NNLO QCD corrections to event-shapes at the LHC

Rene Poncelet

Czakon, Mitov, Poncelet
+Alvarez, Cantero, Llorente
+ Kasabov, Popescu

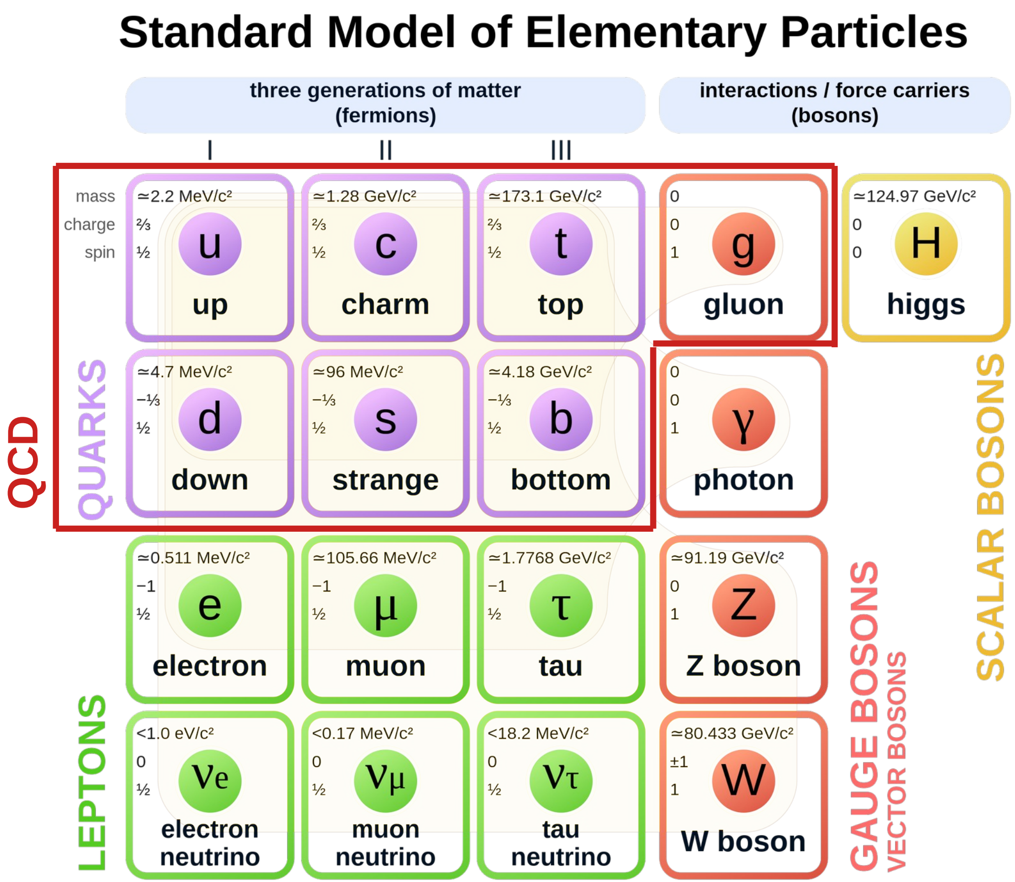
Based on (NNLO three jet) 2106.05331, 2301.01086,
(ATLAS) 2301.09351 and (HighTEA) 2304.05993

LEVERHULME TRUST

Isaac Newton Trust

UNIVERSITY OF CAMBRIDGE

Precision era of the LHC



- Collider data constrains the various interactions in the Standard Model.
- At the LHC QCD is part of any process!
  1) The limiting factor in many analyses is QCD and associated uncertainties.
     → Radiative corrections indispensable
  2) How well we do know QCD? Coupling constant, running, PDFs, …
- The production of high energy jets allow to probe pQCD at high energies directly

$$\mathcal{L}_{\text{QCD}} = \bar{q}_i(\gamma^\mu \mathcal{D}_\mu - m_i)q_i - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a$$

1) Testing the predicted dynamics
2) Extract the coupling constant

Multi-jet observables

NLO theory unc. > experimental unc.

- NNLO QCD needed for precise theory-data comparisons

  → Restricted to two-jet data
  [Currie’17+later][Czakon’19]

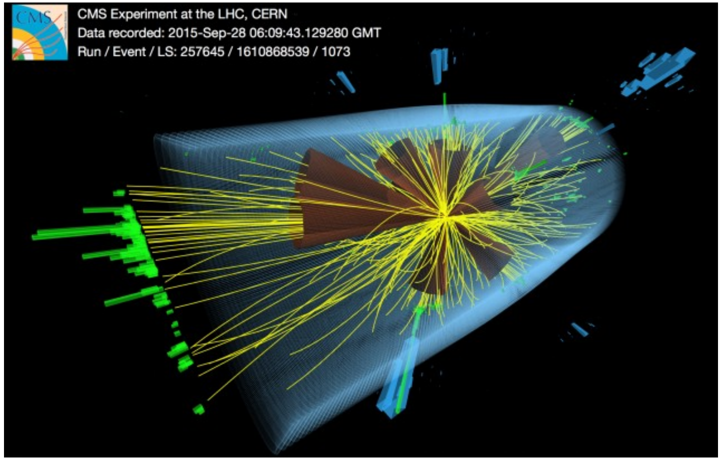
- New NNLO QCD three-jet → access to more observables
  - Jet ratios

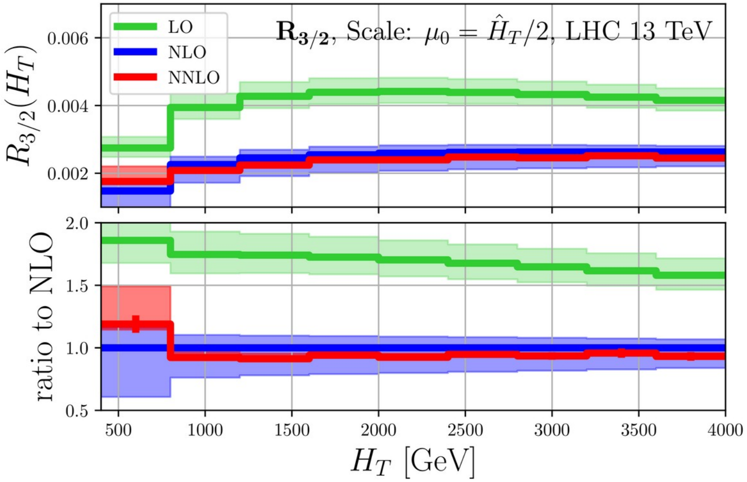
Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC
Czakon, Mitov, **Poncelet** [2106.05331]

$$R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$$

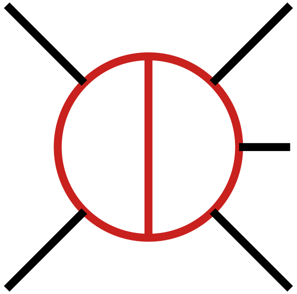
  - Event shapes

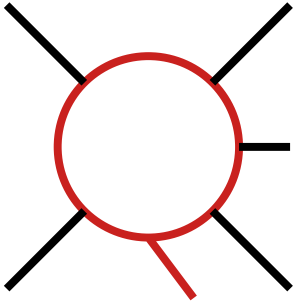
NNLO QCD corrections to event shapes at the LHC
Alvarez, Cantero, Czakon, Llorente, Mitov, **Poncelet** 2301.01086

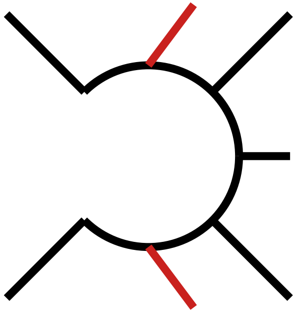




NNLO QCD prediction beyond 2 → 2







Two-loop amplitudes

- (Non-) planar 5 point massless [Chawdry'19'20'21,Abreu'20'21'23,Agarwal'21,Badger'21'23]
  → triggered by efficient MI representation [Chicherin'20]
- 5 point with one external mass [Abreu'20,Syrrakos'20,Canko'20,Badger'21'22,Chicherin'22]
- For three-jets → LC [Abreu'20'21] (checked against NJET [Badger'12'21])

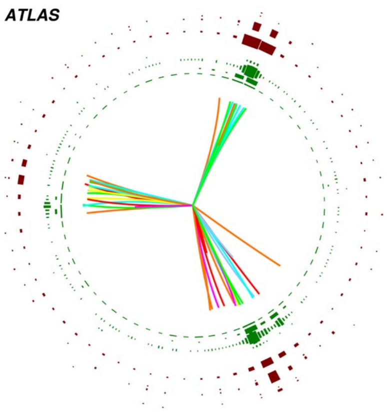
One-loop amplitudes → OpenLoops [Buccioni'19]

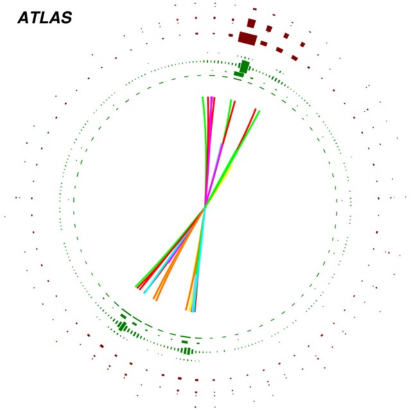
- Many legs and IR stable (soft and collinear limits)

Double-real Born amplitudes → AvHlib[Bury'15]

- IR finite cross-sections → NNLO subtraction schemes
  qT-slicing [Catani'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projetction [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17], Local Analytic [Magnea'18], **Sector-improved residue subtraction** [Czakon'10-'14,'19] STRIPPER

Encoding QCD dynamics in event shapes





Using (global) event information to separate different regimes of QCD event evolution:

- **Thrust & Thrust-Minor**

$$T_\perp = \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}, \quad \text{and} \quad T_m = \frac{\sum_i |\vec{p}_{T,i} \times \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}.$$

- **Energy-energy correlators**

$$\frac{1}{\sigma_2}\frac{d\sigma}{d\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{d\sigma\, x_{\perp,i}x_{\perp,j}}{dx_{\perp,i}dx_{\perp,j}d\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})dx_{\perp,i}dx_{\perp,j}d\cos\Delta\phi_{ij},$$

- → more computed

Separation of energy scales: $H_{T,2} = p_{T,1} + p_{T,2}$

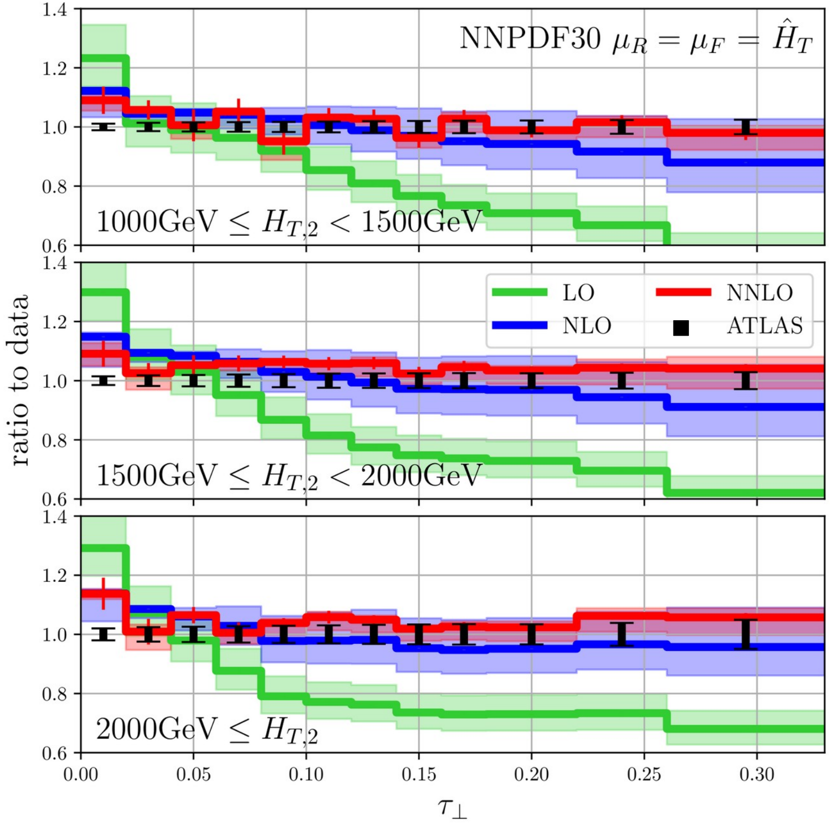
Ratio to 2-jet: $R^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0}) = \frac{d\sigma_3^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}{d\sigma_2^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}$

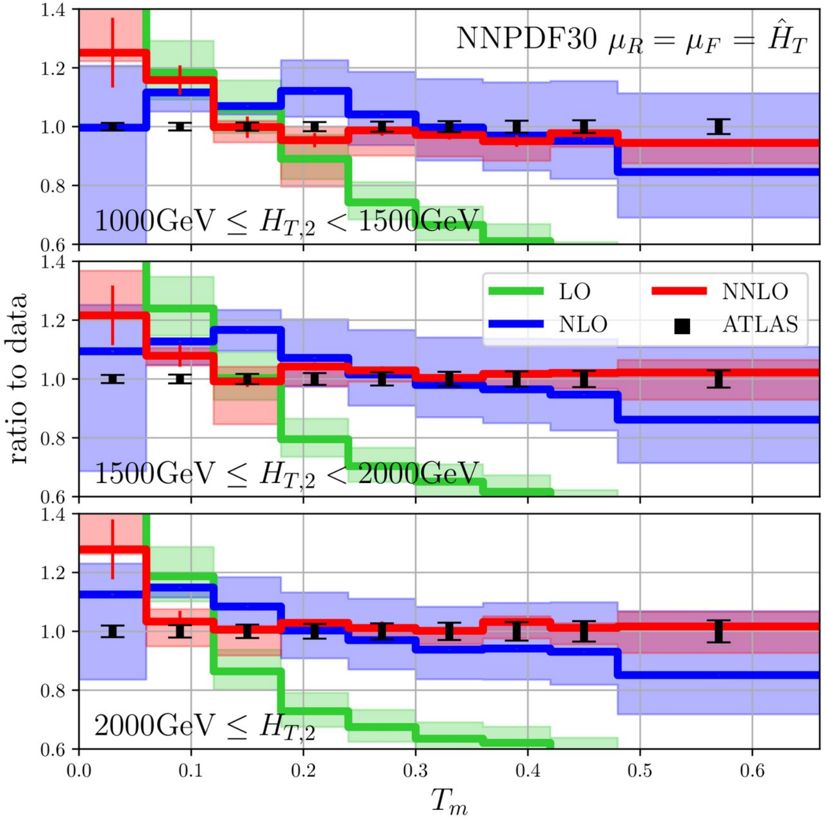
Here: use jets as input → experimentally advantageous (better calibrated, smaller non-pert.)

Transverse Thrust @ NNLO QCD

NNLO QCD corrections to event shapes at the LHC
Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet 2301.01086

ATLAS [2007.12600]

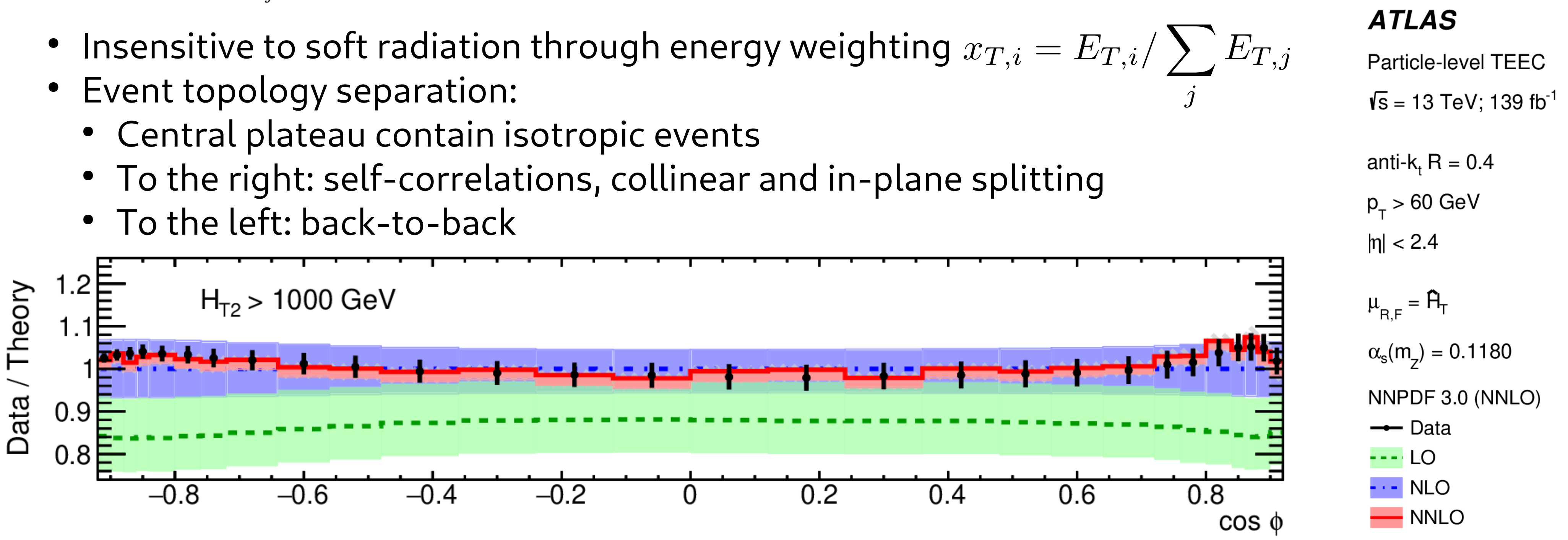




The transverse energy-energy correlator

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$

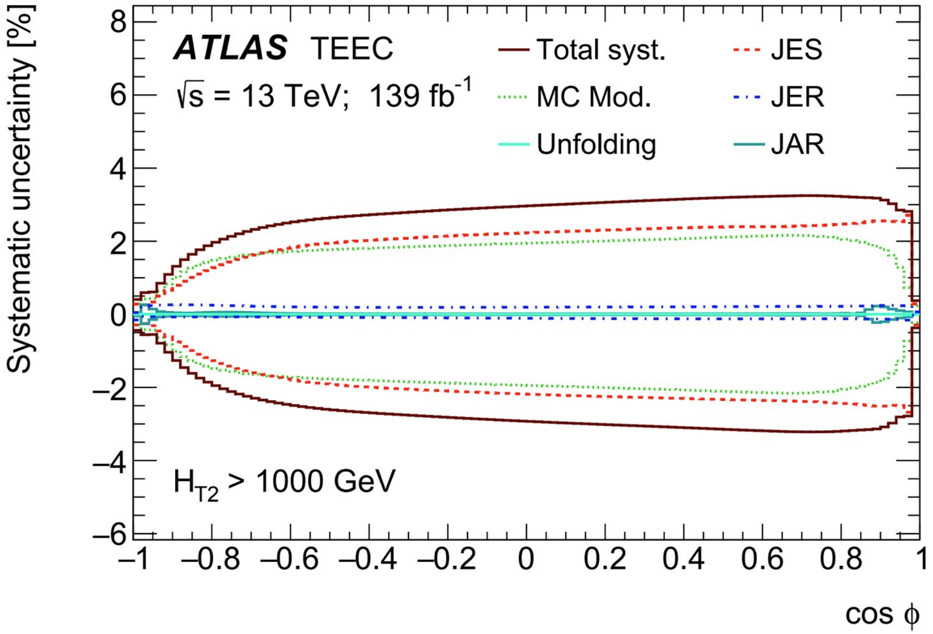
- Insensitive to soft radiation through energy weighting $x_{T,i} = E_{T,i}/\sum_j E_{T,j}$
- Event topology separation:
  - Central plateau contain isotropic events
  - To the right: self-correlations, collinear and in-plane splitting
  - To the left: back-to-back



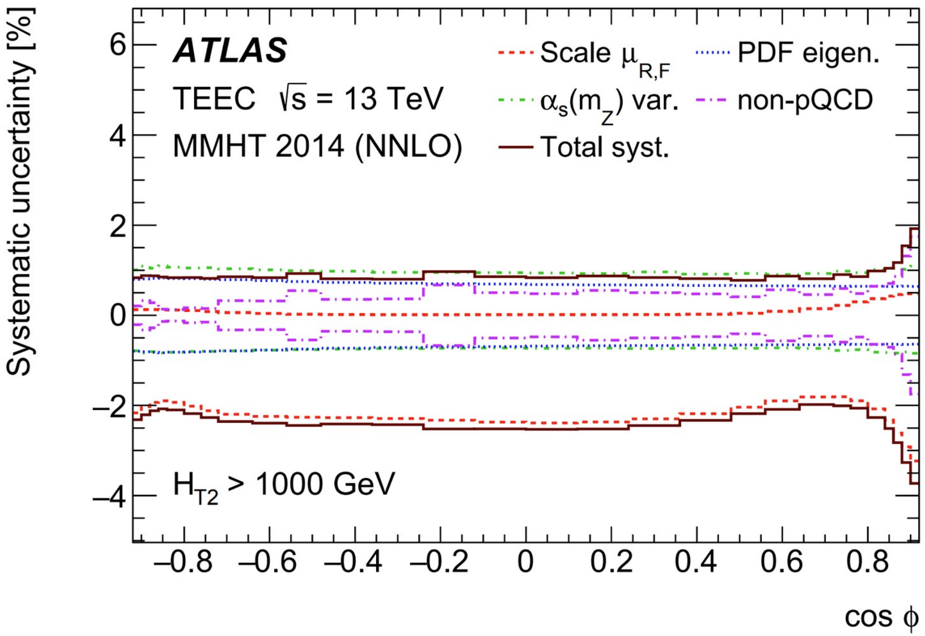
[ATLAS 2301.09351]

Systematic Uncertainties TEEC

Experimental uncertainties



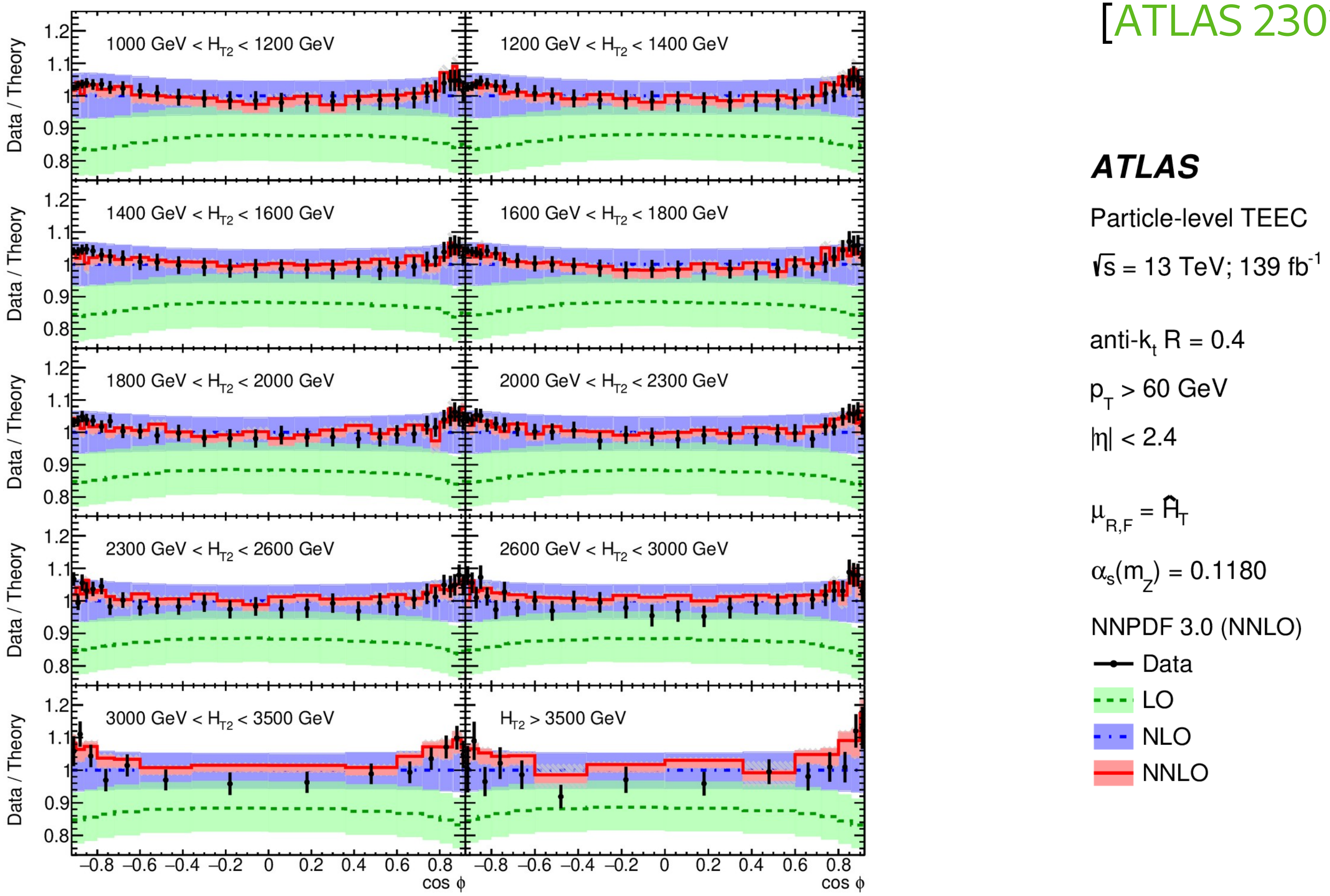
Theory uncertainties



Scale dependence is the dominating uncertainty → NNLO QCD required to match exp.

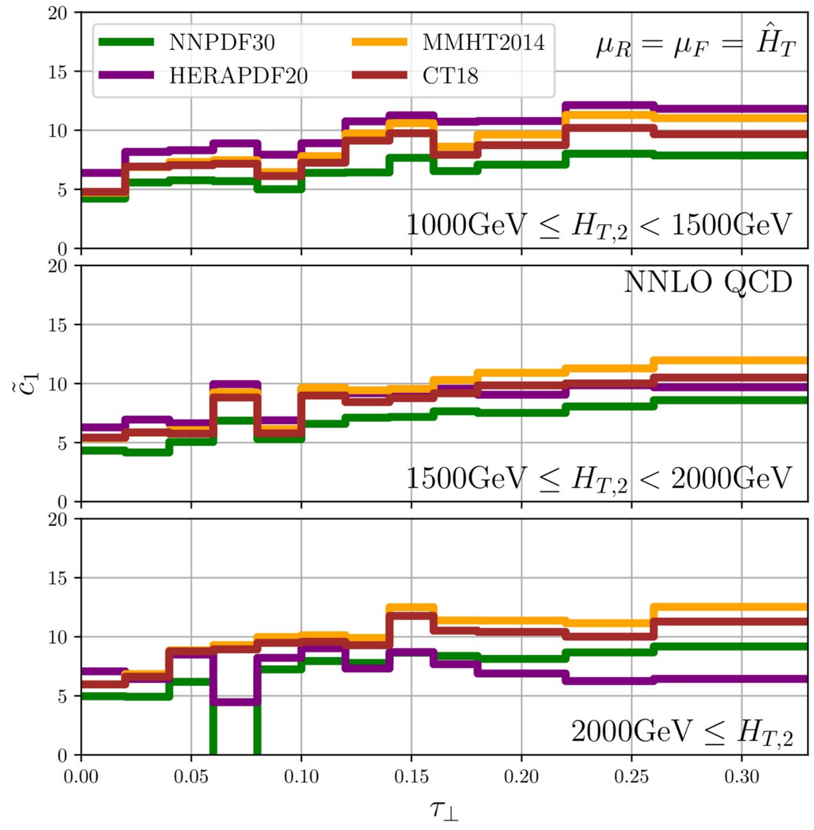
Double differential TEEC

[ATLAS 2301.09351]

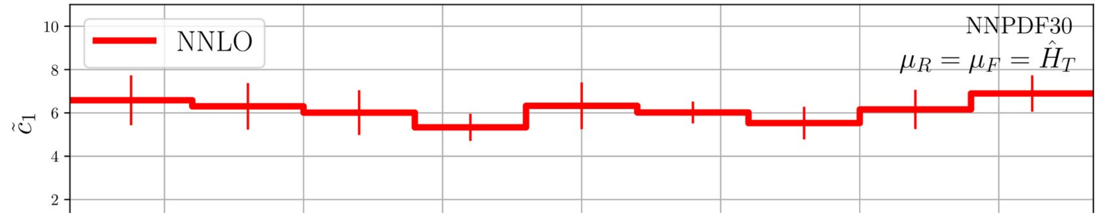


Strong coupling dependence

Thrust



TEEC



$$R^{\text{NNLO,fit}}(\mu, \alpha_{S,0}) = c_0 + c_1(\alpha_{S,0} - 0.118) + c_2(\alpha_{S,0} - 0.118)^2 + c_3(\alpha_{S,0} - 0.118)^3$$

mostly linear dependence

Visualisation of α_S dependence

$$\tilde{c}_1 = \frac{c_1}{R^{\text{NNLO}}(\alpha_{S,0} = 0.118)}$$

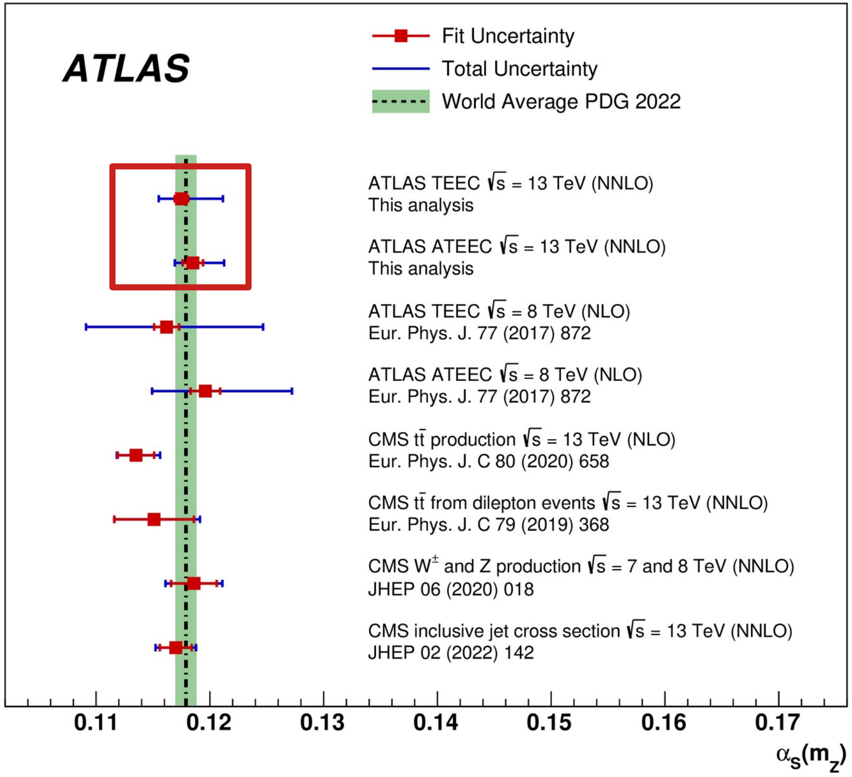
For comparison:
scale dependence (dominant theory uncertainty)

- TEEC ($H_{T,2} > 1$ TeV) : ~2%
- Thrust : ~3-5 %

O(1%) sensitivity

α_S from TEEC @ NNLO by ATLAS

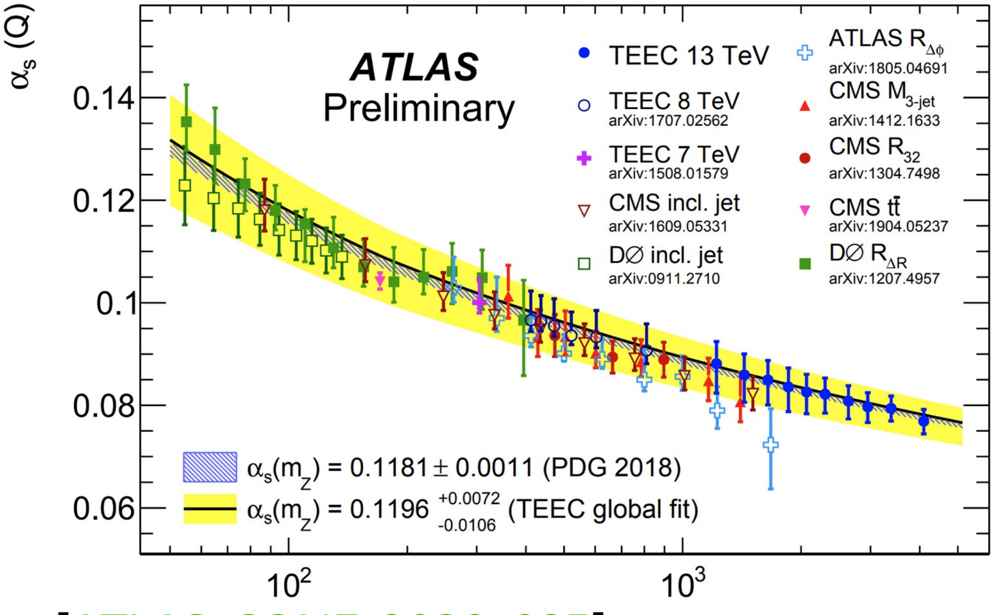
[ATLAS 2301.09351]



- NNLO QCD extraction from multi-jets → will contribute to PDG for the first time
- Significant improvement to 8 TeV → driven by NNLO QCD corrections
- Individual precision large but comparable to top or jets-data.
- However: extraction at high energy scales

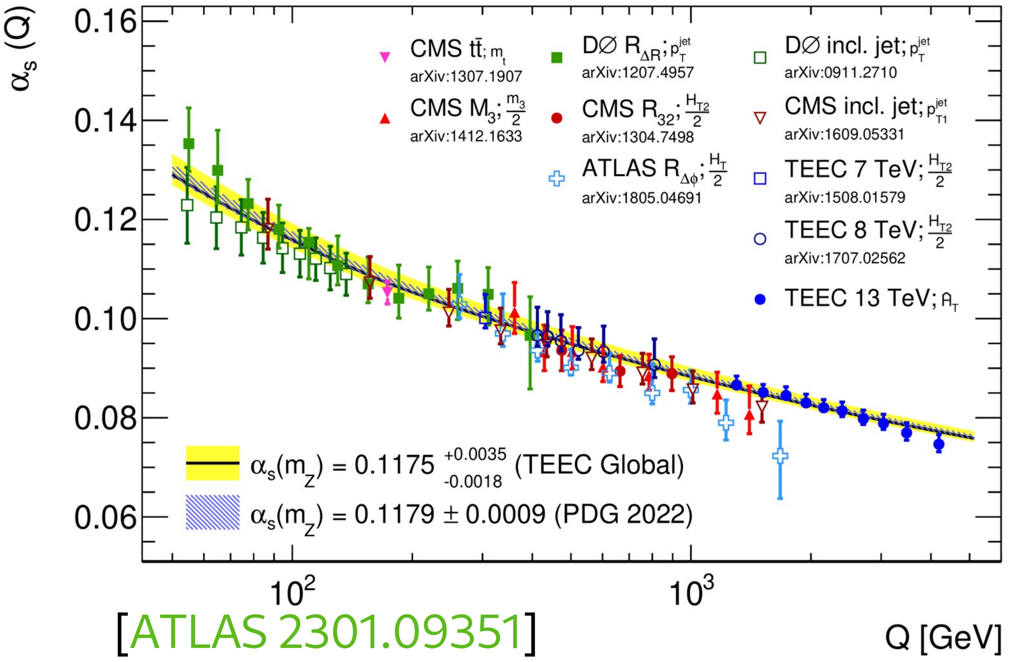
Running of α_S

NLO QCD



[ATLAS-CONF-2020-025]

NNLO QCD



[ATLAS 2301.09351]

Using the running of α_S to probe NP

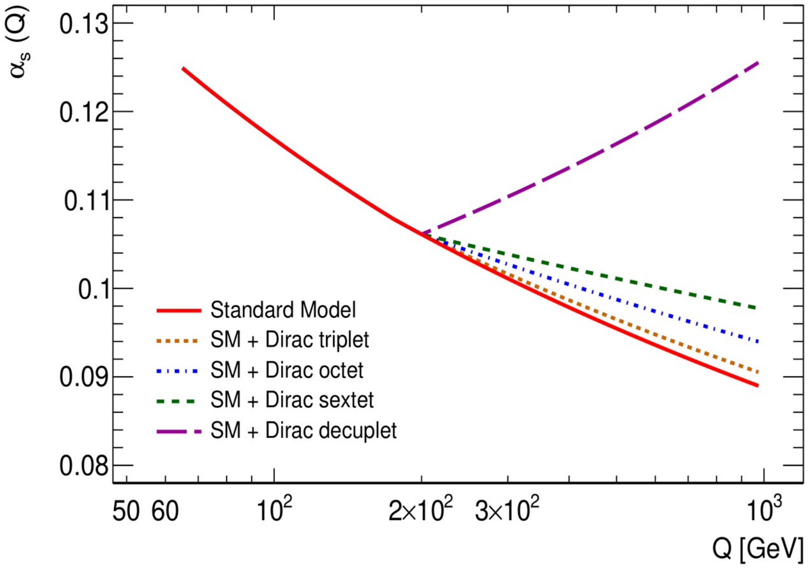
[Llorente, Nachman 1807.00894]

Indirect constraints to NP through modified running:

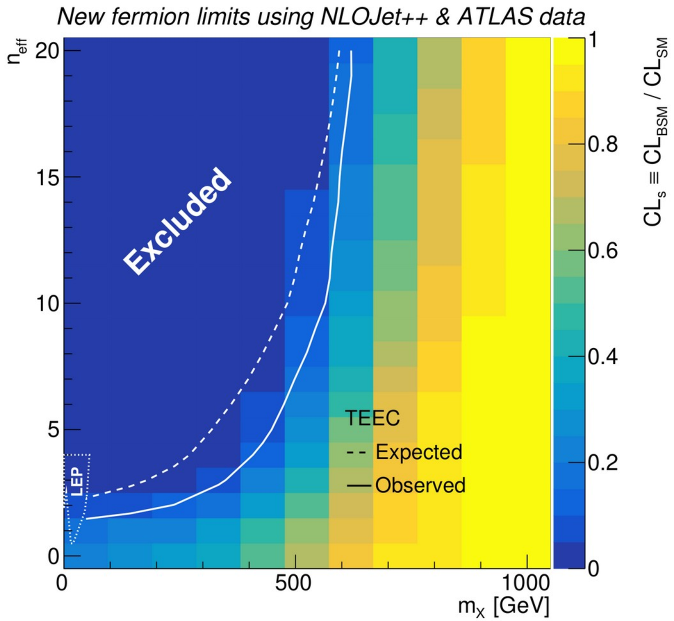
$$\beta_0 = \frac{1}{4\pi}\left(11 - \frac{2}{3}n_f - \frac{4}{3}n_X T_X\right)$$

$$\beta_1 = \frac{1}{(4\pi)^2}\left[102 - \frac{38}{3}n_f - 20 n_X T_X\left(1 + \frac{C_X}{5}\right)\right]$$

$$\alpha_s(Q) = \frac{1}{\beta_0 \log z}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\log(\log z)}{\log z}\right]; \quad z = \frac{Q^2}{\Lambda^2_{\mathrm{QCD}}}$$

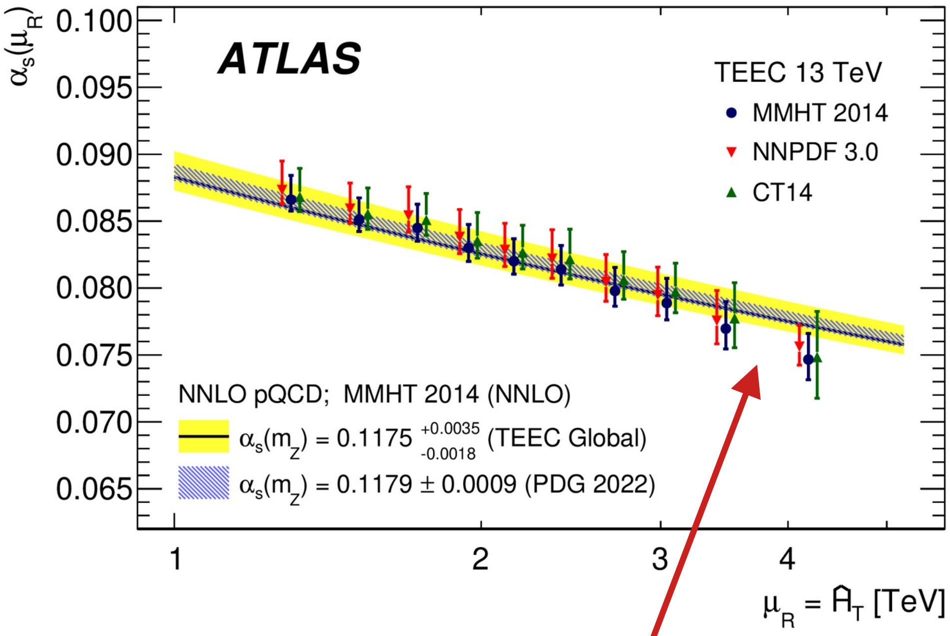


ATLAS
TEEC @ 7 TeV
data



Update with TEEC@13 TeV
→ much improved bounds

… or 'new' SM dynamics



Systematic slope
→ New physics?

Possible SM explanations

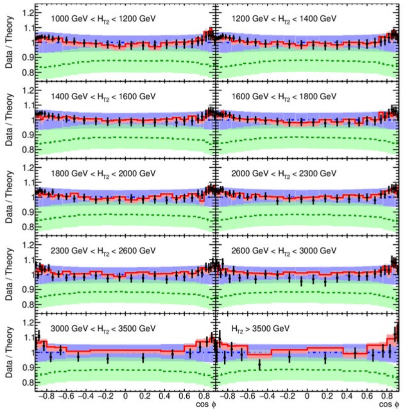
- Residual PDF effects → very high Q^2 ?
- EW corrections?
- Maybe effect from LC approximation in two-loop ME?

$$\mathcal{R}^{(2)}(\mu_R^2) = 2\,\mathrm{Re}\left[\mathcal{M}^{\dagger(0)}\mathcal{F}^{(2)}\right](\mu_R^2) + \left|\mathcal{F}^{(1)}\right|^2(\mu_R^2)$$
$$\equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^{4} c_i \ln^i\left(\frac{\mu_R^2}{s_{12}}\right)$$
$$\mathcal{R}^{(2)}(s_{12}) \approx \mathcal{R}^{(2)l.c.}(s_{12})$$

- Experimental systematics?
- Resummation?

Either case interesting!

HighTEA



= ~100 MCPUh

How to make this more efficient/environment-friendly/accessible/faster?

high tea

for your freshly brewed analysis

https://www.precision.hep.phy.cam.ac.uk/hightea

HighTEA: High energy Theory Event Analyser
[2304.05993]

Michał Czakon,[a] Zahari Kassabov,[b] Alexander Mitov,[c] Rene Poncelet,[c] Andrei Popescu[c]

[a] *Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D-52056 Aachen, Germany*
[b] *DAMTP, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, United Kingdom*
[c] *Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom*

E-mail: mczakon@physik.rwth-aachen.de, zk261@cam.ac.uk, adm74@cam.ac.uk, poncelet@hep.phy.cam.ac.uk, andrei.popescu@cantab.net

Basic idea

- Database of precomputed “Theory Events”
  - **Equivalent to a full fledged computation**
  - Currently this means partonic fixed order events
  - Extensions to included showered/resummed/hadronized events is feasible
- Analysis of the data through an user interface
  - Easy-to-use
  - Fast
  - Flexible:
    - Observables from basic 4-momenta
    - Free specification of bins
    - Renormalization/Factorization Scale variation
    - PDF (member) variation
    - Specify phase space cuts

Not so new idea:
LHE [Alwall et al ‘06],
Ntuple [BlackHat ‘08’13],

(Partially) Unweighting

The hadronic cross section in collinear factorization:

$$\mathrm{d}\sigma(P_1,P_2) = \sum_{ab} \iint_0^1 \mathrm{d}x_1 \mathrm{d}x_2 f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2) \mathrm{d}\hat{\sigma}_{ab}(x_1 P_1, x_2 P_2)$$

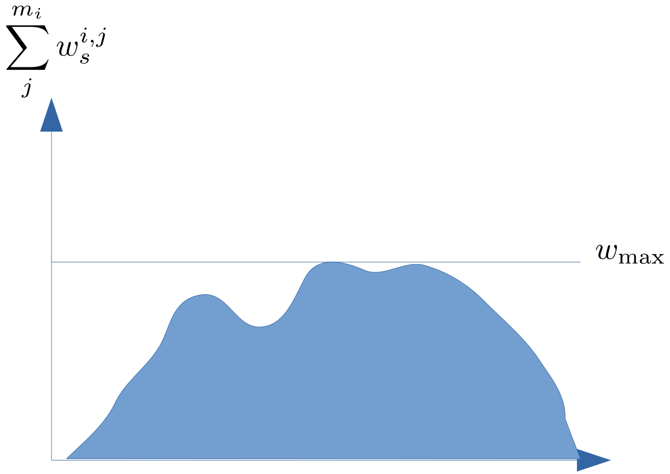
$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

Using MC method for integration:

$$\sigma_{\text{tot}} = \frac{1}{n} \sum_i^n \left(\sum_j^{m_i} w_s^{i,j} \right)$$

Beyond LO events might correspond to more than one kinematic: **Subtraction events!**

Hit-And-Miss Algorithm:



Accept each event i with probability: $\left(\sum_j^{m_i} w_s^{i,j} \right) / w_{\max}$

Store each sub-event with weight: $w_s^{i,j} / \left(\sum_j^{m_i} w_s^{i,j} \right)$

Factorizations

Factorizing renormalization and factorization scale dependence:

$$w_s^{i,j} = w_{\text{PDF}}(\mu_F, x_1, x_2) w_{\alpha_s}(\mu_R) \left(\sum_{i,j} c_{i,j} \ln\left(\mu_R^2\right)^i \ln\left(\mu_F^2\right)^j \right)$$

PDF dependence:

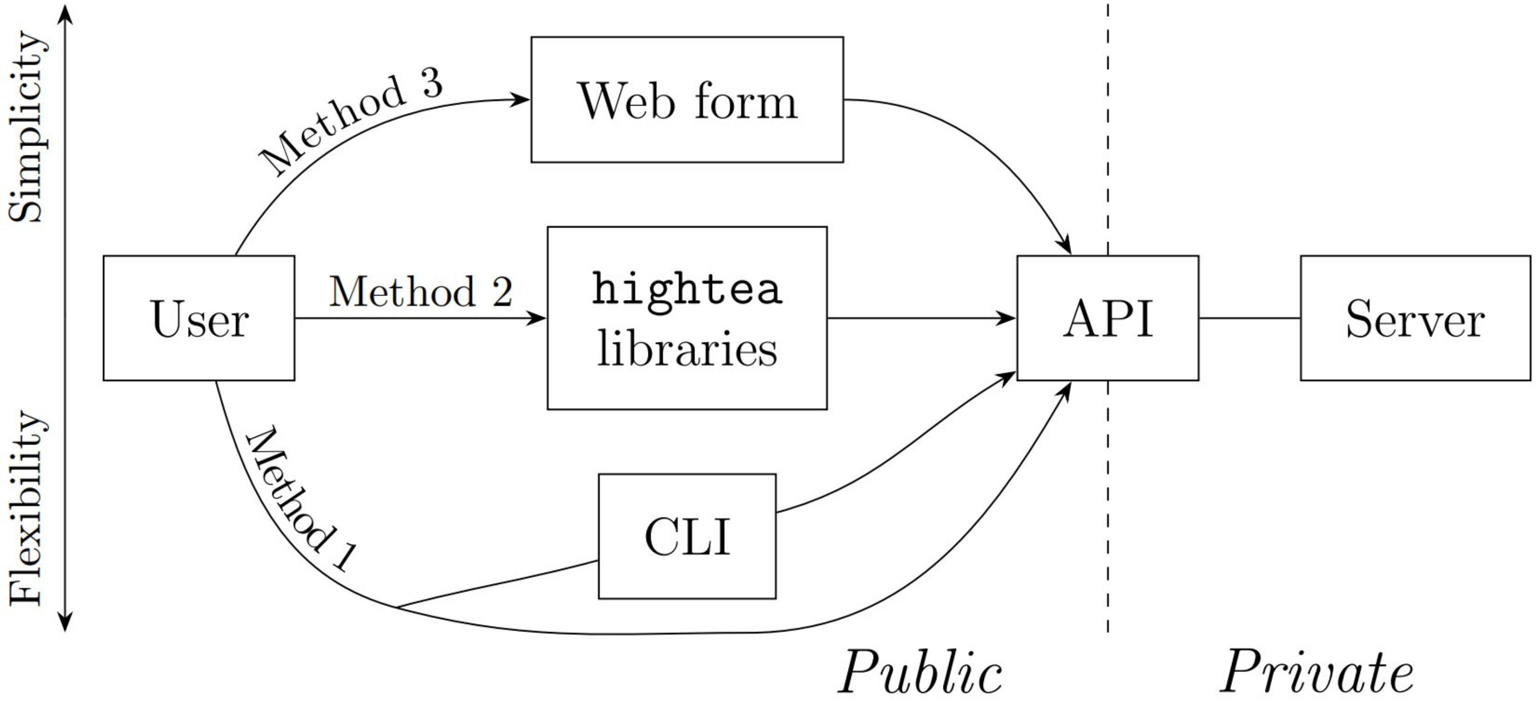
$$w_{\text{PDF}}(\mu, x_1, x_2) = \sum_{ab \in \text{channel}} f_a(x_1, \mu) f_b(x_2, \mu)$$

α_s dependence:

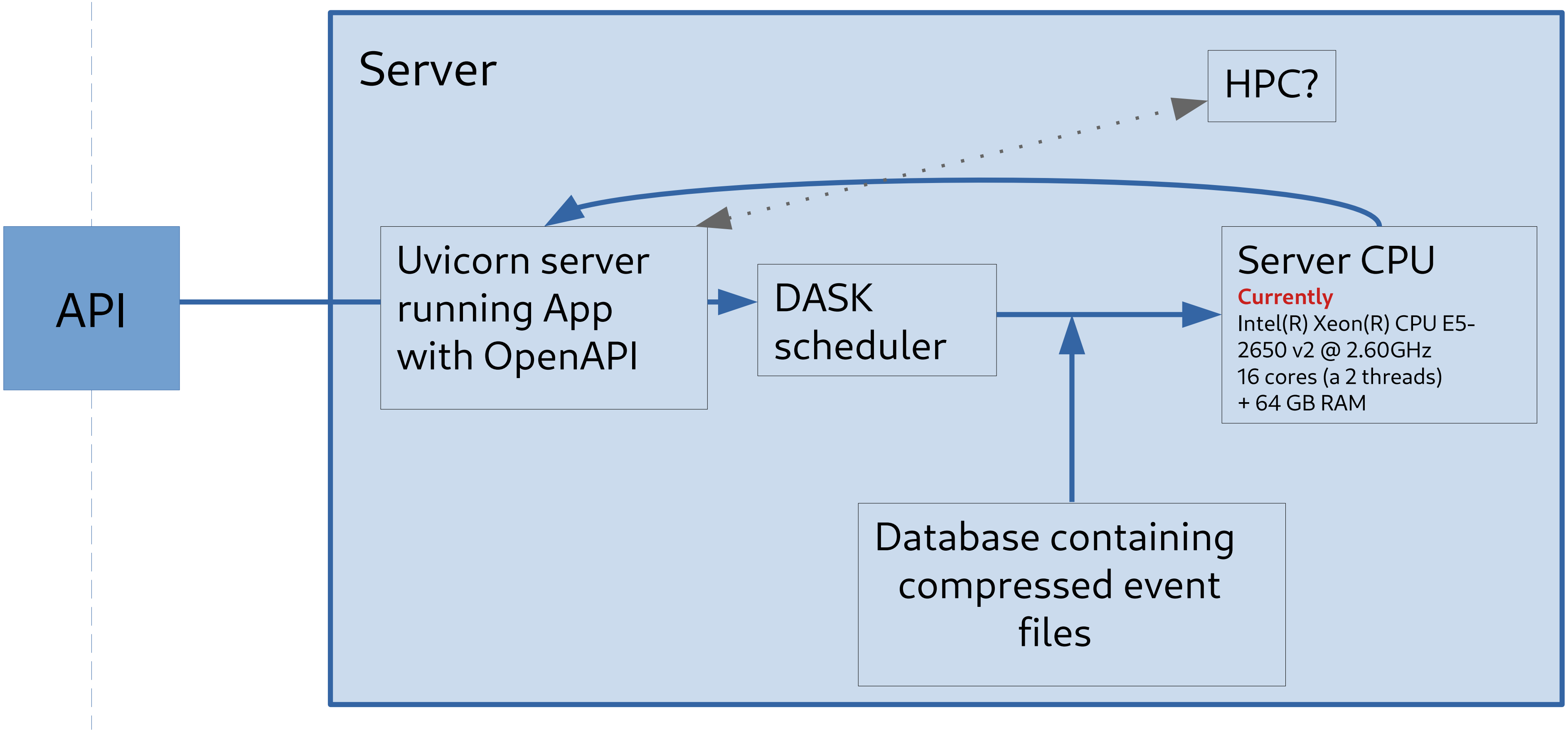
$$w_{\alpha_s}(\mu) = (\alpha_s(\mu))^m$$

Allows **full control over scales and PDF**

HighTEA interface

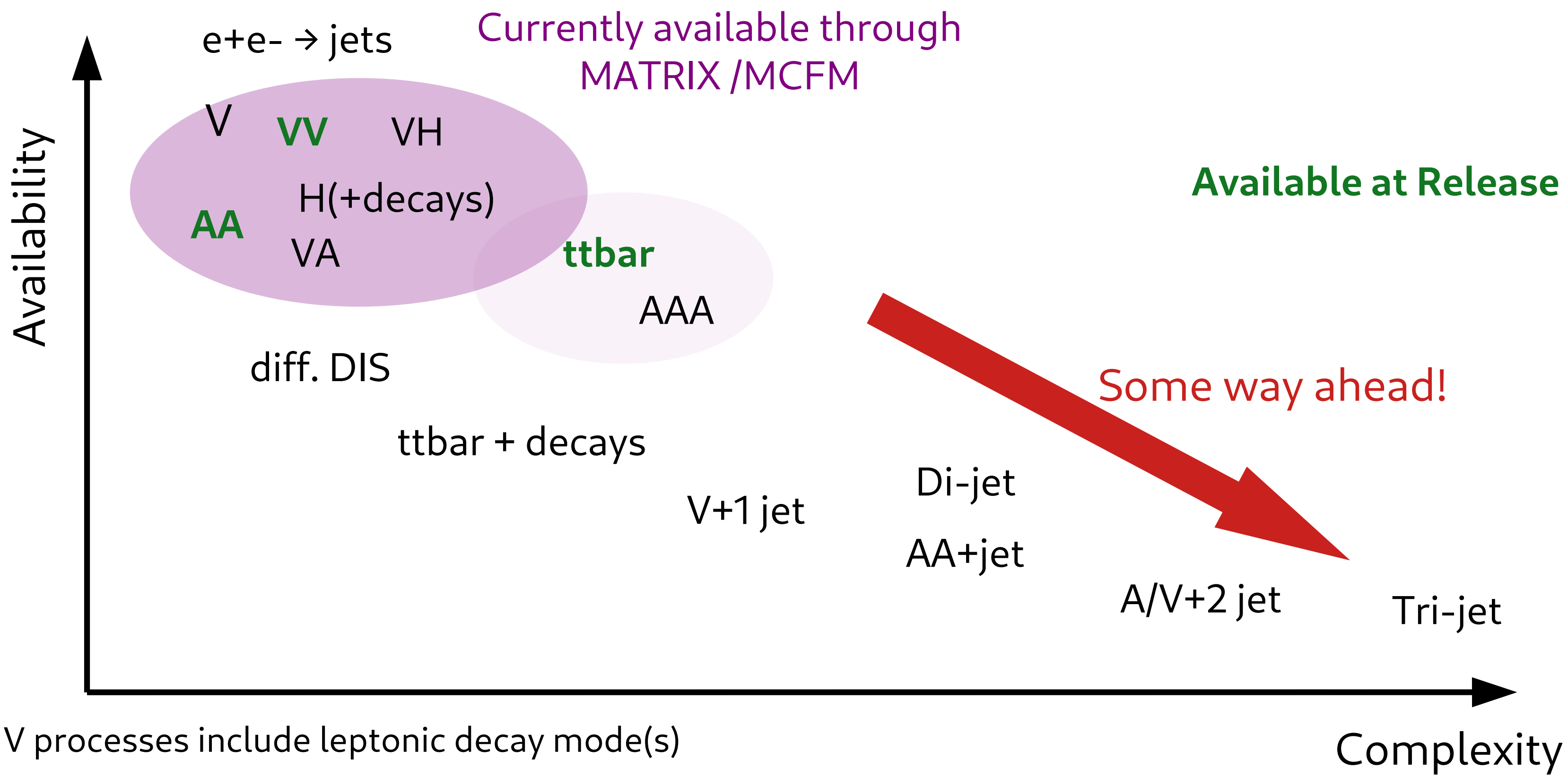


The server


Server
HPC?
API
Uvicorn server running App with OpenAPI
DASK scheduler
Server CPU
Currently
Intel(R) Xeon(R) CPU E5-2650 v2 @ 2.60GHz
16 cores (a 2 threads)
+ 64 GB RAM
Database containing compressed event files

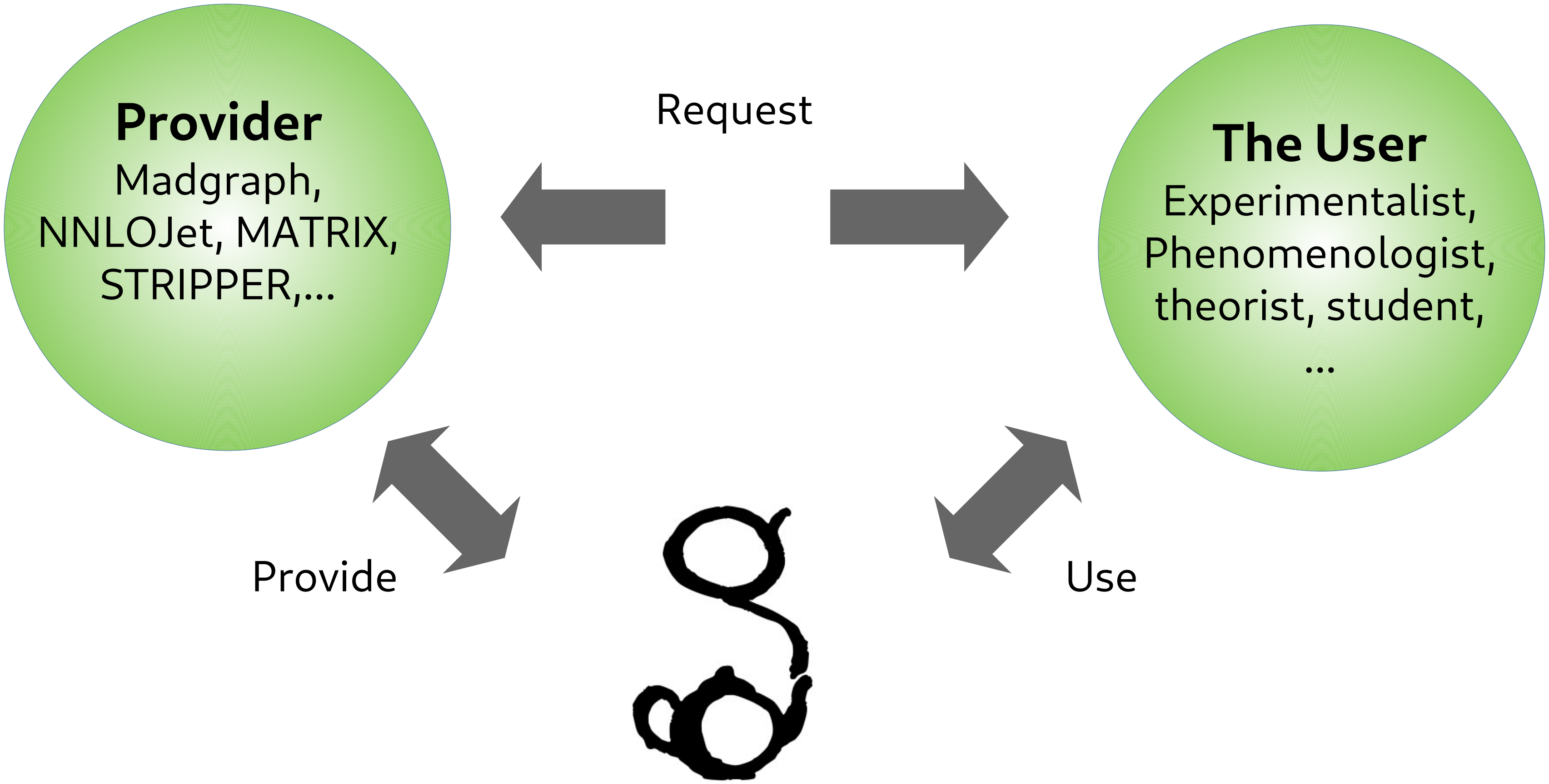
Available Processes

Processes currently implemented in our STRIPPER framework through NNLO QCD



* V processes include leptonic decay mode(s)

The Vision



Provider
Madgraph, NNLOJet, MATRIX, STRIPPER,...
Request
The User
Experimentalist, Phenomenologist, theorist, student, ...
Provide
Use

Summary & Outlook

Summary

- Three jet NNLO QCD predictions allow for precision phenomenology with multi-jet final states
- First predictions for R32 ratios and event shapes
- Extraction of the strong coupling constant from event shapes by ATLAS → will contribute to PDG ave.
- Relatively costly enterprise → effective NNLO QCD tools needed
- HighTEA framework to store and reuse calculations

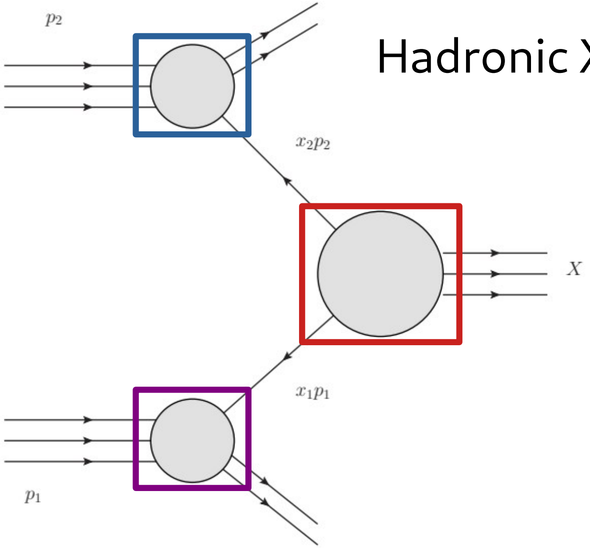
Outlook

- Still improvements to be made on subtractions schemes:
  - Better MC integration techniques → ML community has developed a plethora of tools
  - Technical aspects like form of selector function and phase space mappings

    "3 factors of 2 are also a order of magnitude" → difference between "doable" and "not doable"!
- Progressively extending the capabilities of HighTEA

Backup

Hadronic cross section



Hadronic X-section: $\sigma_{h_1h_2\to X} = \sum_{ij}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\phi_{i,h_1}(x_1,\mu_F^2)\phi_{j/h_2}(x_2,\mu_F^2)\hat\sigma_{ij\to X}(\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)$

Parton distribution functions

Perturbative expansion of partonic cross section:

$$\hat\sigma_{ab\to X} = \hat\sigma^{(0)}_{ab\to X} + \hat\sigma^{(1)}_{ab\to X} + \hat\sigma^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

The NNLO bit: $\hat\sigma^{(2)}_{ab} = \hat\sigma^{\mathrm{RR}}_{ab} + \hat\sigma^{\mathrm{RV}}_{ab} + \hat\sigma^{\mathrm{VV}}_{ab} + \hat\sigma^{\mathrm{C2}}_{ab} + \hat\sigma^{\mathrm{C1}}_{ab}$

Double real radiation

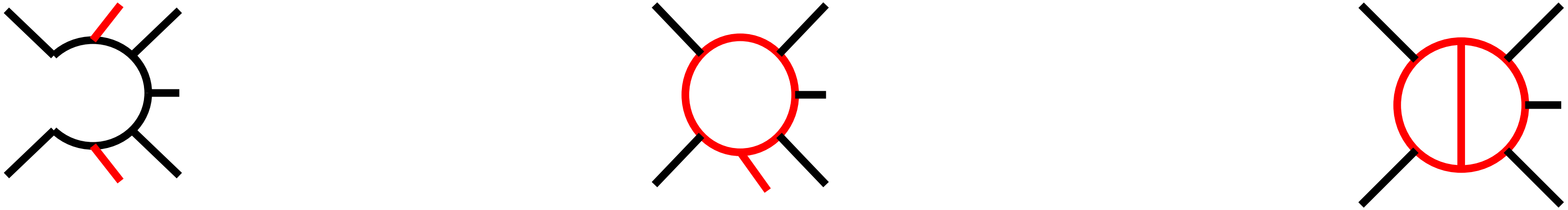
$$\hat\sigma^{\mathrm{RR}}_{ab} = \frac{1}{2\hat s}\int \mathrm{d}\Phi_{n+2}\left\langle \mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2}$$

Real/Virtual correction

$$\hat\sigma^{\mathrm{RV}}_{ab} = \frac{1}{2\hat s}\int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re}\left\langle \mathcal{M}^{(0)}_{n+1}\middle|\mathcal{M}^{(1)}_{n+1}\right\rangle \mathrm{F}_{n+1}$$

Double virtual corrections

$$\hat\sigma^{\mathrm{VV}}_{ab} = \frac{1}{2\hat s}\int \mathrm{d}\Phi_{n}\left(2\mathrm{Re}\left\langle \mathcal{M}^{(0)}_{n}\middle|\mathcal{M}^{(2)}_{n}\right\rangle + \left\langle \mathcal{M}^{(1)}_{n}\middle|\mathcal{M}^{(1)}_{n}\right\rangle\right) \mathrm{F}_{n}$$



Partonic cross section beyond LO

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

Contributions with different multiplicities and # convolutions:

$$\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$$

$$\hat{\sigma}^{\mathrm{RR}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}^{(0)}_{n+2} \middle| \mathcal{M}^{(0)}_{n+2} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}^{\mathrm{RV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(1)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\mathrm{VV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n} \middle| \mathcal{M}^{(2)}_{n} \right\rangle + \left\langle \mathcal{M}^{(1)}_{n} \middle| \mathcal{M}^{(1)}_{n} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}^{\mathrm{C1}}_{ab} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\mathrm{C2}}_{ab} = (\text{double convolution})\, \mathrm{F}_{n}$$

Each term separately IR divergent. But sum is:

→ finite

→ regularization scheme independent

Considering CDR ($d = 4 - 2\epsilon$):

→ Laurent expansion:

$$\hat{\sigma}^{C}_{ab} = \sum_{i=-4}^{0} c_i \epsilon^i + \mathcal{O}(\epsilon)$$

Sector decomposition I

Considering working in CDR:

→ Virtuals are usually done in this regularization

→ Real radiation:

→ Very difficult integrals, analytical impractical (except very simple cases)!

→ Numerics not possible, integrals are divergent: ε-poles!

How to extract these poles? → Sector decomposition!

Divide and conquer the phase space:

$$1 = \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right]$$

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

Sector decomposition II

Divide and conquer the phase space:

→ Each $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$ has simpler divergences.

appearing as $1/s_{ijk} \quad 1/s_{ik}/s_{jl}$

Soft and collinear (w.r.t parton k,l) of partons i and j

→ Parametrization w.r.t. reference parton:

$$\hat{\eta}_i = \frac{1}{2}(1-\cos\theta_{ir}) \in [0,1] \qquad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$$

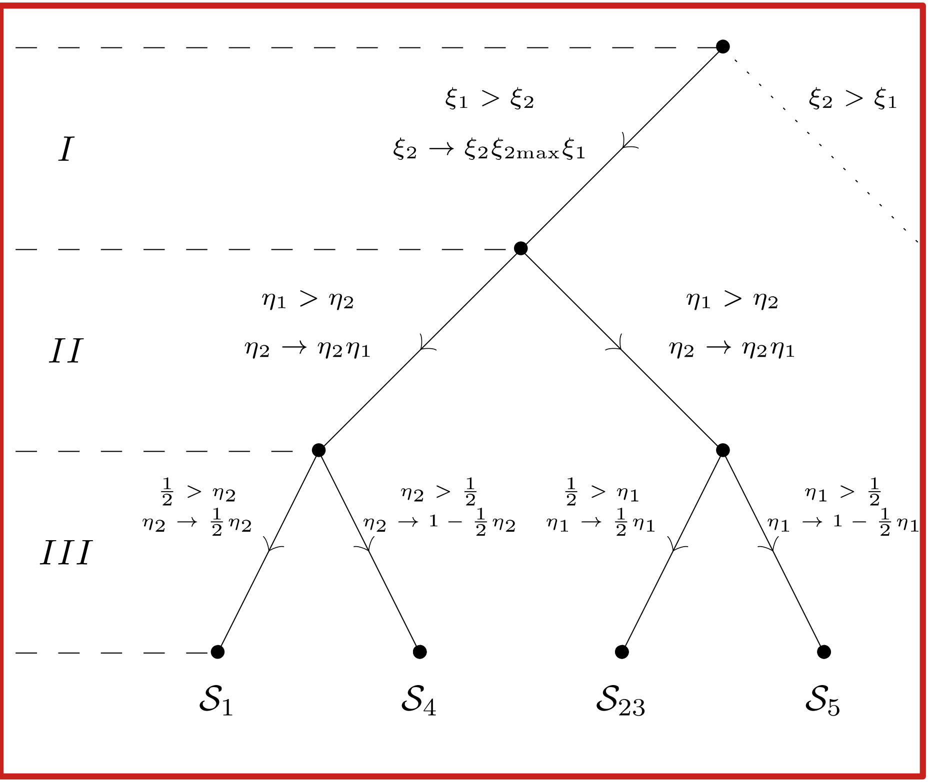
→ Subdivide to factorize divergences

$$s_{u_1u_2k} = (p_k+u_1+u_2)^2 \sim \hat{\eta}_1 u_1^0 + \hat{\eta}_2 u_2^0 + \hat{\eta}_3 u_1^0 u_2^0$$

→ double soft factorization:

$$\theta(u_1^0-u_2^0)+\theta(u_2^0-u_1^0)$$

→ triple collinear factorization



[Czakon'10,Caola'17]

Sector decomposition III

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\,\mathcal{S}_{kl,m}\left\langle \mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2} = \sum_{\text{sub-sec.}}\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i\, \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}}\,\mathrm{d}\tilde{\mu}(\{x_i\})\underbrace{\prod x_i^{a_i+1}\langle \mathcal{M}_{n+2}|\mathcal{M}_{n+2}\rangle}_{\text{regular}}\mathrm{F}_{n+2}$$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 \mathrm{d}x\left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x)-f(0)}{x^{1+b\epsilon}}$$

Finite NNLO cross section

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1} \, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution}) \, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution}) \, \mathrm{F}_{n}$$

sector decomposition and master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

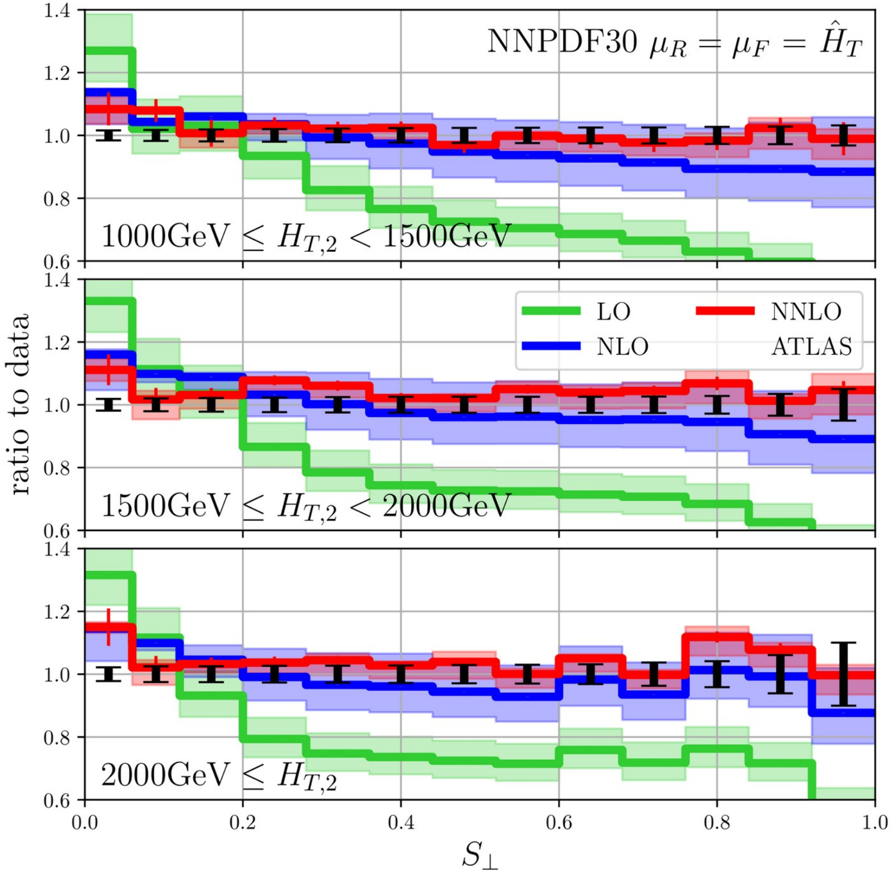
$$\left(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}\right) \quad \left(\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}\right) \quad \left(\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}\right) \quad \left(\sigma_{SU}^{C1}, \sigma_{DU}^{C1}\right) \quad \left(\sigma_{DU}^{C2}, \sigma_{FR}^{C2}\right)$$

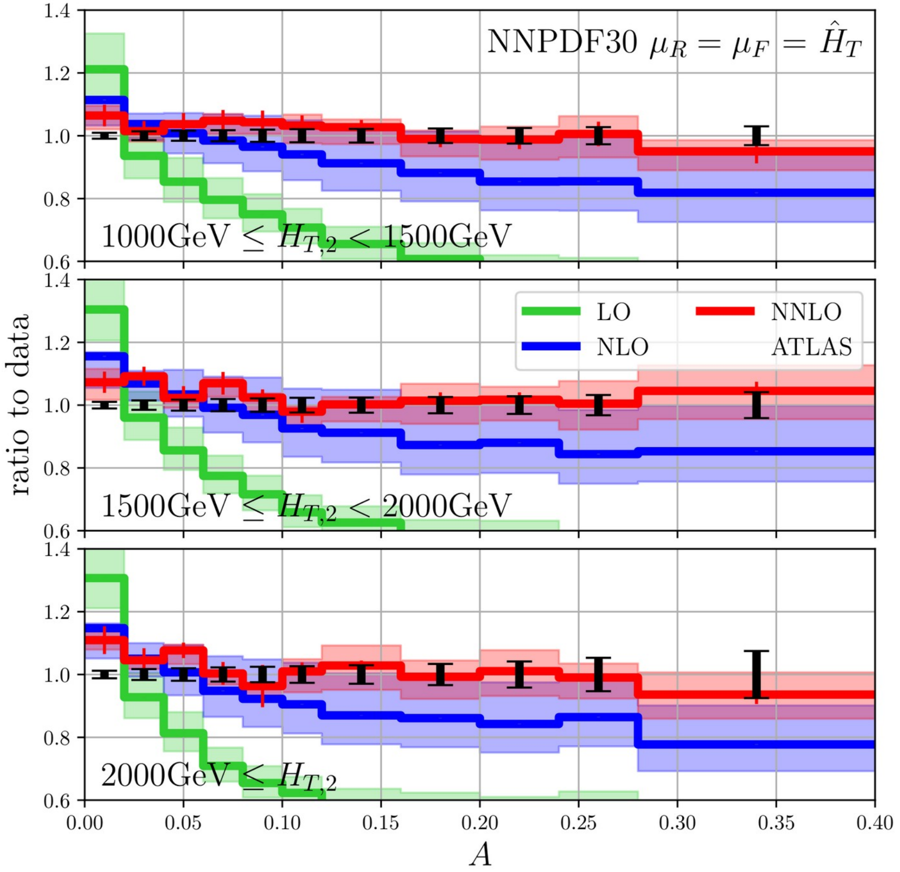
re-arrangement of terms → 4-dim. formulation [Czakon'14, Czakon'19]

$$\underline{\left(\sigma_F^{RR}\right)} \quad \underline{\left(\sigma_F^{RV}\right)} \quad \underline{\left(\sigma_F^{VV}\right)} \quad \underline{\left(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}\right)} \quad \underline{\left(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2}\right)} \quad \underline{\left(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}\right)}$$

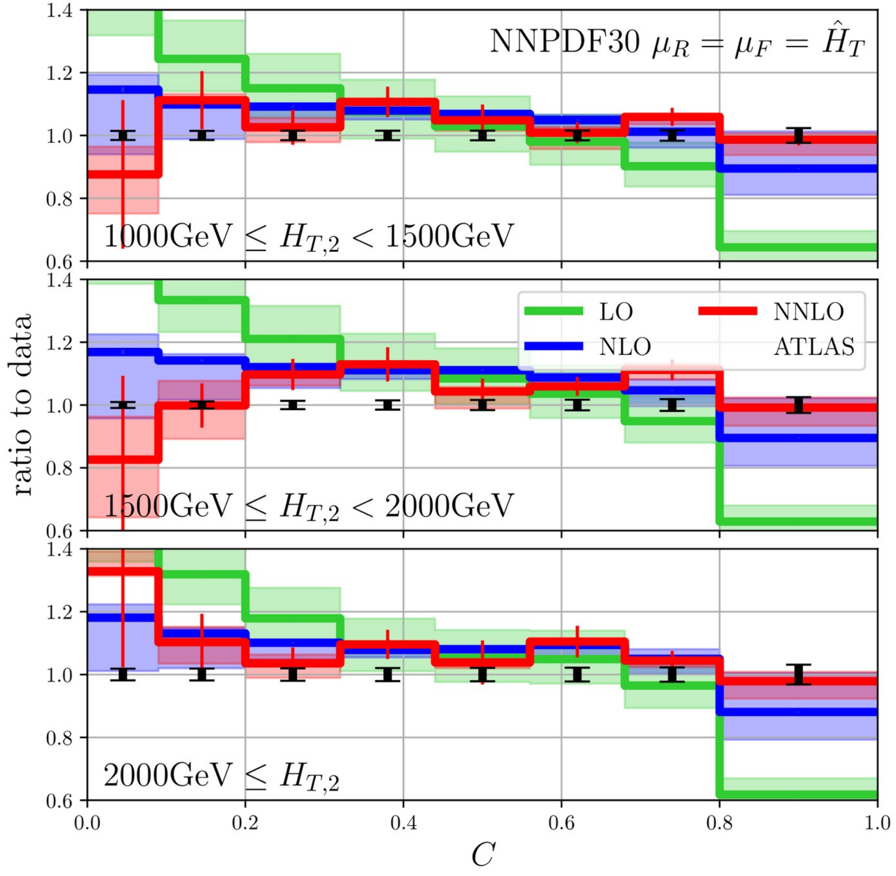
separately finite: ε poles cancel

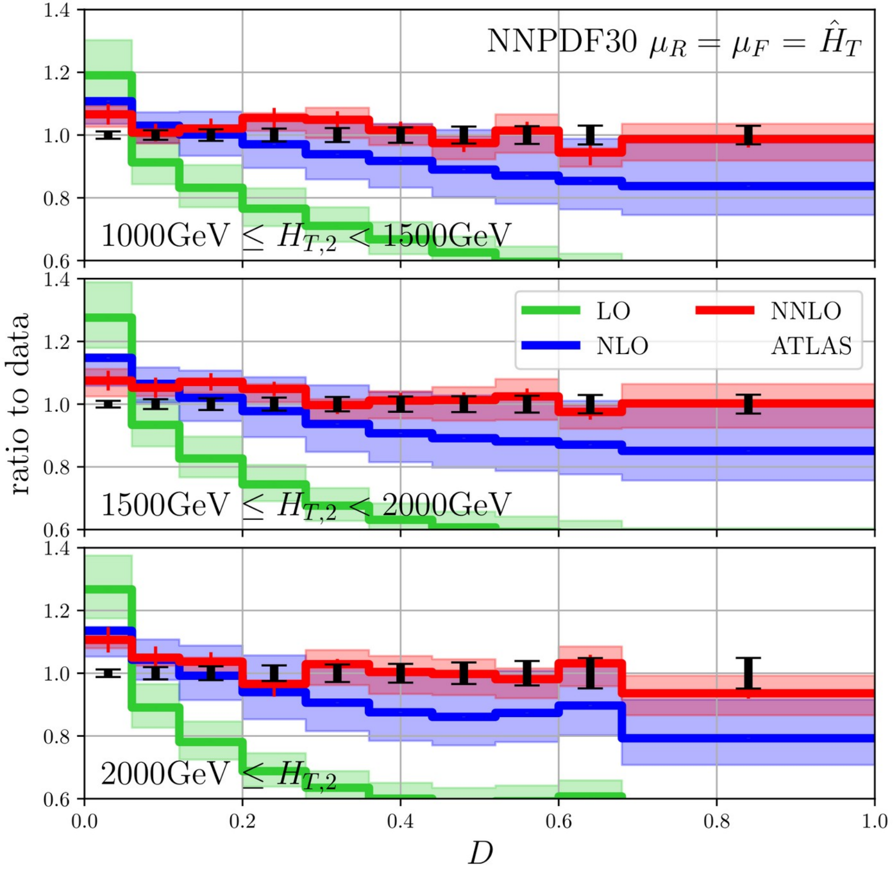
More event-shapes I





More event-shapes II





Event shapes as MC tuning tool