I}^c &= \int d\Phi_{n+1} \left\{ \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} + I_{n+1}^{c,(0)} \right] F_{n+1} + \left[\frac{I_n^{c,(-2)}}{\epsilon^2} + \frac{I_n^{c,(-1)}}{\epsilon} + I_n^{c,(0)} \right] F_n \right\} \\ &\quad - \int d\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1} \Theta_\alpha(\{\alpha_i\}) \\ &= \int d\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)} F_{n+1} + I_n^{c,(-2)} F_n}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)} F_{n+1} + I_n^{c,(-1)} F_n}{\epsilon} \right] (1 - \Theta_\alpha(\{\alpha_i\})) \\ &\quad + \int d\Phi_{n+1} \left[I_{n+1}^{c,(0)} F_{n+1} + I_n^{c,(0)} F_n \right] + \int d\Phi_{n+1} \left[\frac{I_n^{c,(-2)}}{\epsilon^2} + \frac{I_n^{c,(-1)}}{\epsilon} \right] F_n \Theta_\alpha(\{\alpha_i\}) \end{aligned}$$

$$=: \underbrace{Z^c(\alpha)}_{\text{integrable, zero volume for } \alpha \rightarrow 0} + \underbrace{C^c}_{\text{no divergencies}} + \underbrace{N^c(\alpha)}_{\text{only } F_n \rightarrow \text{DU}}$$

t'HV corrections

Looks like slicing, but it is slicing *only* for divergences
→ no actual slicing parameter in result

Powerlog-expansion:

$$N^c(\alpha) = \sum_{k=0}^{\ell_{\max}} \ln^k(\alpha) N_k^c(\alpha)$$

- all $N_k^c(\alpha)$ regular in α
- start expression independent of $\alpha \Rightarrow$ all logs cancel
- only $N_0^c(0)$ relevant

Putting parts together:

$$\sigma_{SU} - \sum_c N_0^c(0) \text{ and } \sigma_{DU} + \sum_c N_0^c(0)$$

are finite in 4 dimension



SU contribution: $\sigma_{SU} - \sum_c N_0^c = \sum_c C^c$
original expression σ_{SU} in 4-dim
without poles, no further ϵ pole
cancellation

C++ framework

- Formulation allows efficient algorithmic implementation → STRIPPER
- **High degree of automation:**
 - Partonic processes (taking into account all symmetries)
 - Sectors and subtraction terms
 - Interfaces to Matrix-element providers + O(100) hardcoded: AvH, OpenLoops, Recola, NJET, HardCoded
 - In practice: Only **two-loop matrix** elements required
- **Broad range of applications** through additional facilities:
 - Narrow-Width & Double-Pole Approximation
 - Fragmentation
 - Polarised intermediate massive bosons
 - (Partial) Unweighting → Event generation for **HighTEA**
 - Interfaces: FastNLO, FastJet

Two-loop five-point amplitudes

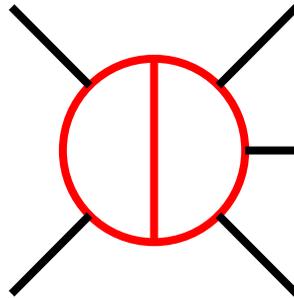
Massless:

[Chawdry'19'20'21] $(3A+2j, 2A+3j)$

[Abreu'20'21] $(3A+2j, 5j)$

[Agarwal'21] $(2A+3j)$

[Badger'21'23] $(5j, gggAA, jjjA)$



1 external mass:

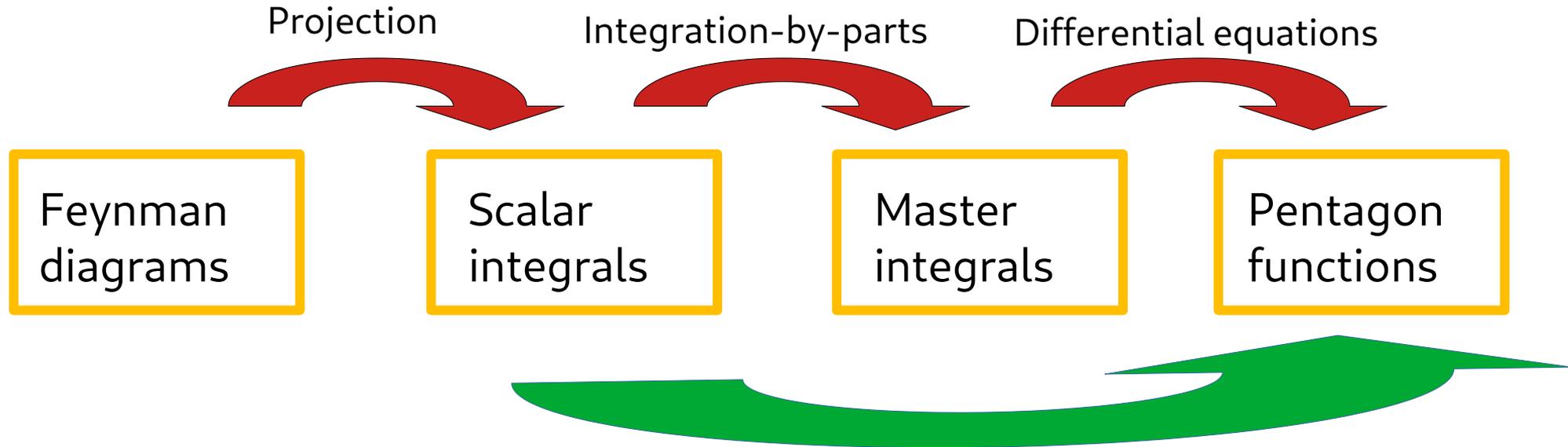
[Abreu'21] $(W+4j)$

[Badger'21'22] $(Hqqgg, W4q, Wajjj)$

[Hartanto'22] $(W4q)$

Overview

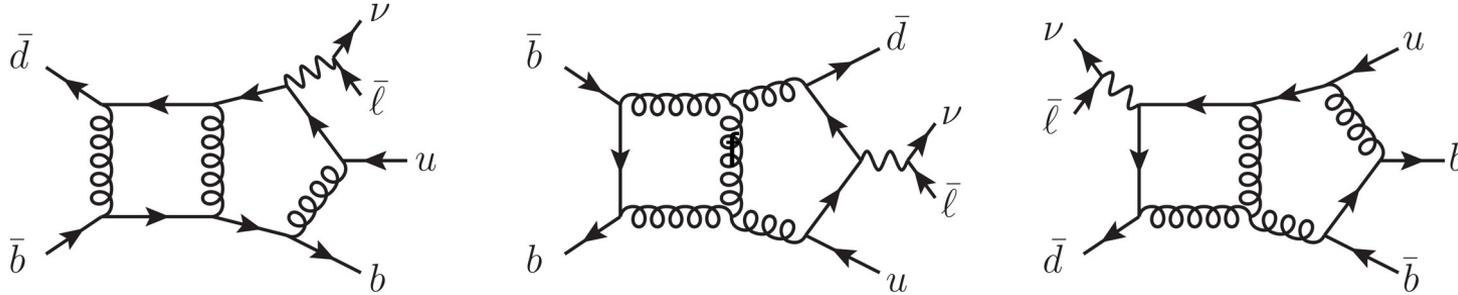
Old school approach:



Automated framework using finite fields
to avoid expression swell based on
FiniteFlow [Peraro'19]

Projection to scalar integrals

Generate diagrams (contributing to leading-colour) with QGRAF



Factorizing decay: $A_6^{(L)} = A_5^{(L)\mu} D_\mu P$

$$M_6^{2(L)} = \sum_{\text{spin}} A_6^{(0)*} A_6^{(L)} = M_5^{(L)\mu\nu} D_{\mu\nu} |P|^2$$

Projection on scalar functions (FORM+Mathematica):
 → anti-commuting γ_5 + Larin prescription

$$M_5^{(L)\mu\nu} = \sum_{i=1}^{16} a_i^{(L)} v_i^{\mu\nu}$$



$$a_i^{(L)} = a_i^{(L),\text{even}} + \text{tr}_5 a_i^{(L),\text{odd}}$$

$$a_i^{(L),p} = \sum_j c_{j,i}(\{p\}, \epsilon) \mathcal{I}(\{p\}, \epsilon)$$

Integration-By-Parts reduction

$$a_i^{(L),p} = \sum_i c_{j,i}(\{p\}, \epsilon) \mathcal{I}(\{p\}, \epsilon) \quad \rightarrow \text{prohibitively large number of integrals}$$

$$\mathcal{I}_i(\{p\}, \epsilon) \equiv \mathcal{I}(\vec{n}_i, \{p\}, \epsilon) = \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\}, \{k\})$$

Integration-By-Parts identities connect different integrals \rightarrow system of equations
 \rightarrow only a small number of independent "master" integrals

$$0 = \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} l^\mu \frac{\partial}{\partial l^\mu} \prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\}, \{k\}) \quad \text{with } l \in \{p\} \cap \{k\}$$

LiteRed (+ Finite Fields)



$$a_i^{(L),p} = \sum_i d_{j,i}(\{p\}, \epsilon) \text{MI}(\{p\}, \epsilon)$$

Master integrals & finite remainder

Differential Equations: $d\vec{M}I = dA(\{p\}, \epsilon)\vec{M}I$

[Remiddi, 97]

[Gehrmann, Remiddi, 99]

[Henn, 13]

Canonical basis: $d\vec{M}I = \epsilon d\tilde{A}(\{p\})\vec{M}I$

Simple iterative solution



$$MI_i = \sum_w \epsilon^w \tilde{M}I_i^w \quad \text{with} \quad \tilde{M}I_i^w = \sum_j c_{i,j} m_j$$

Chen-iterated integrals
"Pentagon"-functions

[Chicherin, Sotnikov, 20]

[Chicherin, Sotnikov, Zoia, 21]

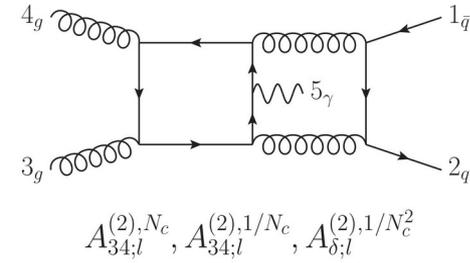
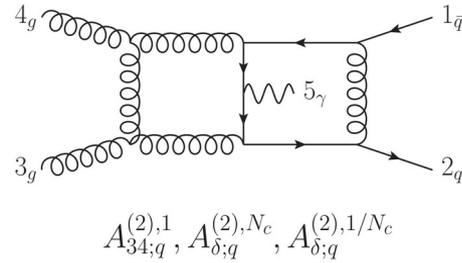
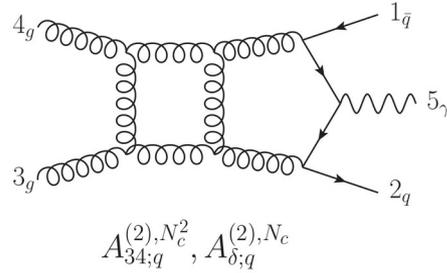
Putting everything together (and removing of IR poles):

$$f_i^{(L),p} = a_i^{(L),p} - \text{poles}$$

$$f_i^{(L),p} = \sum_j c_{i,j}(\{p\}) m_j + \mathcal{O}(\epsilon)$$

Reconstruction of Amplitudes

[Badger'21]



New optimizations

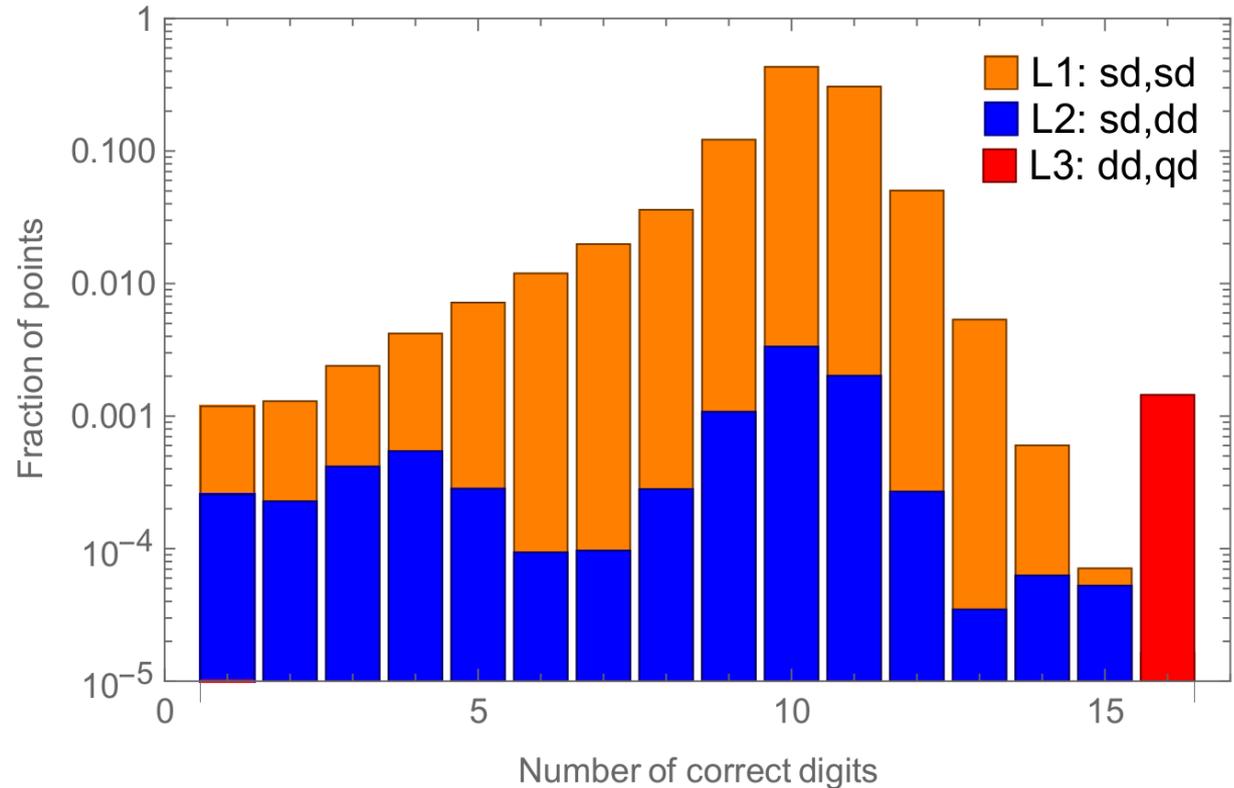
- Syzygy's to simplify IBPs
- Exploitation of Q-linear relations
- Denominator Ansatz
- On-the-fly partial fractioning

amplitude	helicity	original	stage 1	stage 2	stage 3	stage 4
$A_{34;q}^{(2),1}$	- + + - +	94/91	74/71	74/0	22/18	22/0
$A_{34;q}^{(2),1}$	- + - + +	93/89	90/86	90/0	24/14	18/0
$A_{34;q}^{(2),1/N_c^2}$	- + + - +	90/88	73/71	73/0	23/18	22/0
$A_{34;q}^{(2),1/N_c^2}$	- + - + +	90/86	86/82	86/0	24/14	19/0
$A_{34;l}^{(2),1/N_c}$	- + - + +	89/82	74/67	73/0	27/14	20/0
$A_{34;l}^{(2),1/N_c}$	- + + - +	85/81	61/58	60/0	27/18	20/0
$A_{34;q}^{(2),N_c^2}$	- + - + +	58/55	54/51	53/0	20/18	20/0

Massive reduction of complexity

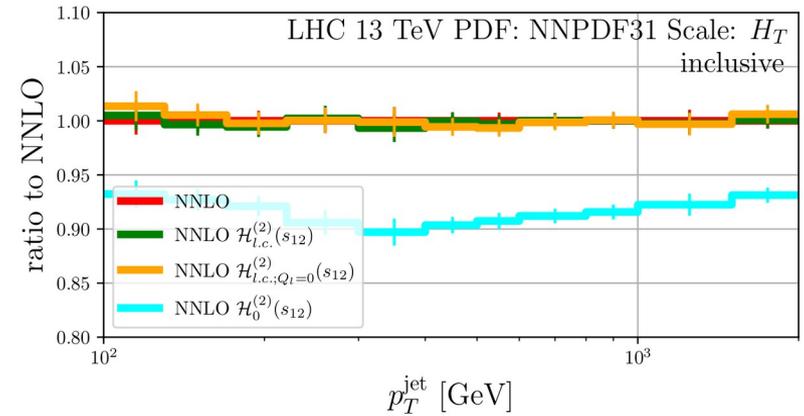
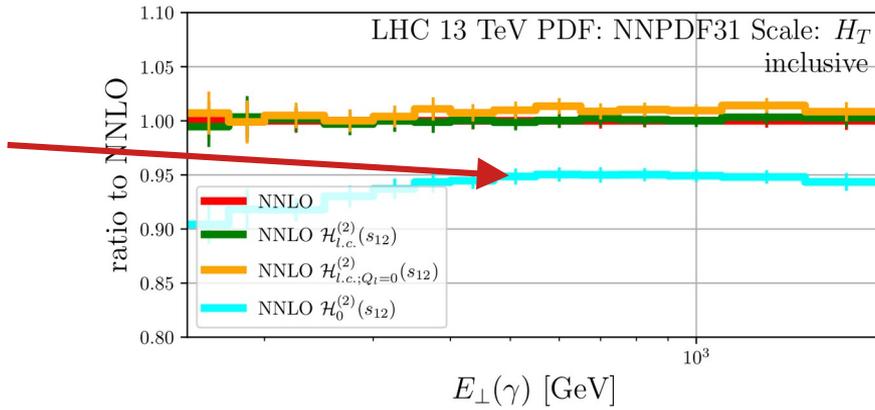
Two-loop matrix element stability

- Stable evaluation requires high floating point precision for rational functions
- In rarer cases higher precision “Pentagon” functions necessary
- 2.2 million events needed → fast evaluation essential

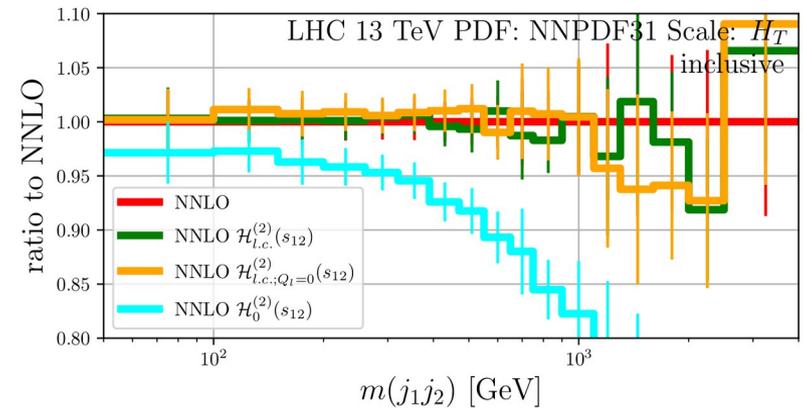
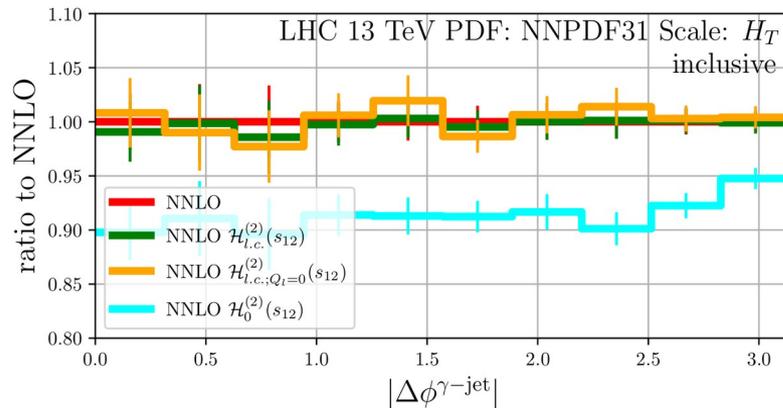


Quality of leading colour the approximation

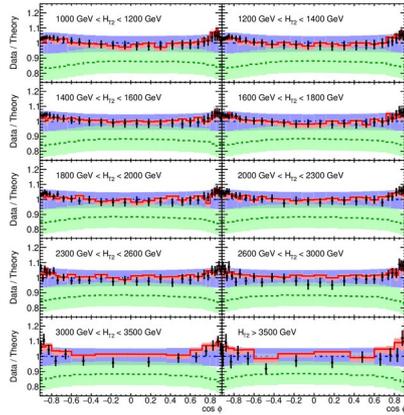
Two-loop contribution
 ~ 5-10%
 wrt. full NNLO
 (scheme dep.)



“Leading colour”
 Approximation
 “Error” = $O(\sim 1\%)$
 wrt full NNLO



HighTEA



= ~100 MCPUh



high tea
for your freshly brewed analysis

<https://www.precision.hep.phy.cam.ac.uk/hightea>

How to make this more
efficient/environment-friendly/
accessible/faster?

HighTEA: High energy Theory Event Analyser
[2304.05993]

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Basic idea

→ Database of precomputed “Theory Events”

- **Equivalent to a full fledged computation**
- Currently this means partonic fixed order events
- Extensions to include showered/resummed/hadronized events is feasible
- (Partially) Unweighting to increase efficiency

Not so new idea:
LHE [Alwall et al '06],
Ntuple [BlackHat '08'13],

→ Analysis of the data through an user interface

- Easy-to-use
- Fast
 - Observables from basic 4-momenta
 - Free specification of bins
- Flexible:
 - Renormalization/Factorization Scale variation
 - PDF (member) variation
 - Specify phase space cuts

Factorizations

Factorizing renormalization and factorization scale dependence:

$$w_s^{i,j} = w_{\text{PDF}}(\mu_F, x_1, x_2) w_{\alpha_s}(\mu_R) \left(\sum_{i,j} c_{i,j} \ln(\mu_R^2)^i \ln(\mu_F^2)^j \right)$$

PDF dependence:

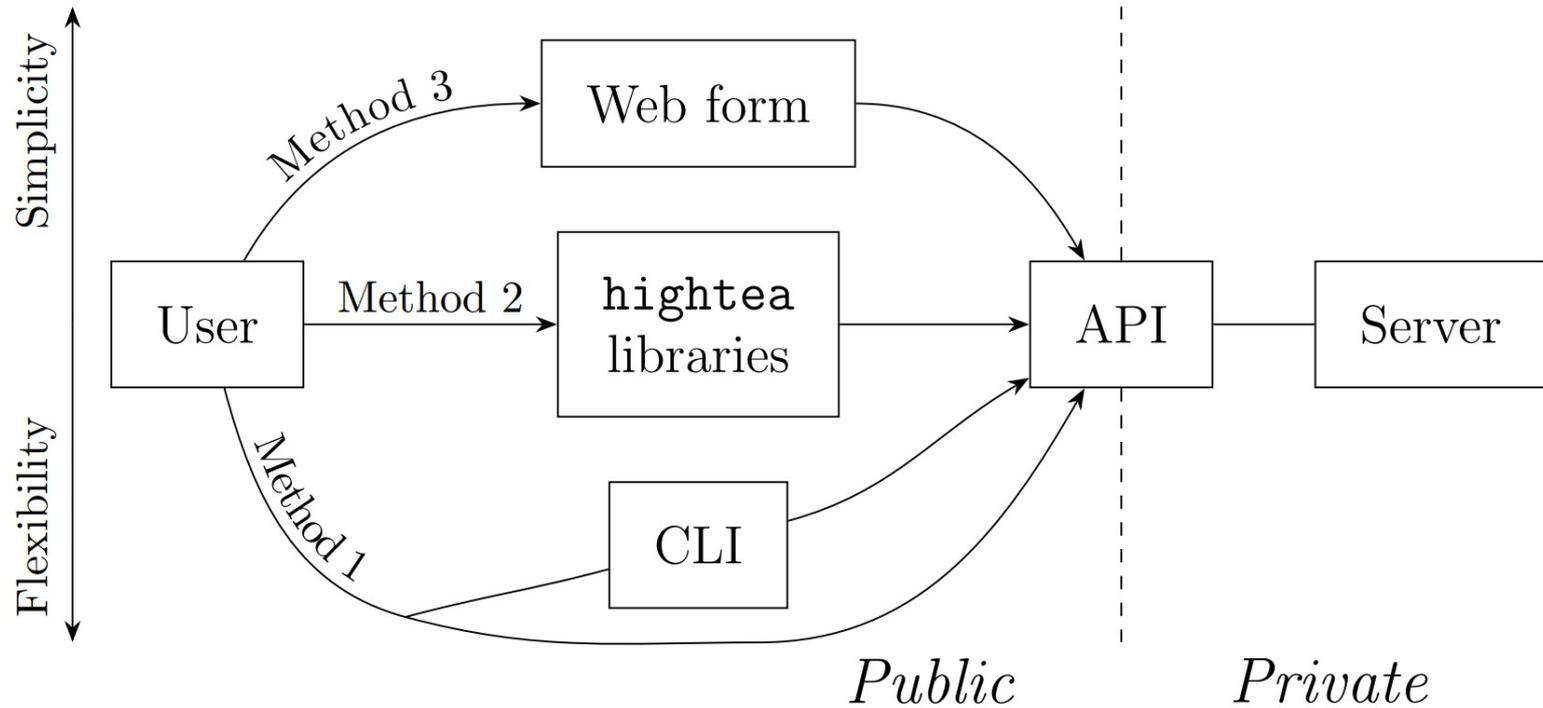
$$w_{\text{PDF}}(\mu, x_1, x_2) = \sum_{ab \in \text{channel}} f_a(x_1, \mu) f_b(x_2, \mu)$$

α_s dependence:

$$w_{\alpha_s}(\mu) = (\alpha_s(\mu))^m$$

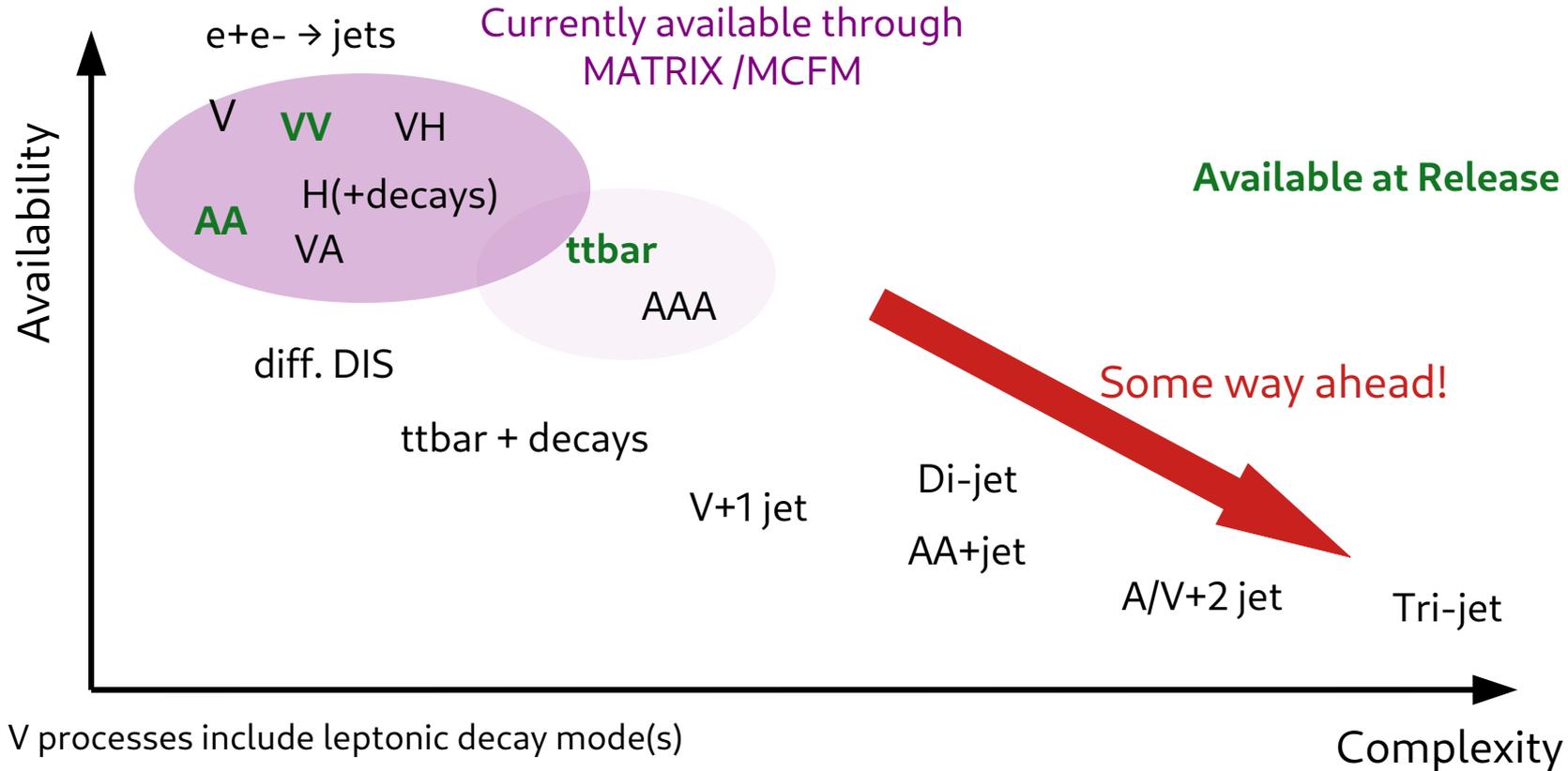
Allows **full control over scales and PDF**

HighTEA interface

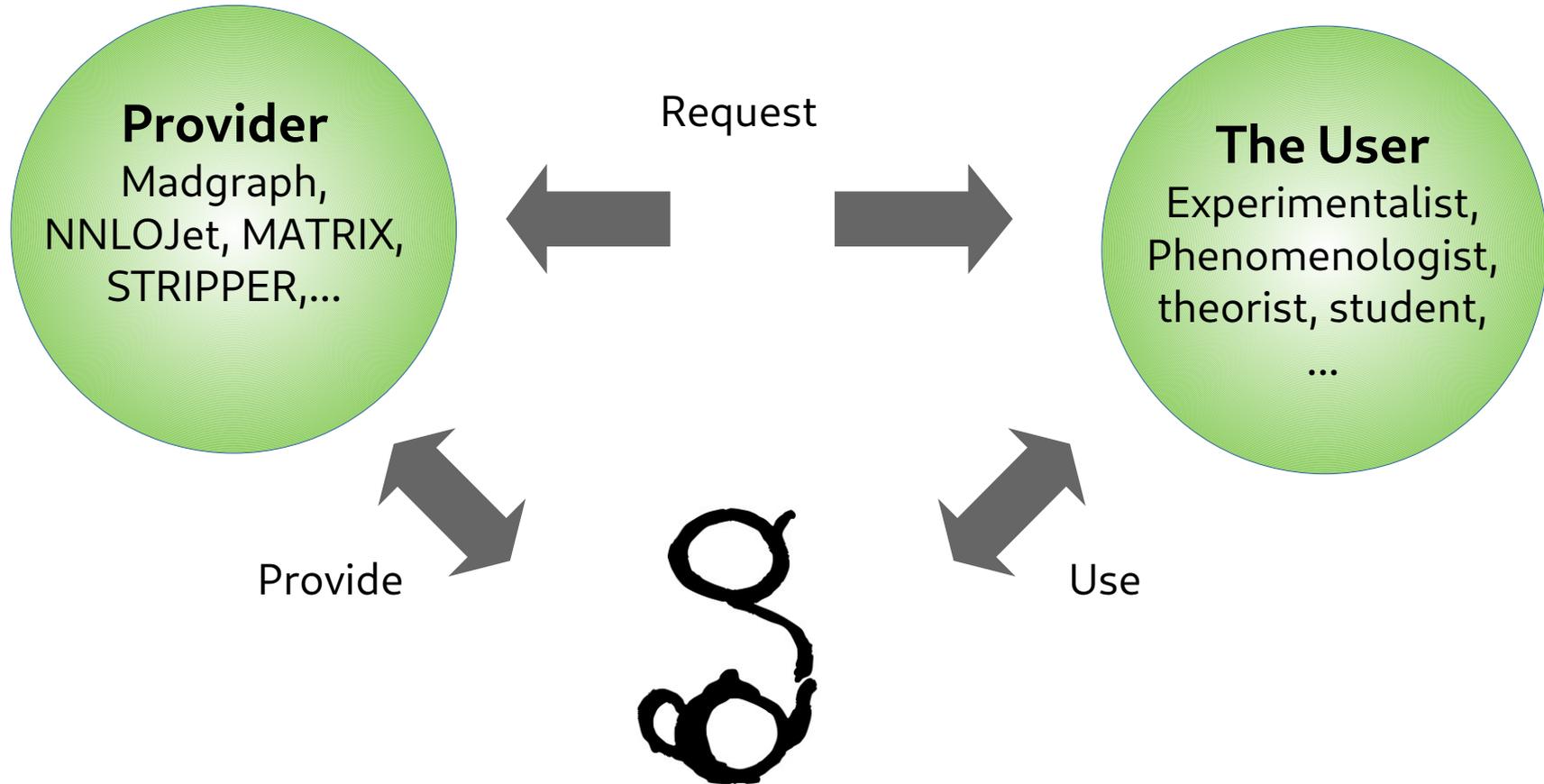


Available Processes

Processes **currently** implemented in our STRIPPER framework through **NNLO QCD**



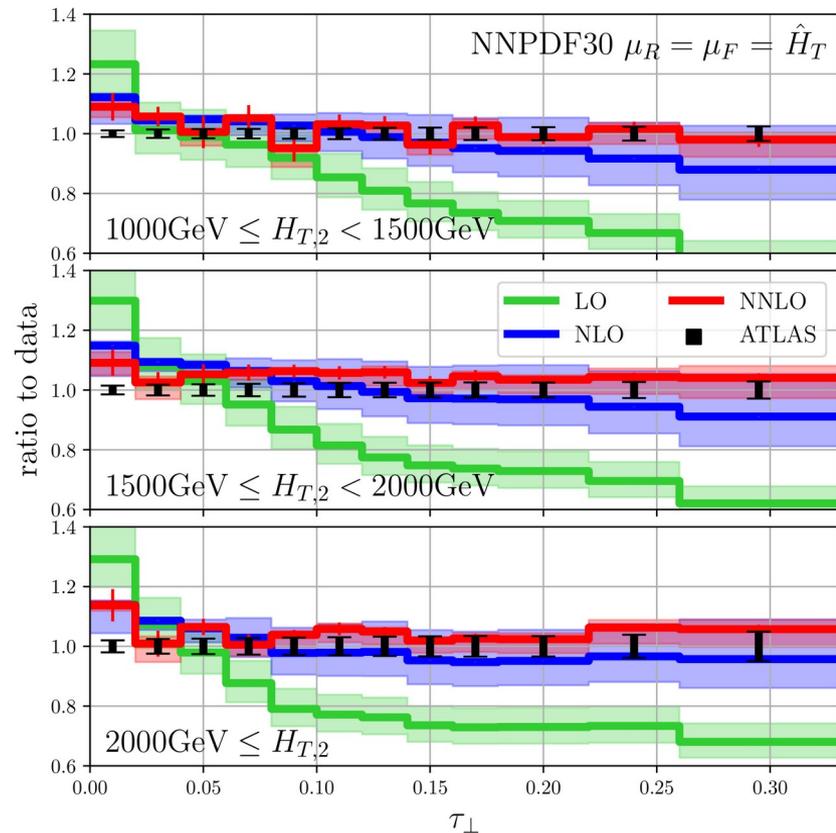
The Vision



Transverse Thrust @ NNLO QCD

NNLO QCD corrections to event shapes at the LHC

Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet [2301.01086]



ATLAS [2007.12600]

