High precision prediction for multi-scale processes at the LHC

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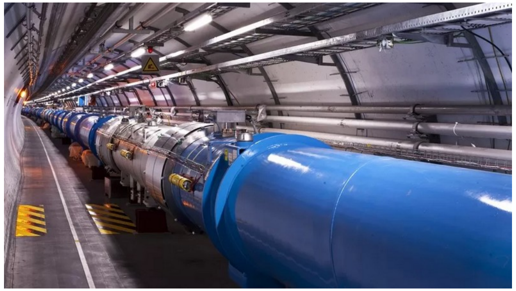
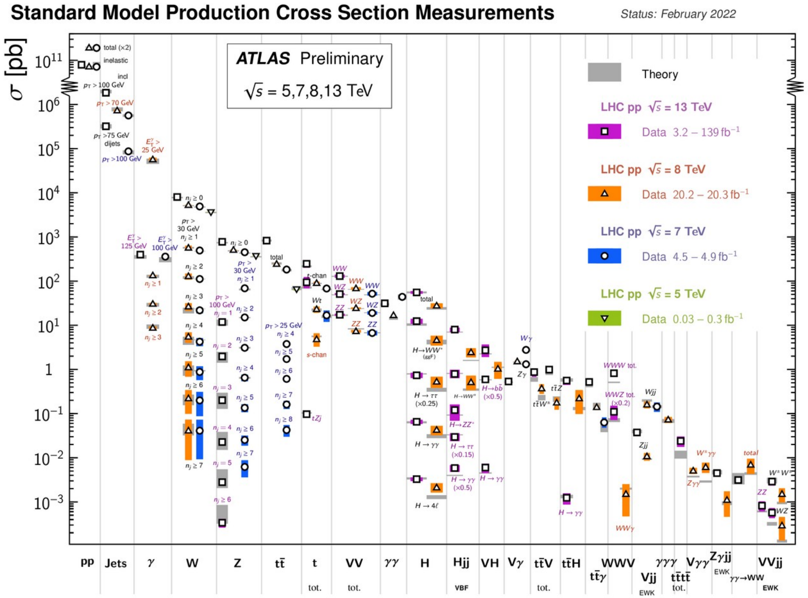
erc European Research Council Established by the European Commission

What are the fundamental building blocks of matter?

Scattering experiments

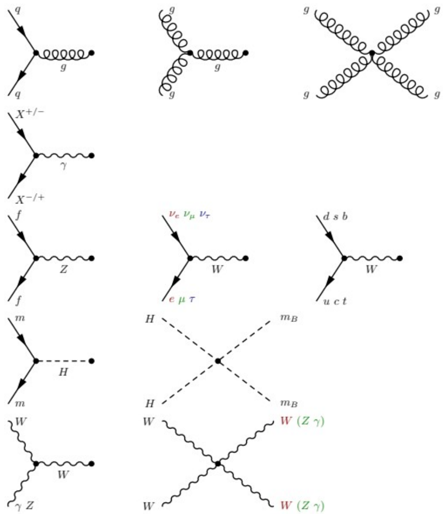


Credit: CERN



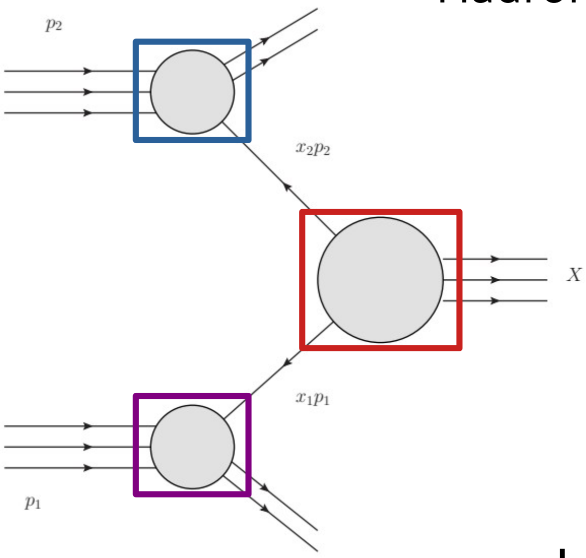
Credit: ATLAS

Theory/Model



Credit: Jack Lindon, CERN

Precision through higher orders



Hadronic cross section:

$$\sigma_{h_1 h_2 \to X} = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \phi_{i,h_1}(x_1, \mu_F^2) \phi_{j/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij \to X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

Parton distribution functions

Perturbative expansion of partonic cross section:

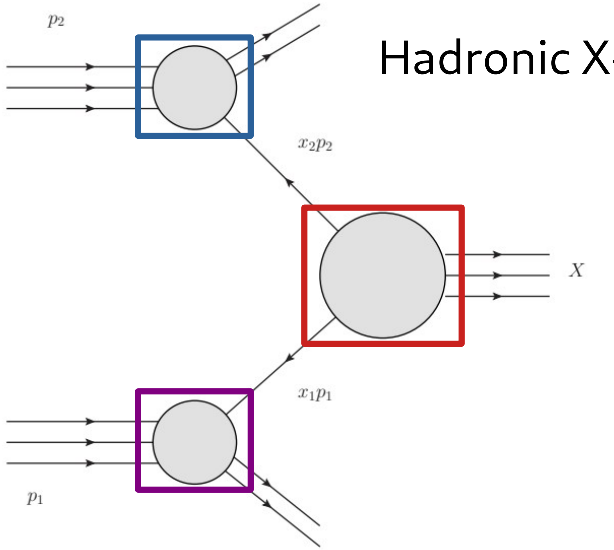
$$\hat{\sigma}_{ab \to X} = \alpha_s^0 \hat{\sigma}^{(0)}_{ab \to X} + \alpha_s^1 \hat{\sigma}^{(1)}_{ab \to X} + \alpha_s^2 \hat{\sigma}^{(2)}_{ab \to X} + \mathcal{O}(\alpha_s^3)$$

Leading order | Next-to-leading order | Next-to-next-to-leading order

Uncertainty: $\alpha_s(m_Z) \approx 0.118$ | Order of magnitude | O(10%) | O(1%)

Next-to-next-to-leading order QCD needed to match experimental precision!
→ In some cases even next-to-next-to-next-to-leading order!

Hadronic cross section in collinear factorization – NNLO QCD



Hadronic X-section: $\sigma_{h_1 h_2 \to X} = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \phi_{i,h_1}(x_1, \mu_F^2) \phi_{j/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij \to X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$

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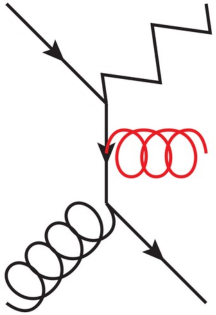
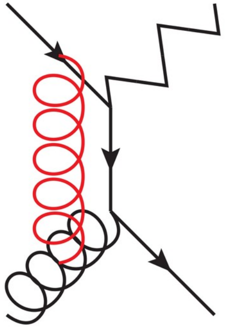
The NLO bit: $\hat{\sigma}^{(1)}_{ab} = \hat{\sigma}^{\mathrm{R}}_{ab} + \hat{\sigma}^{\mathrm{V}}_{ab} + \hat{\sigma}^{\mathrm{C}}_{ab}$

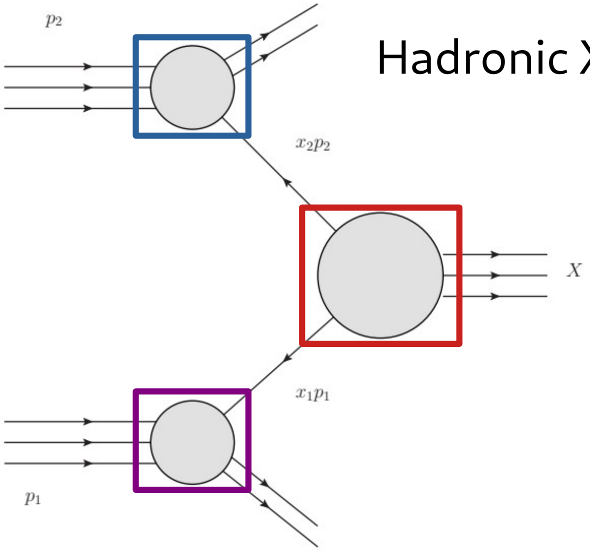
Real radiation



Virtual correction



Hadronic cross section in collinear factorization – NNLO QCD



Hadronic X-section: $\sigma_{h_1h_2\to X} = \sum_{ij}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\phi_{i,h_1}(x_1,\mu_F^2)\phi_{j/h_2}(x_2,\mu_F^2)\hat{\sigma}_{ij\to X}(\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)$

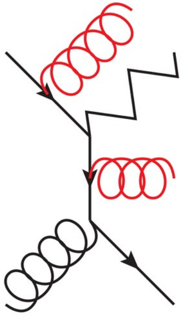
Parton distribution functions

Perturbative expansion of partonic cross section:

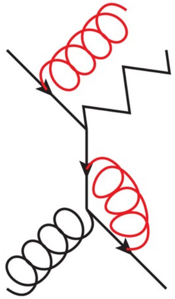
$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

The NNLO bit: $\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$

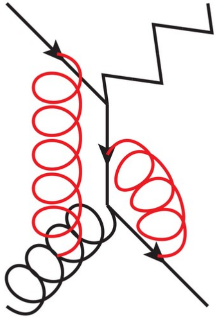
Double real radiation



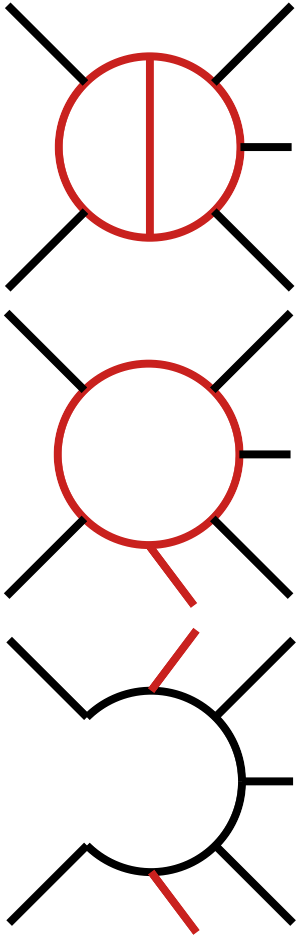
Real/Virtual correction



Double virtual corrections



NNLO QCD for 2→3 processes - inputs



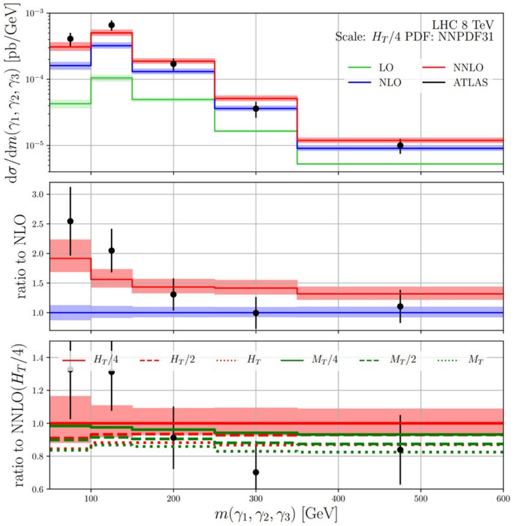
Two-loop amplitudes
- (Non-) planar 5 point massless external states [Chawdry'19'20'21,Abreu'20'21'23,Agarwal'21'23,Badger'21'23]
  → triggered by efficient MI representation [Chicherin'20]
- 5 point with one external mass [Abreu'20,Syrrakos'20,Canko'20,Badger'21'22,Chicherin'22]

One-loop amplitudes → OpenLoops [Buccioni'19]
- Many legs and IR stable (soft and collinear limits)

Double-real Born amplitudes → AvHlib[Bury'15]
- IR finite cross-sections → NNLO subtraction schemes
  qT-slicing [Catani'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projetction [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17], Local Analytic [Magnea'18], **Sector-improved residue subtraction** [Czakon'10-'14,'19]

NNLO QCD cross sections for massless 2 → 3 processes

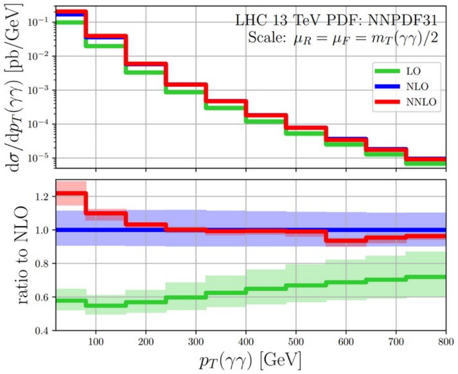
$pp \to \gamma\gamma\gamma$



Chawdhry, Czakon, Mitov, RP [1911.00479]

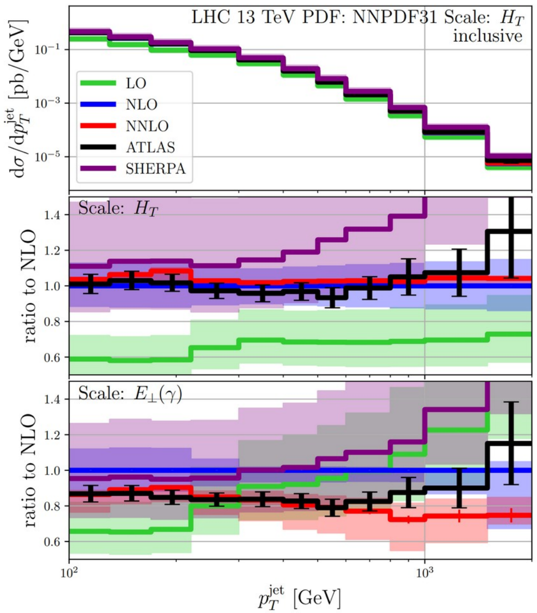
Kallweit, Sotnikov, Wiesemann [2010.04681]

$pp \to \gamma\gamma j$



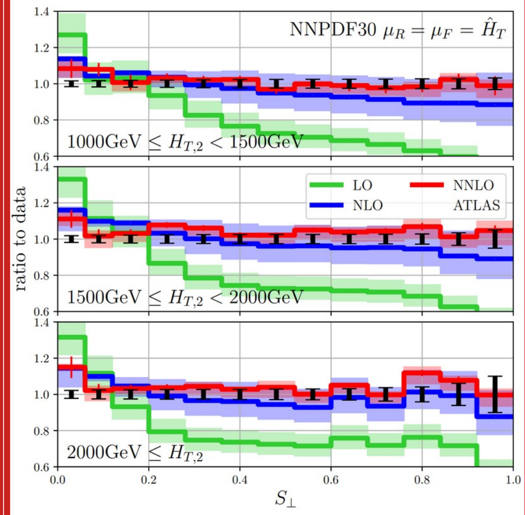
Chawdhry, Czakon, Mitov, RP [2103.04319]

$pp \to \gamma jj$



Badger, Czakon, Hartanto, Moodie, Peraro, RP, Zoia [2304.06682]

$pp \to jjj$



Czakon, Mitov, RP [2106.05331]
+ Alvarez, Cantero, Llorente [2301.01086]

Multi-jet observables

Test of pQCD and extraction of strong coupling constant

NLO theory unc. > experimental unc.

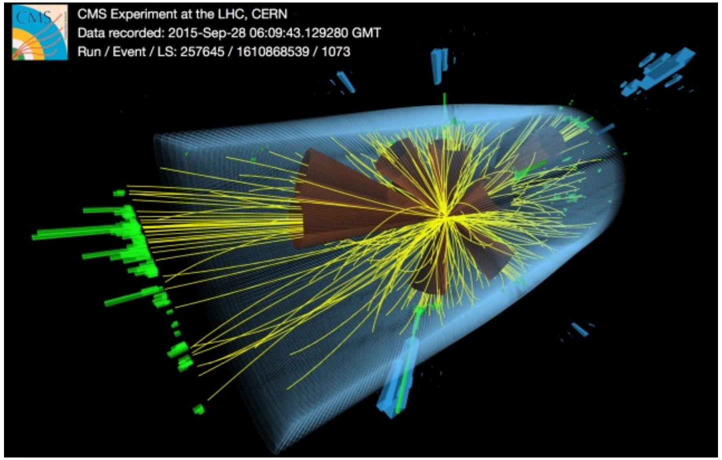
- NNLO QCD needed for precise theory-data comparisons
  → Restricted to two-jet data [Currie'17+later][Czakon'19]
- New NNLO QCD three-jet → access to more observables
  - Jet ratios

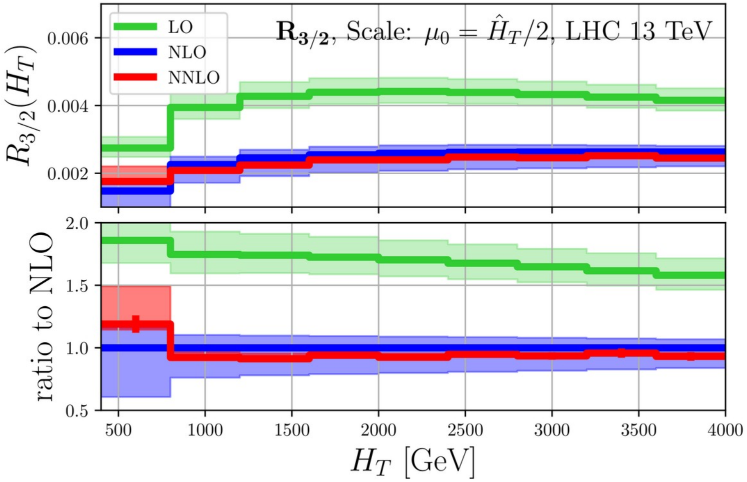
    Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC
    Czakon, Mitov, **Poncelet** [2106.05331]

$$R^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0}) = \frac{d\sigma_3^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}{d\sigma_2^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}$$

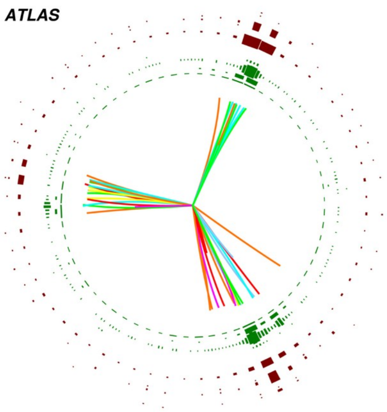
  - Event shapes

    NNLO QCD corrections to event shapes at the LHC
    Alvarez, Cantero, Czakon, Llorente, Mitov, **Poncelet** [2301.01086]





Encoding QCD dynamics in event shapes



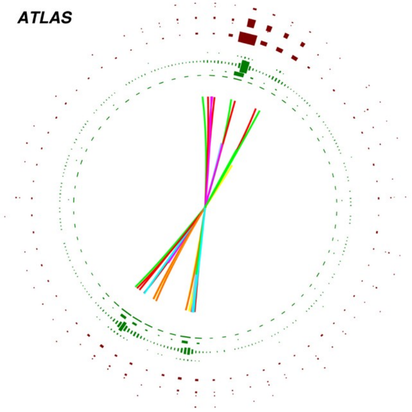
Using (global) event information to separate different regimes of QCD event evolution:

- **Thrust & Thrust-Minor**

$$T_\perp = \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}\,, \quad \text{and} \quad T_m = \frac{\sum_i |\vec{p}_{T,i} \times \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}\,.$$

- **Energy-energy correlators**

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$



Separation of energy scales: $H_{T,2} = p_{T,1} + p_{T,2}$

Ratio to 2-jet:

$$R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$$

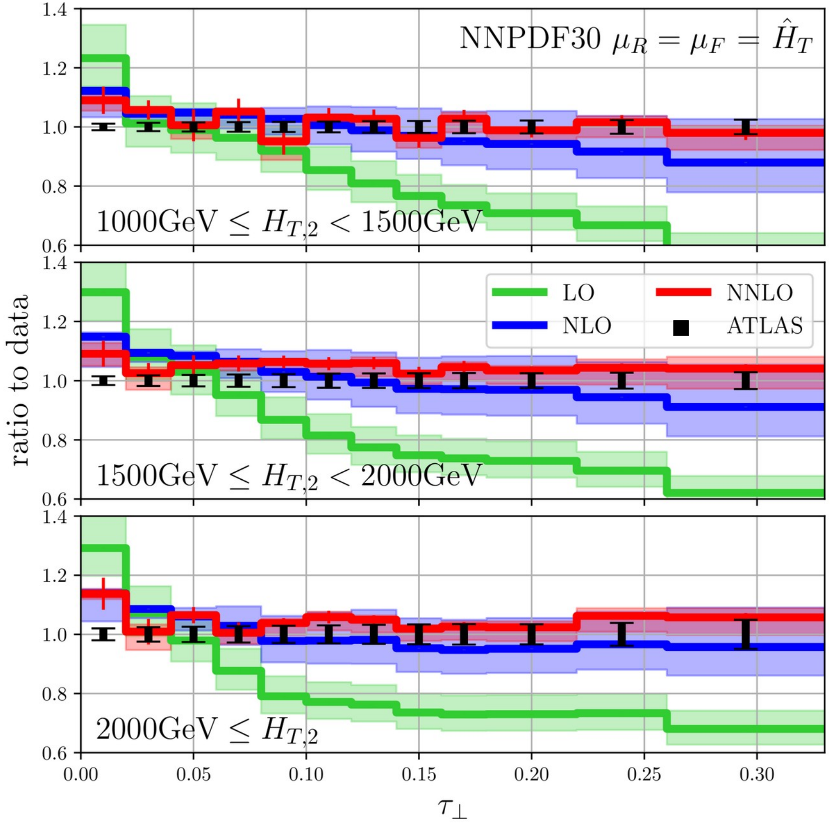
Here: use jets as input → experimentally advantageous (better calibrated, smaller non-pert.)

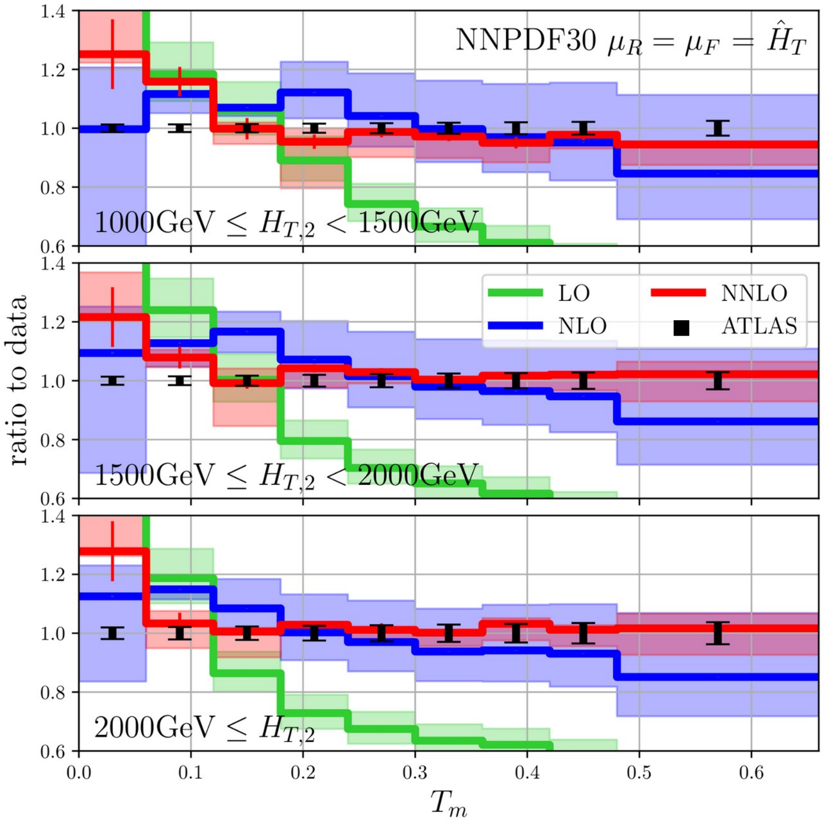
Transverse Thrust @ NNLO QCD

NNLO QCD corrections to event shapes at the LHC

Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet [2301.01086]

ATLAS [2007.12600]

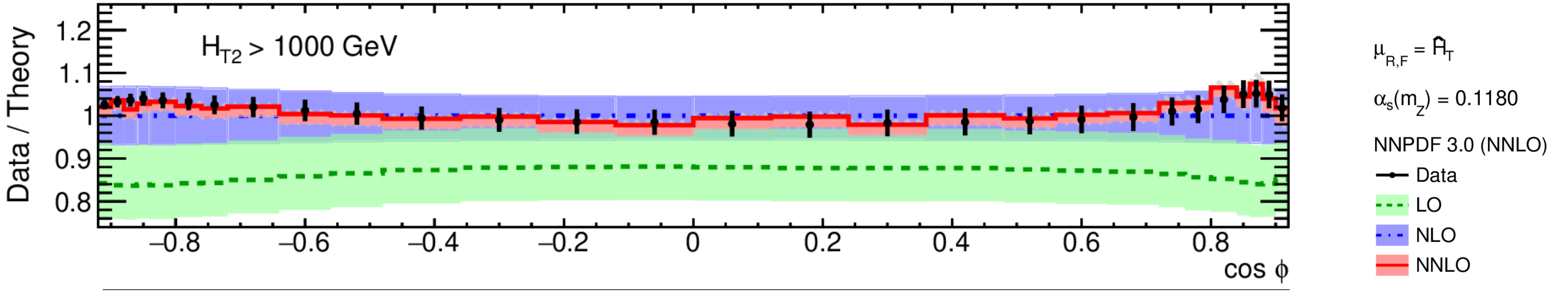




The transverse energy-energy correlator

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$

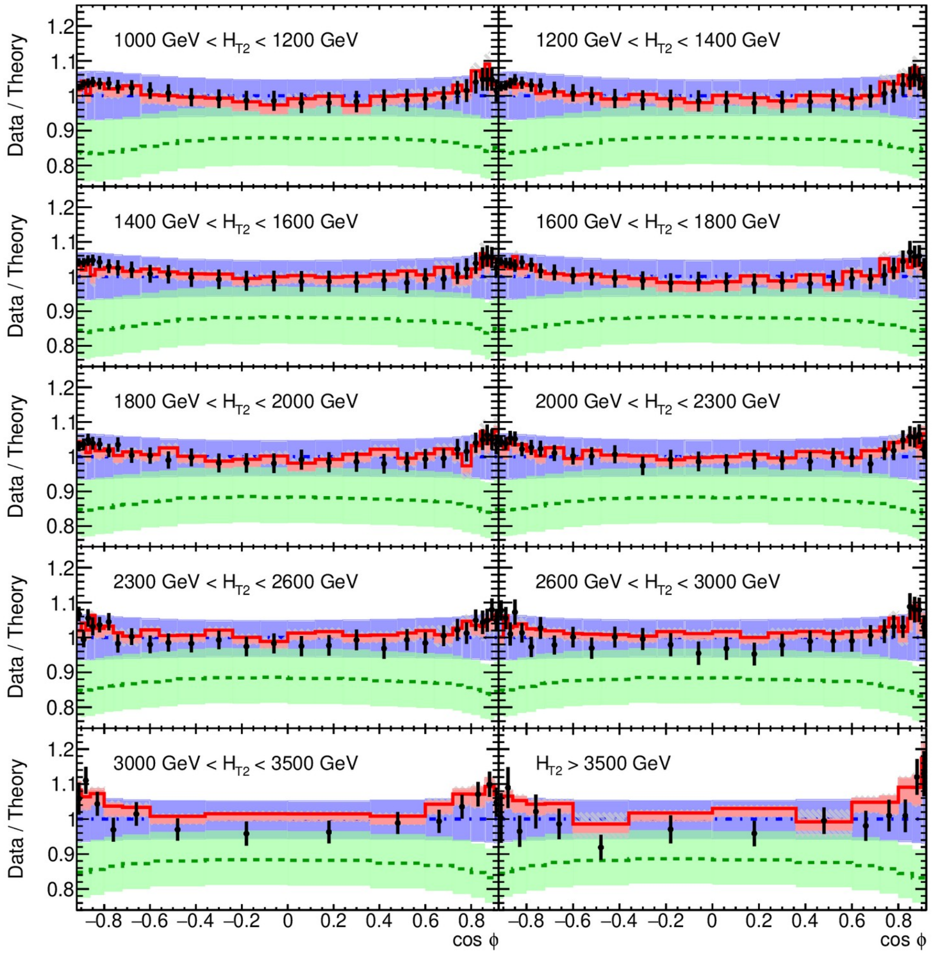
- Insensitive to soft radiation through energy weighting $x_{T,i} = E_{T,i}/\sum_j E_{T,j}$
- Event topology separation:
  - Central plateau contain isotropic events
  - To the right: self-correlations, collinear and in-plane splitting
  - To the left: back-to-back



[ATLAS 2301.09351]

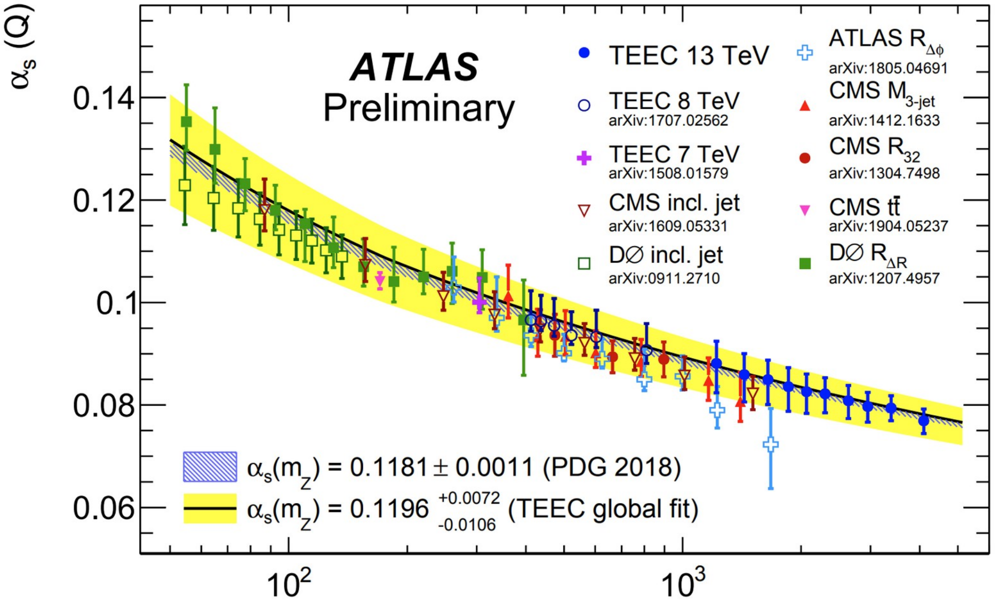
Double differential TEEC

[ATLAS 2301.09351]



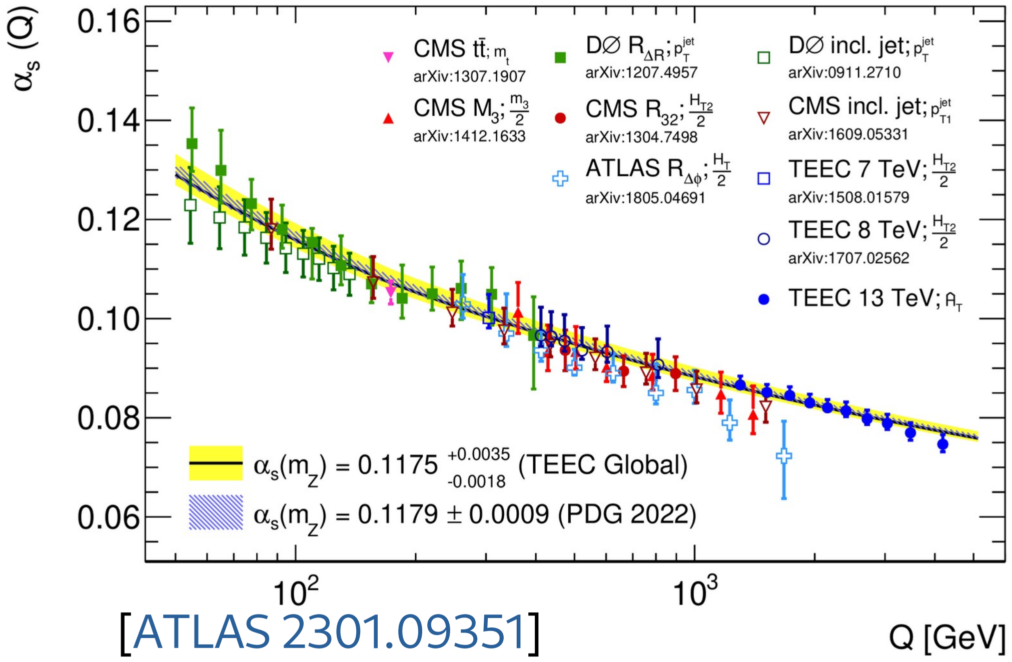
Running of α_S

NLO QCD



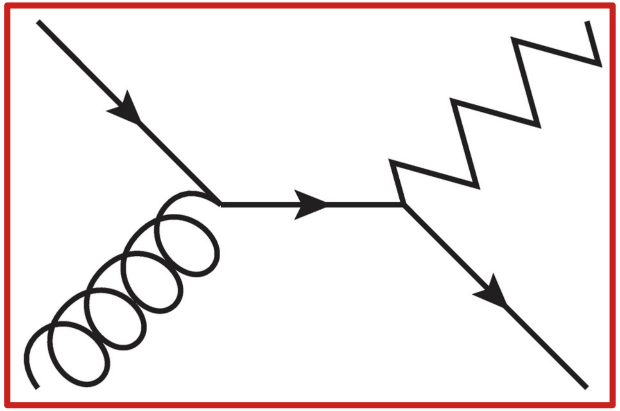
[ATLAS-CONF-2020-025]

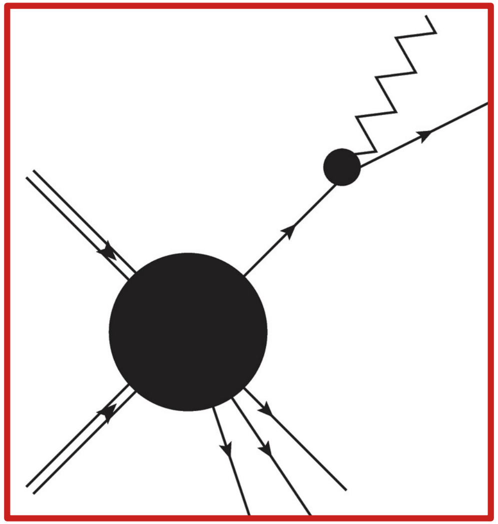
NNLO QCD



[ATLAS 2301.09351]

Prompt photon production

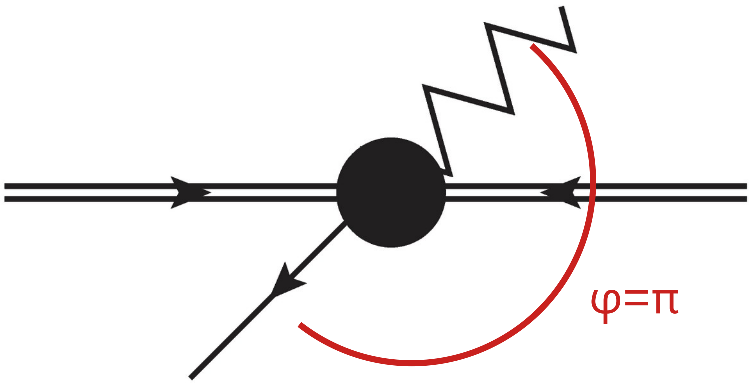


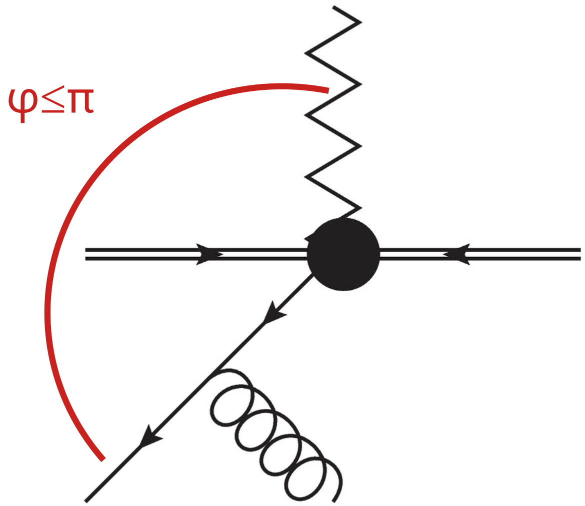


Direct production
- Test of perturbative QCD
- Gluon PDF sensitivity
- Estimates for BSM backgrounds

Fragmentation
- Depends on non-perturbative fragmentation functions
- Separation from "direct" not unique

Why photon plus a jet pair?





- Non-back-to-back Born configurations
  → access to angular correlations between the photon and jets
- Access to different kinematic regimes through distinguishable photon
  → enhance direct, high- or low-z fragmentation
- Background process for BSM: $pp \to \gamma + Y(\to jj)$

Photon plus jet pair

Measurement of isolated-photon plus two-jet production in pp collisions at sqrt(s) = 13 TeV with the ATLAS detector [1912.09866]

Requirements on photon	$E_T^\gamma > 150$ GeV, $\lvert\eta^\gamma\rvert < 2.37$ (excluding $1.37 < \lvert\eta^\gamma\rvert < 1.56$) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 4.8$ GeV (reconstruction level) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 10$ GeV (particle level)		
Requirements on jets	at least two jets using anti-k_t algorithm with $R = 0.4$ $p_T^{\text{jet}} > 100$ GeV, $\lvert y^{\text{jet}}\rvert < 2.5$, $\Delta R^{\gamma-\text{jet}} > 0.8$		
Phase space	**total**	**fragmentation enriched** $E_T^\gamma < p_T^{\text{jet2}}$	**direct enriched** $E_T^\gamma > p_T^{\text{jet1}}$
Number of events	755 270	111 666	386 846

Modelled with hybrid isolation

$$E_\perp(r) \leq E_{\perp\text{max}}(r) = 0.1\, E_\perp(\gamma) \left(\frac{1-\cos(r)}{1-\cos(R_{\text{max}})}\right)^2 \quad \text{for} \quad r \leq R_{\text{max}} = 0.1$$

+

$$E_\perp(r) \leq E_{\perp\text{max}} = 0.0042\, E_\perp(\gamma) + 10 \text{ GeV} \quad \text{for} \quad r \leq R_{\text{max}} = 0.4$$

No fragmentation contribution
→ Purely pQCD through NNLO
→ focus on "inclusive" and "direct" PS

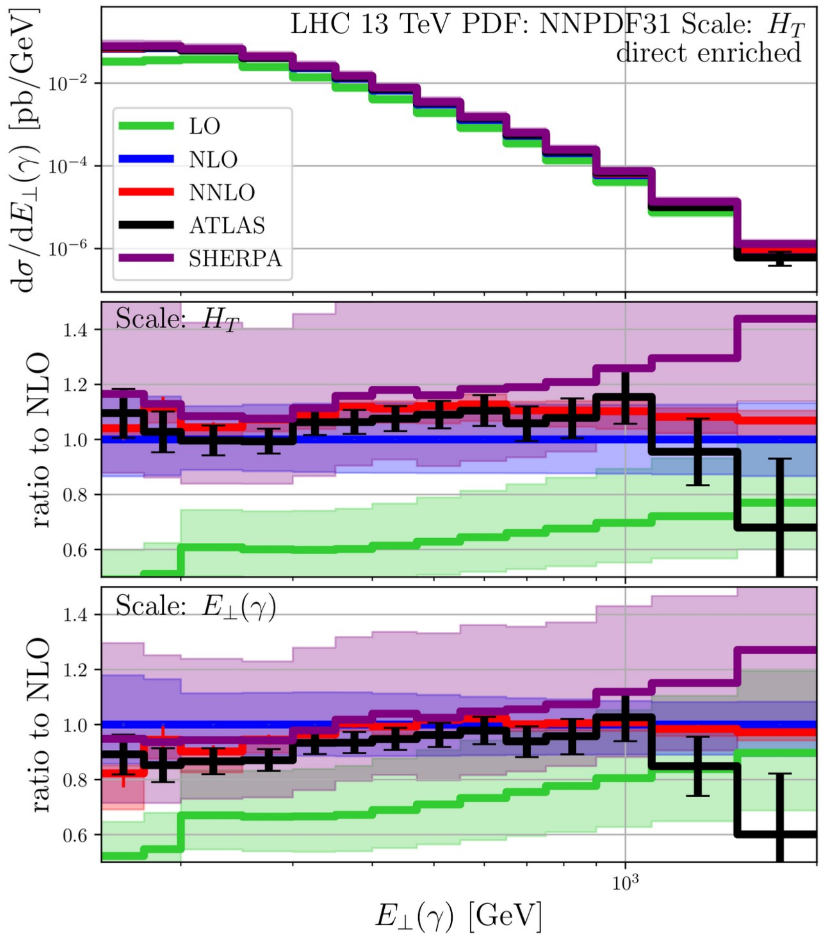
Theory - data comparisons

NNLO QCD

- Describes data well
- Improvements on the shape
- Small corrections
- Small remaining scale dependence

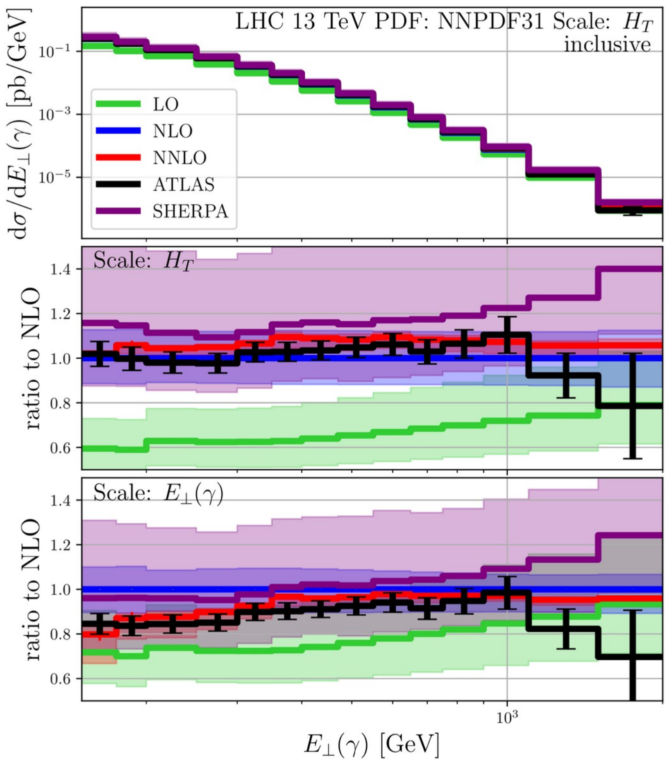
Comment on the SHERPA predictions

- Large NLO scale uncertainties
- The shape is not well described
- Maybe an artefact of multi-jet merging?

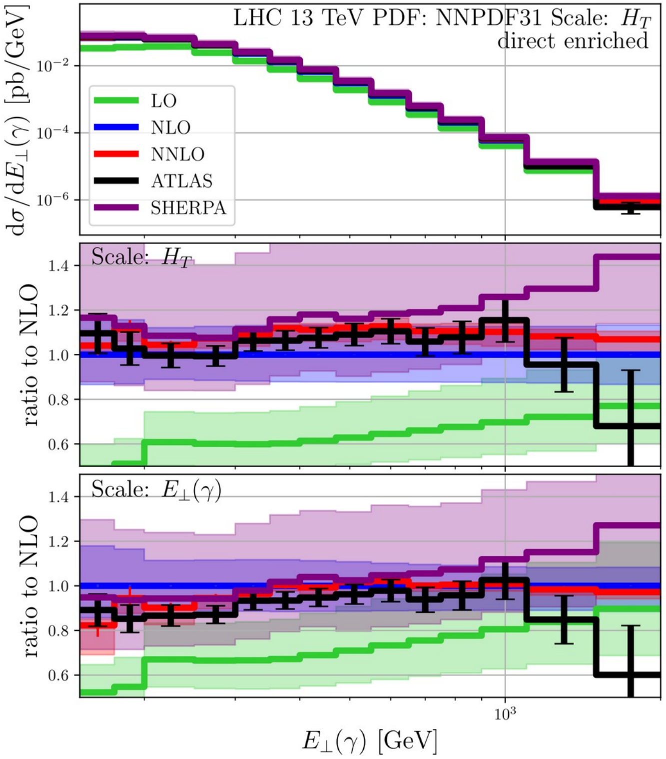


Inclusive vs. direct vs. fragmentation

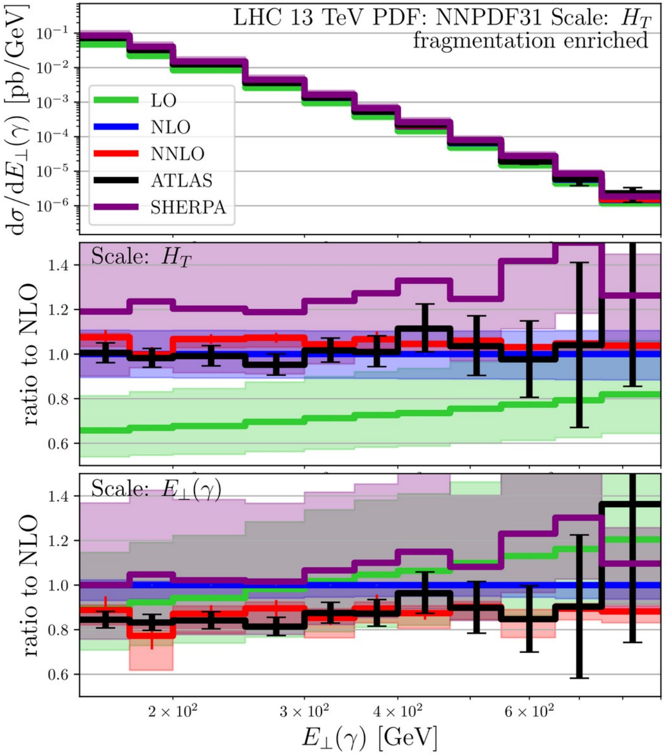
Inclusive



Direct-enriched



Fragmentation



Transverse photon energy

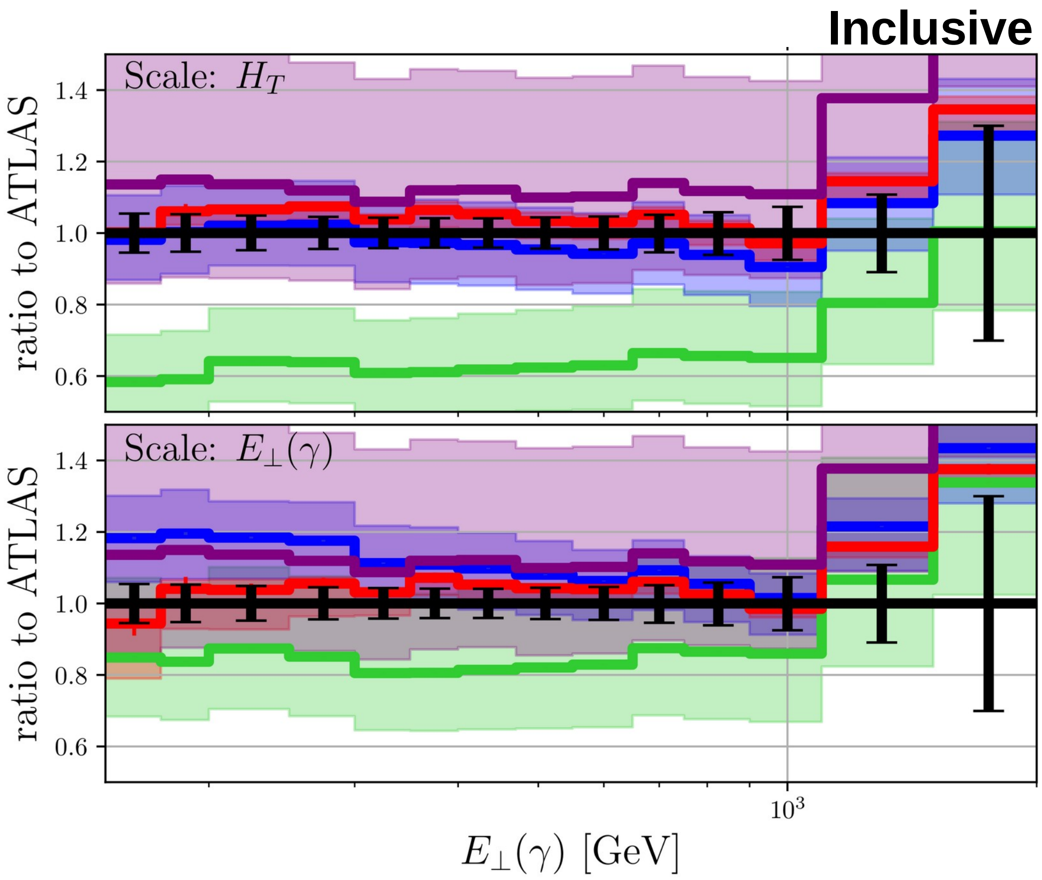
Scale choice

Full tree kinematics

$$\mu_R = \mu_F = H_T = E_\perp(\gamma) + p_T(j_1) + p_T(j_2)$$

$$\mu_R = \mu_F = E_\perp(\gamma)\ ,$$

Only photon



Perturbative convergence

NNLO result similar **but** $E_\perp(\gamma)$

- Larger (negative) NNLO corrections
- Larger scale dependence (for jet obs.)

Scale choice

Full tree kinematics

$$\mu_R = \mu_F = H_T = E_\perp(\gamma) + p_T(j_1) + p_T(j_2)$$

$$\mu_R = \mu_F = E_\perp(\gamma) \; ,$$

Only photon

Perturbative convergence

NNLO result similar **but** $E_\perp(\gamma)$

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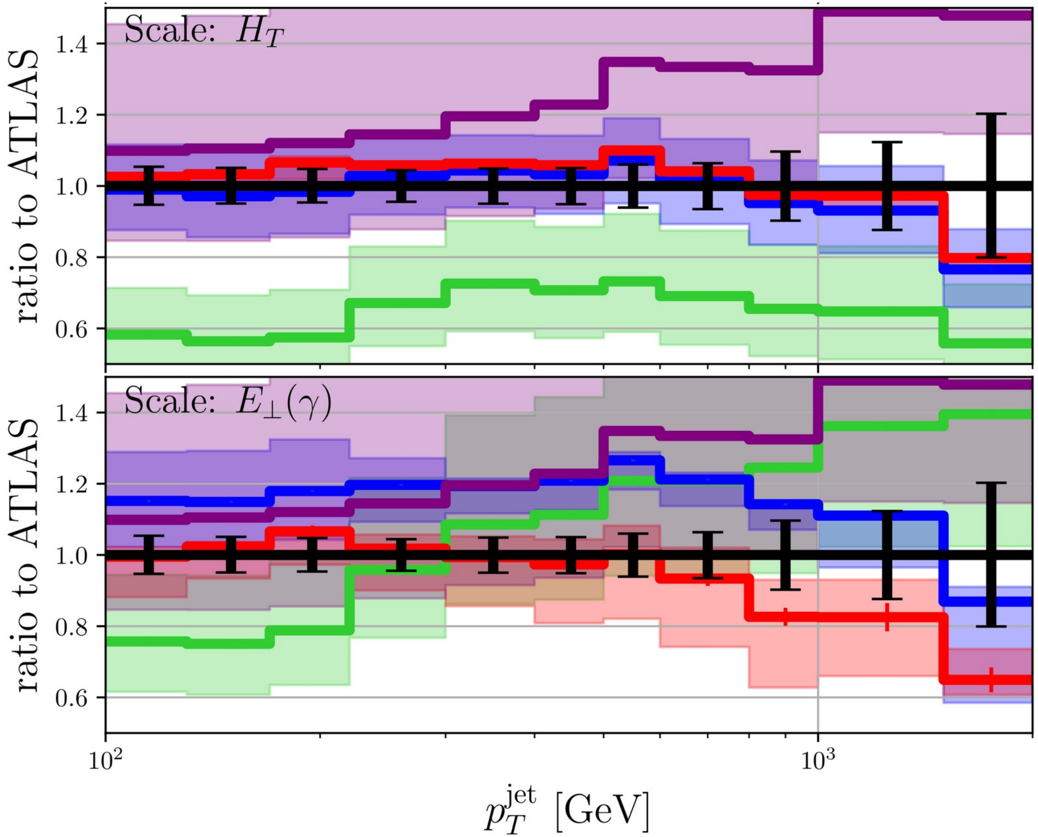
Inclusive

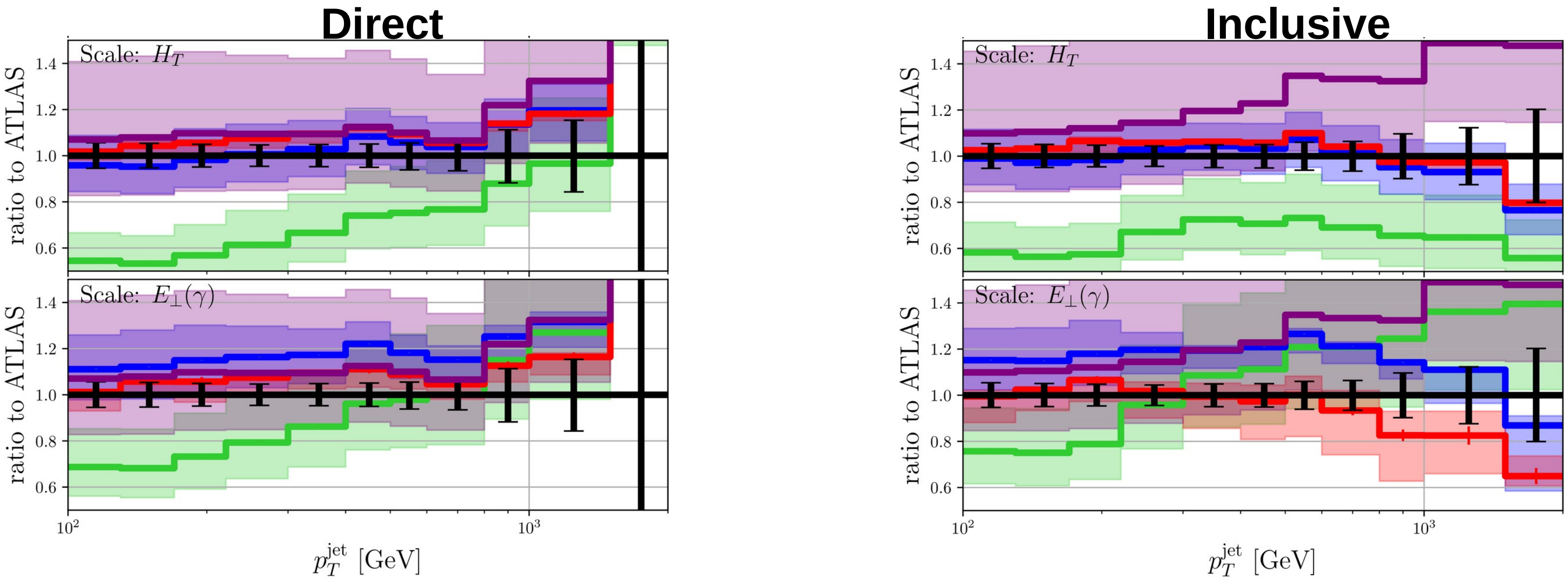


Scale choice

→ $E_\perp(\gamma)$ **does not capture relevant scales** for $pp \to \gamma + 2j$

- Better for "direct" enriched phase space $p_T(\gamma) > p_T(j_1)$
  → $E_\perp(\gamma)$ closer to $H_T = p_T(\gamma) + p_T(j_1) + p_T(j_2)$

NNLO QCD needed for this conclusion



Take home messages

- Very good description of data using NNLO QCD
  → Significantly improved theory uncertainty estimates
  → First phenomenological applications: extraction of the strong coupling constant

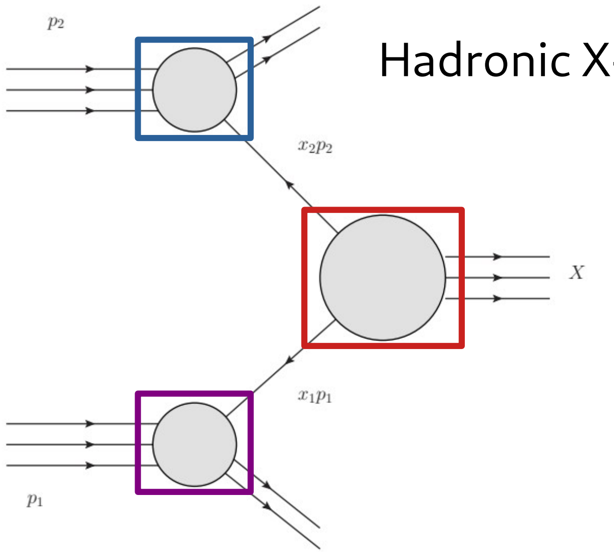
- Completion of massless 2→ 3 processes at hadron colliders through NNLO QCD

$$pp \to \gamma\gamma\gamma \qquad pp \to \gamma\gamma j \qquad pp \to \gamma j j \qquad pp \to j j j$$

- Most important bottlenecks:
  → Monte Carlo integration of real radiation contributions → improved methods needed!
  → Two-loop amplitudes
  (including external/internal masses are the current frontier)

Backup

Hadronic cross section



Hadronic X-section: $\sigma_{h_1h_2\to X} = \sum_{ij}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\phi_{i,h_1}(x_1,\mu_F^2)\phi_{j/h_2}(x_2,\mu_F^2)\hat{\sigma}_{ij\to X}(\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)$

Parton distribution functions

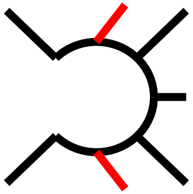
Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

The NNLO bit: $\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$

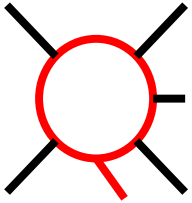
Double real radiation

$$\hat{\sigma}^{\mathrm{RR}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\left\langle \mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2}$$



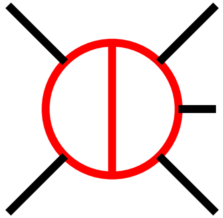
Real/Virtual correction

$$\hat{\sigma}^{\mathrm{RV}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re}\left\langle \mathcal{M}^{(0)}_{n+1}\middle|\mathcal{M}^{(1)}_{n+1}\right\rangle \mathrm{F}_{n+1}$$



Double virtual corrections

$$\hat{\sigma}^{\mathrm{VV}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n}\left(2\mathrm{Re}\left\langle \mathcal{M}^{(0)}_{n}\middle|\mathcal{M}^{(2)}_{n}\right\rangle + \left\langle \mathcal{M}^{(1)}_{n}\middle|\mathcal{M}^{(1)}_{n}\right\rangle\right) \mathrm{F}_{n}$$



Partonic cross section beyond LO

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

Contributions with different multiplicities and # convolutions:

$$\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$$

Each term separately IR divergent. But sum is:

→ finite

→ regularization scheme independent

Considering CDR ($d = 4 - 2\epsilon$):

→ Laurent expansion: $\hat{\sigma}^{C}_{ab} = \sum_{i=-4}^{0} c_i \epsilon^i + \mathcal{O}(\epsilon)$

$$\hat{\sigma}^{\mathrm{RR}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}^{(0)}_{n+2} \middle| \mathcal{M}^{(0)}_{n+2} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}^{\mathrm{RV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(1)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\mathrm{VV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n} \middle| \mathcal{M}^{(2)}_{n} \right\rangle + \left\langle \mathcal{M}^{(1)}_{n} \middle| \mathcal{M}^{(1)}_{n} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}^{\mathrm{C1}}_{ab} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\mathrm{C2}}_{ab} = (\text{double convolution})\, \mathrm{F}_{n}$$

Sector decomposition I

Considering working in CDR:

→ Virtuals are usually done in this regularization

→ Real radiation:

→ Very difficult integrals, analytical impractical (except very simple cases)!

→ Numerics not possible, integrals are divergent: ε-poles!

How to extract these poles? → Sector decomposition!

Divide and conquer the phase space:

$$1 = \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right]$$

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

Sector decomposition II

Divide and conquer the phase space:

→ Each $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$ has simpler divergences.

appearing as $1/s_{ijk} \quad 1/s_{ik}/s_{jl}$

Soft and collinear (w.r.t parton k,l) of partons i and j

→ Parametrization w.r.t. reference parton:

$$\hat{\eta}_i = \frac{1}{2}(1 - \cos\theta_{ir}) \in [0,1] \qquad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$$

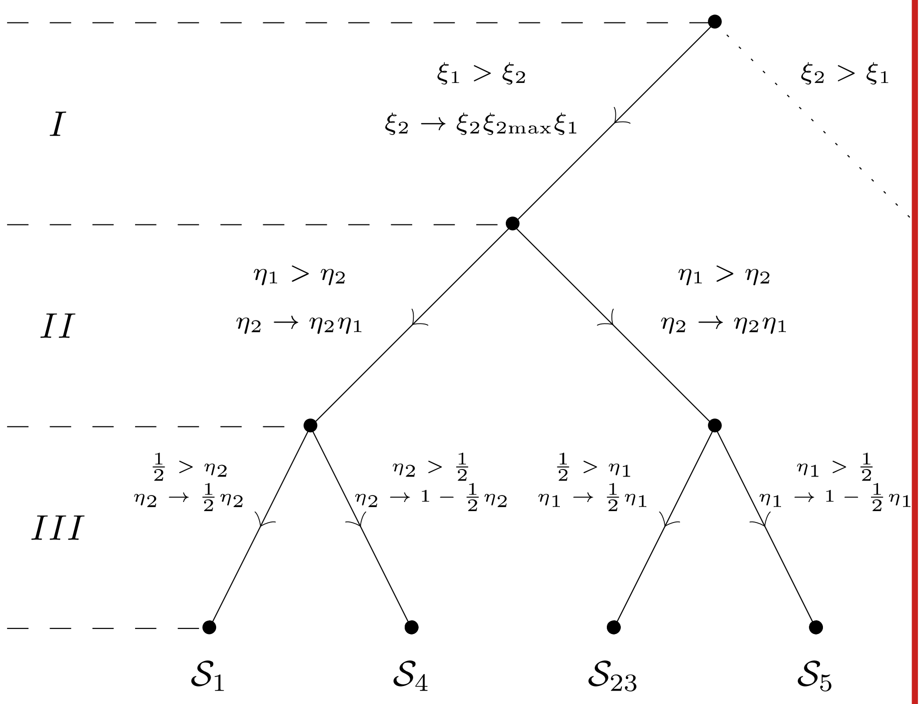
→ Subdivide to factorize divergences

$$s_{u_1 u_2 k} = (p_k + u_1 + u_2)^2 \sim \hat{\eta}_1 u_1^0 + \hat{\eta}_2 u_2^0 + \hat{\eta}_3 u_1^0 u_2^0$$

→ double soft factorization:

$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

→ triple collinear factorization



[Czakon'10,Caola'17]

Sector decomposition III

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2}\, \mathcal{S}_{kl,m} \left\langle \mathcal{M}^{(0)}_{n+2} \middle| \mathcal{M}^{(0)}_{n+2} \right\rangle \mathrm{F}_{n+2} = \sum_{\text{sub-sec.}} \int \mathrm{d}\Phi_n \prod \mathrm{d}x_i \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}} \mathrm{d}\tilde{\mu}(\{x_i\}) \underbrace{\prod x_i^{a_i+1} \langle \mathcal{M}_{n+2} | \mathcal{M}_{n+2} \rangle}_{\text{regular}} \mathrm{F}_{n+2}$$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 \mathrm{d}x \left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x) - f(0)}{x^{1+b\epsilon}}$$

Finite NNLO cross section

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1} \, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution}) \, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution}) \, \mathrm{F}_{n}$$

↓ sector decomposition and master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\left(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}\right) \quad \left(\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}\right) \quad \left(\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}\right) \quad \left(\sigma_{SU}^{C1}, \sigma_{DU}^{C1}\right) \quad \left(\sigma_{DU}^{C2}, \sigma_{FR}^{C2}\right)$$

↓ re-arrangement of terms → 4-dim. formulation [Czakon'14, Czakon'19]

$$\underline{\left(\sigma_F^{RR}\right)} \quad \underline{\left(\sigma_F^{RV}\right)} \quad \underline{\left(\sigma_F^{VV}\right)} \quad \underline{\left(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}\right)} \quad \underline{\left(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2}\right)} \quad \underline{\left(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}\right)}$$

separately finite: ε poles cancel

C++ framework

- Formulation allows efficient algorithmic implementation
- **High degree of automation**:
  - Partonic processes (taking into account all symmetries)
  - Sectors and subtraction terms
  - Interfaces to Matrix-element providers:
    AvH, OpenLoops, Recola, NJET, HardCoded

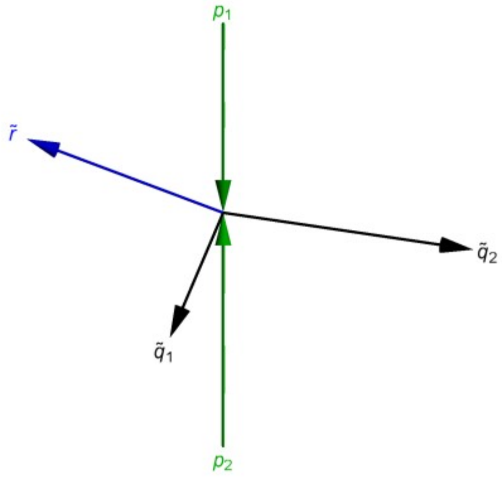
    → Only two-loop matrix elements required
- **Broad range of applications** through additional facilities:
  - Narrow-Width & Double-Pole Approximation
  - Fragmentation
  - Polarised intermediate massive bosons
  - (Partial) Unweighting → Event generation for **HighTEA**
  - Interfaces: FastNLO, FastJet

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed : u_i $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,,\ \ \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

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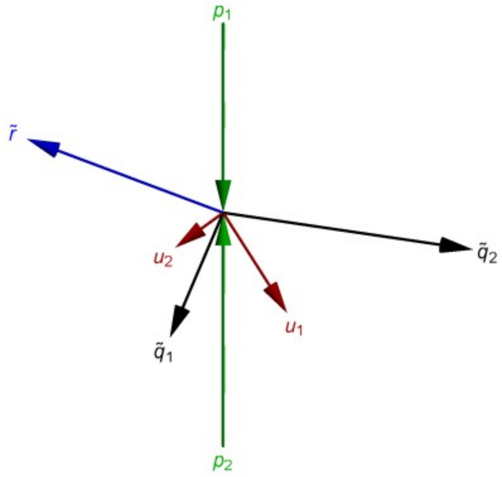

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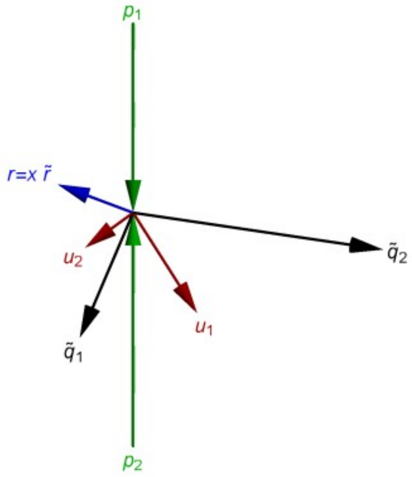

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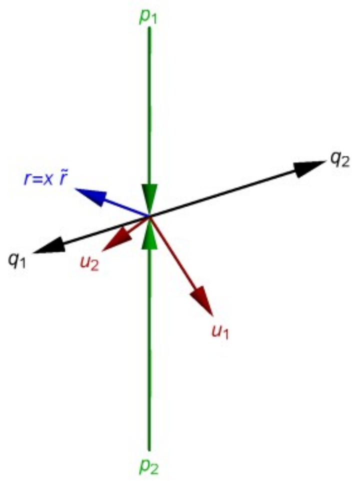

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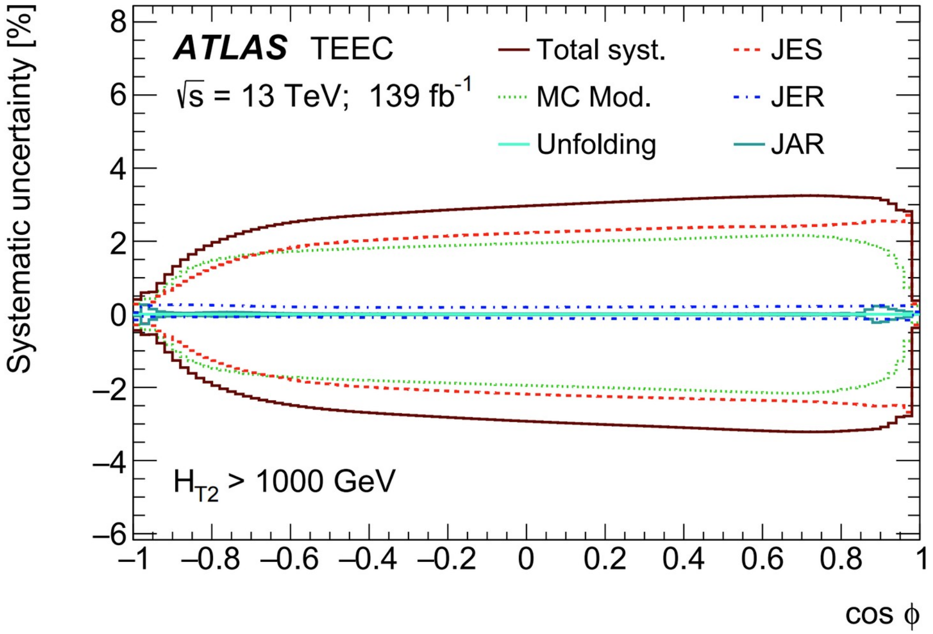
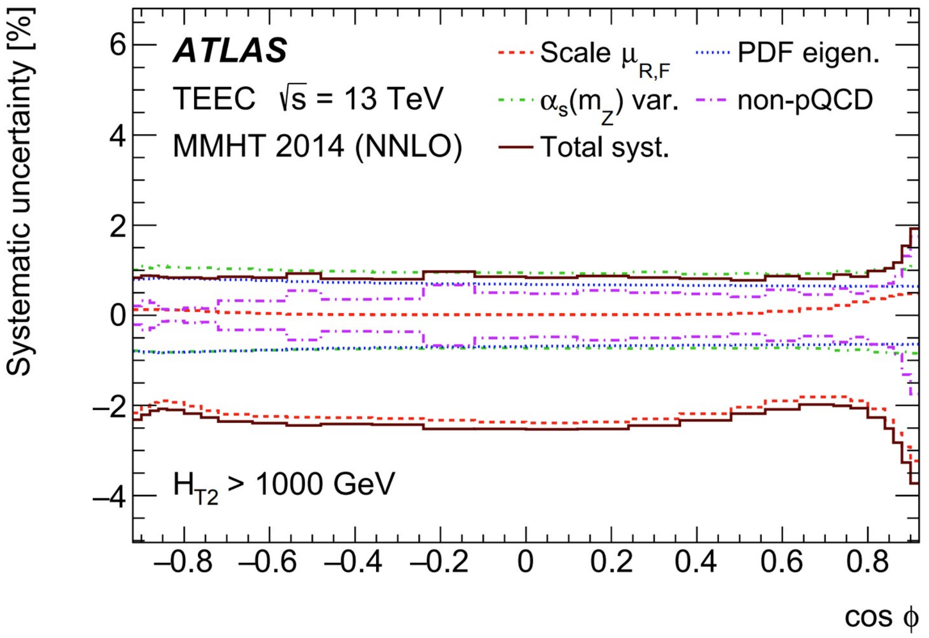
Systematic Uncertainties TEEC

Experimental uncertainties

Theory uncertainties

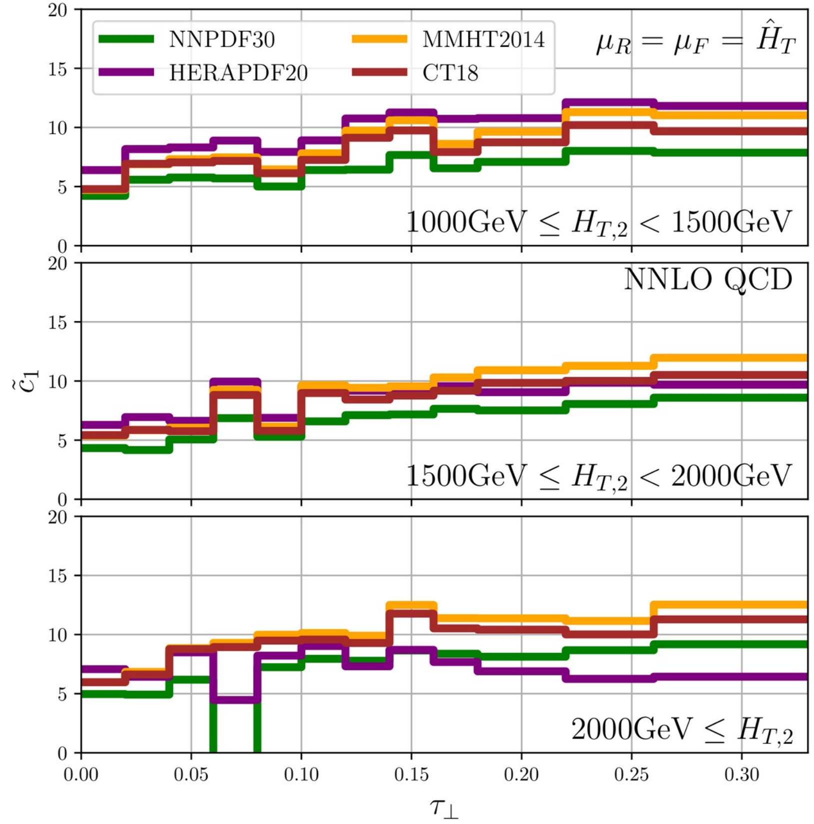




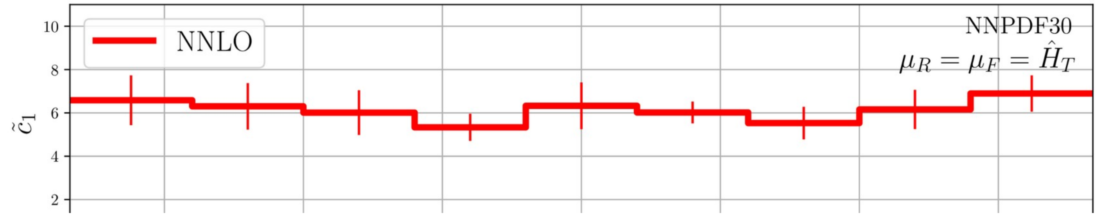
Scale dependence is the dominating uncertainty → NNLO QCD required to match exp.

Strong coupling dependence

Thrust



TEEC



$$R^{\text{NNLO,fit}}(\mu, \alpha_{S,0}) = c_0 + c_1(\alpha_{S,0} - 0.118) + c_2(\alpha_{S,0} - 0.118)^2 + c_3(\alpha_{S,0} - 0.118)^3$$

mostly linear dependence

Visualisation of α_S dependence

$$\tilde{c}_1 = \frac{c_1}{R^{\text{NNLO}}(\alpha_{S,0} = 0.118)}$$

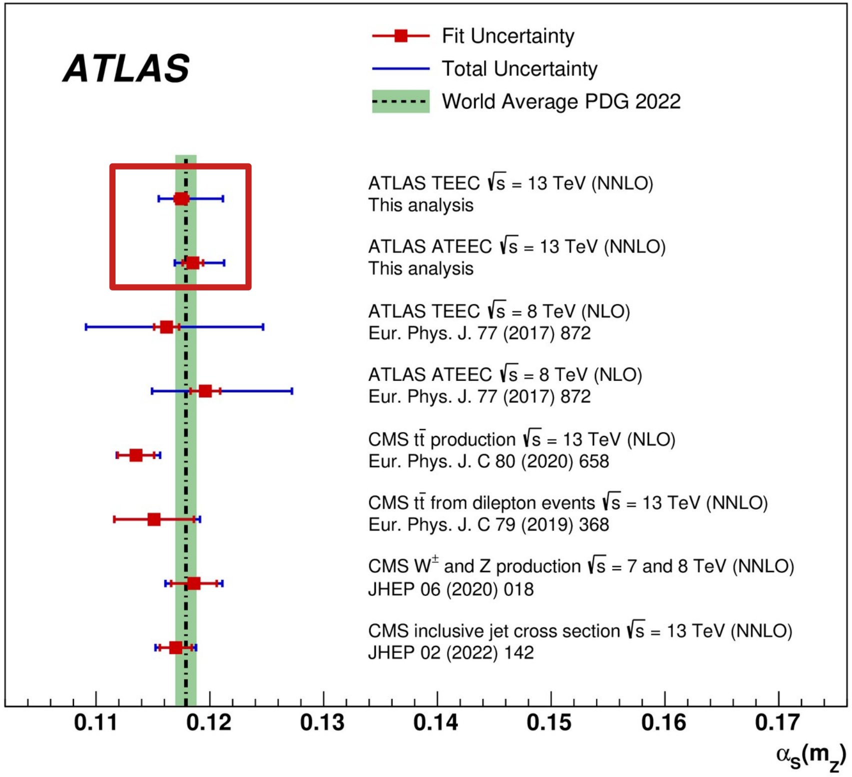
For comparison:
scale dependence (dominant theory uncertainty)

- TEEC ($H_{T,2} > 1\ \text{TeV}$) : ~2%
- Thrust : ~3-5 %

O(1%) sensitivity

α_S from TEEC @ NNLO by ATLAS

[ATLAS 2301.09351]



- NNLO QCD extraction from multi-jets → will contribute to PDG for the first time
- Significant improvement to 8 TeV → driven by NNLO QCD corrections
- Individual precision large but comparable to top or jets-data.
- However: extraction at high energy scales

Using the running of α_S to probe NP

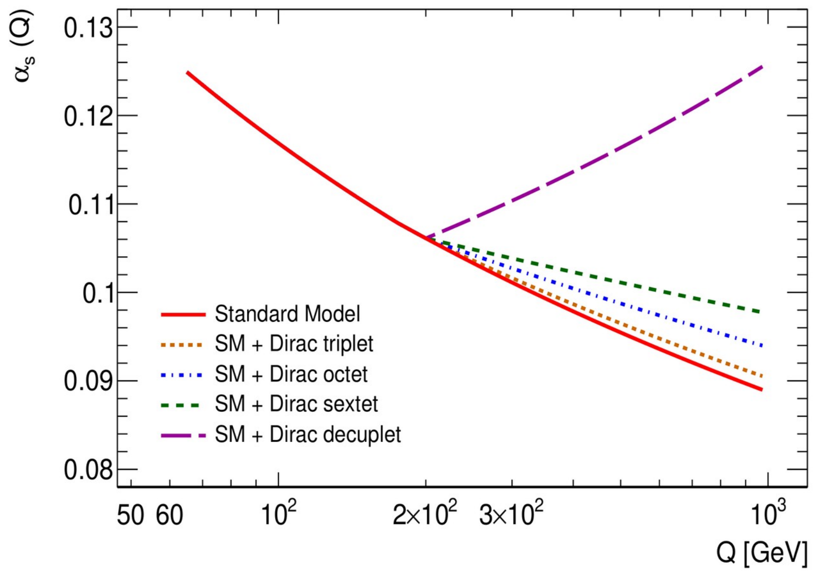
[Llorente, Nachman 1807.00894]

Indirect constraints to NP through modified running:

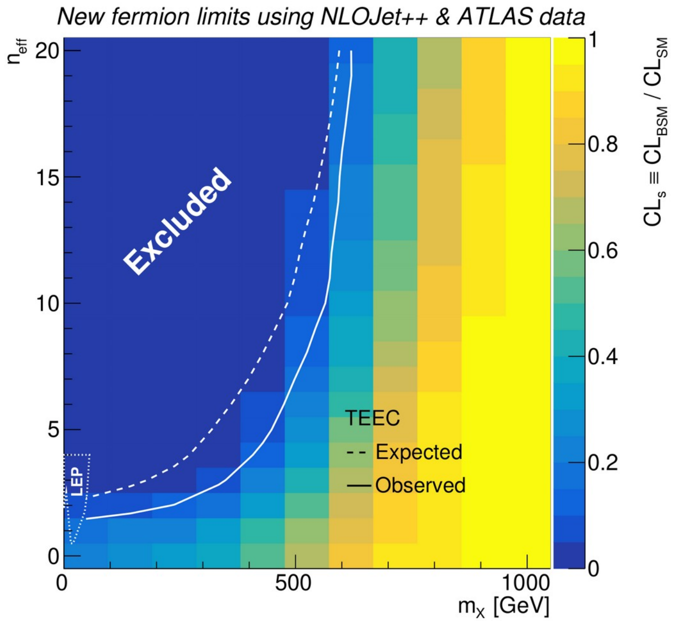
$$\beta_0 = \frac{1}{4\pi}\left(11 - \frac{2}{3}n_f - \frac{4}{3}n_X T_X\right)$$

$$\beta_1 = \frac{1}{(4\pi)^2}\left[102 - \frac{38}{3}n_f - 20 n_X T_X\left(1 + \frac{C_X}{5}\right)\right]$$

$$\alpha_s(Q) = \frac{1}{\beta_0 \log z}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\log(\log z)}{\log z}\right]; \quad z = \frac{Q^2}{\Lambda_{QCD}^2}$$

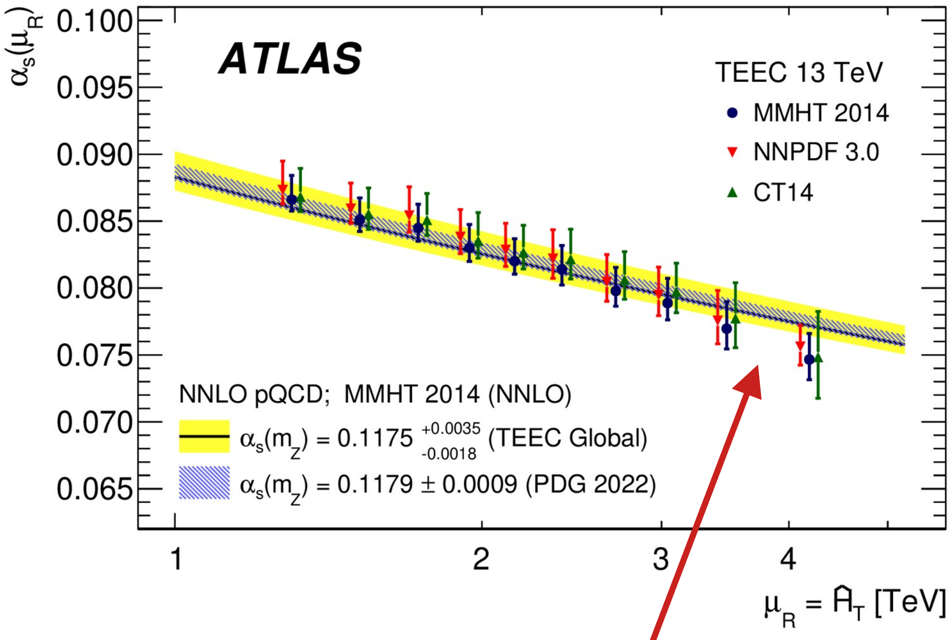


ATLAS TEEC @ 7 TeV data



Update with TEEC@13 TeV
→ much improved bounds

… or 'new' SM dynamics



Systematic slope
→ New physics?

Possible SM explanations

- Residual PDF effects → high x,Q^2 ?
- EW corrections?
- Maybe effect from LC approximation in two-loop ME?

$$\mathcal{R}^{(2)}(\mu_R^2) = 2\,\mathrm{Re}\left[\mathcal{M}^{\dagger(0)}\mathcal{F}^{(2)}\right](\mu_R^2) + \left|\mathcal{F}^{(1)}\right|^2(\mu_R^2)$$

$$\equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^{4} c_i \ln^i\left(\frac{\mu_R^2}{s_{12}}\right)$$

$$\mathcal{R}^{(2)}(s_{12}) \approx \mathcal{R}^{(2)l.c.}(s_{12})$$

- Experimental systematics?
- Resummation?

Either case interesting!

Photon isolation

Hard cone

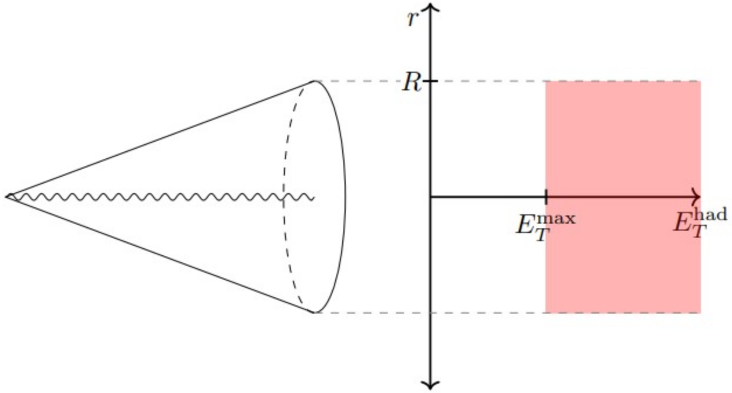
- Experimental hard cone:

$$E_{\perp}(r) \leq E_{\perp\max} = 0.0042\, E_{\perp}(\gamma) + 10\ \text{GeV} \quad \text{for} \quad r \leq R_{\max} = 0.4$$

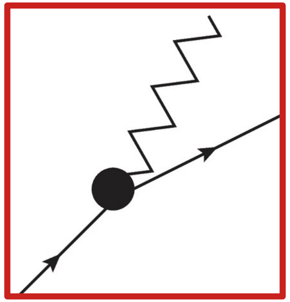
- Theory perspective:

  Not collinear safe in perturbative QCD
  due to q → qγ splittings

  → Non-vanishing fragmentation contribution
  (NNLO QCD with frag. [2201.06982][2205.01516])



Credit: Marius Hoefer (talk@SM@LHC22)



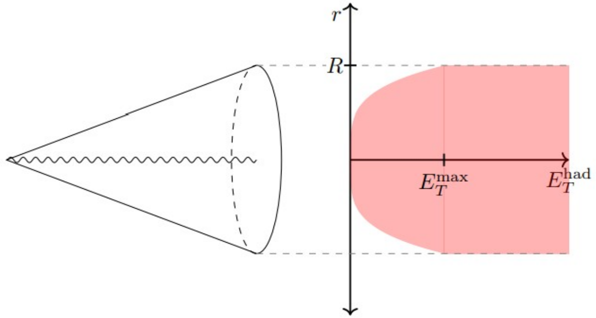
Photon isolation

Smooth cone

- by Frixione [hep-ph/9801442]

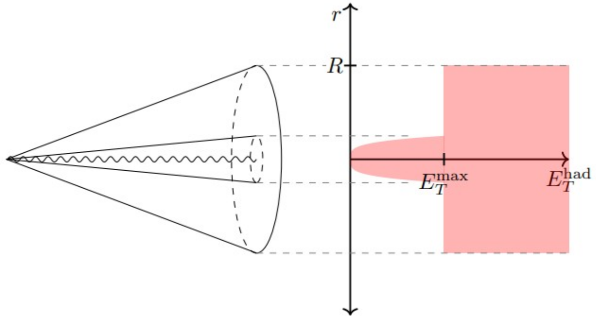
$$E_\perp(r) \leq E_{\perp\max}(r) = 0.1\, E_\perp(\gamma) \left(\frac{1-\cos(r)}{1-\cos(R_{\max})} \right)^2 \quad \text{for} \quad r \leq R_{\max} = 0.1$$

→ Theoretically convenient
→ Removes fragmentation contribution
→ Experimentally limited by detector resolution



Hybrid cone

- [1611.07226][2205.01516]
  - Combines smooth & hard cone
  - Fair approx. to hard cone [2205.01516]

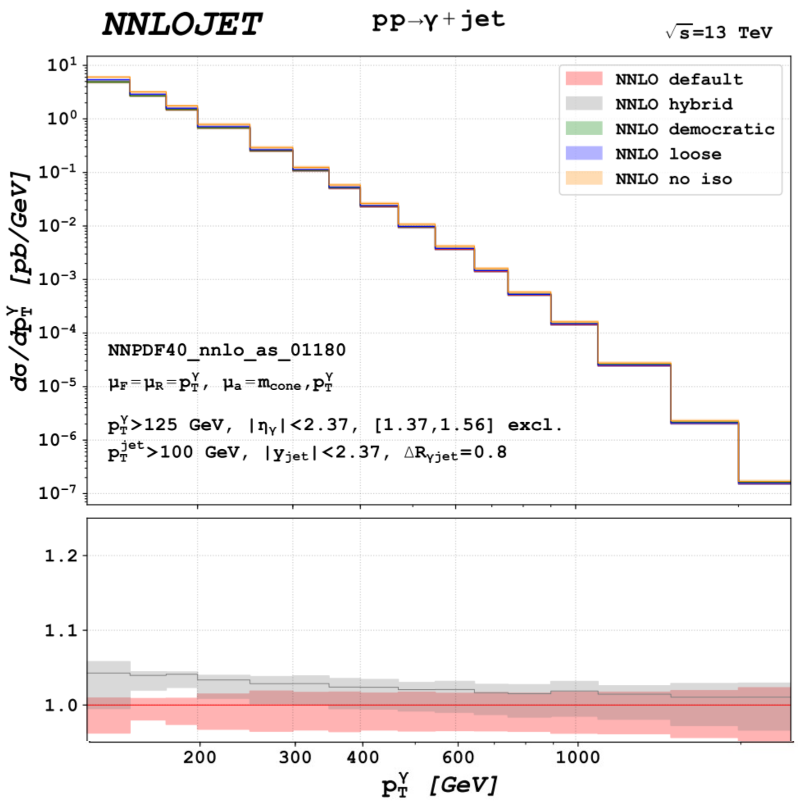


Credit: Marius Hoefer (talk@SM@LHC22)

Fragmentation contribution

[2205.01516]

- ATLAS photon requirements (same as for $pp \to \gamma + 2j$)
- Comparison between:
  - "default" NNLO with fragmentation
  - "hybrid" NNLO with hybrid isolation
- Fragmentation contr.
  - ~5% at small $E_T(\gamma)$
  - ~<1% at high $E_T(\gamma)$



Missing effects

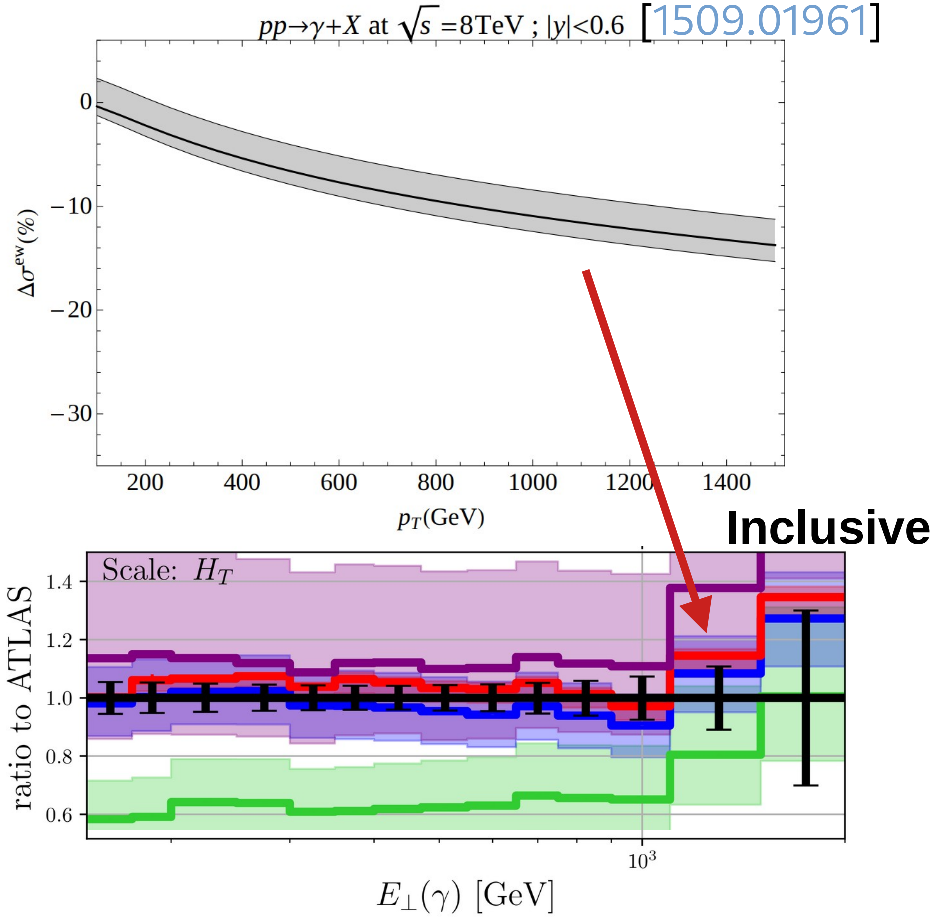
Electro-weak corrections

- EW Sudakov logs at high $E_\perp(\gamma)$
- ~O(-10%) above 1 TeV
- Further improvement of theory/data

Fragmentation

- More relevant at small $E_\perp(\gamma)$
- For $pp \to \gamma + X:\ \ \sigma(\text{hybrid}) > \sigma(\text{frag.})$
- Inclusion might cure slightly high normalisation

[1509.01961]



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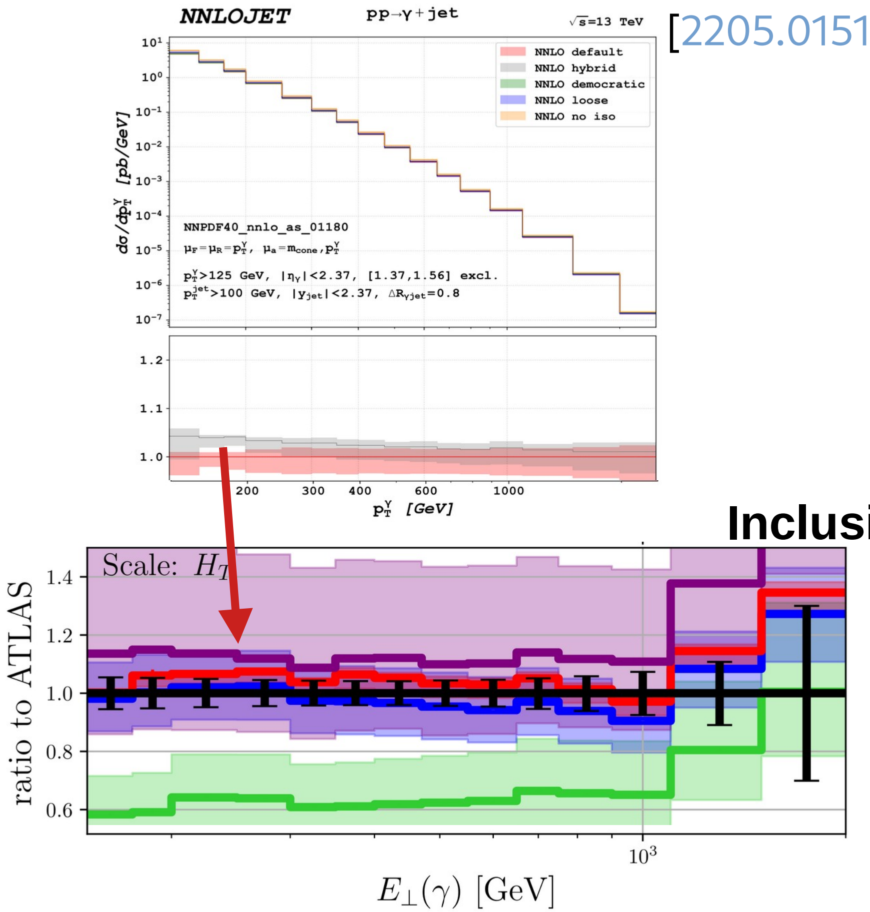

Inclusive