

# Precision phenomenology with multi-jet final states at the LHC

Dr. rer. nat. Rene Poncelet

LEVERHULME  
TRUST



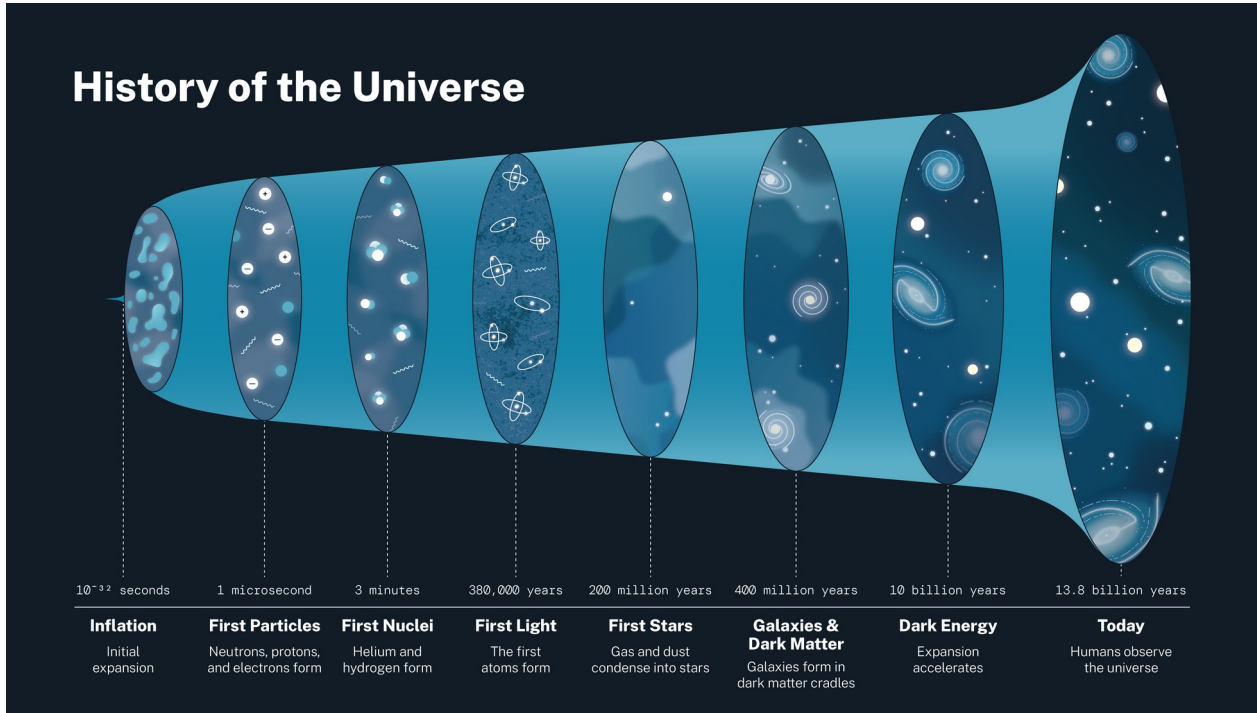
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IFJ PAN, Krakow
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RWTH Aachen University, Aachen
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Georg-August University, Göttingen

# Outline

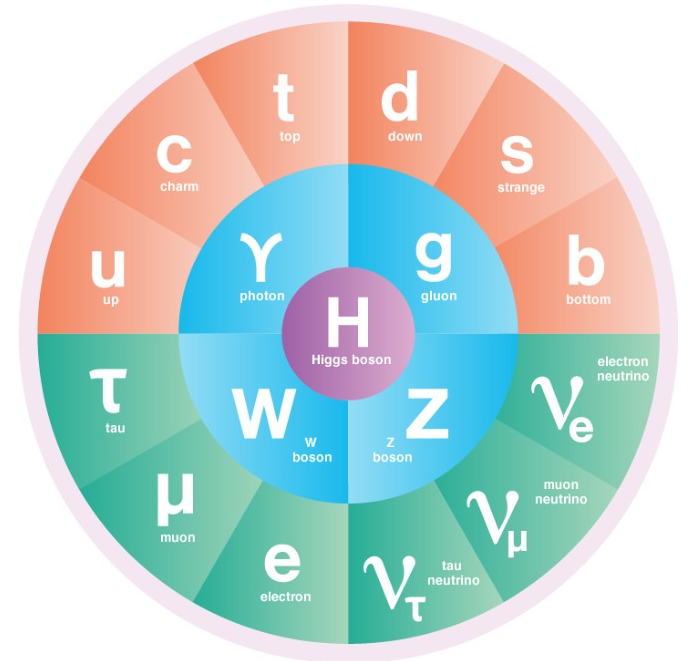
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- Introduction
- NNLO QCD predictions for multi-jet observables and event shapes
- HighTEA
- Sector-improved residue subtraction scheme
- Wider research context

# What is the universe made of and where does it come from?



Credit: NASA

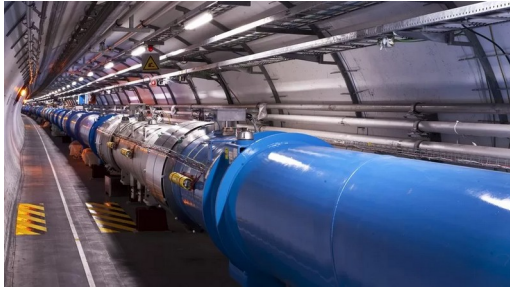


Credit: SymmetryMagazine

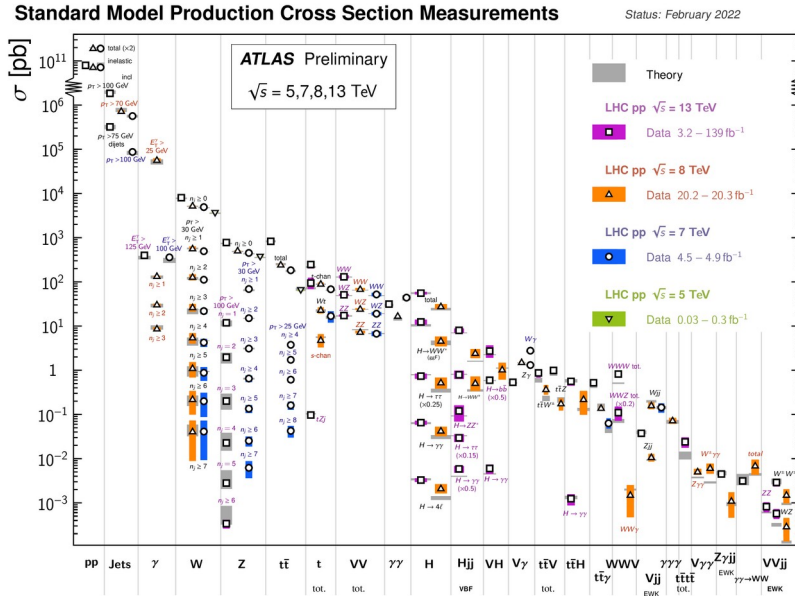
● QUARKS   
 ● LEPTONS   
 ● BOSONS   
 ● HIGGS BOSON

# What are the fundamental building blocks of matter?

Scattering experiments

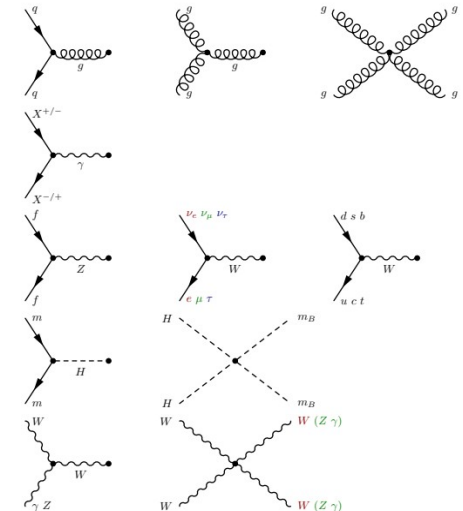


Credit: CERN



Credit: ATLAS

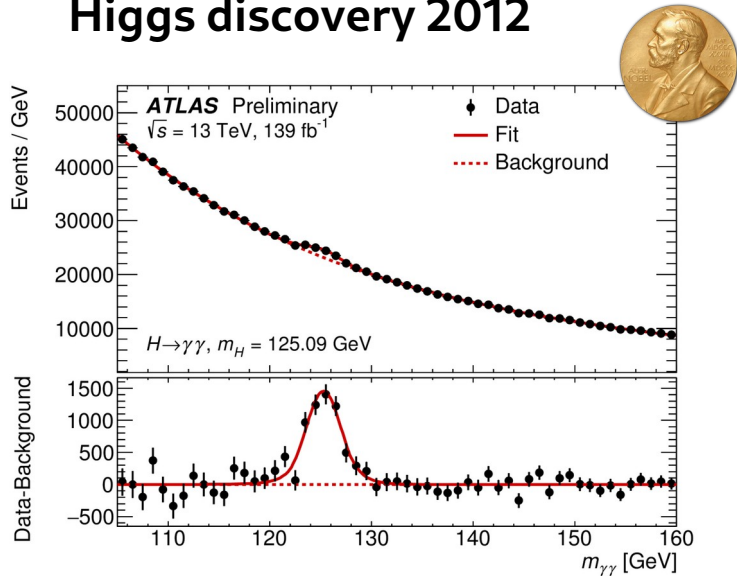
Theory/Model



Credit: Jack Lindon, CERN

# Standard Model of Particle Physics and beyond

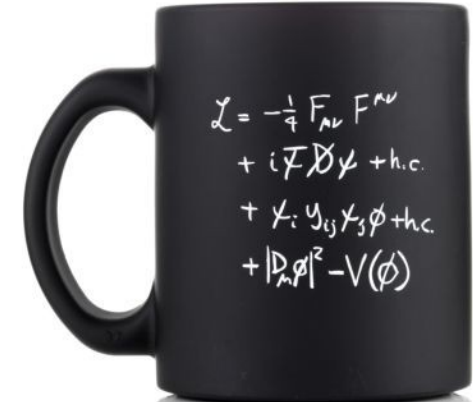
## Higgs discovery 2012



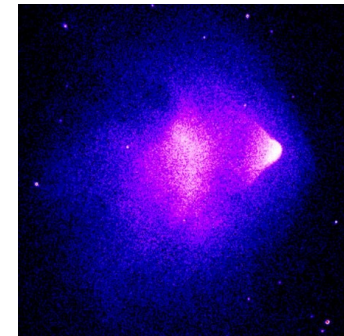
Credit: ATLAS

**BUT:**

- Is the Higgs a fundamental scalar?
- What is dark matter?
- Why is there a matter-anti-matter asymmetry?
- ...

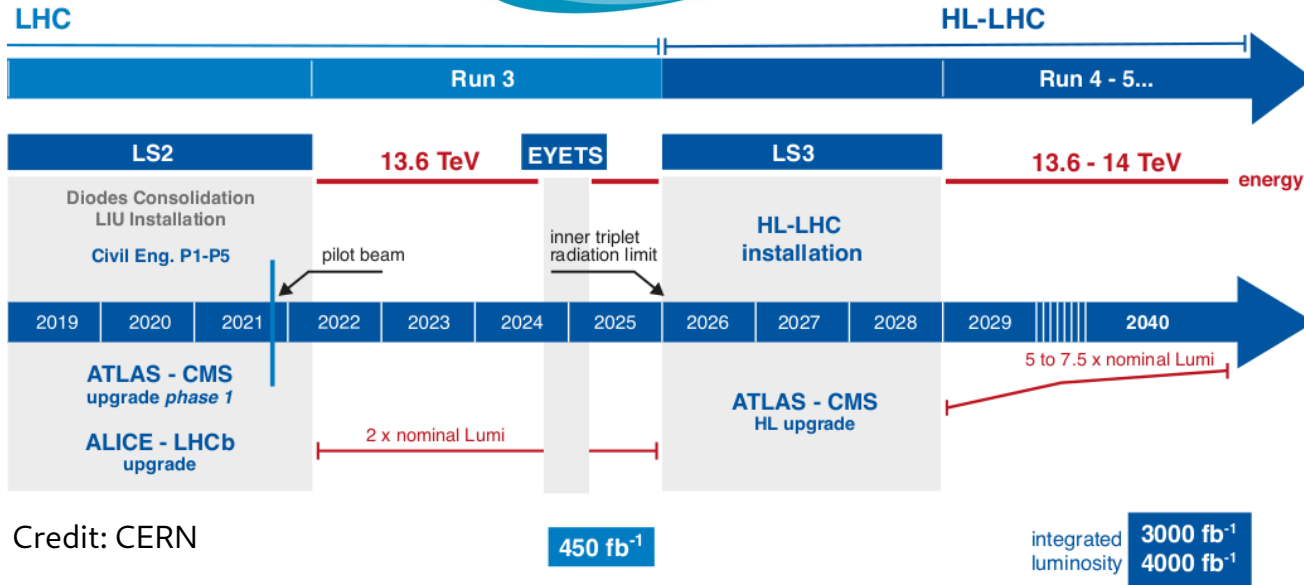


Credit: CERN



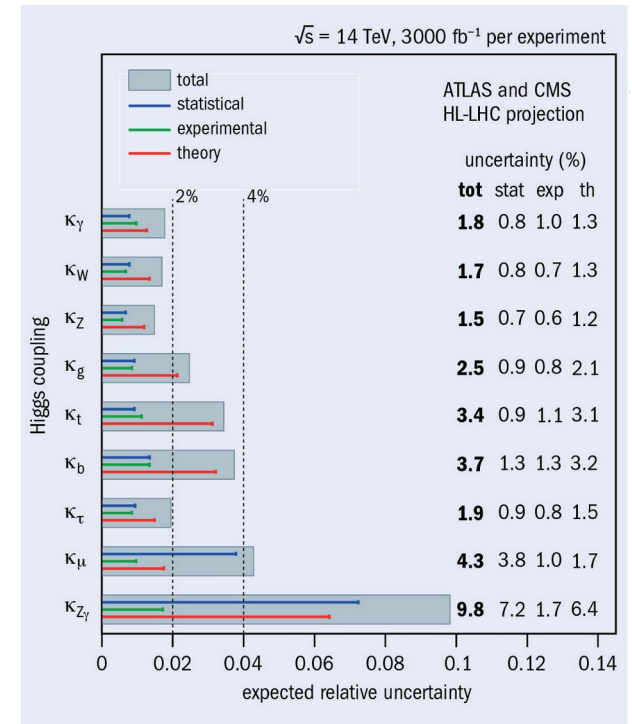
Credit: NASA

# LHC Precision era and future experiments



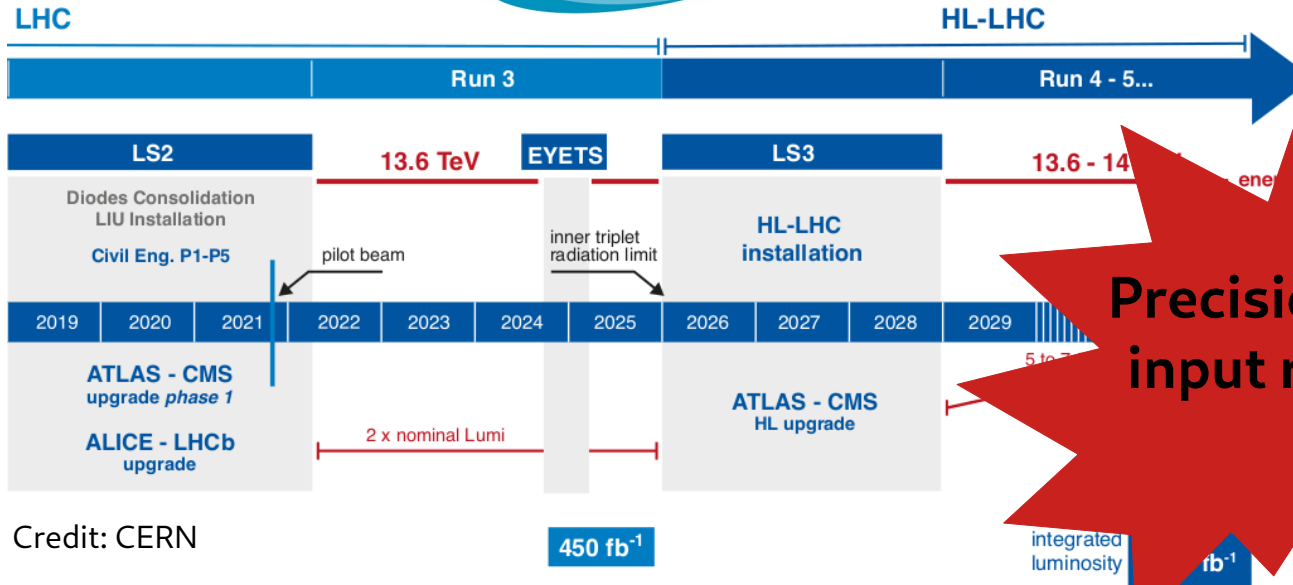
Credit: CERN

## Example: Projected Higgs coupling measurements



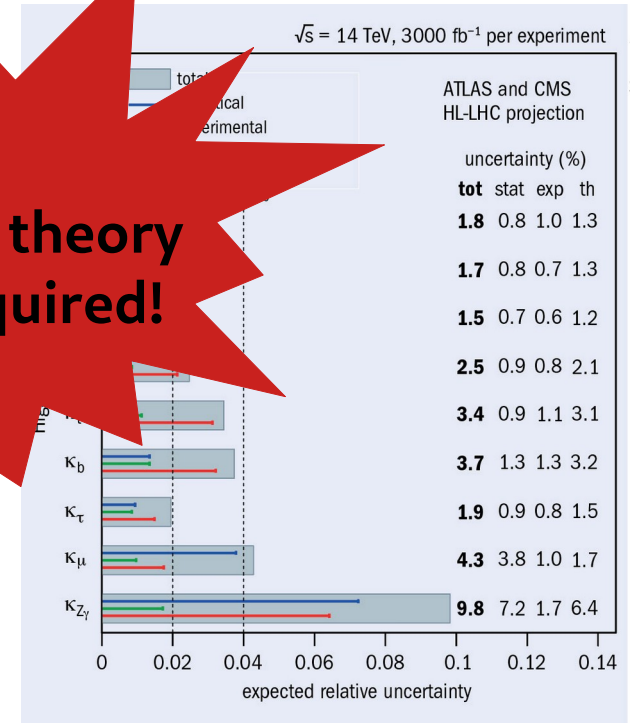
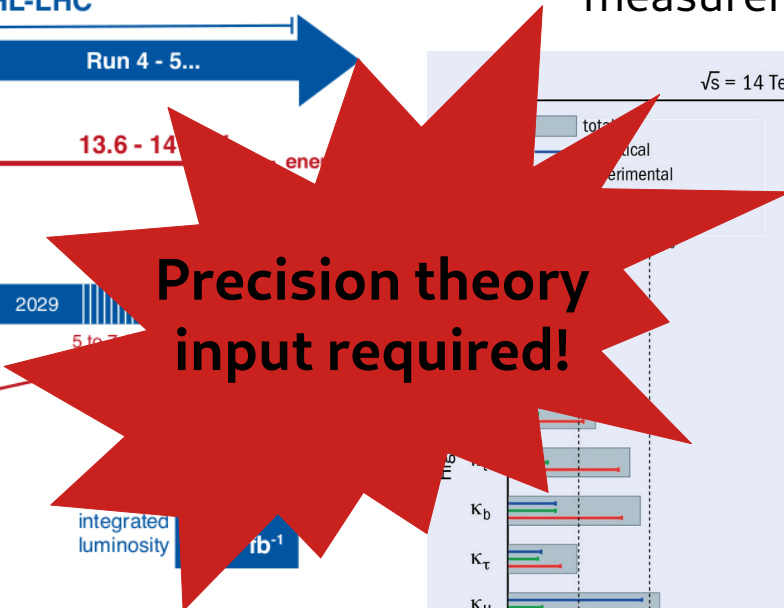
[1902.00134]

# LHC Precision era and future experiments



Credit: CERN

Example:  
Projected Higgs coupling measurements



[1902.00134]

# Precision through higher orders

Hadronic cross section:

$$\sigma_{h_1 h_2 \rightarrow X} = \sum_{ij} \int_0^1 \int_0^1 dx_1 dx_2 \underbrace{\phi_{i,h_1}(x_1, \mu_F^2)}_{\text{Parton distribution functions}} \underbrace{\phi_{j/h_2}(x_2, \mu_F^2)}_{\text{Parton distribution functions}} \underbrace{\hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)}_{\text{Partonic cross section}}$$

Parton distribution functions

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab \rightarrow X} = \underbrace{\alpha_s^0 \hat{\sigma}_{ab \rightarrow X}^{(0)}}_{\text{Leading order}} + \underbrace{\alpha_s^1 \hat{\sigma}_{ab \rightarrow X}^{(1)}}_{\text{Next-to-leading order}} + \underbrace{\alpha_s^2 \hat{\sigma}_{ab \rightarrow X}^{(2)}}_{\text{Next-to-next-to-leading order}} + \mathcal{O}(\alpha_s^3)$$

Leading order

Next-to-leading order

Next-to-next-to-leading order

Uncertainty:  
 $\alpha_s(m_Z) \approx 0.118$

Order of magnitude

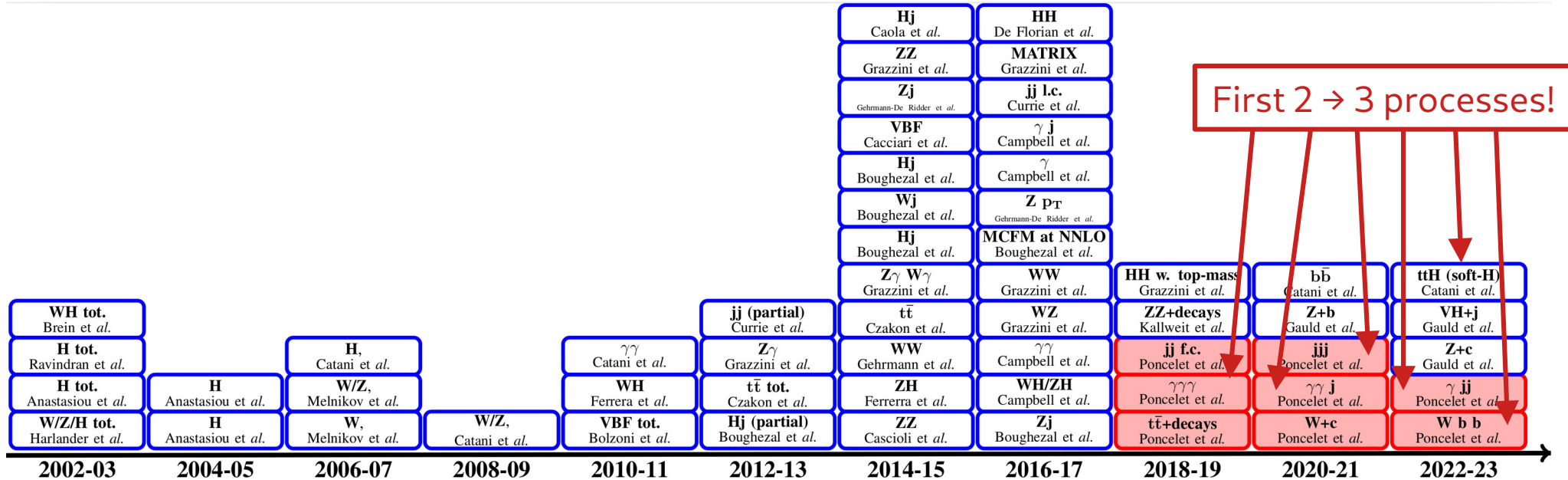
O(10%)

O(1%)

**Next-to-next-to-leading order QCD needed to match experimental precision!**  
**→ In some cases even next-to-next-to-next-to-leading order!**



# The NNLO QCD revolution



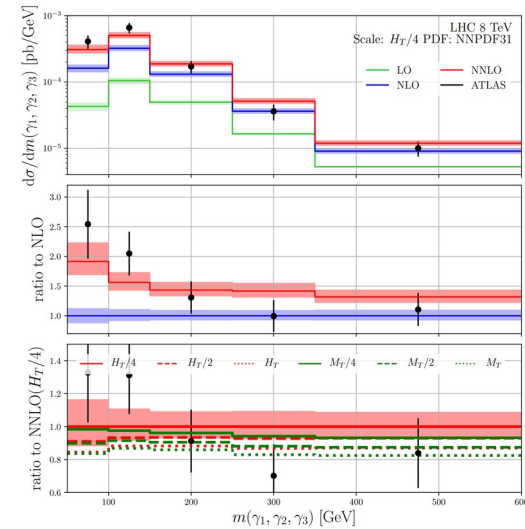
# NNLO QCD for 2 → 3 processes

$$pp \rightarrow \gamma\gamma\gamma$$

$$pp \rightarrow \gamma\gamma j$$

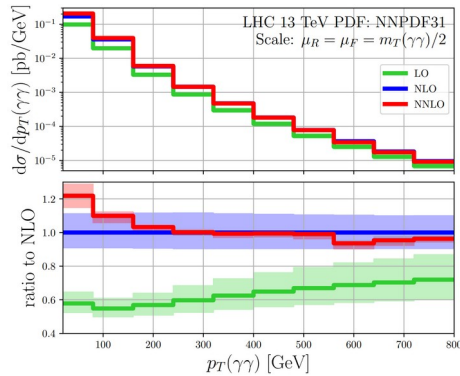
$$pp \rightarrow \gamma j j$$

$$pp \rightarrow j j j$$

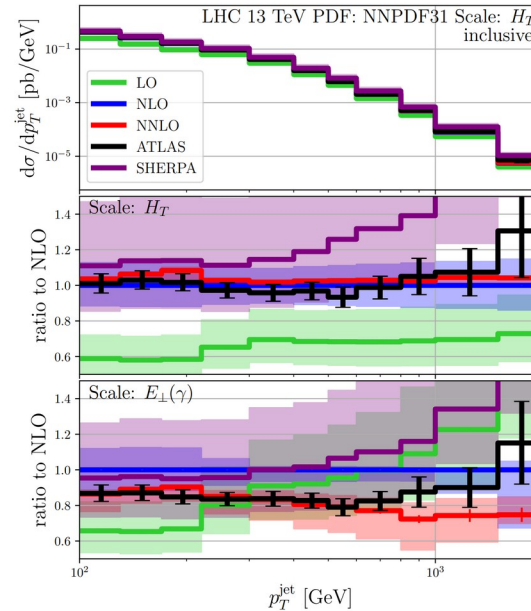


Chawdhry, Czakon, Mitov,  
Poncelet [[1911.00479](#)]

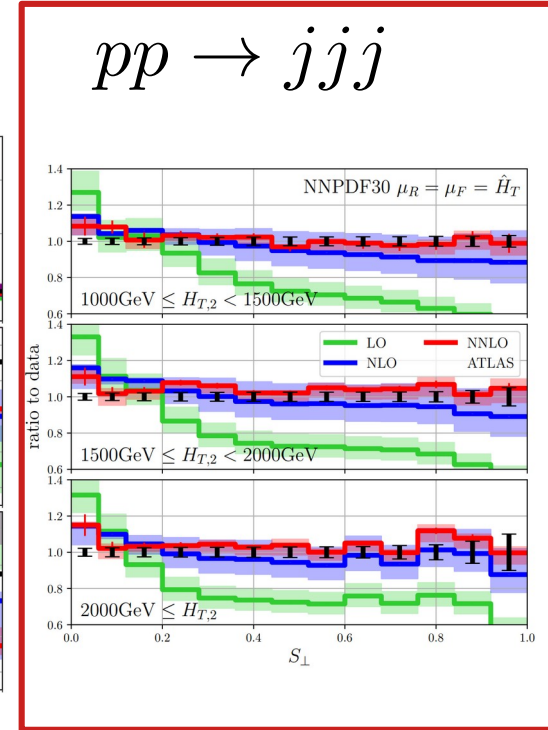
Kallweit, Sotnikov,  
Wiesemann [[2010.04681](#)]



Chawdhry, Czakon, Mitov,  
Poncelet [[2103.04319](#)]

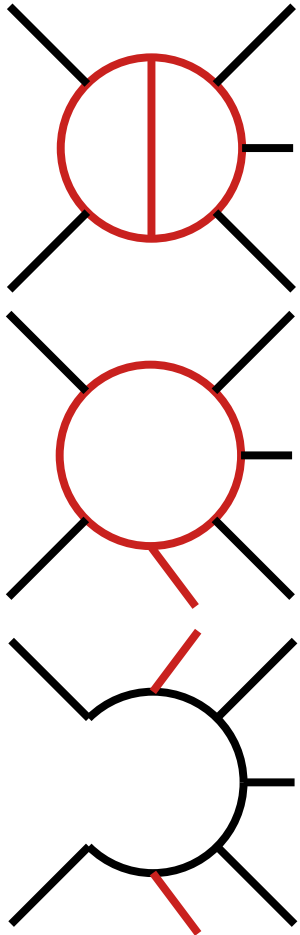


Badger, Czakon, Hartanto,  
Moodie, Peraro, Poncelet,  
Zoia [[2304.06682](#)]



Czakon, Mitov, Poncelet  
[[2106.05331](#)]  
+ Alvarez, Cantero, Llorente  
[[2301.01086](#)]

# NNLO QCD for 2→3 processes - inputs



## Two-loop amplitudes

- (Non-) planar 5 point massless [[Chawdry'19'20'21](#),[Abreu'20'21'23](#),[Agarwal'21](#),[Badger'21'23](#)] → triggered by efficient MI representation [[Chicherin'20](#)]
- 5 point with one external mass [[Abreu'20](#),[Syrrakos'20](#),[Canko'20](#),[Badger'21'22](#),[Chicherin'22](#)]
- **For three-jets** → LC [[Abreu'20'21](#)] (checked against NJET [[Badger'12'21](#)])

## One-loop amplitudes → OpenLoops [[Buccioni'19](#)]

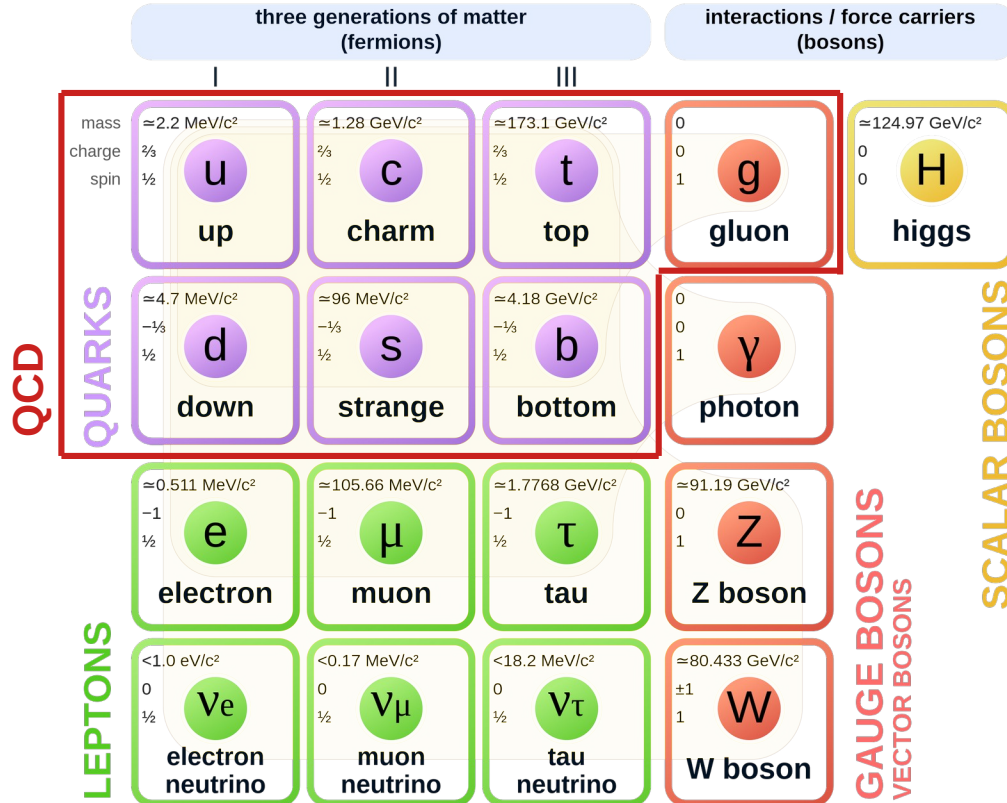
- Many legs and IR stable (soft and collinear limits)

## Double-real Born amplitudes → AvHlib [[Bury'15](#)]

- IR finite cross-sections → NNLO subtraction schemes  
qT-slicing [[Catani'07](#)], N-jettiness slicing [[Gaunt'15/Boughezal'15](#)], Antenna [[Gehrmann'05-'08](#)], Colorful [[DelDuca'05-'15](#)], Projctction [[Cacciari'15](#)], Geometric [[Herzog'18](#)], Unsubtraction [[Aguilera-Verdugo'19](#)], Nested collinear [[Caola'17](#)], Local Analytic [[Magnea'18](#)], **Sector-improved residue subtraction** [[Czakon'10-'14,'19](#)]

# Precision tests of QCD at the LHC

## Standard Model of Elementary Particles



- At the LHC **QCD is part of any process!**
  - The limiting factor in many analyses is QCD and associated uncertainties.  
→ **Radiative corrections indispensable**
  - How well we do know QCD? Coupling constant, running, PDFs, ...
- The production of high energy jets allow to **probe pQCD at high energies** directly

$$\mathcal{L}_{\text{QCD}} = \bar{q}_i (\gamma^\mu \mathcal{D}_\mu - m_i) q_i - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a$$

- Testing the predicted dynamics
- Extract the coupling constant

# Multi-jet observables

NLO theory unc. > experimental unc.

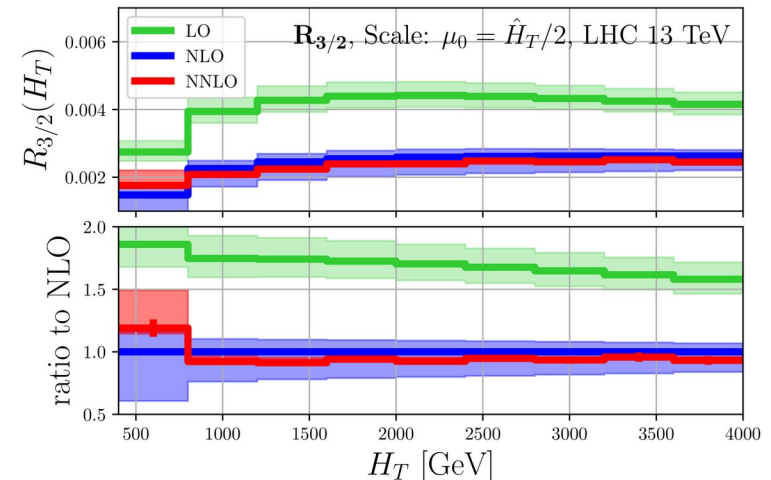
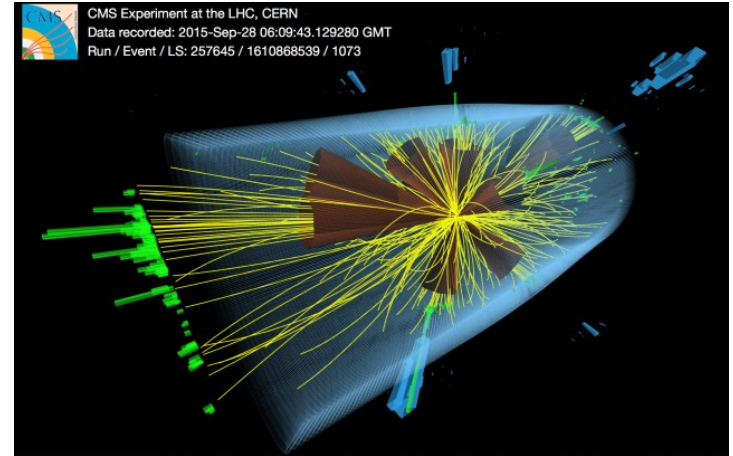
- **NNLO QCD needed for precise theory-data comparisons**  
→ Restricted to two-jet data  
[Currie'17+later][Czakon'19]
- **New NNLO QCD three-jet** → access to more observables
  - Jet ratios

**Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC**  
Czakon, Mitov, Poncelet [[2106.05331](#)]

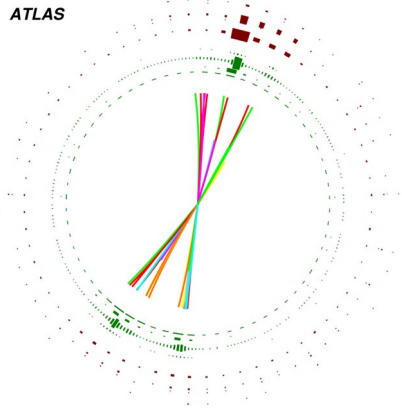
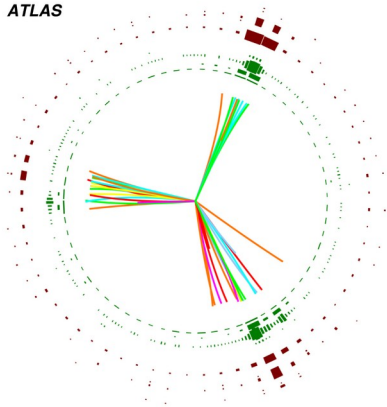
$$R^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0}) = \frac{d\sigma_3^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}{d\sigma_2^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}$$

- Event shapes

**NNLO QCD corrections to event shapes at the LHC**  
Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet [[2301.01086](#)]



# Encoding QCD dynamics in event shapes



Using (global) event information to separate different regimes of QCD event evolution:

- **Thrust & Thrust-Minor**  $T_{\perp} = \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}_{\perp}|}{\sum_i |\vec{p}_{T,i}|}$ , and  $T_m = \frac{\sum_i |\vec{p}_{T,i} \times \hat{n}_{\perp}|}{\sum_i |\vec{p}_{T,i}|}$ .

- **Energy-energy correlators**

$$\frac{1}{\sigma_2} \frac{d\sigma}{d \cos \Delta\phi} = \frac{1}{\sigma_2} \sum_{ij} \int \frac{d\sigma x_{\perp,i} x_{\perp,j}}{dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij}} \delta(\cos \Delta\phi - \cos \Delta\phi_{ij}) dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij},$$

- more computed:  
aplanarity, sphericity, C and D variables

Separation of energy scales:  $H_{T,2} = p_{T,1} + p_{T,2}$

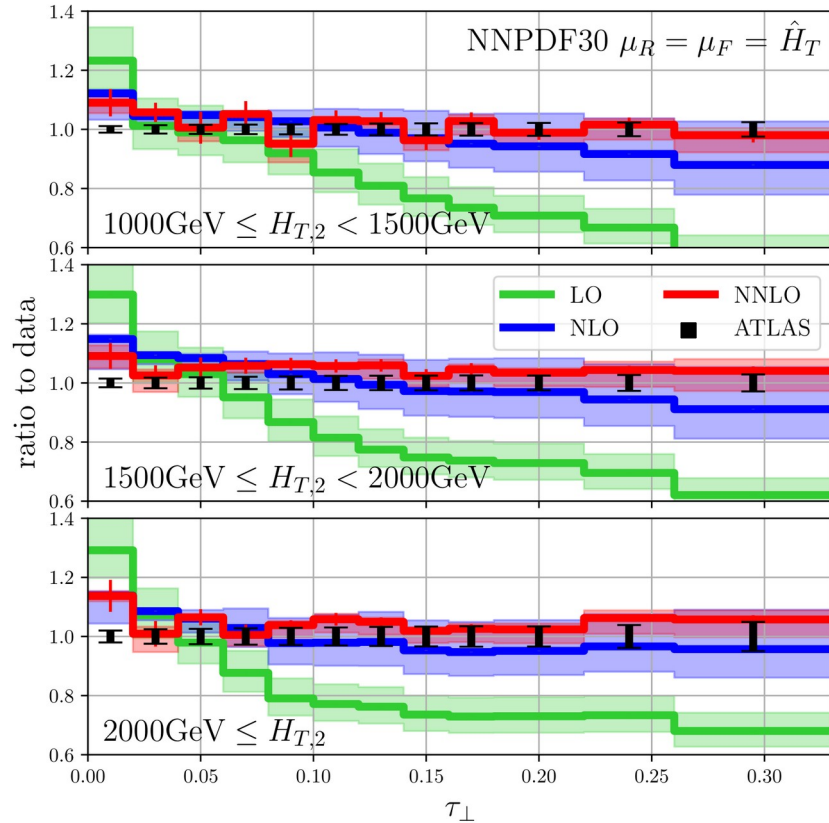
Ratio to 2-jet:  $R^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0}) = \frac{d\sigma_3^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}{d\sigma_2^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}$

Here: **use jets as input** → experimentally advantageous  
(better calibrated, smaller non-pert.)

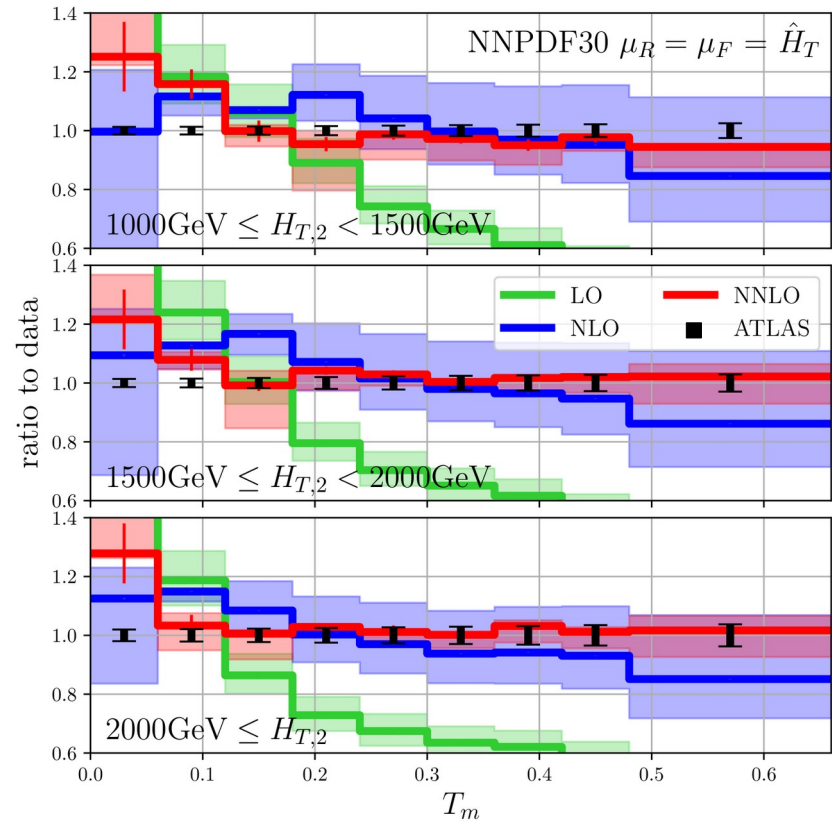
# Transverse Thrust @ NNLO QCD

## NNLO QCD corrections to event shapes at the LHC

Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet [2301.01086]



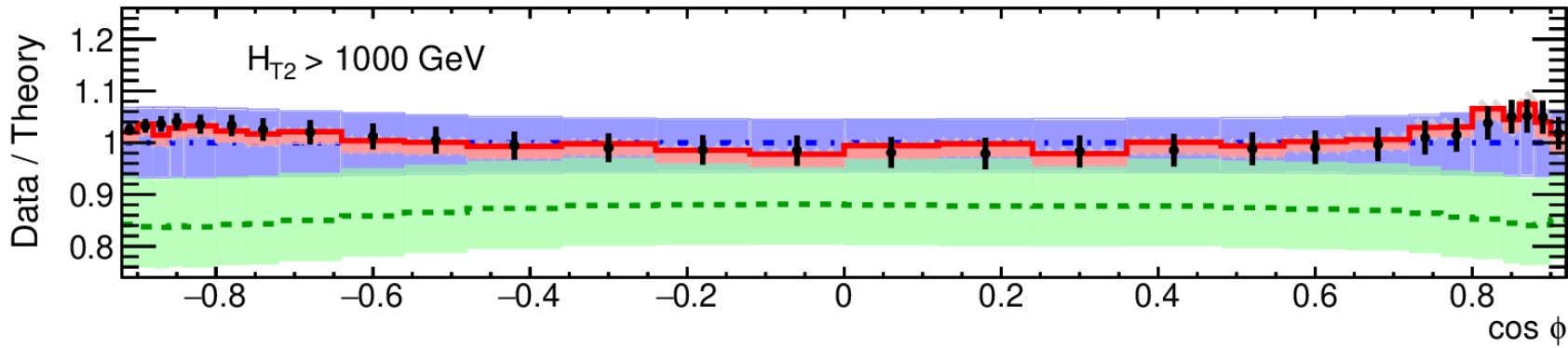
ATLAS [2007.12600]



# The transverse energy-energy correlator

$$\frac{1}{\sigma_2} \frac{d\sigma}{d \cos \Delta\phi} = \frac{1}{\sigma_2} \sum_{ij} \int \frac{d\sigma x_{\perp,i} x_{\perp,j}}{dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij}} \delta(\cos \Delta\phi - \cos \Delta\phi_{ij}) dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij},$$

- Insensitive to soft radiation through energy weighting  $x_{T,i} = E_{T,i} / \sum_j E_{T,j}$
- Event topology separation:
  - Central plateau contain isotropic events
  - To the right: self-correlations, collinear and in-plane splitting
  - To the left: back-to-back



[ATLAS 2301.09351]

**ATLAS**

Particle-level TEEC

$\sqrt{s} = 13 \text{ TeV}; 139 \text{ fb}^{-1}$

anti- $k_t$   $R = 0.4$

$p_T > 60 \text{ GeV}$

$|\eta| < 2.4$

$\mu_{R,F} = \hat{P}_T$

$\alpha_s(m_Z) = 0.1180$

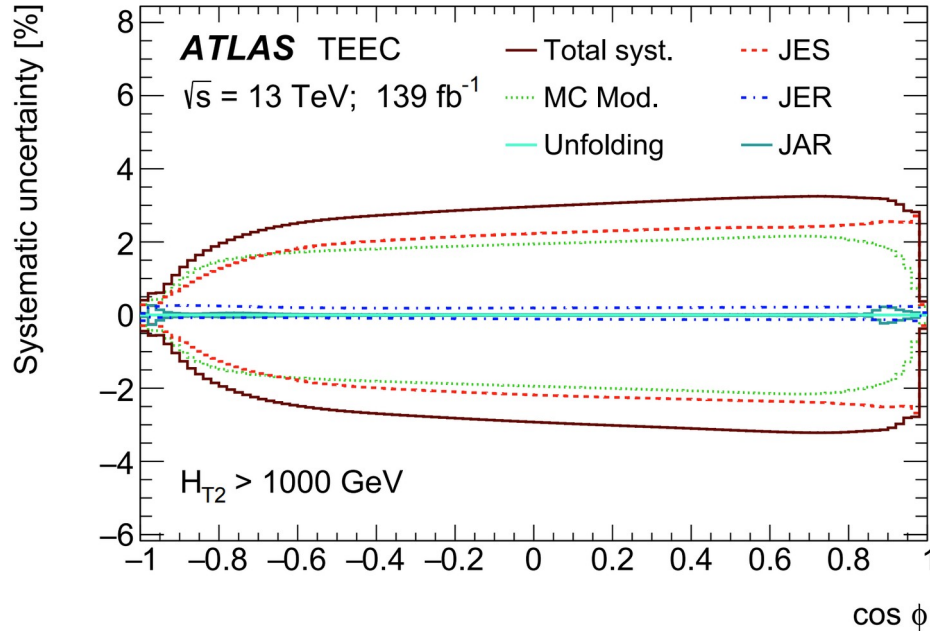
NNPDF 3.0 (NNLO)

—•— Data  
 - - - LO  
 - · - NLO  
 - - - NNLO

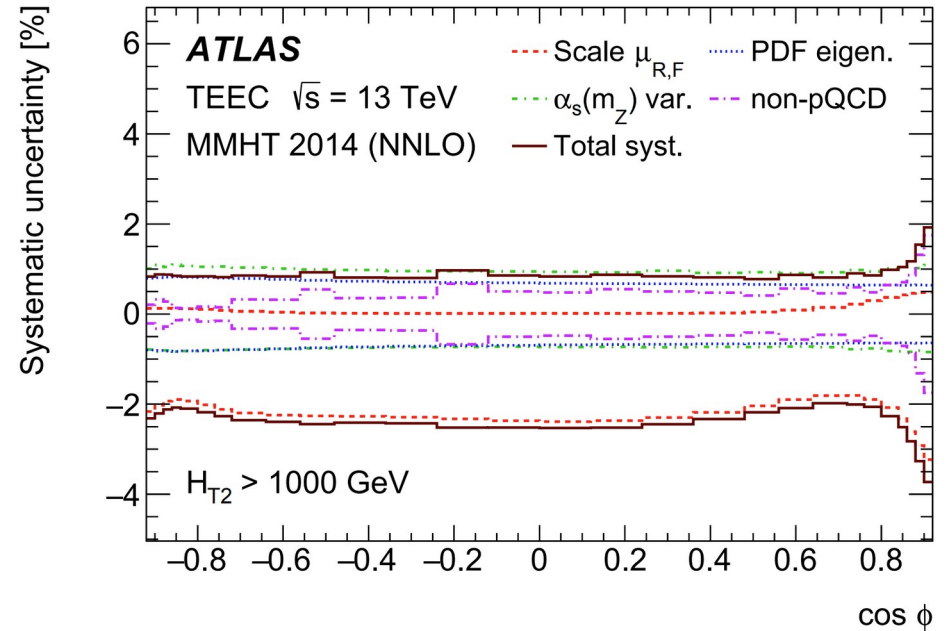


# Systematic Uncertainties TEEC

## Experimental uncertainties



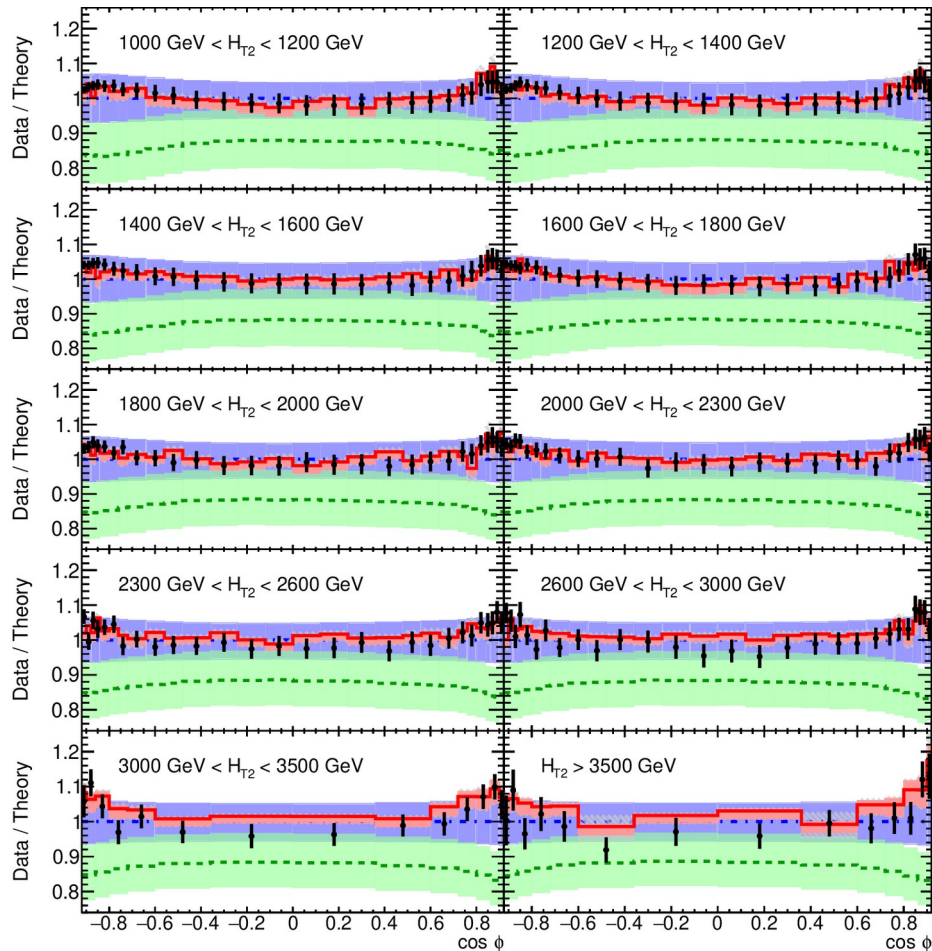
## Theory uncertainties



Scale dependence is the dominating uncertainty  $\rightarrow$  **NNLO QCD required to match exp.**

# Double differential TEEC

Cern Courier  
3<sup>rd</sup> March 23



[ATLAS 2301.09351]

**ATLAS**

Particle-level TEEC

$\sqrt{s} = 13 \text{ TeV}; 139 \text{ fb}^{-1}$

anti- $k_t$   $R = 0.4$

$p_T > 60 \text{ GeV}$

$|\eta| < 2.4$

$\mu_{R,F} = \hat{p}_T$

$\alpha_s(m_Z) = 0.1180$

NNPDF 3.0 (NNLO)

— Data

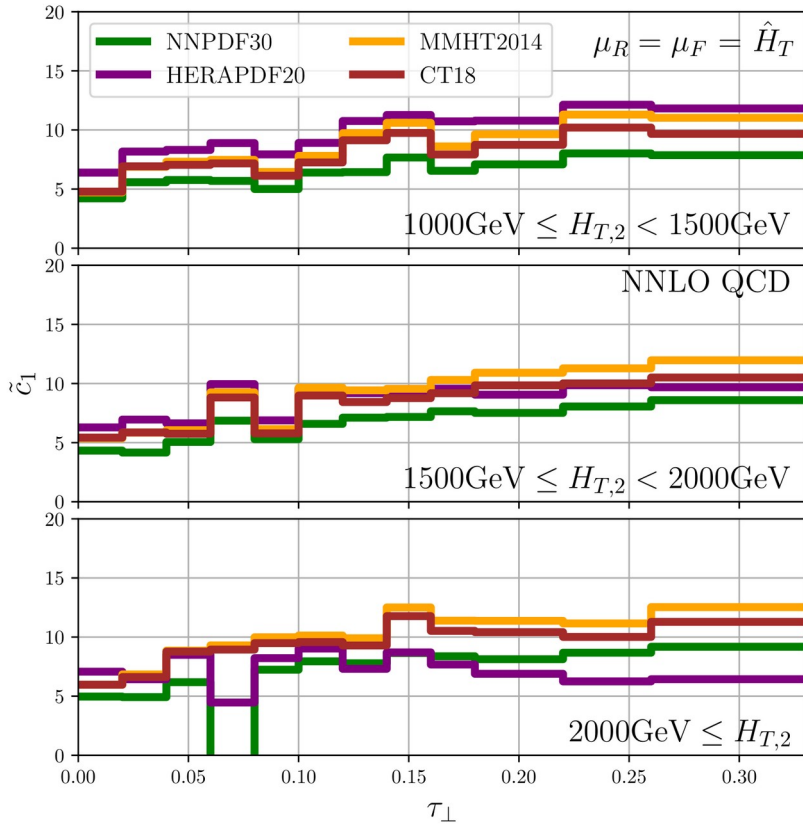
--- LO

--- NLO

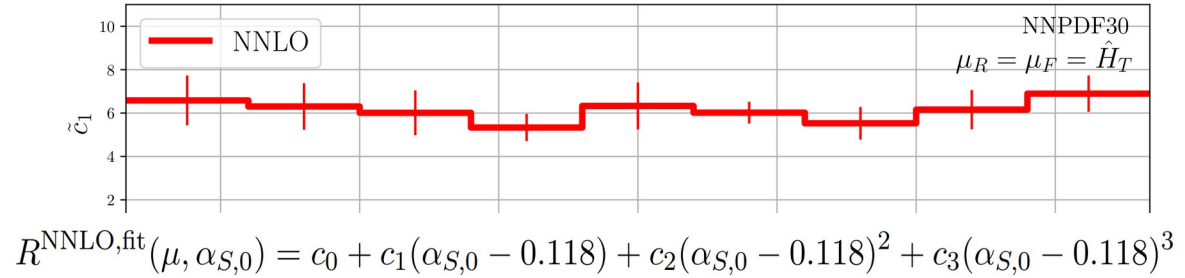
--- NNLO

# Strong coupling dependence

## Thrust



## TEEC



Visualisation of  $\alpha_S$  dependence

mostly linear dependence

$$\tilde{c}_1 = \frac{c_1}{R^{\text{NNLO}}(\alpha_{S,0} = 0.118)}$$

For comparison:

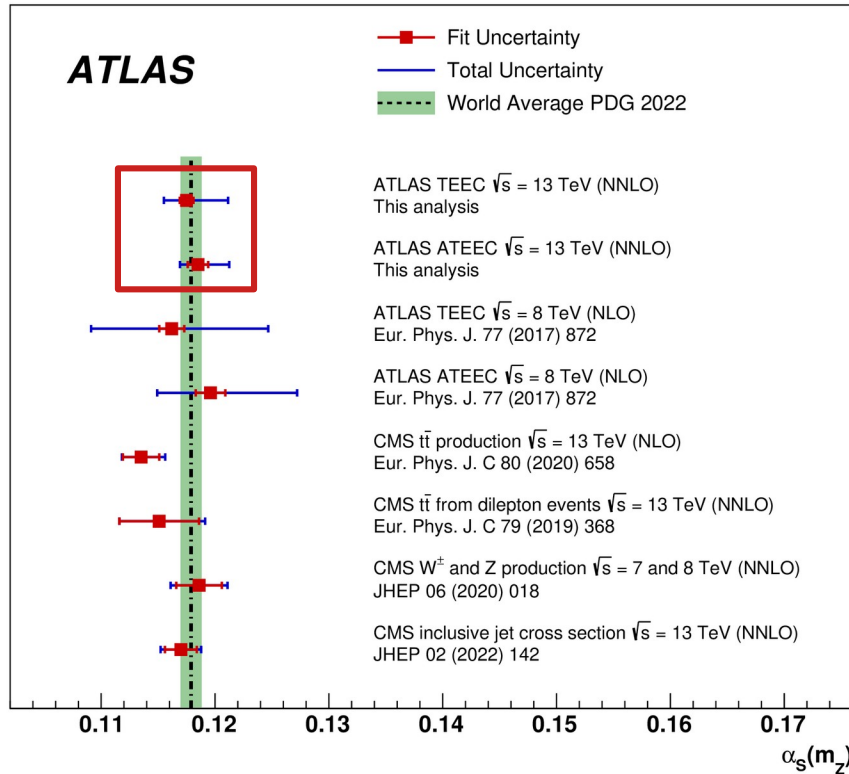
scale dependence (dominant theory uncertainty)

- TEEC ( $H_{T,2} > 1 \text{ TeV}$ ) : ~2%
- Thrust : ~3-5 %

}  **$O(1\%)$  sensitivity**

# $\alpha_s$ from TEEC @ NNLO by ATLAS

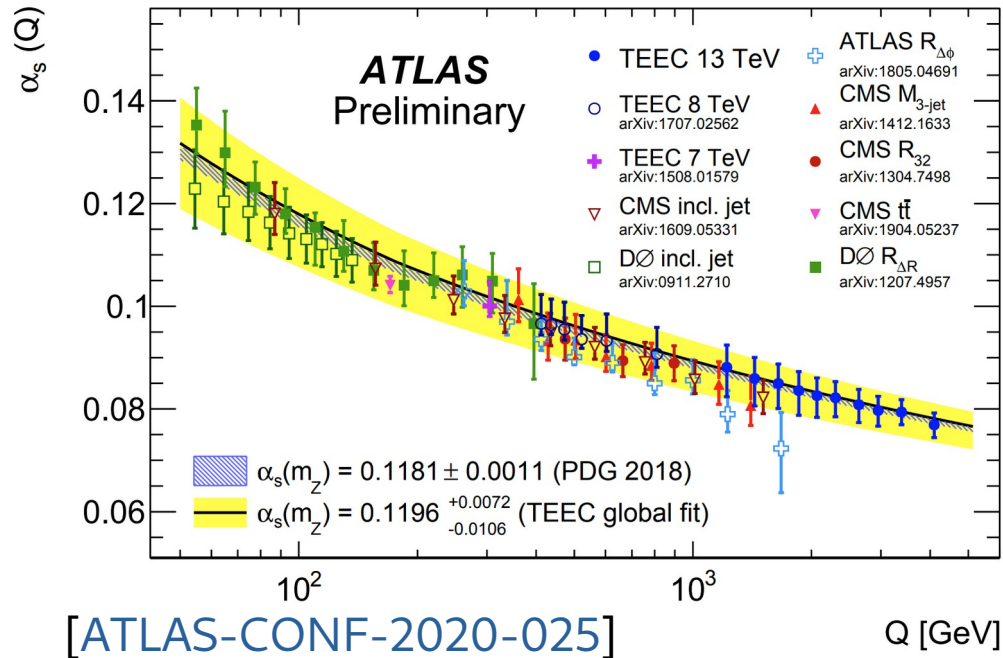
[ATLAS 2301.09351]



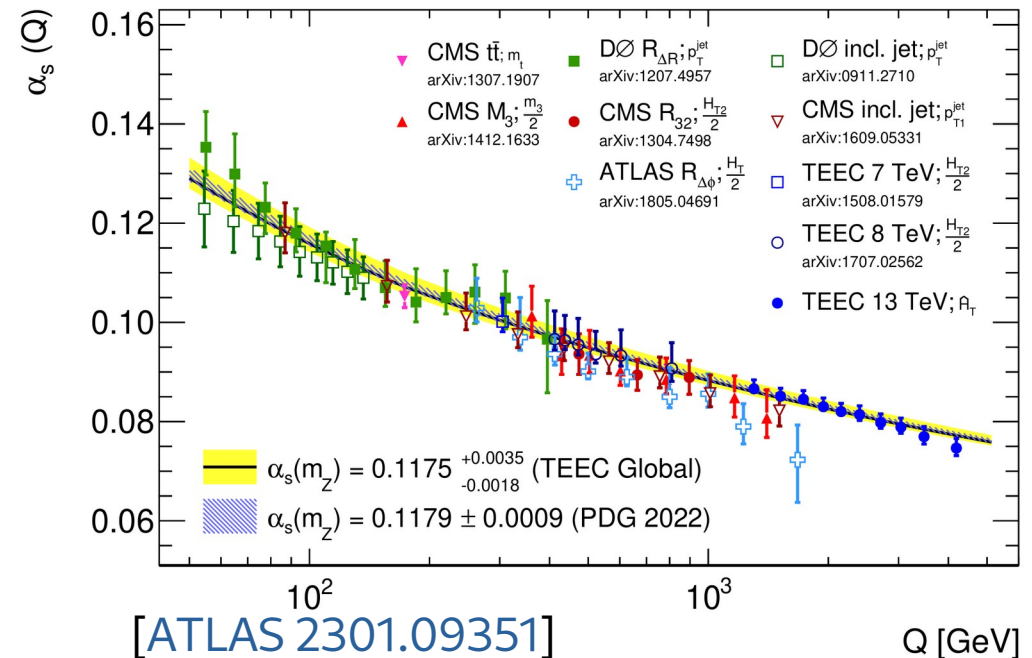
- NNLO QCD extraction from multi-jets  $\rightarrow$  will contribute to **PDG for the first time**
- **Significant improvement to 8 TeV**  
 $\rightarrow$  driven by **NNLO QCD corrections**
- Individual precision large but comparable to top or jets-data.
- However: extraction at high energy scales

# Running of $\alpha_s$

NLO QCD



NNLO QCD

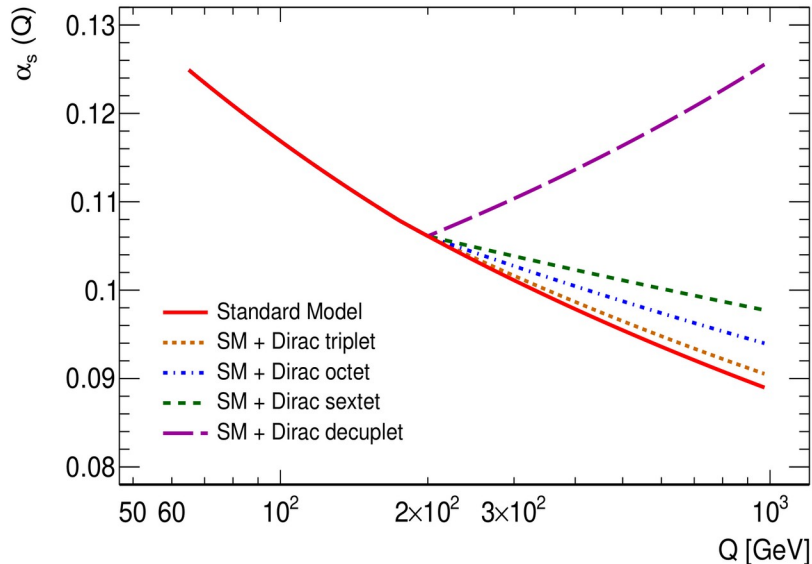


# Using the running of $\alpha_S$ to probe NP

[Llorente, Nachman 1807.00894]

Indirect constraints to NP through modified running:

$$\alpha_s(Q) = \frac{1}{\beta_0 \log z} \left[ 1 - \frac{\beta_1 \log(\log z)}{\beta_0^2 \log z} \right]; \quad z = \frac{Q^2}{\Lambda_{\text{QCD}}^2}$$

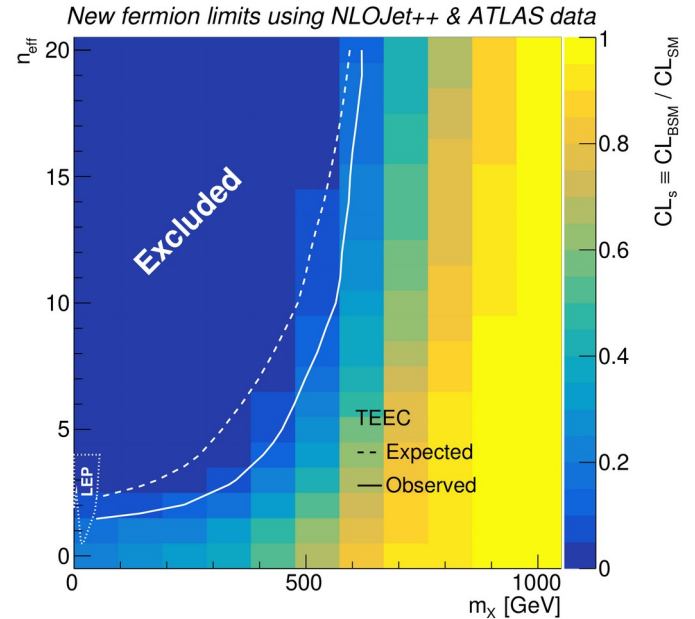


ATLAS  
TEEC @ 7 TeV  
data



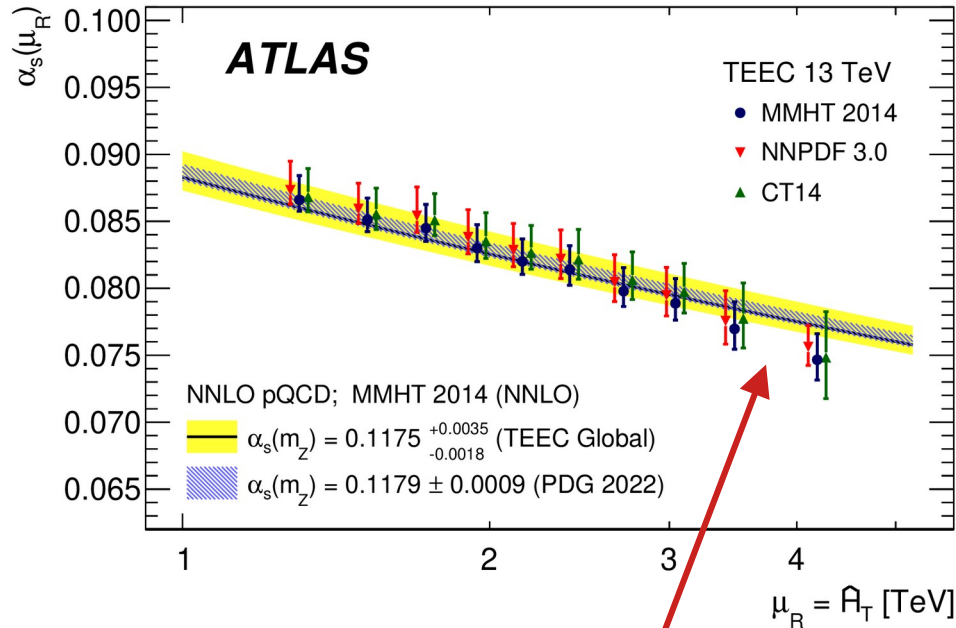
$$\beta_0 = \frac{1}{4\pi} \left( 11 - \frac{2}{3}n_f - \frac{4}{3}n_X T_X \right)$$

$$\beta_1 = \frac{1}{(4\pi)^2} \left[ 102 - \frac{38}{3}n_f - 20n_X T_X \left( 1 + \frac{C_X}{5} \right) \right]$$



Update with TEEC@13 TeV  
→ much improved bounds

# ... or 'new' SM dynamics



Systematic slope  
→ New physics?

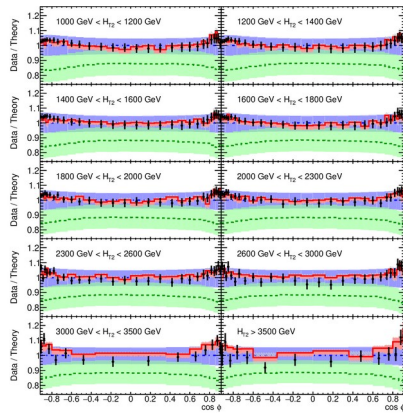
## Possible SM explanations

- Residual PDF effects → high  $x, Q^2$  ?
- EW corrections?
- Maybe effect from LC approximation in two-loop ME?

$$\begin{aligned} \mathcal{R}^{(2)}(\mu_R^2) &= 2 \operatorname{Re} \left[ \mathcal{M}^{\dagger(0)} \mathcal{F}^{(2)} \right] (\mu_R^2) + |\mathcal{F}^{(1)}|^2(\mu_R^2) \\ &\equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^4 c_i \ln^i \left( \frac{\mu_R^2}{s_{12}} \right) \\ \mathcal{R}^{(2)}(s_{12}) &\approx \mathcal{R}^{(2)l.c.}(s_{12}) \end{aligned}$$

- Experimental systematics?
- Resummation?

**Either case interesting!**



= ~100 MCPUh



How to make this more  
efficient/environment-friendly/  
accessible/faster?

high tea  
for your freshly brewed analysis

<https://www.precision.hep.phy.cam.ac.uk/hightea>

HighTEA: High energy Theory Event Analyser  
[2304.05993]

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[poncelet@hep.phy.cam.ac.uk](mailto:poncelet@hep.phy.cam.ac.uk), [andrei.popescu@cantab.net](mailto:andrei.popescu@cantab.net)



# Basic idea

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## → Database of precomputed “Theory Events”

- **Equivalent to a full fledged computation**
- Currently this means partonic fixed order events
- Extensions to include showered/resummed/hadronized events is feasible
- (Partially) Unweighting to increase efficiency

Not so new idea:  
LHE [Alwall et al '06],  
Ntuple [BlackHat '08'13],

## → Analysis of the data through an user interface

- Easy-to-use
- Fast
  - Observables from basic 4-momenta
  - Free specification of bins
- Flexible:
  - Renormalization/Factorization Scale variation
  - PDF (member) variation
  - Specify phase space cuts

# Factorizations

---

Factorizing renormalization and factorization scale dependence:

$$w_s^{i,j} = w_{\text{PDF}}(\mu_F, x_1, x_2) w_{\alpha_s}(\mu_R) \left( \sum_{i,j} c_{i,j} \ln(\mu_R^2)^i \ln(\mu_F^2)^j \right)$$

PDF dependence:

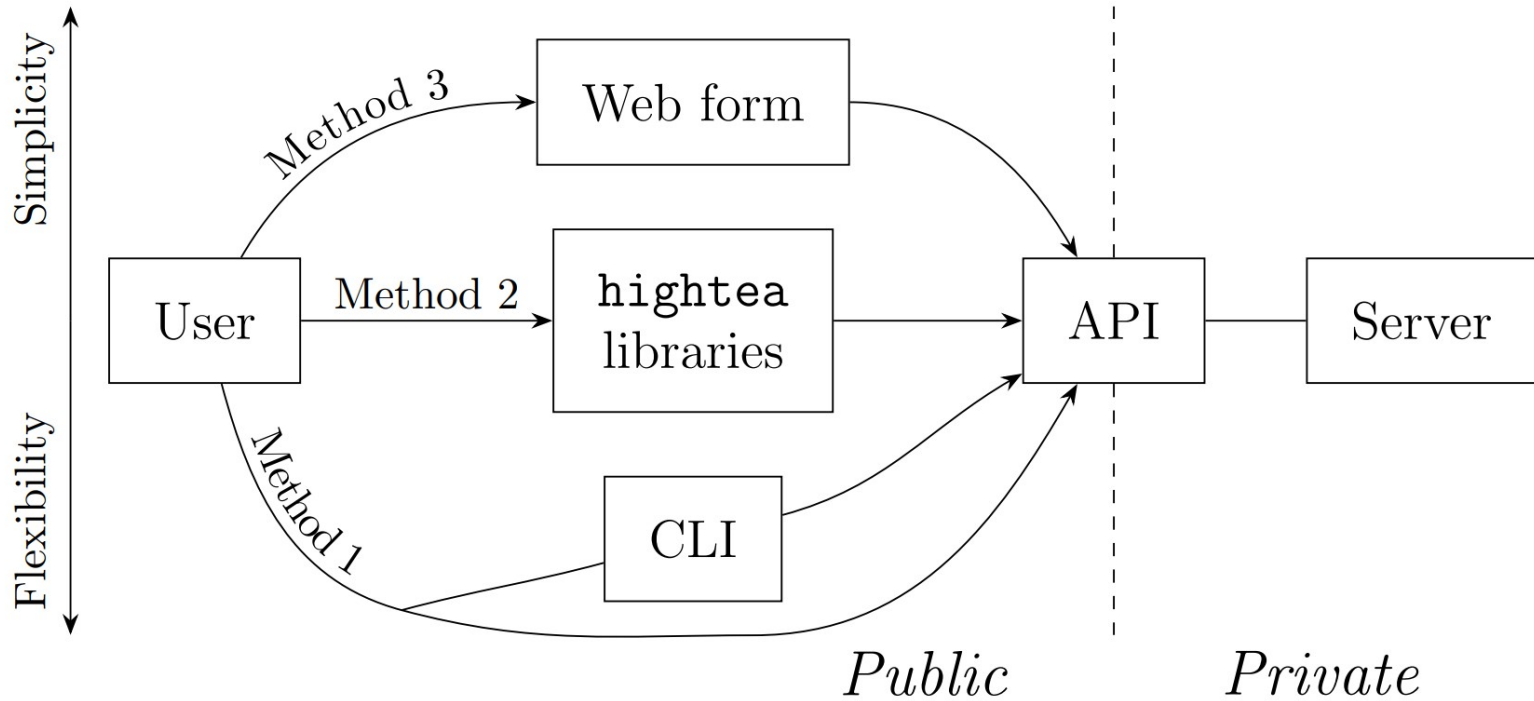
$$w_{\text{PDF}}(\mu, x_1, x_2) = \sum_{ab \in \text{channel}} f_a(x_1, \mu) f_b(x_2, \mu)$$

$\alpha_s$  dependence:

$$w_{\alpha_s}(\mu) = (\alpha_s(\mu))^m$$

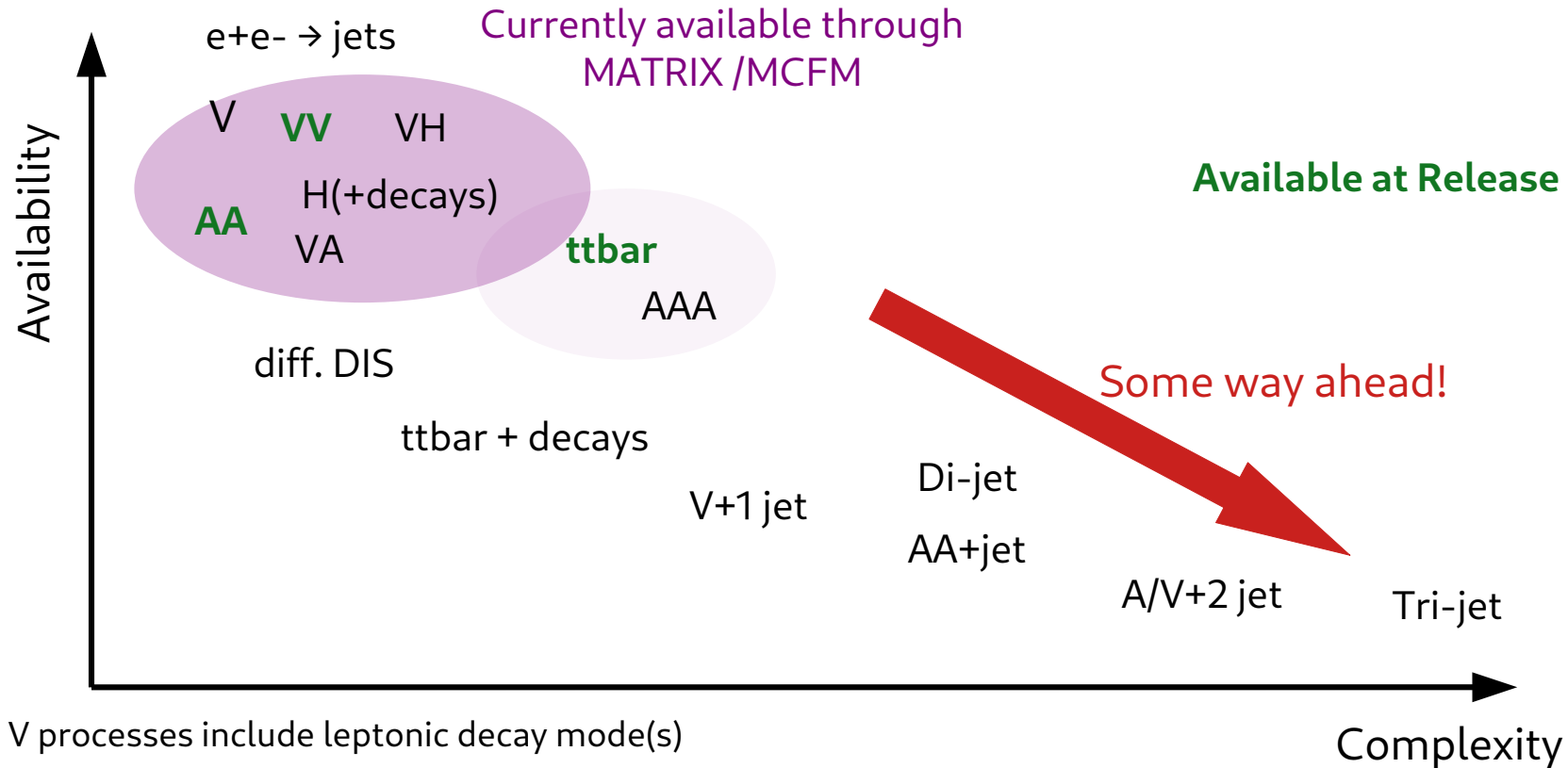
Allows **full control over scales and PDF**

# HighTEA interface



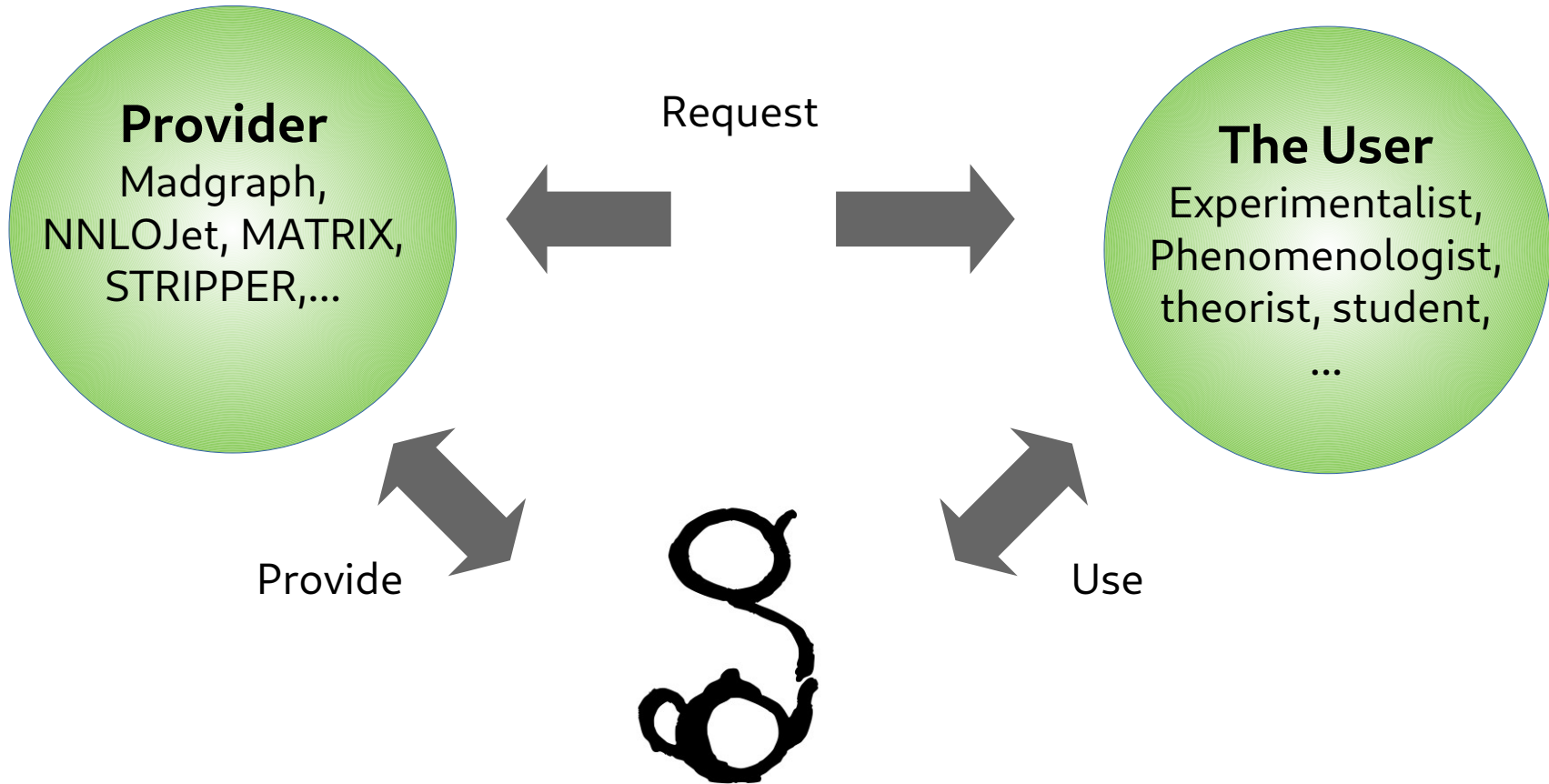
# Available Processes

Processes **currently** implemented in our STRIPPER framework through **NNLO QCD**

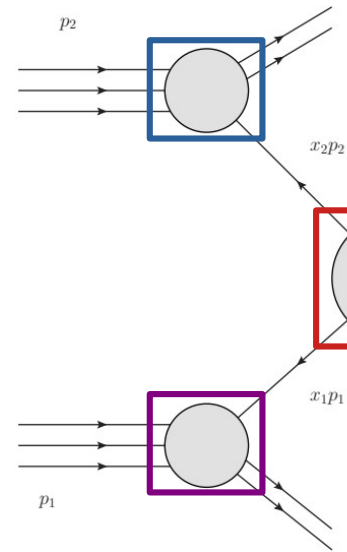


# The Vision

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# Hadronic cross section



Hadronic X-section: 
$$\sigma_{h_1 h_2 \rightarrow X} = \sum_{ij} \int_0^1 \int_0^1 dx_1 dx_2 \underbrace{\phi_{i/h_1}(x_1, \mu_F^2)}_{\text{parton distribution functions}} \underbrace{\phi_{j/h_2}(x_2, \mu_F^2)}_{\text{parton distribution functions}} \underbrace{\hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)}_{\text{partonic cross section}}$$

Parton distribution functions

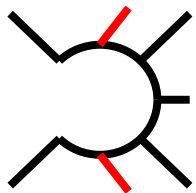
Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \hat{\sigma}_{ab \rightarrow X}^{(1)} + \hat{\sigma}_{ab \rightarrow X}^{(2)} + \mathcal{O}(\alpha_s^3)$$

The NNLO bit: 
$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\text{RR}} + \hat{\sigma}_{ab}^{\text{RV}} + \hat{\sigma}_{ab}^{\text{VV}} + \hat{\sigma}_{ab}^{\text{C2}} + \hat{\sigma}_{ab}^{\text{C1}}$$

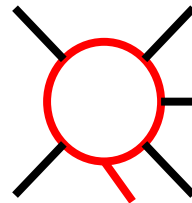
Double real radiation

$$\hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2}$$



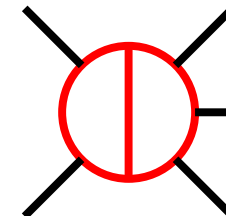
Real/Virtual correction

$$\hat{\sigma}_{ab}^{\text{RV}} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(1)} \rangle F_{n+1}$$



Double virtual corrections

$$\hat{\sigma}_{ab}^{\text{VV}} = \frac{1}{2\hat{s}} \int d\Phi_n \left( 2\text{Re} \langle \mathcal{M}_n^{(0)} | \mathcal{M}_n^{(2)} \rangle + \langle \mathcal{M}_n^{(1)} | \mathcal{M}_n^{(1)} \rangle \right) F_n$$



# Partonic cross section beyond LO

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \hat{\sigma}_{ab \rightarrow X}^{(1)} + \hat{\sigma}_{ab \rightarrow X}^{(2)} + \mathcal{O}(\alpha_s^3)$$

Contributions with different multiplicities and # convolutions:

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\text{RR}} + \hat{\sigma}_{ab}^{\text{RV}} + \hat{\sigma}_{ab}^{\text{VV}} + \hat{\sigma}_{ab}^{\text{C2}} + \hat{\sigma}_{ab}^{\text{C1}}$$



Each term separately IR divergent. But sum is:

→ finite

→ regularization scheme independent

Considering CDR ( $d = 4 - 2\epsilon$ ):

→ Laurent expansion:

$$\hat{\sigma}_{ab}^{\text{C}} = \sum_{i=-4}^0 c_i \epsilon^i + \mathcal{O}(\epsilon)$$

$$\hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2}$$

$$\hat{\sigma}_{ab}^{\text{RV}} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(1)} \rangle F_{n+1}$$

$$\hat{\sigma}_{ab}^{\text{VV}} = \frac{1}{2\hat{s}} \int d\Phi_n \left( 2\text{Re} \langle \mathcal{M}_n^{(0)} | \mathcal{M}_n^{(2)} \rangle + \langle \mathcal{M}_n^{(1)} | \mathcal{M}_n^{(1)} \rangle \right) F_n$$

$$\hat{\sigma}_{ab}^{\text{C1}} = (\text{single convolution}) F_{n+1}$$

$$\hat{\sigma}_{ab}^{\text{C2}} = (\text{double convolution}) F_n$$

# Sector decomposition I

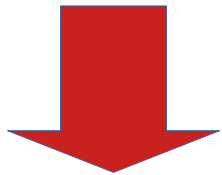
Considering working in CDR:

→ Virtuals are usually done in this regularization

→ Real radiation:

→ Very difficult integrals, analytical impractical (except very simple cases)!

→ Numerics not possible, integrals are divergent:  $\varepsilon$ -poles!



How to extract these poles? → Sector decomposition!

**Divide and conquer** the phase space:

$$1 = \sum_{i,j} \left[ \sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \longrightarrow \hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \sum_{i,j} \left[ \sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle_{\text{F}_{n+2}}$$



# Sector decomposition II

Divide and conquer the phase space:

→ Each  $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$  has simpler divergences.

appearing as  $1/s_{ijk}$   $1/s_{ik}/s_{jl}$

Soft and collinear (w.r.t parton k,l) of partons i and j

→ Parametrization w.r.t. reference parton:

$$\hat{\eta}_i = \frac{1}{2}(1 - \cos \theta_{ir}) \in [0, 1] \quad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0, 1]$$

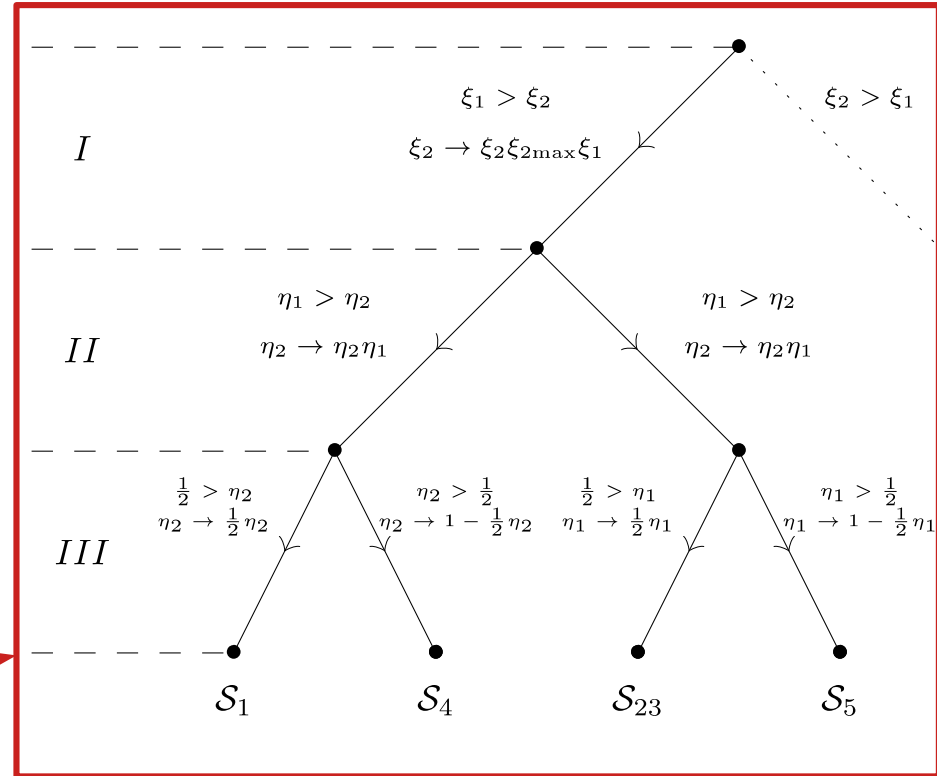
→ Subdivide to factorize divergences

$$s_{u_1 u_2 k} = (p_k + u_1 + u_2)^2 \sim \hat{\eta}_1 u_1^0 + \hat{\eta}_2 u_2^0 + \hat{\eta}_3 u_1^0 u_2^0$$

→ double soft factorization:

$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

→ triple collinear factorization



[Czakon'10, Caola'17]

# Sector decomposition III

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}} \int d\Phi_{n+2} \mathcal{S}_{kl,m} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2} = \sum_{\text{sub-sec.}} \int d\Phi_n \prod dx_i \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}} d\tilde{\mu}(\{x_i\}) \underbrace{\prod x_i^{a_i+1} \langle \mathcal{M}_{n+2} | \mathcal{M}_{n+2} \rangle}_{\text{regular}} F_{n+2}$$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{[x^{-1-b\epsilon}]_+}_{\text{reg. + sub.}} \quad \int_0^1 dx [x^{-1-b\epsilon}]_+ f(x) = \int_0^1 \frac{f(x) - f(0)}{x^{1+b\epsilon}}$$

# Finite NNLO cross section

$$\hat{\sigma}_{ab}^{RR} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2}$$

$$\hat{\sigma}_{ab}^{C1} = (\text{single convolution}) F_{n+1}$$

$$\hat{\sigma}_{ab}^{RV} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(1)} \rangle F_{n+1}$$

$$\hat{\sigma}_{ab}^{C2} = (\text{double convolution}) F_n$$

$$\hat{\sigma}_{ab}^{VV} = \frac{1}{2\hat{s}} \int d\Phi_n \left( 2\text{Re} \langle \mathcal{M}_n^{(0)} | \mathcal{M}_n^{(2)} \rangle + \langle \mathcal{M}_n^{(1)} | \mathcal{M}_n^{(1)} \rangle \right) F_n$$



sector decomposition and master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{[x^{-1-b\epsilon}]_+}_{\text{reg. + sub.}}$$

$$(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}) \quad (\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}) \quad (\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}) \quad (\sigma_{SU}^{C1}, \sigma_{DU}^{C1}) \quad (\sigma_{DU}^{C2}, \sigma_{FR}^{C2})$$



re-arrangement of terms  $\rightarrow$  4-dim. formulation [Czakon'14, Czakon'19]

$$\underline{(\sigma_F^{RR})} \quad \underline{(\sigma_F^{RV})} \quad \underline{(\sigma_F^{VV})} \quad \underline{(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1})} \quad \underline{(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2})} \quad \underline{(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2})}$$

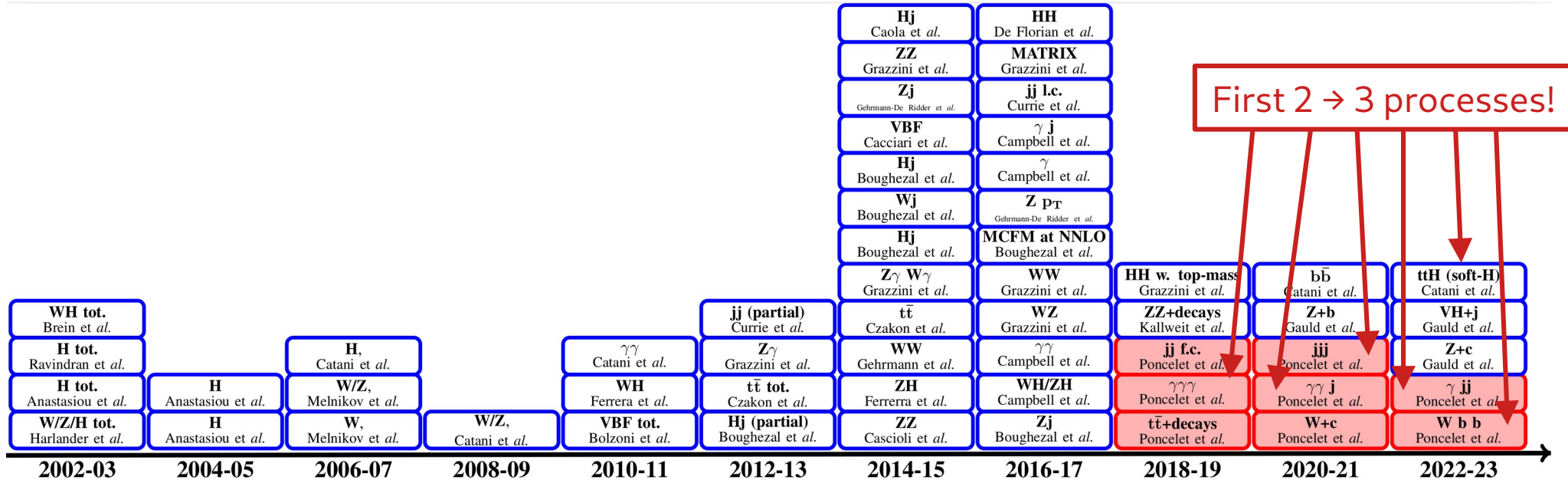
separately finite:  $\epsilon$  poles cancel

# C++ framework

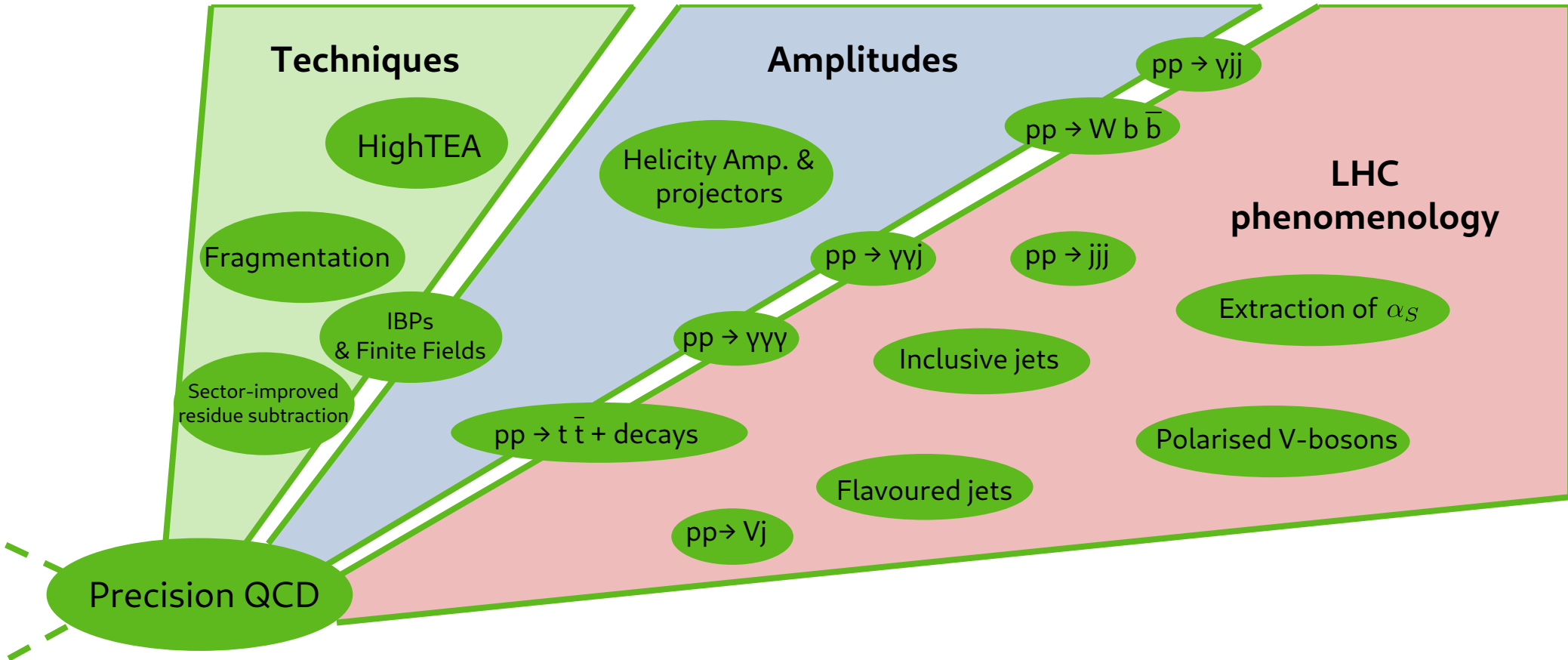
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- Formulation allows efficient algorithmic implementation
- **High degree of automation:**
  - Partonic processes (taking into account all symmetries)
  - Sectors and subtraction terms
  - Interfaces to Matrix-element providers:  
AvH, OpenLoops, Recola, NJET, HardCoded  
→ Only two-loop matrix elements required
- **Broad range of applications** through additional facilities:
  - Narrow-Width & Double-Pole Approximation
  - Fragmentation
  - Polarised intermediate massive bosons
  - (Partial) Unweighting → Event generation for **HighTEA**
  - Interfaces: FastNLO, FastJet

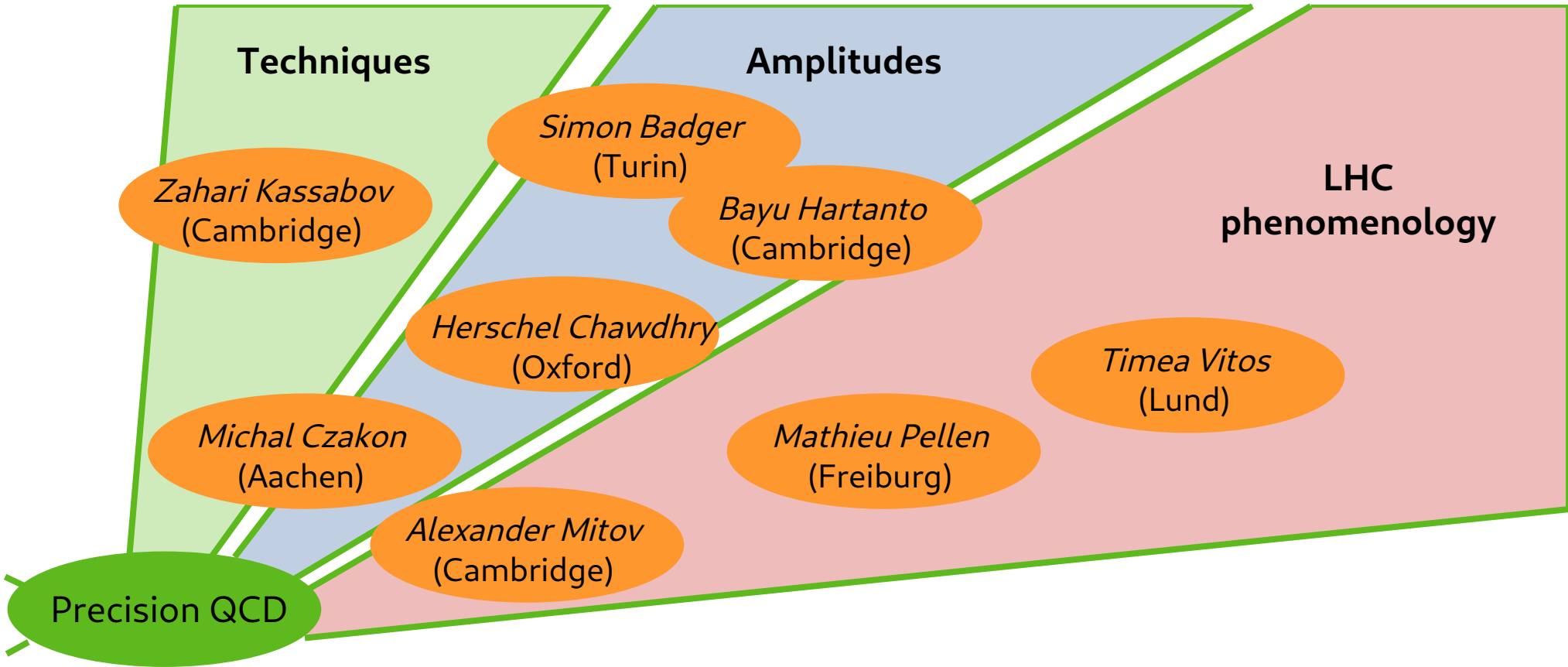
# The NNLO QCD revolution



# Research context



# Research network



# Experimental collaborations

## NNLO QCD computations

- **Top-quark pair production and leptonic decays**

[1901.05407] [2008.11133]

+ b-quark fragmentation: [2102.08267] [2210.06078]

- **Vector + jets**

W + charm-jet [2011.01011] [2212.00467]

Z + b-jet [2205.11879]

- **Polarised vector-bosons**

W W [2102.13583]

W+jet [2109.14336] [2204.12394]

- **Inclusive jets**

[1907.12911]

- **"2 → 3" processes**

pp →  $\gamma\gamma\gamma$  [1911.00479]

pp →  $\gamma\gamma j$  [2105.06940]

pp →  $j j j$  [2106.05331] [2301.01086]

pp →  $\gamma j j$  [2304.06682]

pp → W + 2 b-jets [2205.01687] [2209.03280]

## Exp. collaborations

DESY CMS top-quark group  
(*Behnke, Aldaya Martin*)  
→ [CMS-PAS-TOP-20-006]

Top spin-correlations in ATLAS  
(*Howard*) → [1903.07570]

W+charm CMS measurement  
(*Hernandez*) → [2308.02285]

Approved COST network  
COMETA

ATLAS multi-jet group at CERN  
(*Llorente, Roloff, LeBlanc*)  
→  $\alpha_S$  from TEEC [2301.09351]  
→ More to appear



# Future directions

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## The provider for precision QCD predictions

- extending process portfolio
- branching out to other colliders
- development of public tool like HighTEA
- towards **full higher-order event simulations**



Experimental communities  
LHC, EIC, RHIC, FCC (-ee,-hh,-eh)



Theory/Amplitudes  
community

# Near future projects

---

## Modern MC integration/sampling

- Interdisciplinary work with *Steffen Schumann* (Göttingen) & *David Yallup* (Kalvi Institute Cambridge)
- 1) “Nested sampling” → phase space explorations
- 2) “Normalising flows” → phase space sampler using Neural Networks

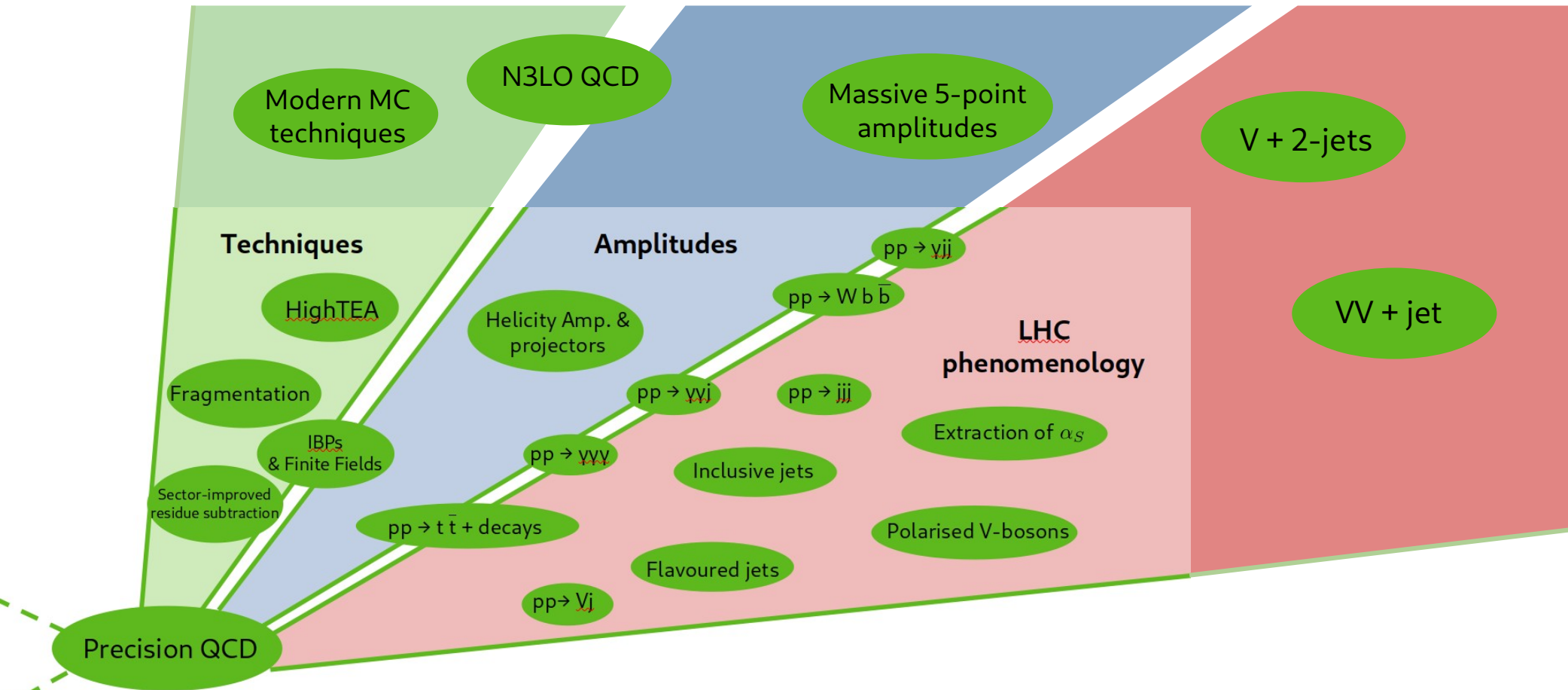
## NNLO with massive bosons: $V + 2\text{-jet}$ , $VV + 1\text{-jet}$

- A lot to do: amplitudes + cross sections → rich phenomenology!

## N3LO QCD for $2 \rightarrow 2$ processes

- First with slicing methods then towards local N3LO subtraction schemes (→ *Sebastian Sapeta* (Cracow))

# Summary



# Backup

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# (Partially) Unweighting

The hadronic cross section in collinear factorization:

$$d\sigma(P_1, P_2) = \sum_{ab} \int \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) d\hat{\sigma}_{ab}(x_1 P_1, x_2 P_2)$$

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \hat{\sigma}_{ab \rightarrow X}^{(1)} + \hat{\sigma}_{ab \rightarrow X}^{(2)} + \mathcal{O}(\alpha_s^3)$$

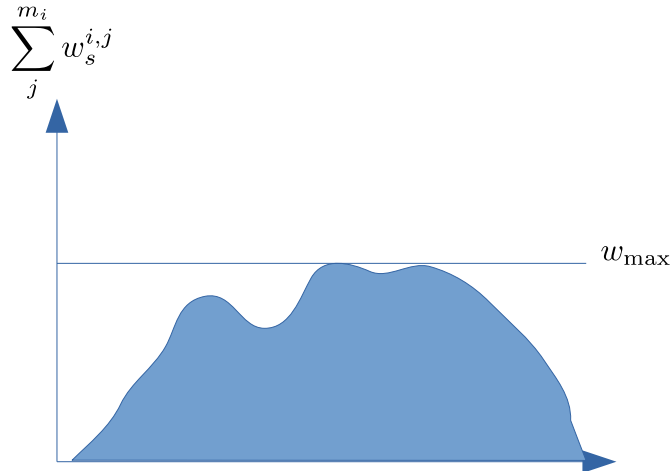
Using MC method for integration:

$$\sigma_{\text{tot}} = \frac{1}{n} \sum_i^n \left( \sum_j^{m_i} w_s^{i,j} \right)$$

Beyond LO events might correspond to more than one kinematic:

**Subtraction events!**

Hit-And-Miss Algorithm:



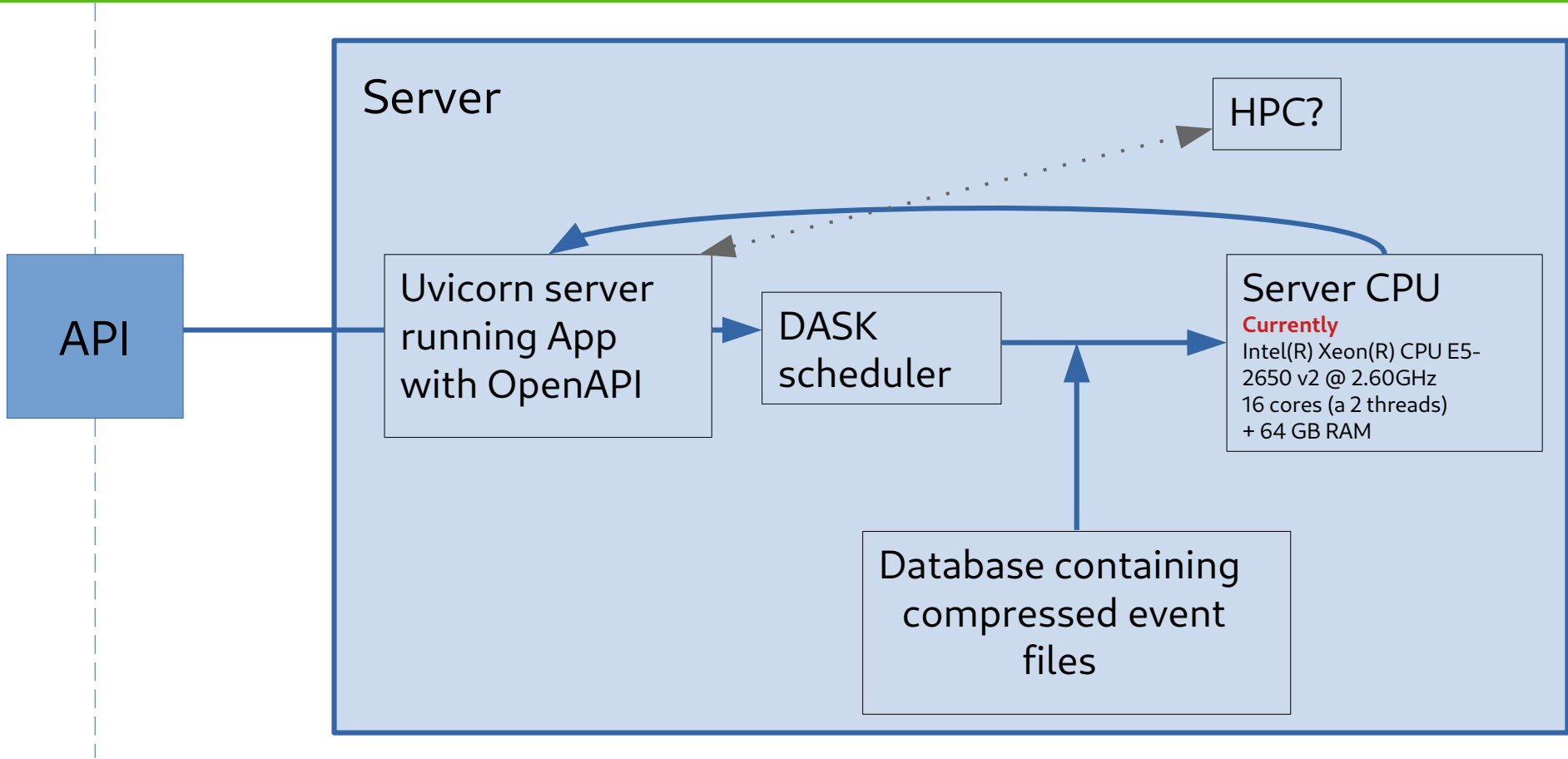
Accept each event i with probability:

$$\left( \sum_j^{m_i} w_s^{i,j} \right) / w_{\text{max}}$$

Store each sub-event with weight:

$$w_s^{i,j} / \left( \sum_j^{m_i} w_s^{i,j} \right)$$

# The server



# Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

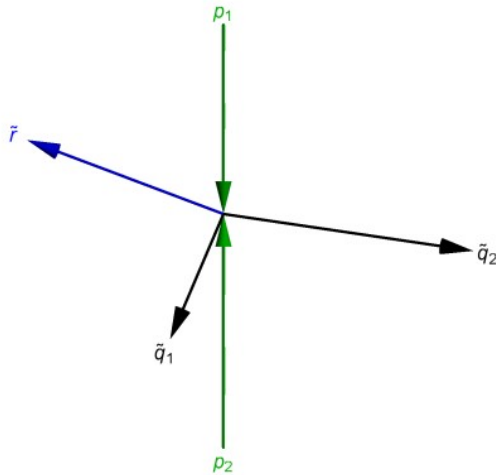
Mapping from  $n+2$  to  $n$  particle phase space:  $\{P, r_j, u_k\} \rightarrow \{\tilde{P}, \tilde{r}_j\}$

Requirements:

- Keep direction of reference  $r$  fixed
- Invertible for fixed  $u_i$  :  $\{\tilde{P}, \tilde{r}_j, u_k\} \rightarrow \{P, r_j, u_k\}$
- Preserve Born invariant mass:  $q^2 = \tilde{q}^2$ ,  $\tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born configuration
- Generate unresolved partons  $u_i$
- Rescale reference momentum  $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration



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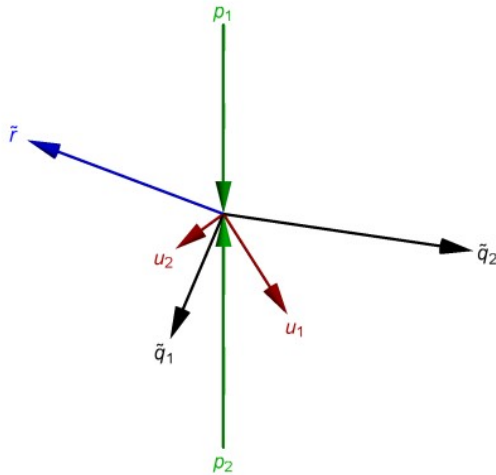
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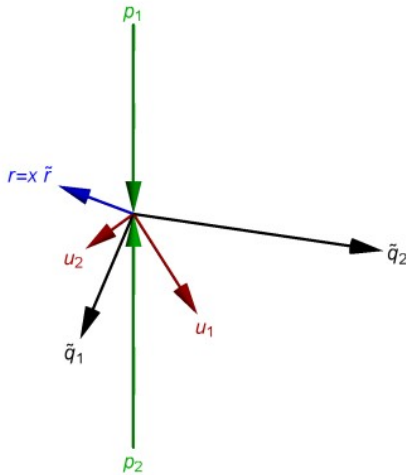
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