Isolated photon production in association with a jet pair through next-to-next-to-leading order in QCD

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in collaboration with
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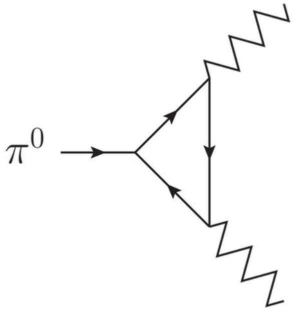
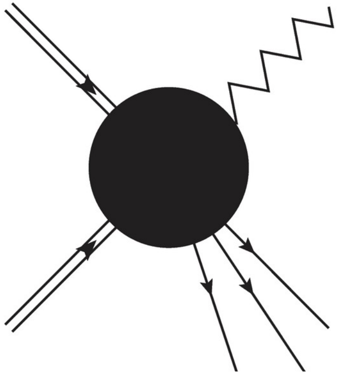
Photon production @ colliders

Hadron decays

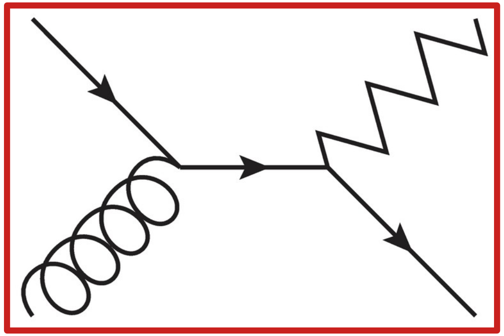


Prompt production

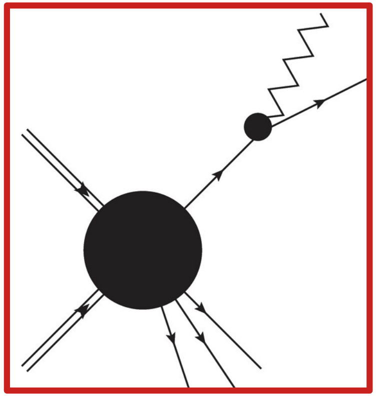


Photon isolation

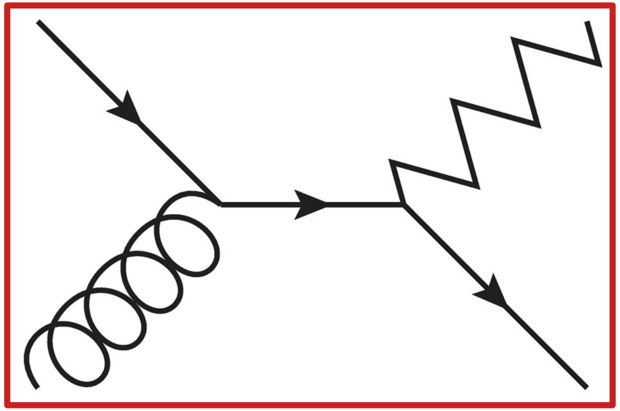
Direct

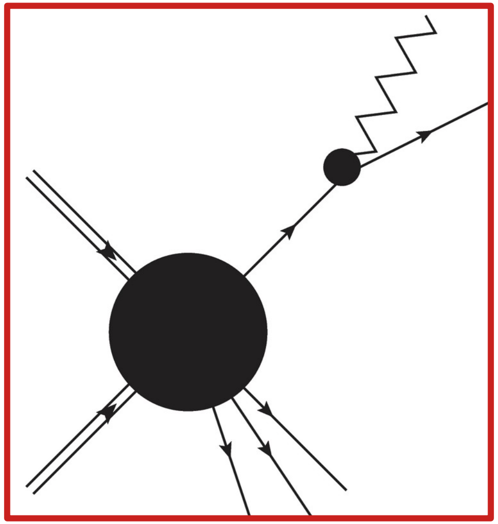


Fragmentation



Prompt photon production





Direct production
- Test of perturbative QCD
- Gluon PDF sensitivity
- Estimates for BSM backgrounds

Fragmentation
- Depends on non-perturbative fragmentation functions
- Separation from “direct” not unique

LHC Experimental/theory status

CMS

- 8 TeV: $pp \to \gamma + j$ [1505.06520]
- 13 TeV: $pp \to \gamma + j/X$ [1907.08155] [1807.00782]

ATLAS

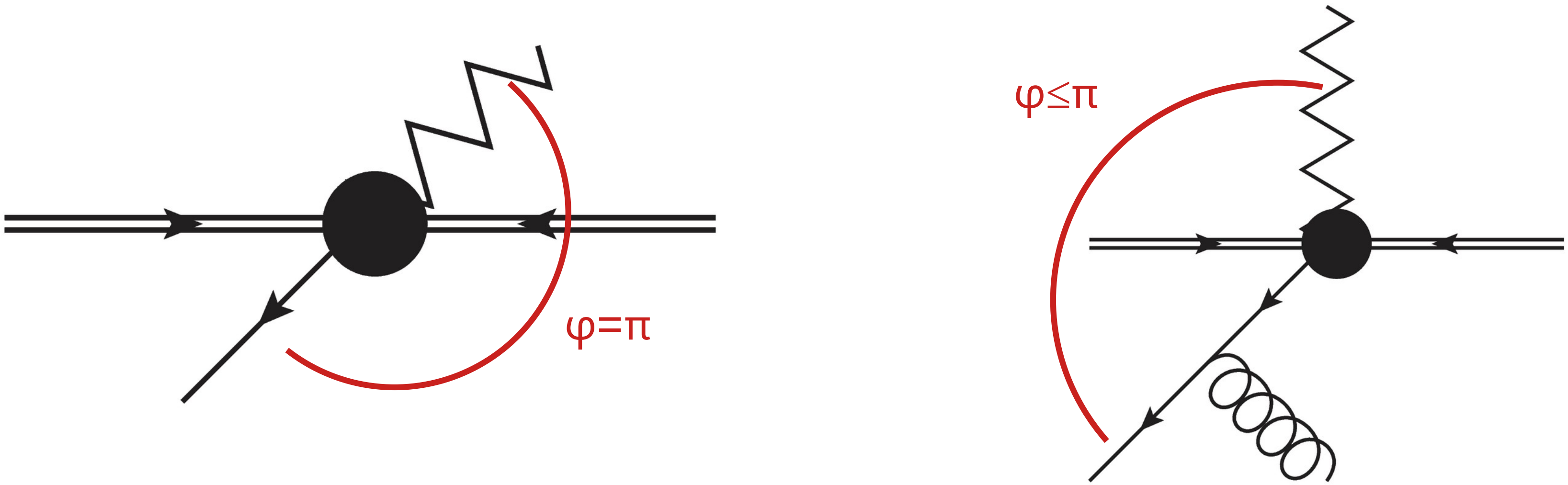
- 8 TeV: $pp \to \gamma + X$ [1605.03495]
- 13 TeV: $pp \to \gamma + j$ [1801.00112]
- 13 TeV/ 8 TeV: $pp \to \gamma + X$ [1901.10075]
- 13 TeV: $pp \to \gamma + X$ [1908.02746]
- 13 TeV: $pp \to \gamma + 2j$ [1912.09866]

→ O(%) uncertainties
→ typically dominated by systematics

- NLO QCD in public codes
  → large uncertainties O(10%)
- NNLO QCD for $pp \to \gamma + j/X$
  - **MCFM** [1703.10109]
  - **NNLOJET** [1904.01044]
    Fragmentation [2201.06982]
    Hybrid isolation [2205.01516]
- NNLO QCD for $pp \to \gamma + 2j$
  - **STRIPPER** [2304.06682]

This talk

Why photon plus a jet pair?



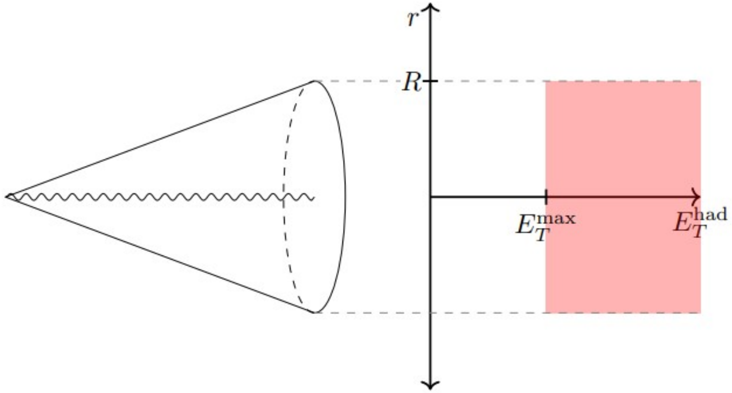
- Non-back-to-back Born configurations
  → access to angular correlations between the photon and jets
- Access to different kinematic regimes through distinguishable photon
  → enhance direct, high- or low-z fragmentation
- Background process for BSM: $pp \to \gamma + Y(\to jj)$

Photon isolation

Hard cone

- Experimental hard cone:

$$E_\perp(r) \leq E_{\perp\max} = 0.0042\, E_\perp(\gamma) + 10\ \text{GeV} \quad \text{for} \quad r \leq R_{\max} = 0.4$$

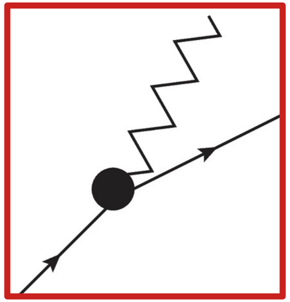


Credit: Marius Hoefer (talk@SM@LHC22)

- Theory perspective:

  Not collinear safe in perturbative QCD
  due to q → qγ splittings

  → Non-vanishing fragmentation contribution
  (NNLO QCD with frag. [2201.06982][2205.01516])



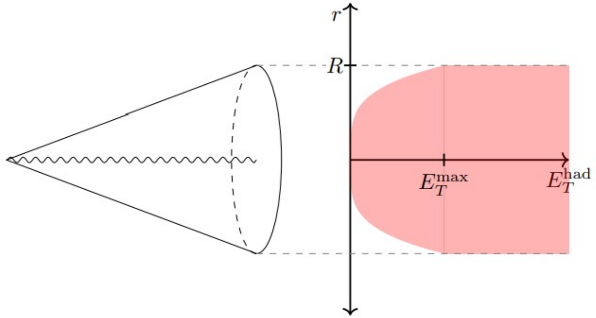
Photon isolation

Smooth cone

- by Frixione [hep-ph/9801442]

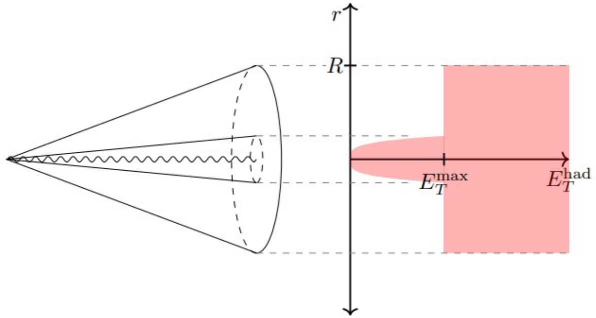
$$E_\perp(r) \leq E_{\perp\max}(r) = 0.1\, E_\perp(\gamma)\left(\frac{1-\cos(r)}{1-\cos(R_{\max})}\right)^2 \quad \text{for} \quad r \leq R_{\max} = 0.1$$

→ Theoretically convenient
→ Removes fragmentation contribution
→ Experimentally limited by detector resolution



Hybrid cone

- [1611.07226][2205.01516]
  - Combines smooth & hard cone
  - Fair approx. to hard cone [2205.01516]

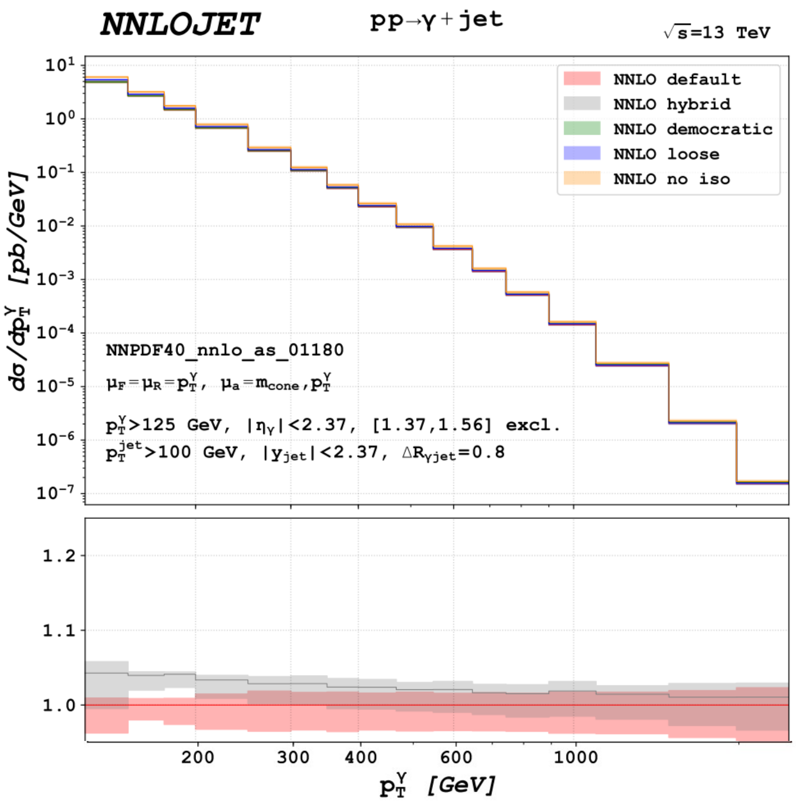


Credit: Marius Hoefer (talk@SM@LHC22)

Fragmentation contribution

[2205.01516]

- ATLAS photon requirements (same as for $pp \to \gamma + 2j$)
- Comparison between:
  - "default" NNLO with fragmentation
  - "hybrid" NNLO with hybrid isolation
- Fragmentation contr.
  - ~5% at small $E_T(\gamma)$
  - ~<1% at high $E_T(\gamma)$



Photon plus jet pair

Measurement of isolated-photon plus two-jet production in pp collisions at sqrt(s) = 13 TeV with the ATLAS detector [1912.09866]

Requirements on photon	$E_T^\gamma > 150$ GeV, $\lvert\eta^\gamma\rvert < 2.37$ (excluding $1.37 < \lvert\eta^\gamma\rvert < 1.56$) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 4.8$ GeV (reconstruction level) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 10$ GeV (particle level)		
Requirements on jets	at least two jets using anti-k_t algorithm with $R = 0.4$ $p_T^{\text{jet}} > 100$ GeV, $\lvert y^{\text{jet}}\rvert < 2.5$, $\Delta R^{\gamma-\text{jet}} > 0.8$		
Phase space	**total**	**fragmentation enriched** $E_T^\gamma < p_T^{\text{jet2}}$	**direct enriched** $E_T^\gamma > p_T^{\text{jet1}}$
Number of events	755 270	111 666	386 846

Modelled with hybrid isolation

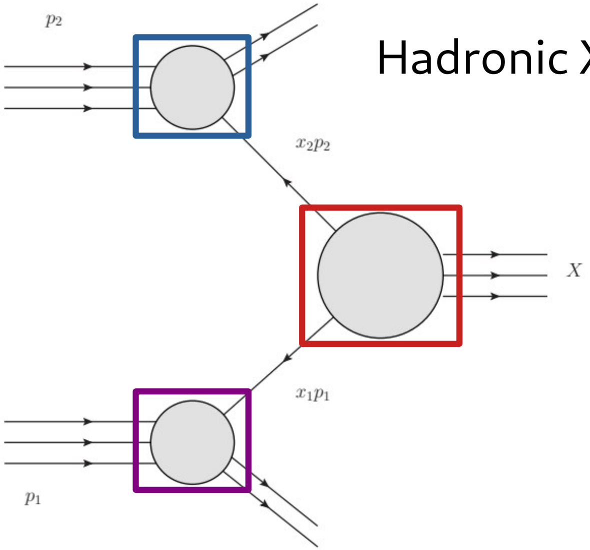
$$E_\perp(r) \leq E_{\perp\max}(r) = 0.1\, E_\perp(\gamma)\left(\frac{1-\cos(r)}{1-\cos(R_{\max})}\right)^2 \quad \text{for} \quad r \leq R_{\max} = 0.1$$

+

$$E_\perp(r) \leq E_{\perp\max} = 0.0042\, E_\perp(\gamma) + 10\ \text{GeV} \quad \text{for} \quad r \leq R_{\max} = 0.4$$

No fragmentation contribution
→ Purely pQCD through NNLO
→ focus on “inclusive” and “direct” PS

Perturbative QCD



Hadronic X-section: $\sigma_{h_1h_2\to X} = \sum_{ij}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\phi_{i,h_1}(x_1,\mu_F^2)\phi_{j/h_2}(x_2,\mu_F^2)\hat{\sigma}_{ij\to X}(\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)$

Parton distribution functions

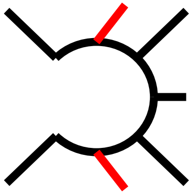
Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

The NNLO bit: $\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \mathrm{coll.\ counter\ terms}$

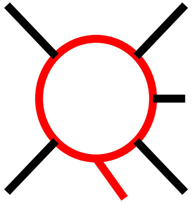
Double real radiation

$$\hat{\sigma}^{\mathrm{RR}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\left\langle\mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2}$$



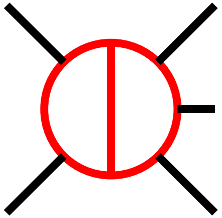
Real/Virtual correction

$$\hat{\sigma}^{\mathrm{RV}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re}\left\langle\mathcal{M}^{(0)}_{n+1}\middle|\mathcal{M}^{(1)}_{n+1}\right\rangle \mathrm{F}_{n+1}$$



Double virtual corrections

$$\hat{\sigma}^{\mathrm{VV}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n}\left(2\mathrm{Re}\left\langle\mathcal{M}^{(0)}_{n}\middle|\mathcal{M}^{(2)}_{n}\right\rangle + \left\langle\mathcal{M}^{(1)}_{n}\middle|\mathcal{M}^{(1)}_{n}\right\rangle\right) \mathrm{F}_{n}$$



Observables

Binned for 1st and 2nd leading jet

1. $E_\perp(\gamma)$: photon transverse energy,
2. p_T^{jet}: jet transverse momentum,
3. y^{jet}: jet pseudorapidity,
4. $|\Delta y^{\gamma-\text{jet}}|$: absolute value of the pseudorapidity difference between the photon and the jet,
5. $|\Delta\phi^{\gamma-\text{jet}}|$: azimuthal-angle difference between the photon and the jet,
6. $|\Delta y^{j_1-j_2}|$: absolute value of the pseudorapidity difference between the leading and sub-leading jet,
7. $|\Delta\phi^{j_1-j_2}|$: azimuthal-angle difference between the leading and sub-leading jet,
8. $m(j_1 j_2)$: invariant mass of the leading and sub-leading jet,
9. $m(\gamma j_1 j_2)$: invariant mass of the photon, leading and sub-leading jet.

Angular correlations

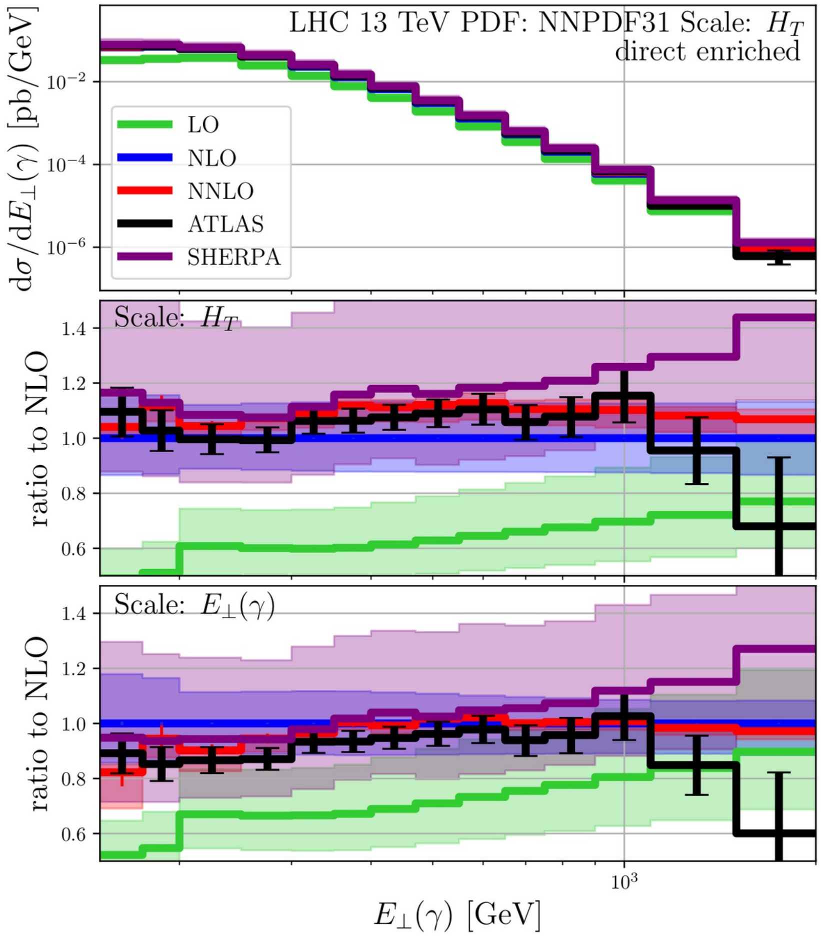
Theory - data comparisons

NNLO QCD

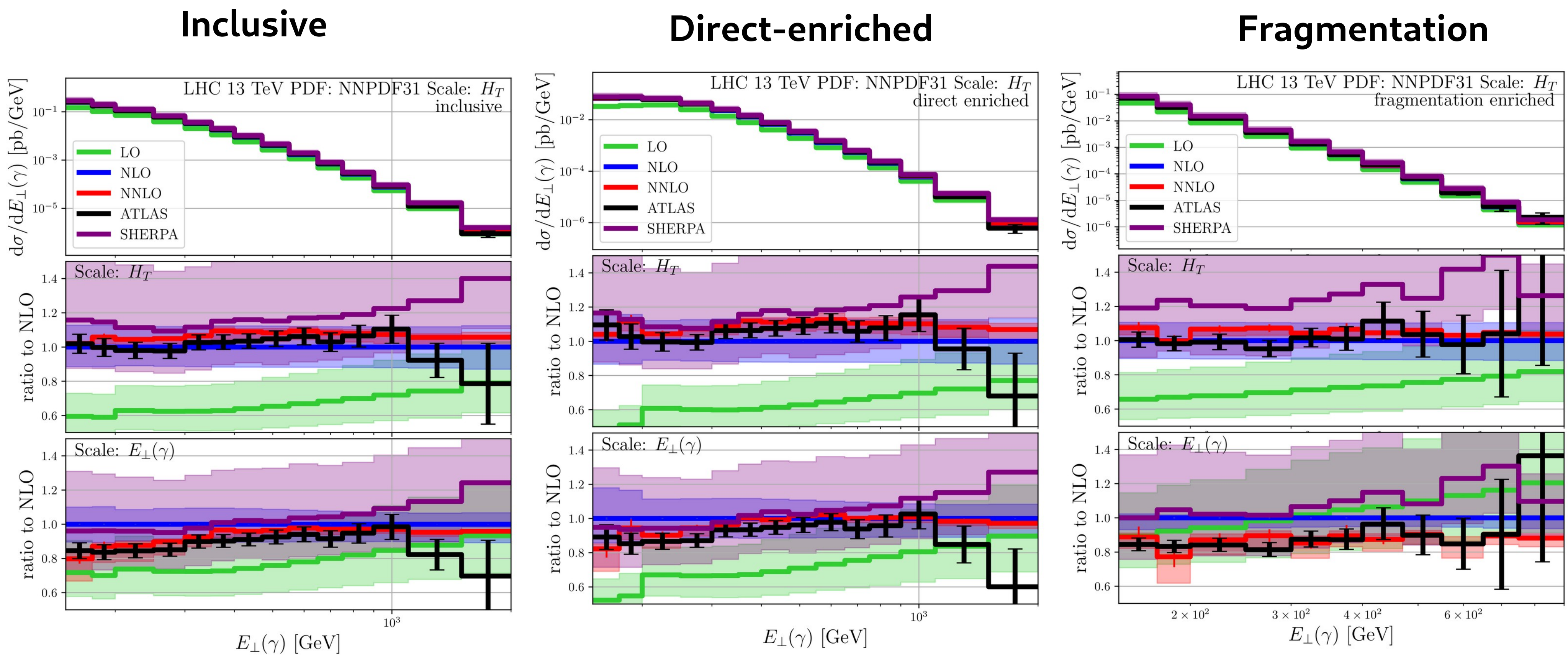
- Describes data well
- Improvements on the shape
- Small corrections
- Small remaining scale dependence

Comment on the SHERPA predictions

- Large NLO scale uncertainties
- The shape is not well described
- Maybe an artefact of multi-jet merging?



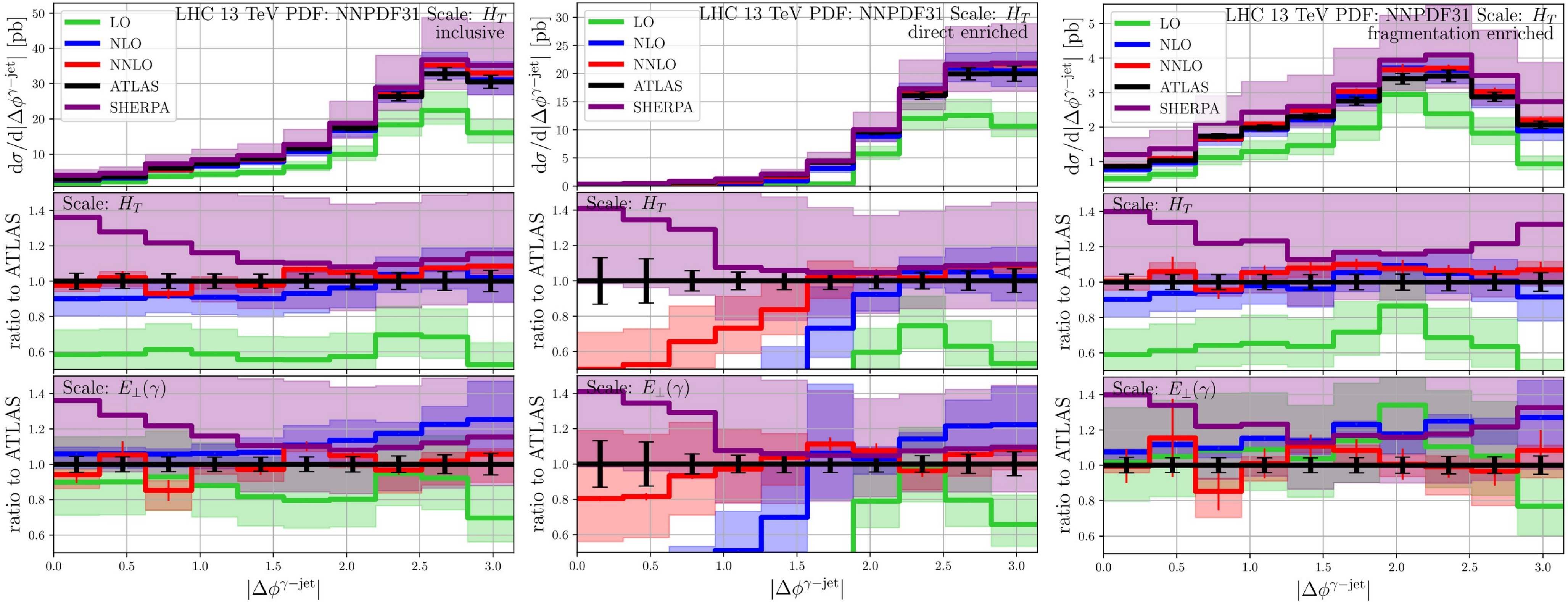
Inclusive vs. direct vs. fragmentation



Transverse photon energy

Inclusive vs. direct vs. fragmentation

Inclusive | **Direct-enriched** | **Fragmentation**



Azimuthal separation photon - jet

Scale choice

Full tree kinematics

$$\mu_R = \mu_F = H_T = E_\perp(\gamma) + p_T(j_1) + p_T(j_2)$$

$$\mu_R = \mu_F = E_\perp(\gamma)\ ,$$

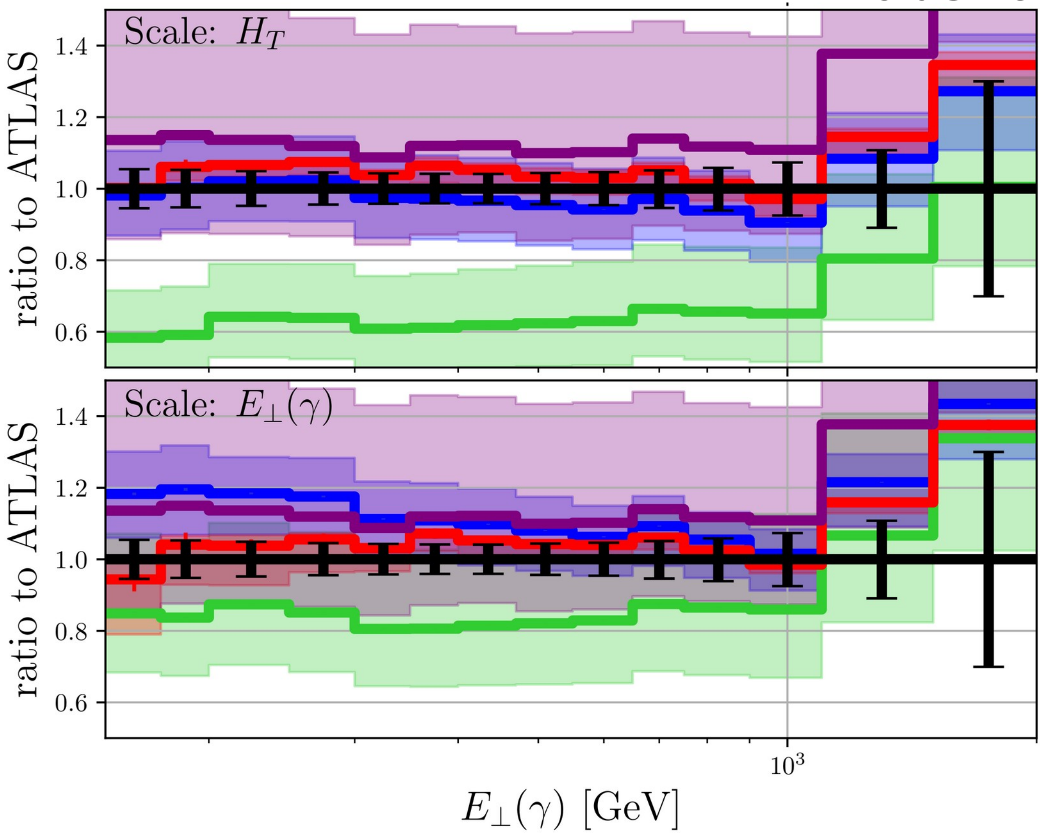
Only photon

Perturbative convergence

NNLO result similar **but** $E_\perp(\gamma)$

- Larger (negative) NNLO corrections
- Larger scale dependence (for jet obs.)

Inclusive



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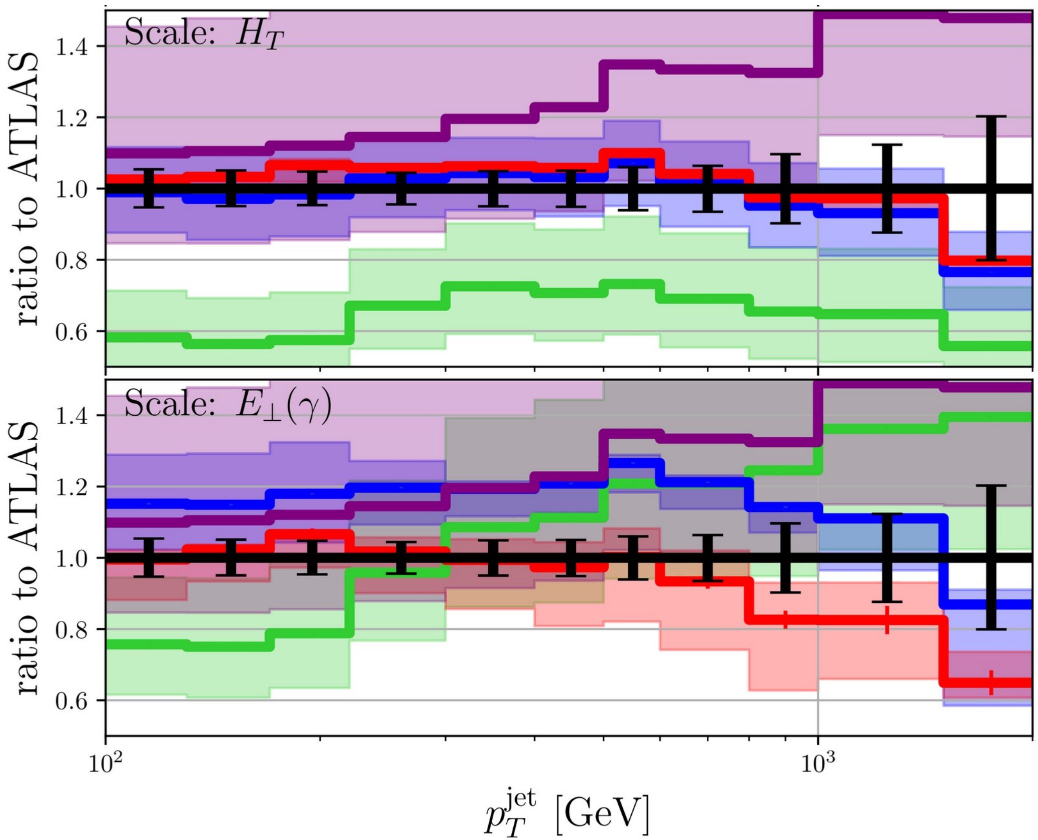
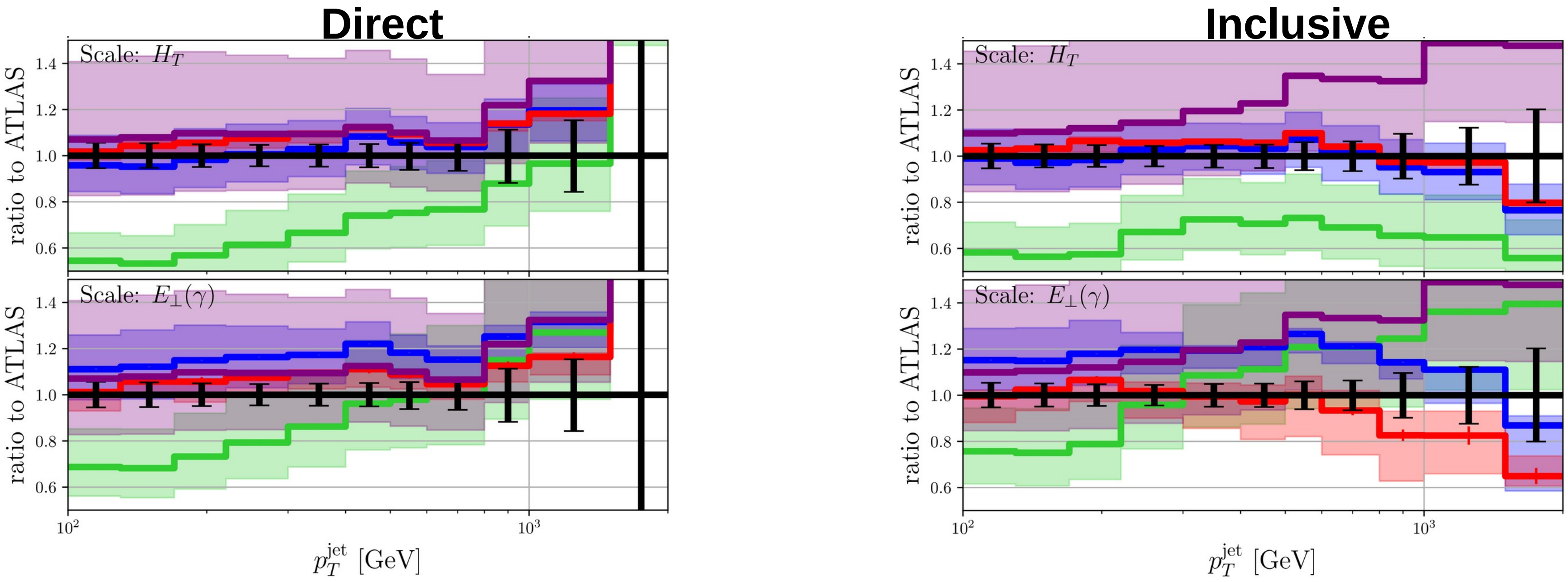


Scale choice

→ $E_\perp(\gamma)$ **does not capture relevant scales** for $pp \to \gamma + 2j$

- Better for "direct" enriched phase space $p_T(\gamma) > p_T(j_1)$
  → $E_\perp(\gamma)$ closer to $H_T = p_T(\gamma) + p_T(j_1) + p_T(j_2)$



Missing effects

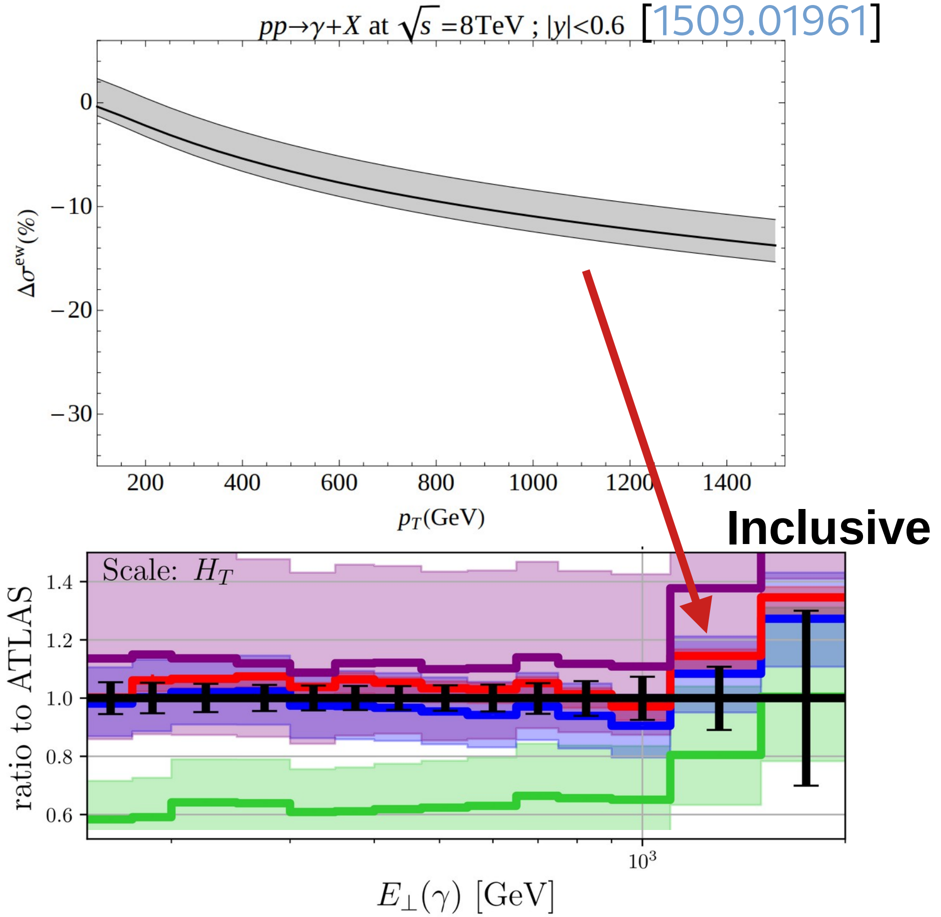
Electro-weak corrections

- EW Sudakov logs at high $E_\perp(\gamma)$
- ~O(-10%) above 1 TeV
- Further improvement of theory/data

Fragmentation

- More relevant at small $E_\perp(\gamma)$
- For $pp \to \gamma + X:\ \ \sigma(\text{hybrid}) > \sigma(\text{frag.})$
- Inclusion might cure slightly high normalisation

[1509.01961]



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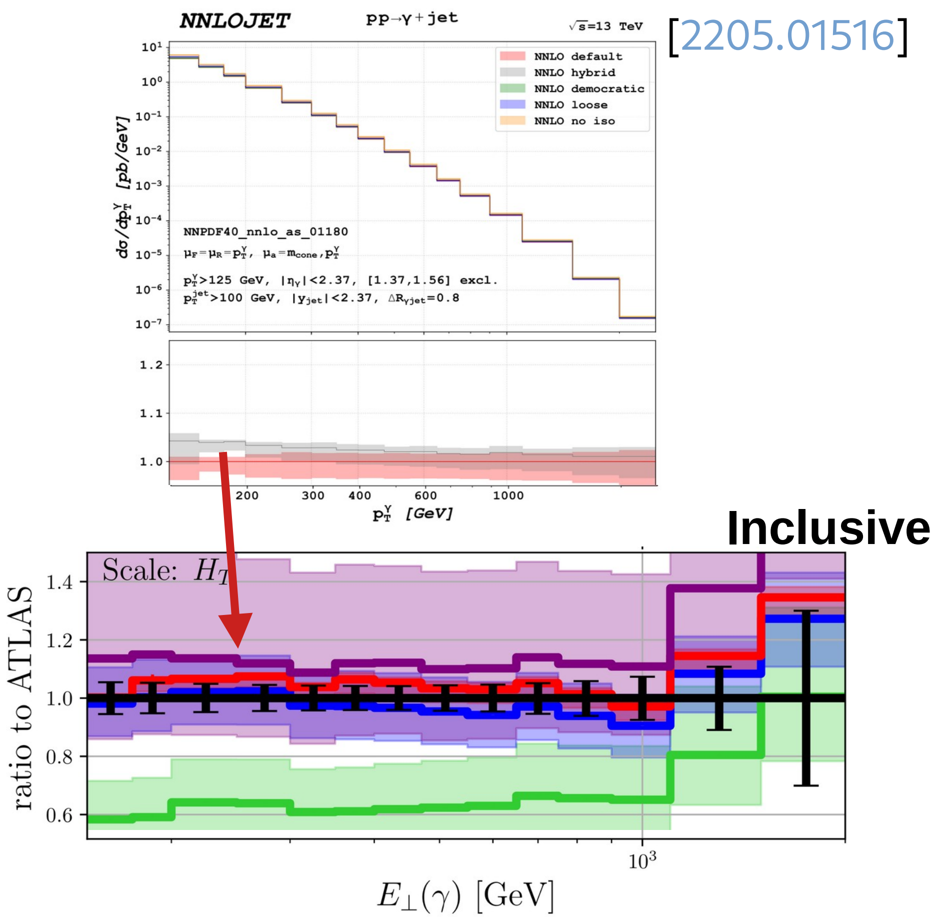
[2205.01516]



Overview 2 → 3 massless computations

$pp \to \gamma\gamma\gamma$

STRIPPER [1911.00479]
MATRIX [2010.04681]

$pp \to \gamma\gamma j$

STRIPPER [2105.06940]

$pp \to jjj$

STRIPPER [2106.05331]
NNLOJET (gluons only) [2203.13531]
STRIPPER [2301.01086]

Cross sections

$pp \to \gamma jj$

STRIPPER [2304.06682]

Overview 2 → 3 massless amplitudes

$pp \to \gamma\gamma j$
[2102.01820]
[2103.04319]
[2105.04585]

$pp \to \gamma\gamma\gamma$
[1911.00479]
[2010.15834]
[2012.13553]

$pp \to jjj$

[1904.00945]
[2102.13609]

Integral representation
in terms of
“Pentagon functions”
[2009.07803]

Amplitudes

Full colour

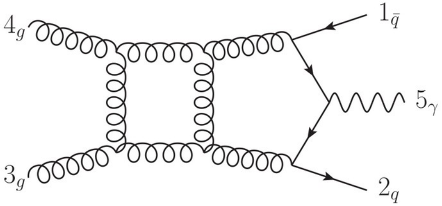
$pp \to \gamma jj$
[2304.06682]

→ $pp \to \gamma jj$ **first computation with full colour two-loop** matrix elements

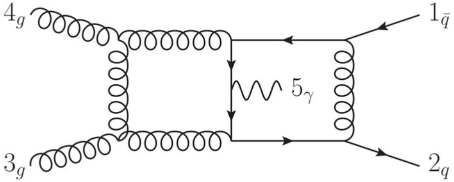
Virtual amplitudes

[2304.06682]

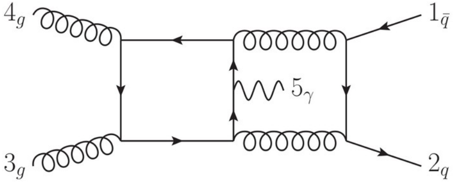
Sample diagrams



$A^{(2),N_c^2}_{34;q}, A^{(2),N_c}_{\delta;q}$



$A^{(2),1}_{34;q}, A^{(2),N_c}_{\delta;q}, A^{(2),1/N_c}_{\delta;q}$



$A^{(2),N_c}_{34;l}, A^{(2),1/N_c}_{34;l}, A^{(2),1/N_c^2}_{\delta;l}$

Decomposition:

Colour structures

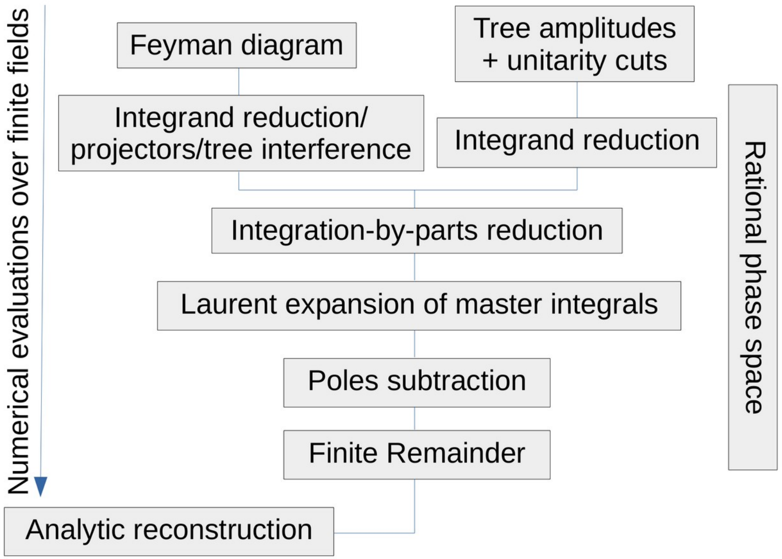
$$\mathcal{M}^{(L)}(1_{\bar{q}},2_q,3_g,4_g,5_\gamma) = \sqrt{2}\, e\, g_s^2\, n^L \Big\{ (t^{a_3}t^{a_4})_{i_2}^{\;\bar{i}_1} \mathcal{A}^{(L)}_{34}(1_{\bar{q}},2_q,3_g,4_g,5_\gamma) + (t^{a_4}t^{a_3})_{i_2}^{\;\bar{i}_1} \mathcal{A}^{(L)}_{43}(1_{\bar{q}},2_q,3_g,4_g,5_\gamma) + \delta_{i_2}^{\;\bar{i}_1}\delta^{a_3a_4} \mathcal{A}^{(L)}_{\delta}(1_{\bar{q}},2_q,4_g,3_g,5_\gamma) \Big\}$$

Independent partial amplitudes
→ different gauge couplings &
Nc/nf

$$\mathcal{A}^{(2)}_{34} = \mathcal{Q}_q N_c^2 A^{(2),N_c^2}_{34;q} + \mathcal{Q}_q A^{(2),1}_{34;q} + \mathcal{Q}_q \frac{1}{N_c^2} A^{(1),1/N_c^2}_{34;q} + \mathcal{Q}_q N_c n_f A^{(2),N_c n_f}_{34;q} + \mathcal{Q}_q \frac{n_f}{N_c} A^{(2),n_f/N_c}_{34;q} + \mathcal{Q}_q n_f^2 A^{(2),n_f^2}_{34;q} + \Big(\sum_l \mathcal{Q}_l\Big) N_c A^{(2),N_c}_{34;l} + \Big(\sum_l \mathcal{Q}_l\Big) \frac{1}{N_c} A^{(2),1/N_c}_{34;l} + \Big(\sum_l \mathcal{Q}_l\Big) n_f A^{(2),n_f}_{34;l},$$

Reconstruction of Amplitudes

Workflow [Badger,Bronnum-Hansen,Hartanto,Moodie,Peraro,Krys,Zoia]



QGRAF[Nogueira], FORM[Vermaseren,etal]
MATHEMATICA, SPINNEY[Cullen,etal]

Credit: Bayu

finite field framework: FINITEFLOW[Peraro(2019)]

IBP identities generated using LITERED[Lee(2012)]
solved numerically in FINITEFLOW using
Laporta algorithm[Laporta(2000)]

Mature technology + new optimizations

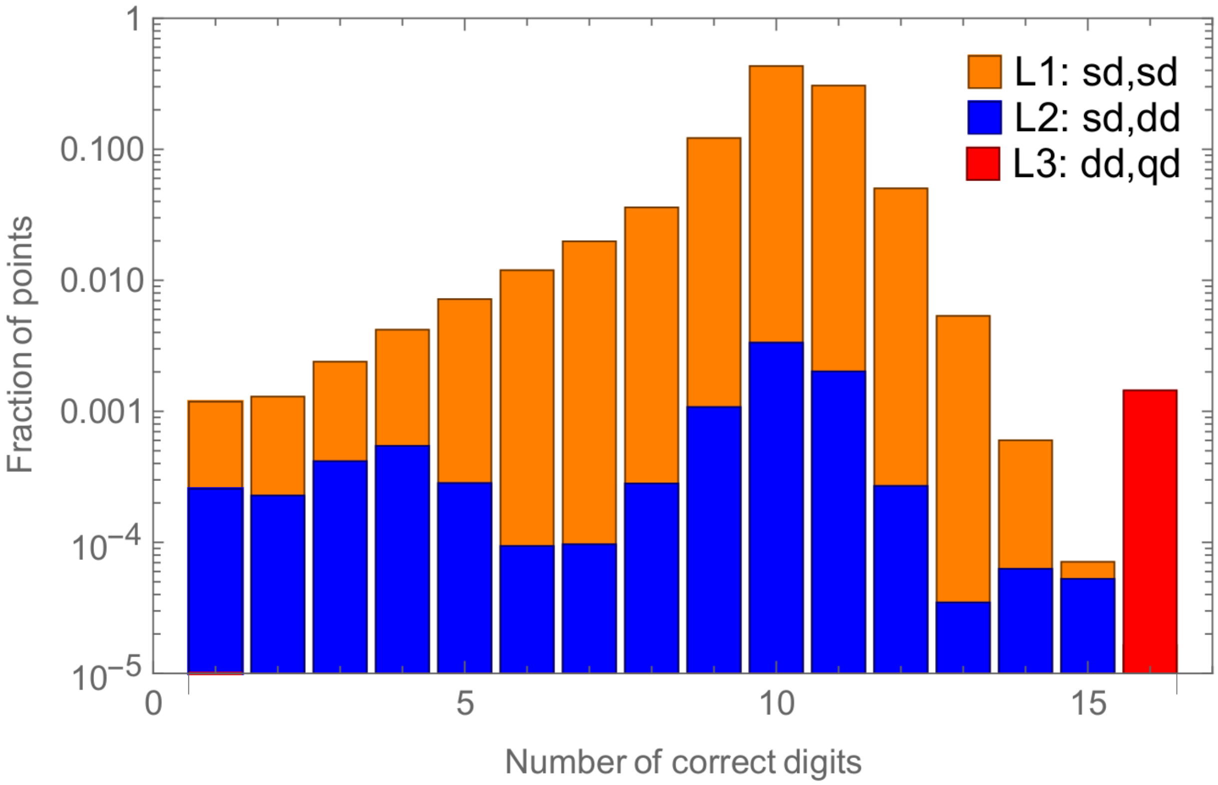
- Syzygy's to simplify IBPs
- Exploitation of Q-linear relations
- Denominator Ansaetze
- On-the-fly partial fractioning

amplitude	helicity	original	stage 1	stage 2	stage 3	stage 4
$A^{(2),1}_{34;q}$	$-++-+$	94/91	74/71	74/0	22/18	22/0
$A^{(2),1}_{34;q}$	$-+-++$	93/89	90/86	90/0	24/14	18/0
$A^{(2),1/N_c^2}_{34;q}$	$-++-+$	90/88	73/71	73/0	23/18	22/0
$A^{(2),1/N_c^2}_{34;q}$	$-+-++$	90/86	86/82	86/0	24/14	19/0
$A^{(2),1/N_c}_{34;l}$	$-+-++$	89/82	74/67	73/0	27/14	20/0
$A^{(2),1/N_c}_{34;l}$	$-++-+$	85/81	61/58	60/0	27/18	20/0
$A^{(2),N_c^2}_{34;q}$	$-+-++$	58/55	54/51	53/0	20/16	20/0

Massive reduction of complexity

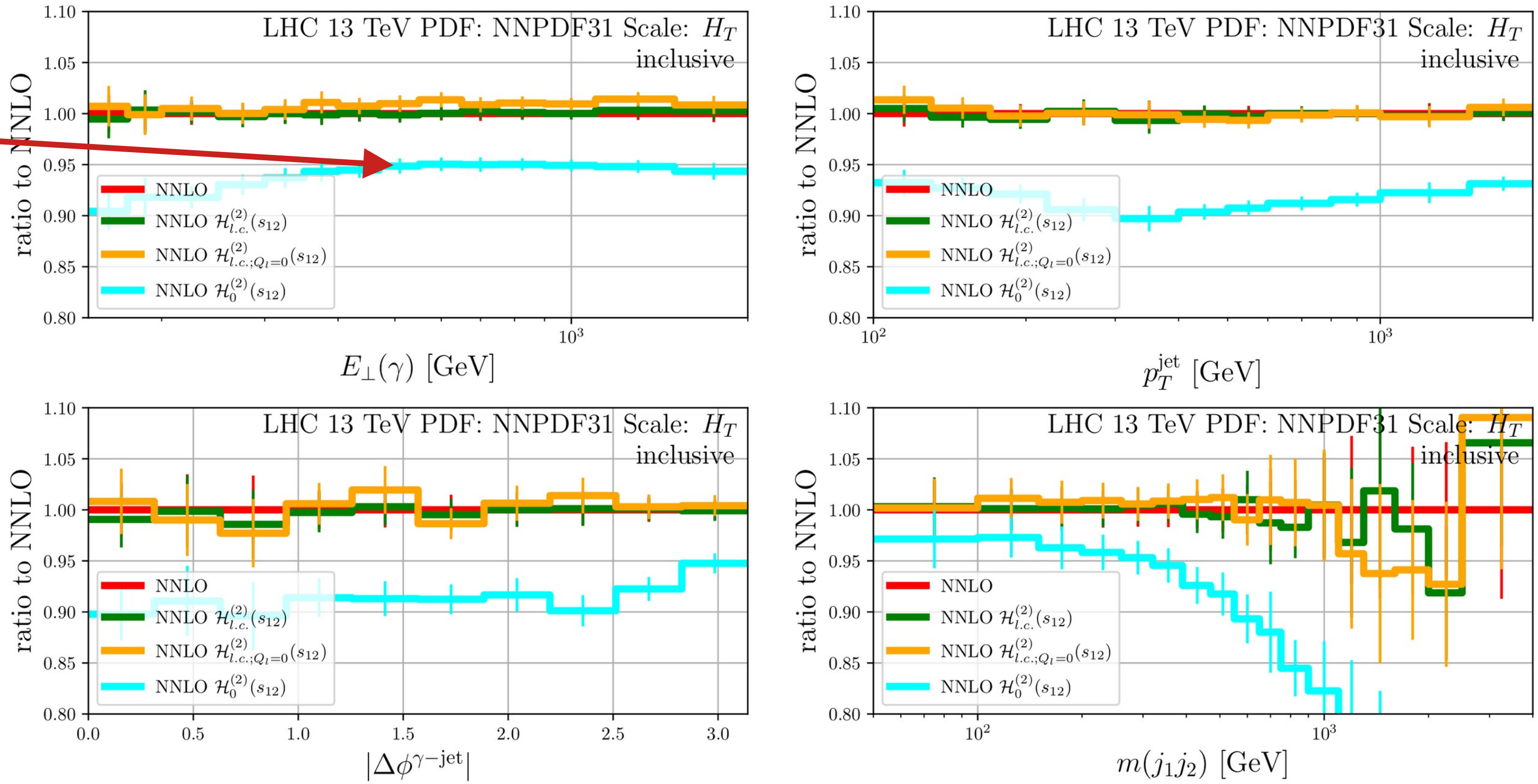
Two-loop matrix element stability

- Stable evaluation requires high floating point precision for rational functions
- In rarer cases higher precision “Pentagon” functions necessary
- 2.2 million events needed → fast evaluation essential



Quality of leading colour the approximation

Two-loop contribution
~ 5-10%
wrt. full NNLO
(scheme dep.)



"Leading colour"
Approximation
"Error" = O(~1%)
wrt full NNLO

Summary & Outlook

Summary

- Good description of pp→ γ j j ATLAS data using NNLO pQCD
  → Issues with ATLAS MC setup: multi-jet merging?
- Completion of all massless 2→ 3 processes at NNLO QCD
- First cross section with full-colour double virtual corrections
  → validated the expected quality of leading-colour approx. O(1%) of the cross section

Outlook

- Inclusion of fragmentation contributions
  → extension of hadron fragmentation to photon fragmentation
- Electro-weak corrections
- Completion of all full-colour two-loop matrix elements in the massless 2→3 case