Precision phenomenology
with multi-jet final states at the LHC

Rene Poncelet

based on 2106.05331, 2301.01086 and 2301.09351 (ATLAS)

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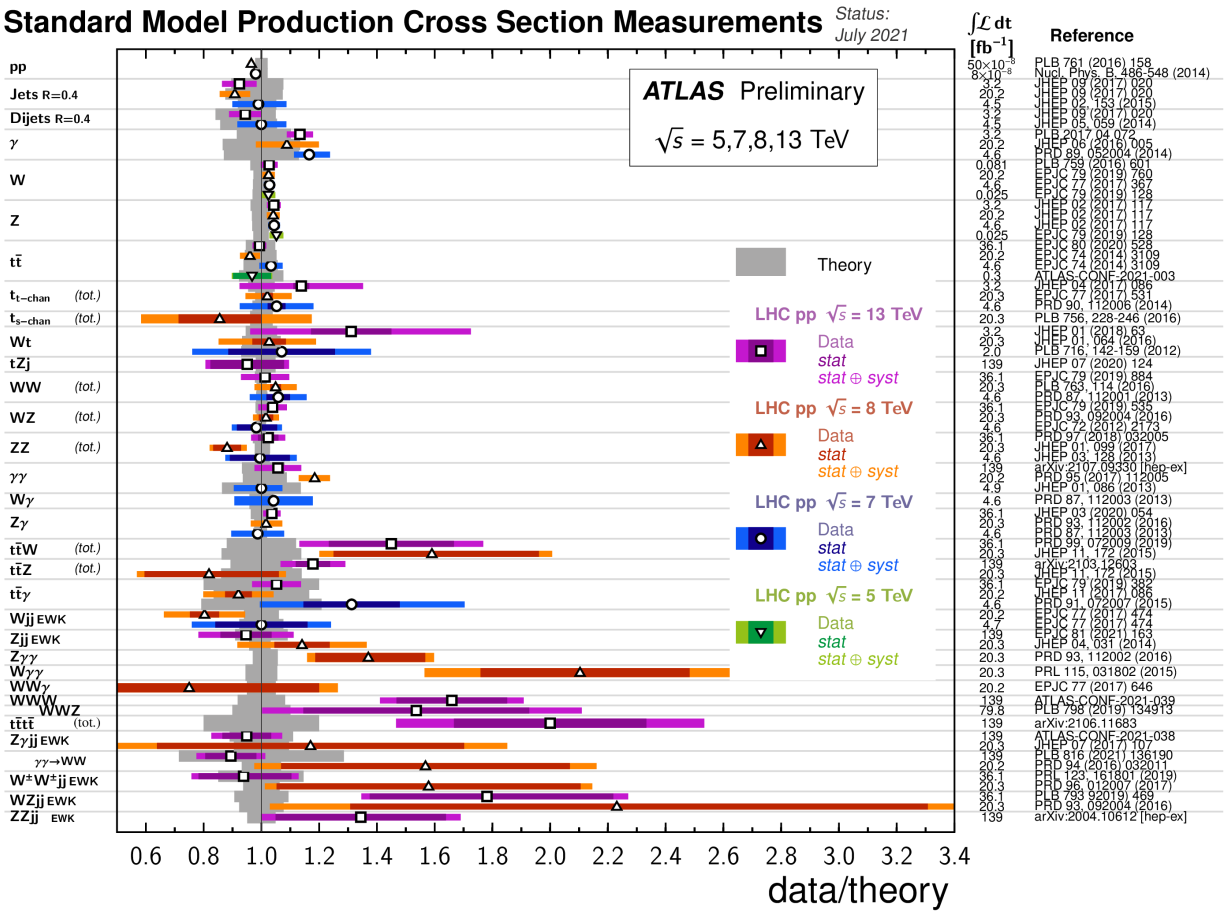
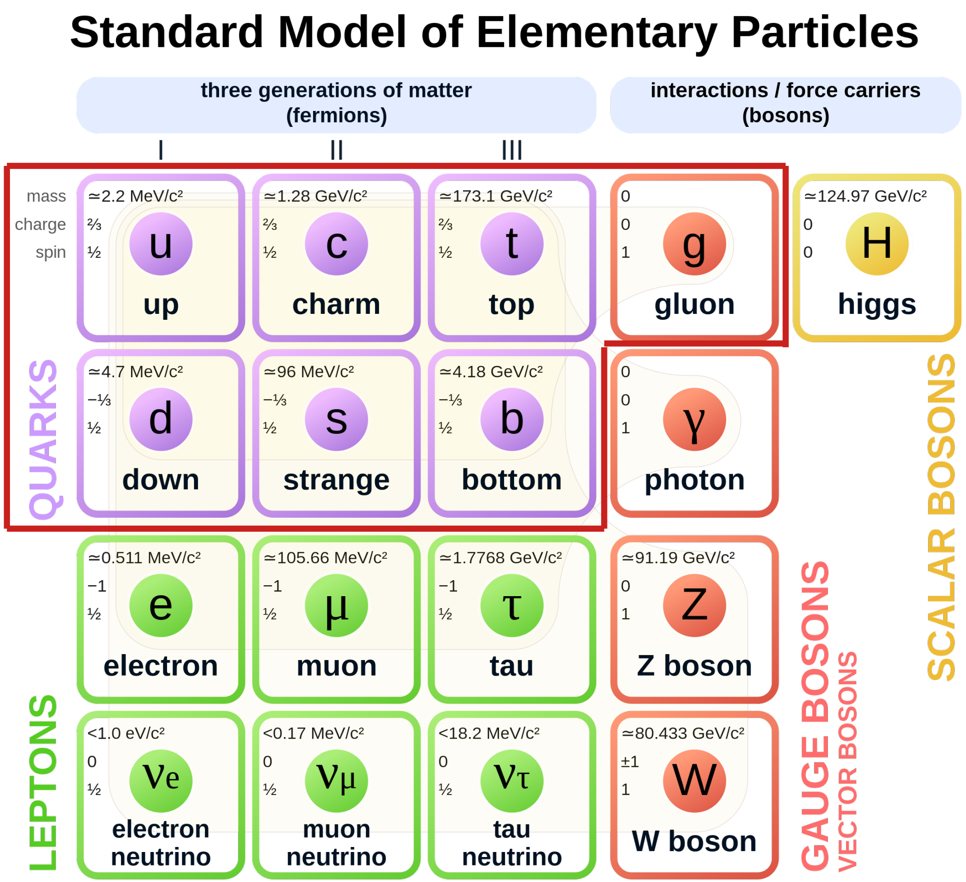
Outline

- Introduction
- Multi-jet observables/event shapes at hadron colliders
- The strong coupling constant
- NNLO QCD with STRIPPER
- Summary and conclusion

Precision era of the LHC



Precision era of the LHC

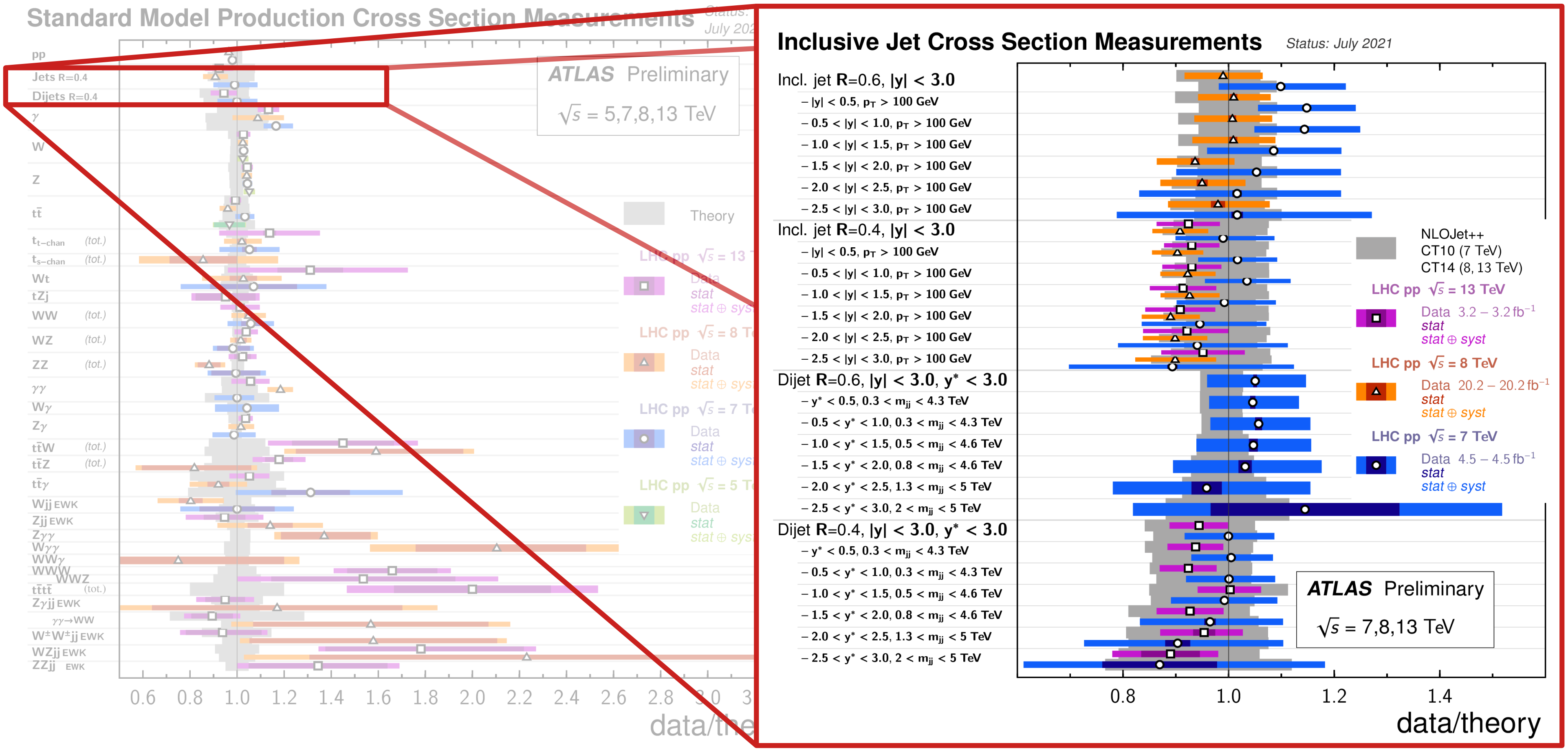


- Collider data constrains the various interactions in the Standard Model.
- At the LHC QCD is part of any process!
  1) The limiting factor in many analyses is QCD and associated uncertainties.
     → Radiative corrections indispensable
  2) How well we do know QCD? Coupling constant, running, PDFs, ...
- The production of high energy jets allow to probe pQCD at high energies directly

$$\mathcal{L}_{\text{QCD}} = \bar{q}_i(\gamma^\mu \mathcal{D}_\mu - m_i)q_i - \frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}$$

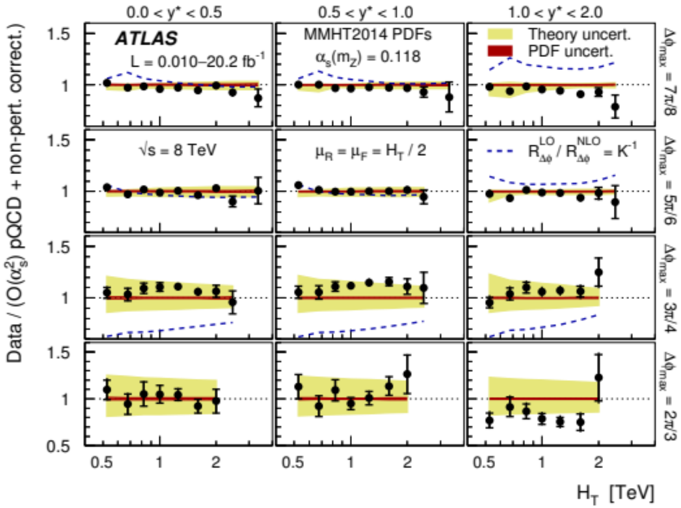
1) Testing the predicted dynamics
2) Extract the coupling constant

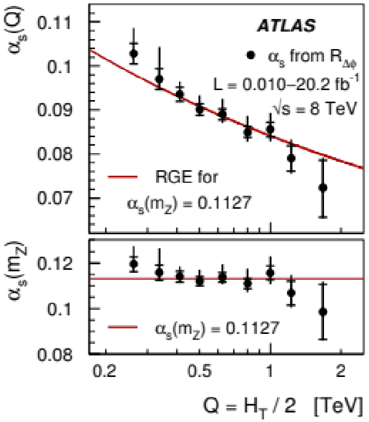
Jet measurements at the LHC



Phenomenology with jet observables

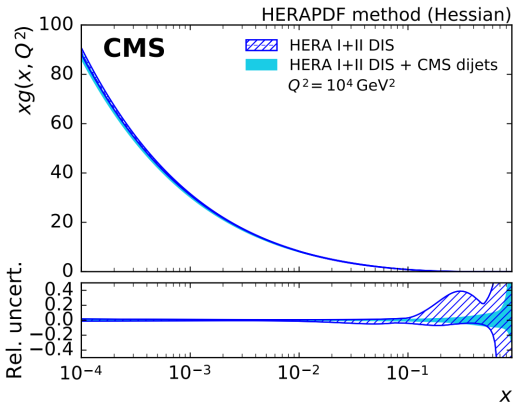
Tests of pQCD, α_s extraction:
R32 ratios, event-shapes





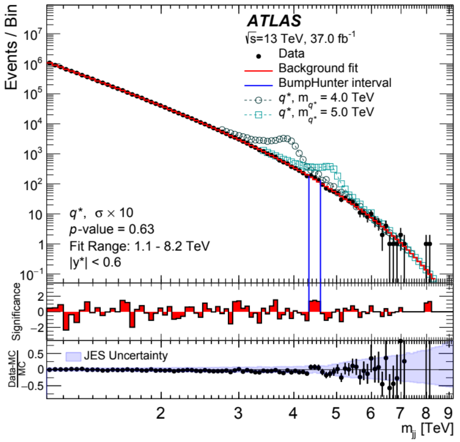
[1805.04691]

PDF determination:
Single inclusive,
Multi-differential dijet



[1705.02628]

Direct BSM:
dijet mass



[1703.09127]

Precision theory required!

Data driven

Multi-jet observables (more than 2 ...)

Jet-production processes have relatively large theory uncertainty compared to experimental uncertainties.

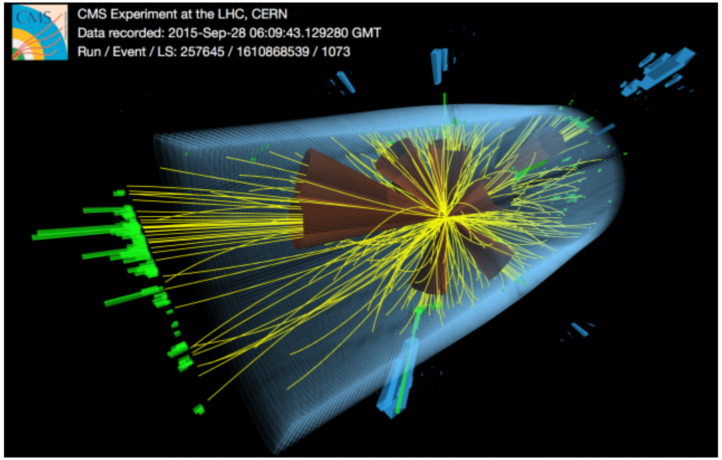
- NNLO QCD needed for precise theory-data comparisons
- Restricted precision QCD studies to incl. or di-jet data
- New NNLO QCD three-jet computations give access to many more observables!

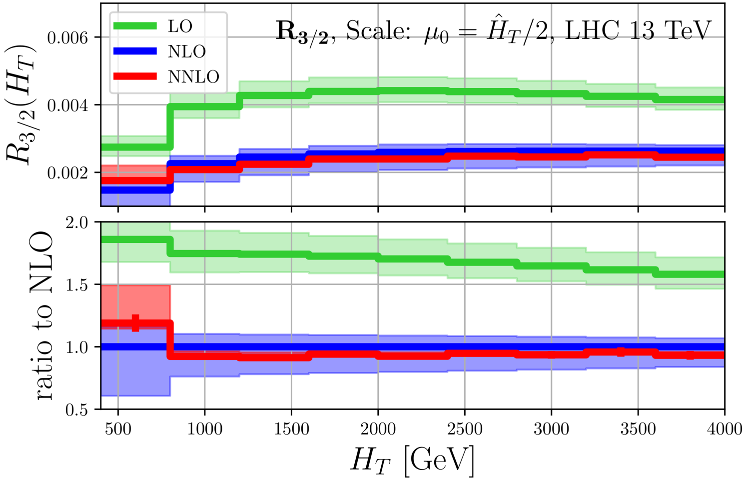
  Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC
  Czakon, Mitov, Poncelet [2106.05331]

- (From my view point) there are basically two groups:
  - Three-to-two-jet ratios

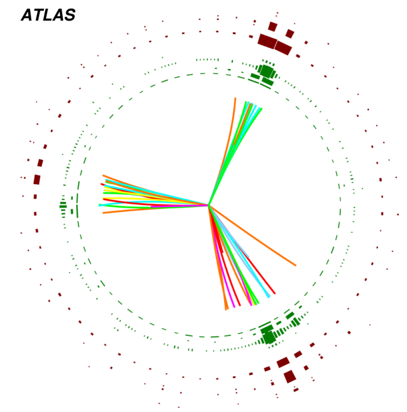
$$R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$$

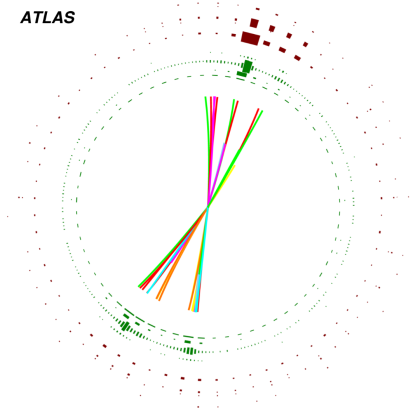
  - Event shapes (based on particles or jets)





Encoding QCD dynamics in event shapes





Using (global) event information to separate different regimes of QCD event evolution

- Thrust & Thrust-Minor

$$T_\perp = \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}, \quad \text{and} \quad T_m = \frac{\sum_i |\vec{p}_{T,i} \times \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}.$$

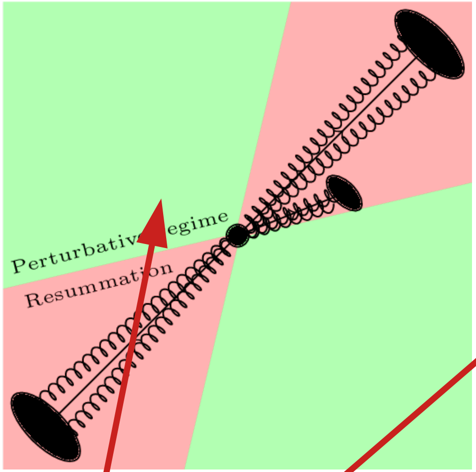
- (Transverse) Linearised Sphericity Tensor

$$\mathcal{M}_{xyz} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{1}{|\vec{p}_i|} \begin{pmatrix} p_{x,i}^2 & p_{x,i}p_{y,i} & p_{x,i}p_{z,i} \\ p_{y,i}p_{x,i} & p_{y,i}^2 & p_{y,i}p_{z,i} \\ p_{z,i}p_{x,i} & p_{z,i}p_{y,i} & p_{z,i}^2 \end{pmatrix}$$

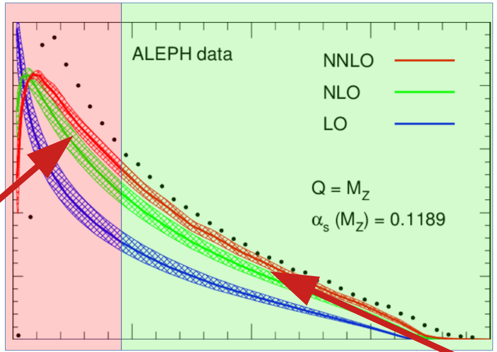
- Energy-energy correlators
- N-Jettiness
- Generalised event shapes → Earth-Mover Distance
- Many observables used in jet-substructure

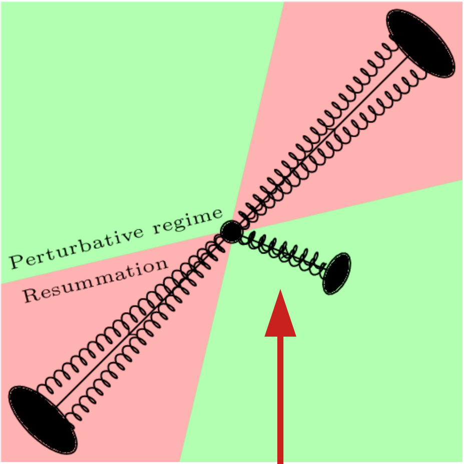
Resummation

Example: 1-Thrust at LEP



[0711.4711]





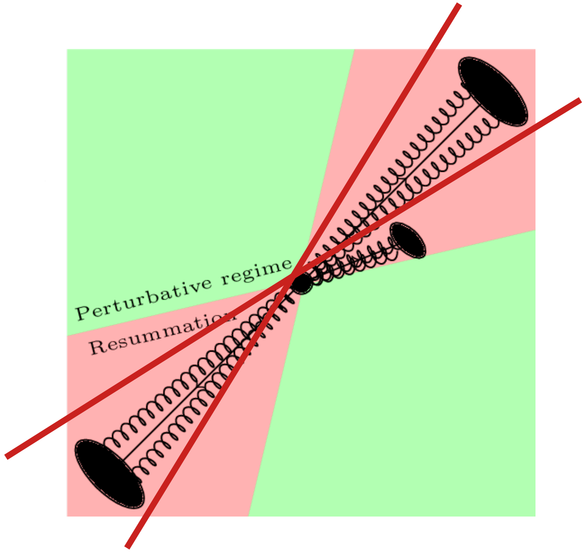
Anisotropic di-jet topology:
Sensitivity to resummation,
non-perturbative effects

Isotropic multi-jets:
Sensitive to hard
matrix elements

Nice overview: **Phenomenology of event shapes at hadron colliders,** Banfi, Salam, Zanderighi [1001.4082]

Resummation & jets

Example: 1-Thrust at LEP



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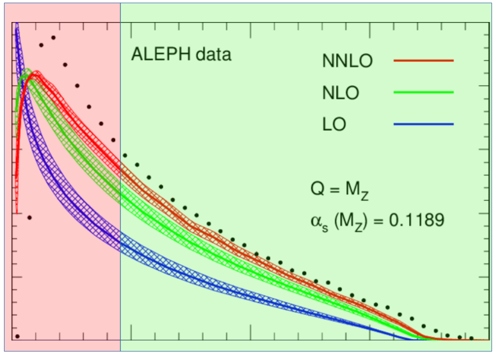
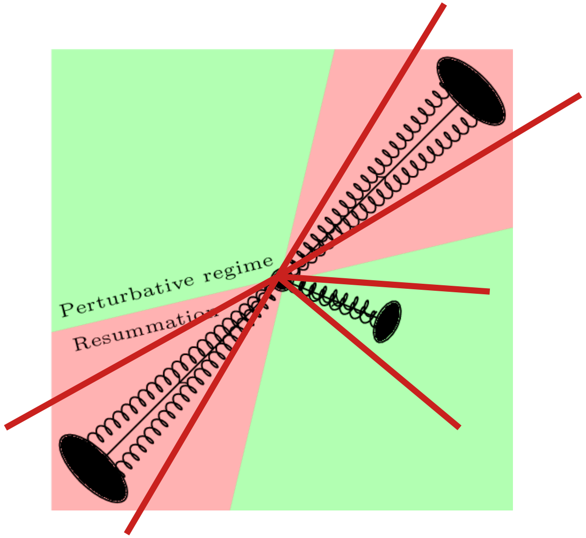




For the result presented we define event shapes in terms of jets

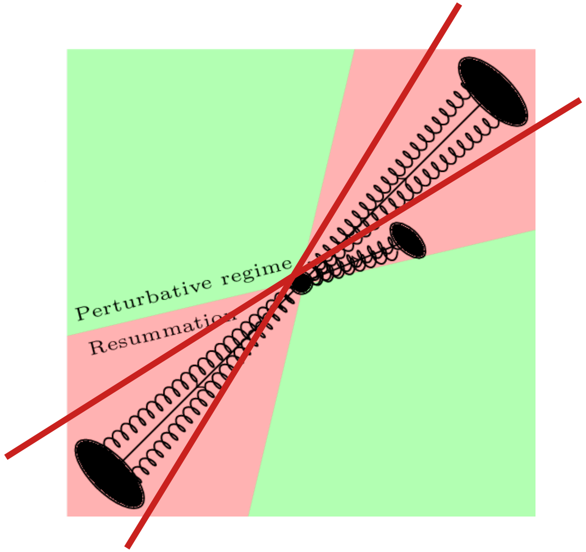
✔ Suppression of non perturbative effects

✔ Higher experimental resolution

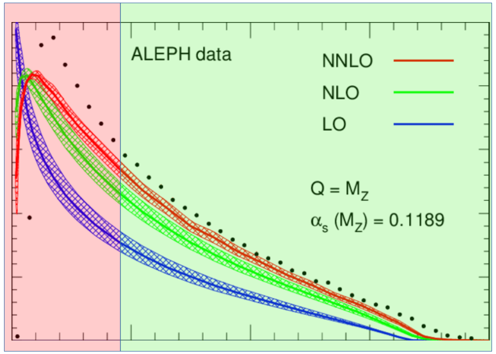
✘ But also introduce non-global logarithms

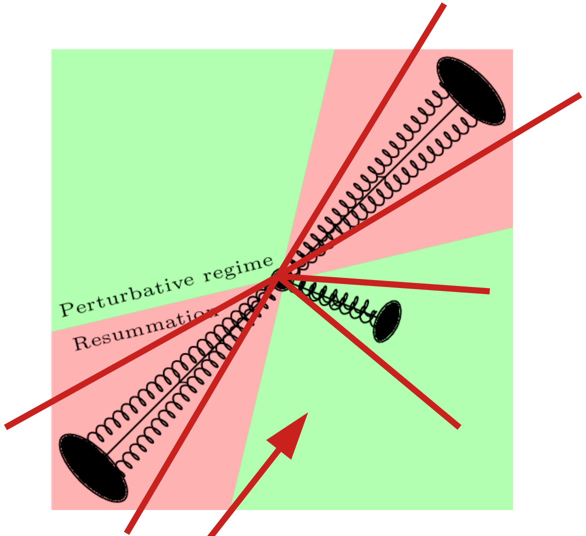
Resummation of non-global logarithms

Example: 1-Thrust at LEP



[0711.4711]





The usage of jet-algorithms implies vetoed phase space regions
→ leading to non-cancellation of soft-radiation
→ logarithmic enhancements: non-gobal logarithms
→ Resummation tricky but active field of research
→ For complicated observables → PS simulations

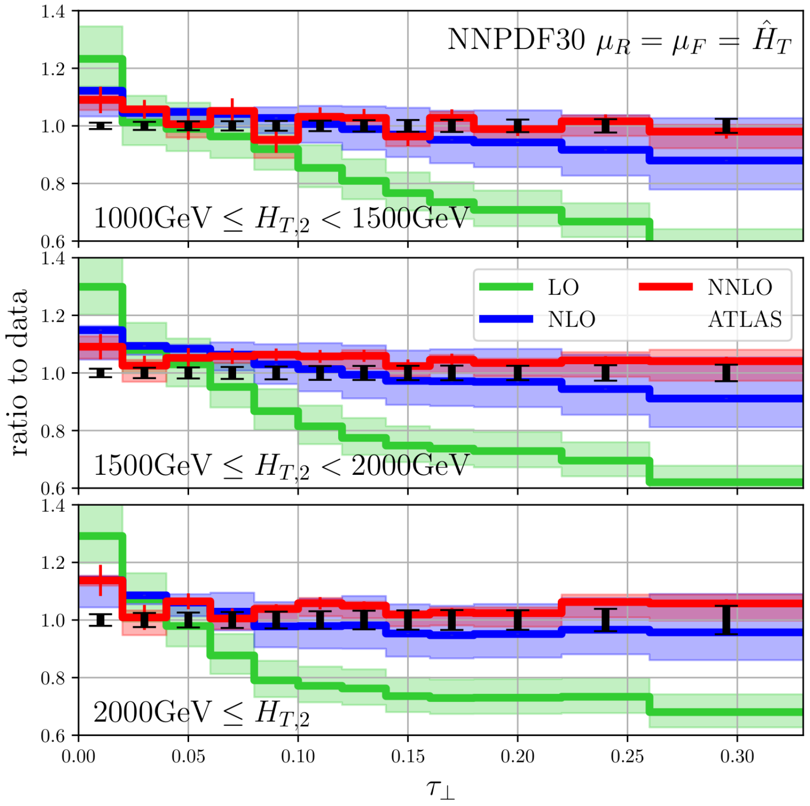
Resummation of nonglobal QCD observables
Dasgupta, Salam [0104277]

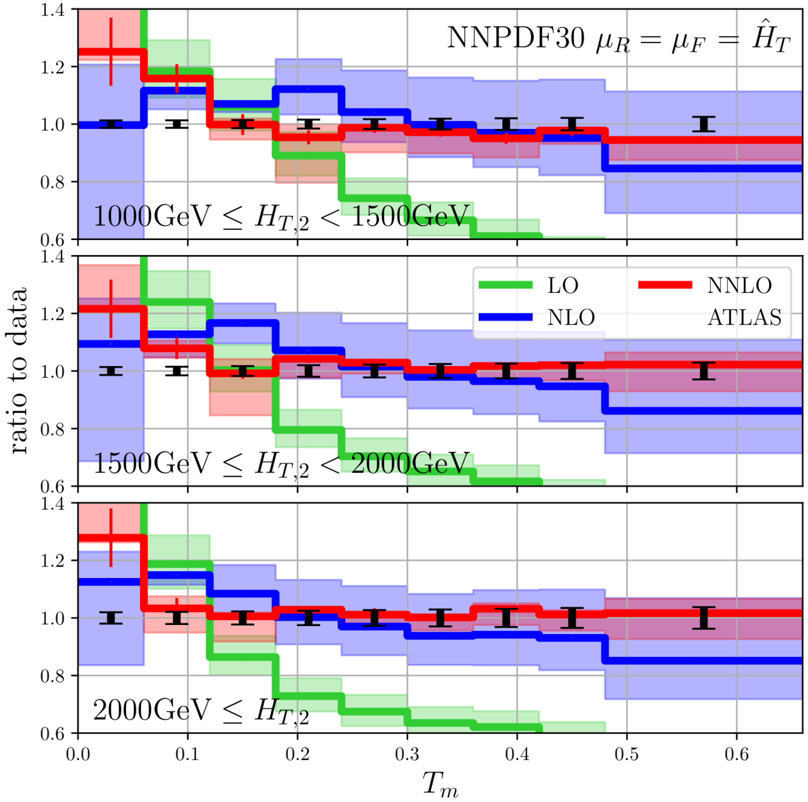
NNLO QCD three jets meets ATLAS data

NNLO QCD event shapes

Thrust & Thrust-Minor

NNLO QCD corrections to event shapes at the LHC
Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet 2301.01086

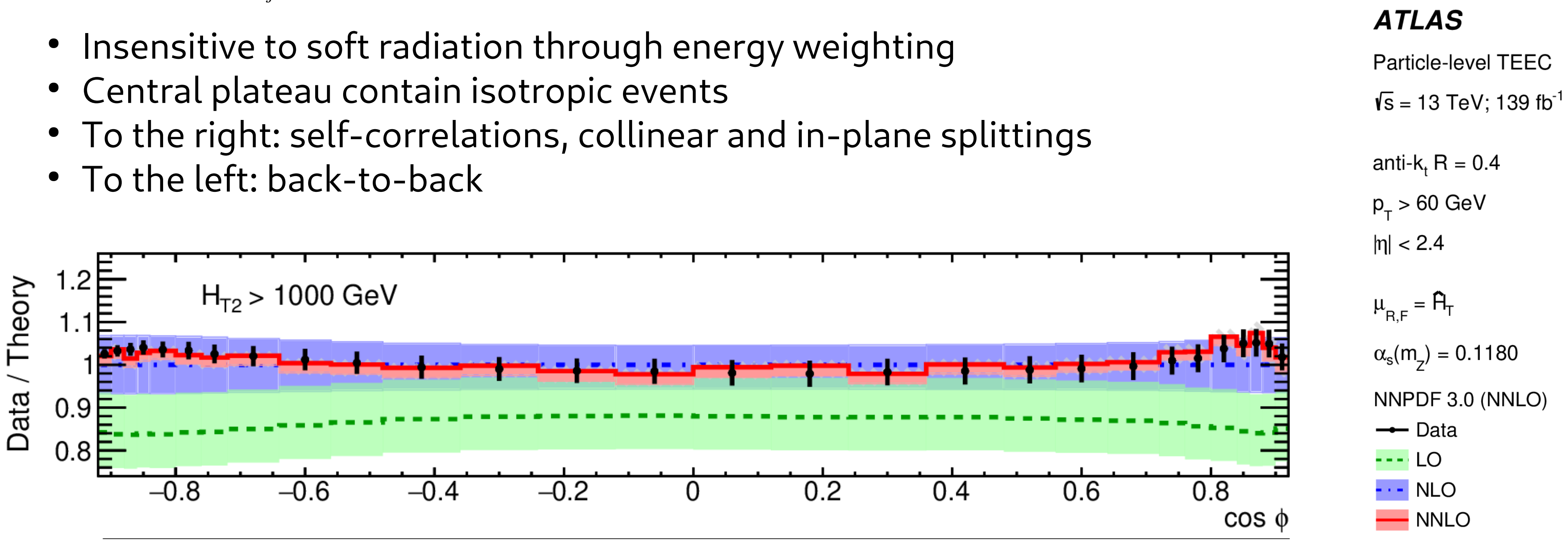




The transverse energy-energy correlator

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$

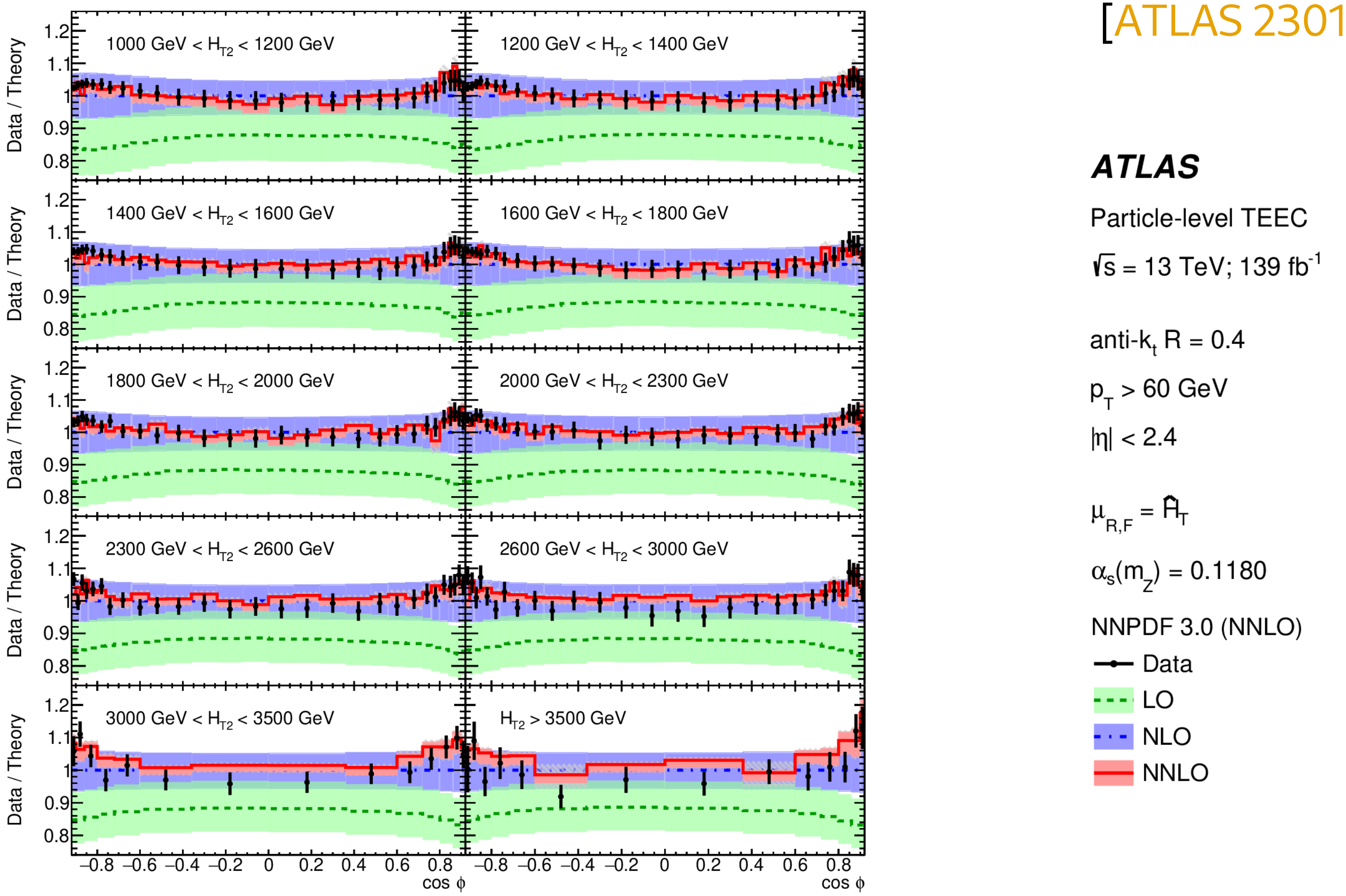
- Insensitive to soft radiation through energy weighting
- Central plateau contain isotropic events
- To the right: self-correlations, collinear and in-plane splittings
- To the left: back-to-back



[ATLAS 2301.09351]

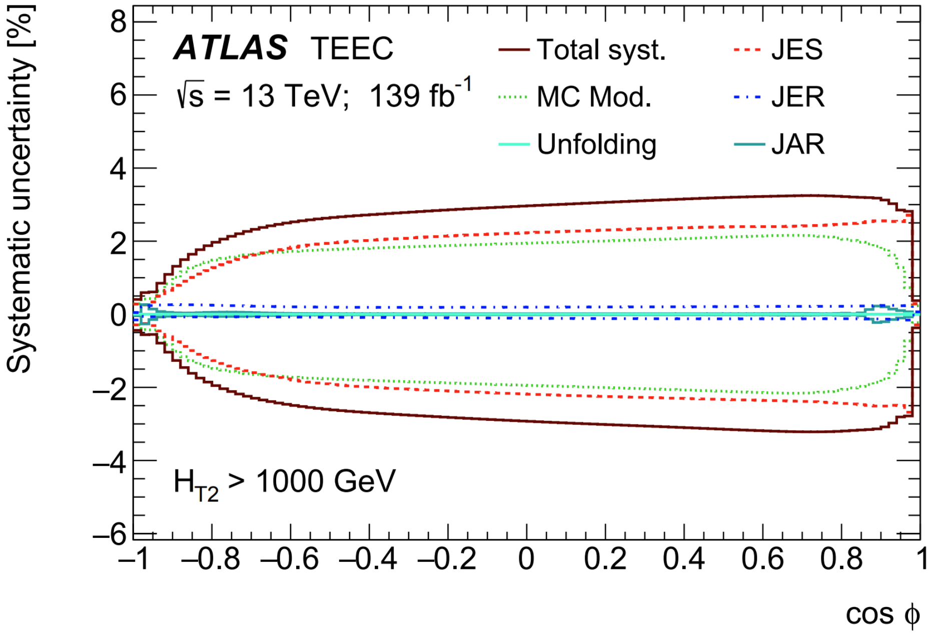
Double differential TEEC

[ATLAS 2301.09351]

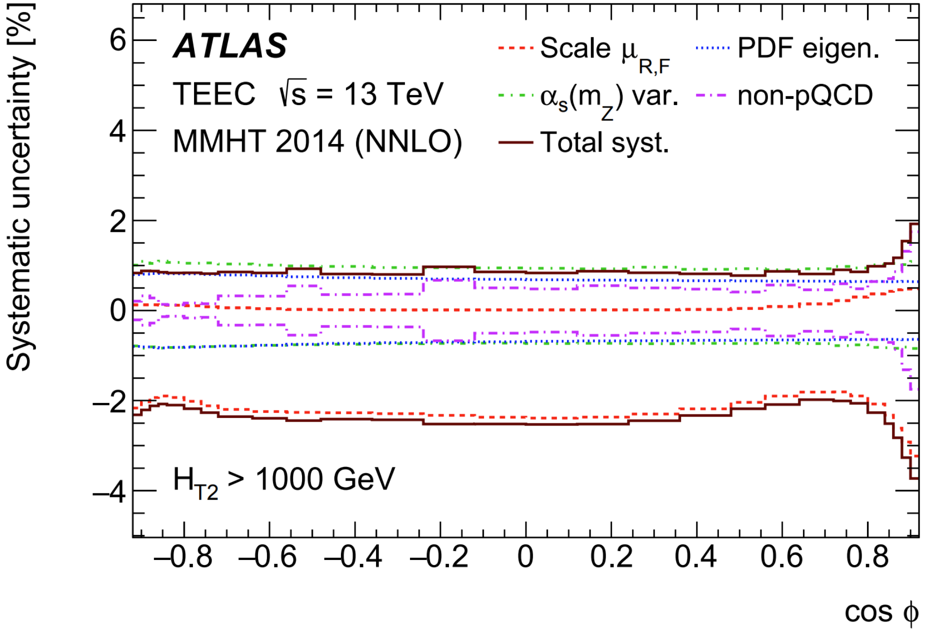


Systematic Uncertainties TEEC

Experimental uncertainties



Theory uncertainties



Scale dependence is the dominating uncertainty → NNLO QCD required to match exp.

Extraction of the strong coupling constant

Sensitivity to the strong coupling constant

- R32 ratio: $$R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$$
- Using the strong coupling's running: $$\alpha_S(\mu_R, \alpha_{S,0}) = \alpha_{S,0}\left(1 - \alpha_{S,0} b_0 \ln\left(\frac{\mu_R^2}{m_Z^2}\right) + \mathcal{O}({\alpha_{S,0}}^2)\right)$$
- Absorb running in the perturbative expansion → linear dependence

$$\begin{aligned} R^{\mathrm{NNLO}}(\mu, \alpha_{S,0}) &= \frac{\mathrm{d}\sigma_3^{\mathrm{NNLO}}(\mu, \alpha_{S,0})}{\mathrm{d}\sigma_2^{\mathrm{NNLO}}(\mu, \alpha_{S,0})} \\ &= \frac{{\alpha_{S,0}}^3\left(\mathrm{d}\tilde{\sigma}_3^{(0)}(\mu) + \alpha_{S,0}\mathrm{d}\tilde{\sigma}_3^{(1)}(\mu) + {\alpha_{S,0}}^2\mathrm{d}\tilde{\sigma}_3^{(2)}(\mu) + \mathcal{O}({\alpha_{S,0}}^3)\right)}{{\alpha_{S,0}}^2\left(\mathrm{d}\tilde{\sigma}_2^{(0)}(\mu) + \alpha_{S,0}\mathrm{d}\tilde{\sigma}_2^{(1)}(\mu) + {\alpha_{S,0}}^2\mathrm{d}\tilde{\sigma}_2^{(2)}(\mu) + \mathcal{O}({\alpha_{S,0}}^3)\right)} . \end{aligned}$$

- In practise using LHAPDF running and perform fit to Taylor expansion around $\alpha_s = 0.118$:

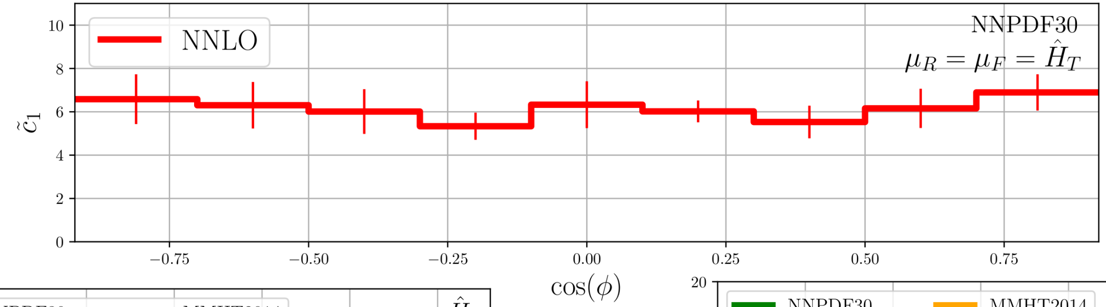
$$R^{\mathrm{NNLO,fit}}(\mu, \alpha_{S,0}) = c_0 + c_1(\alpha_{S,0} - 0.118) + c_2(\alpha_{S,0} - 0.118)^2 + c_3(\alpha_{S,0} - 0.118)^3$$

↖ dependence mostly linear

Strong coupling dependence (differential)

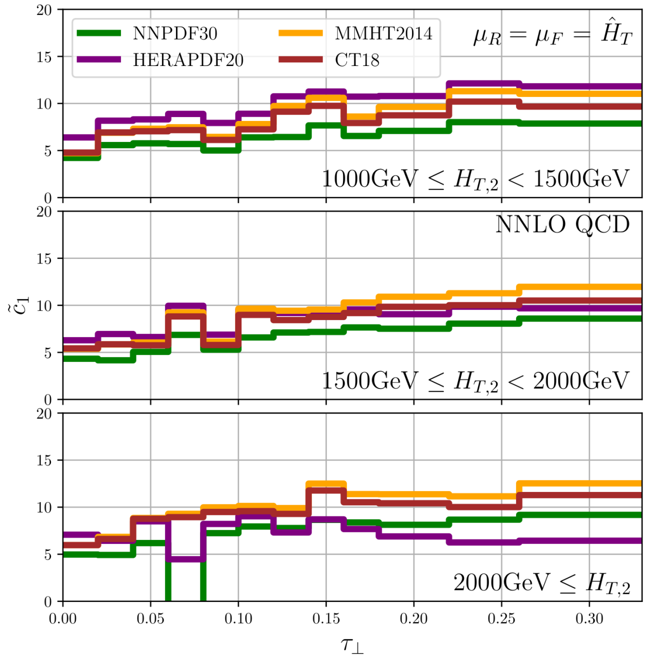
For visualisation:

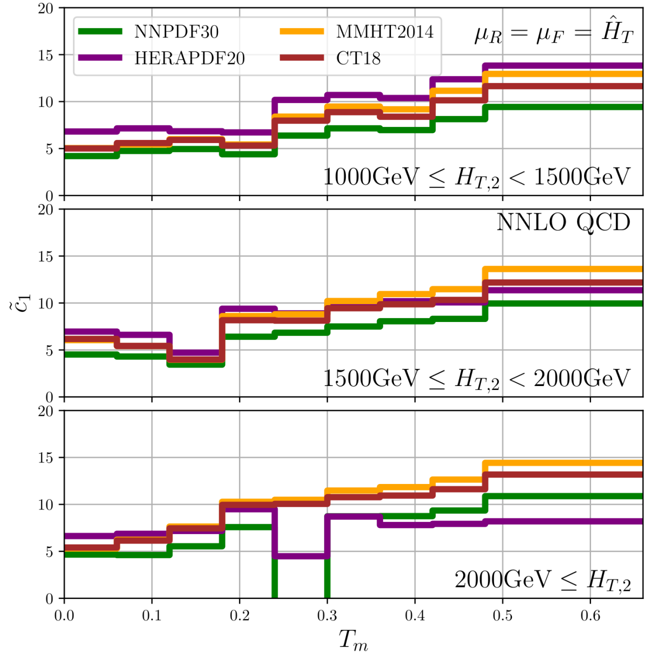
$$\tilde{c}_1 = \frac{c_1}{R^{\text{NNLO}}(\alpha_{S,0} = 0.118)}$$



TEEC
scale unc. ~2%

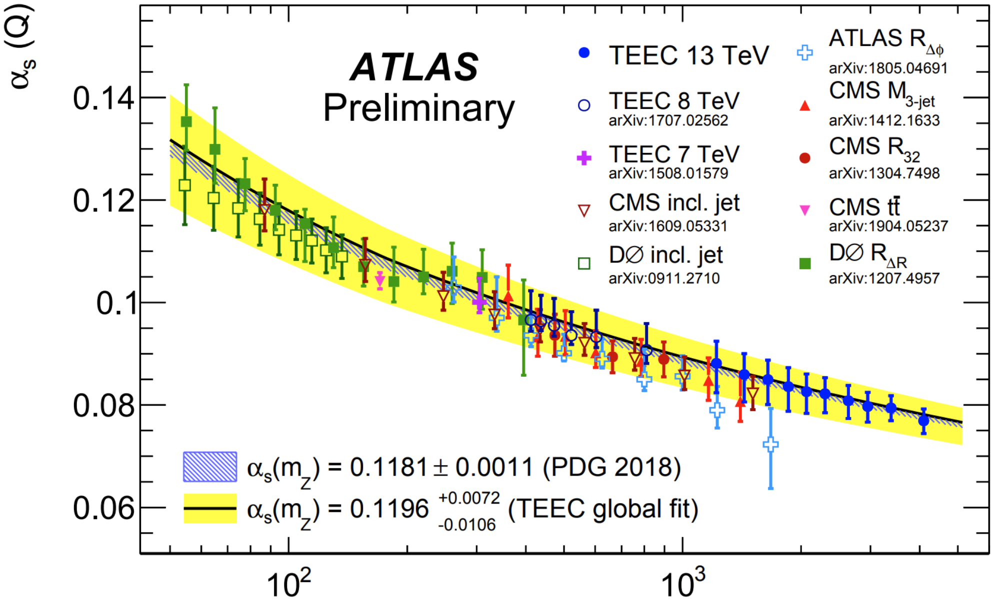
Thrust &
Thrust-Minor
scale unc. ~3-5%





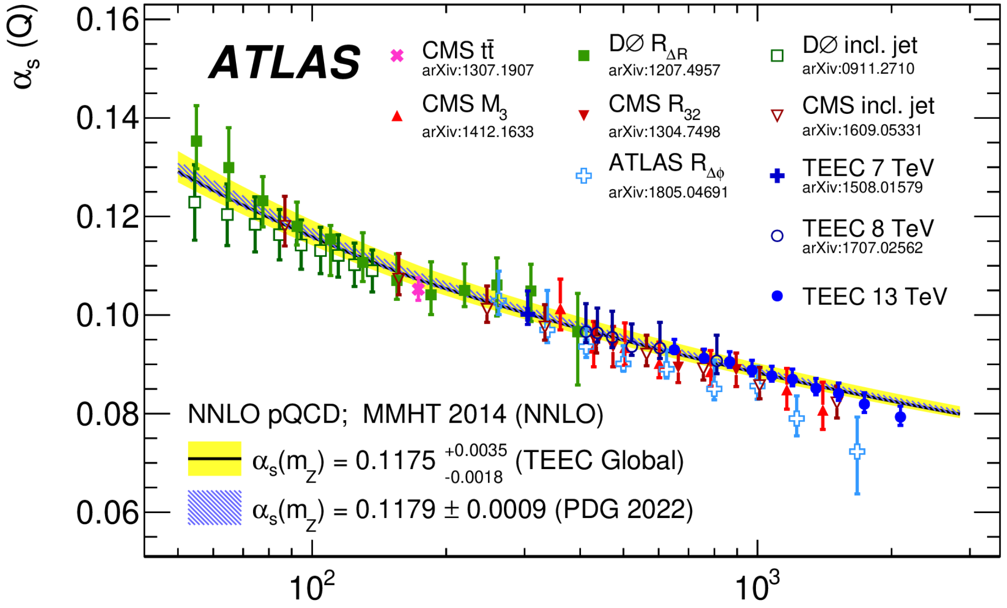
Alphas from TEEC (ATLAS)

NLO QCD



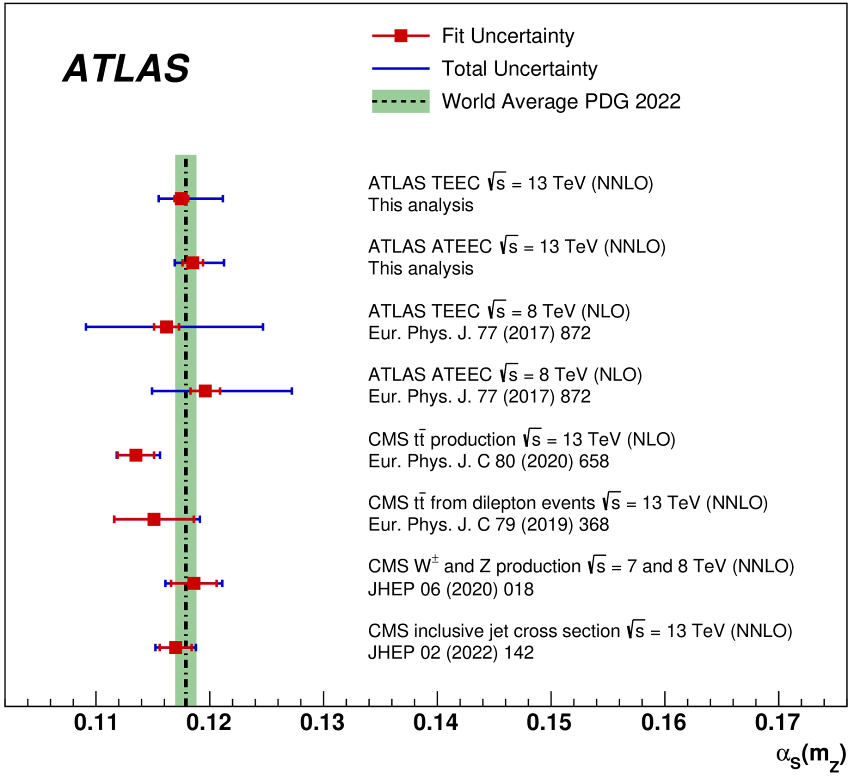
[ATLAS-CONF-2020-025]

NNLO QCD



[ATLAS 2301.09351]

Comparison against other measurements



- NNLO QCD extraction from multi-jets → will contribute to the PDG average for the first time.
- Significant improvement to 8 TeV result mainly driven by NNLO QCD corrections
- Individual precision comparable to other measurements which include DIS and top or jets-data.

Using the running of alphaS to probe NP

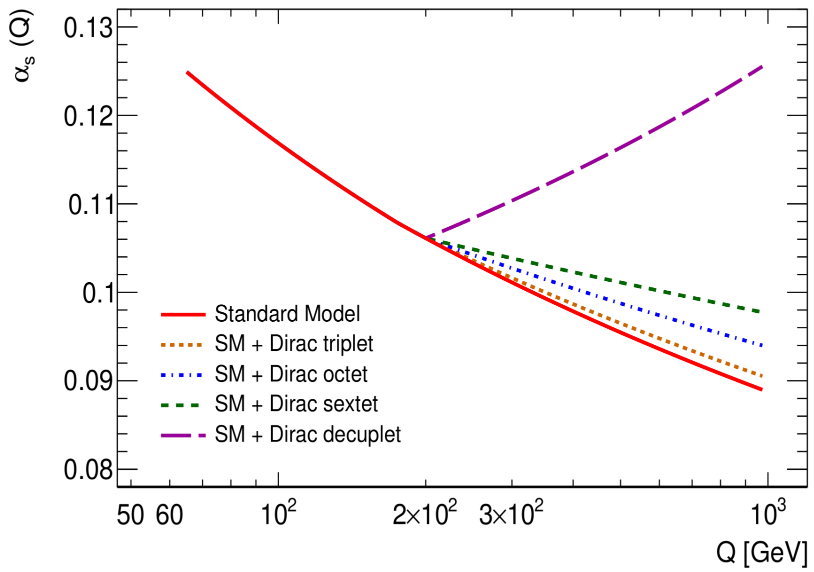
[Llorente, Nachman 1807.00894]

Indirect constraints to NP through modified running:

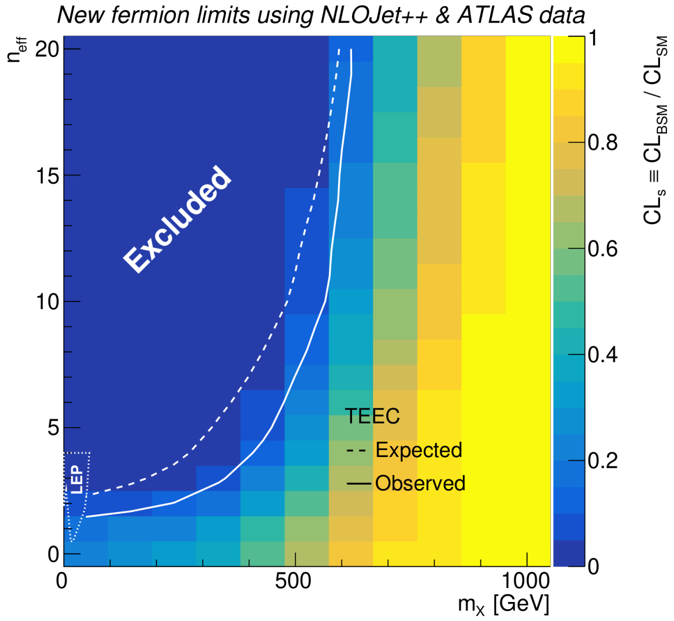
$$\beta_0 = \frac{1}{4\pi}\left(11 - \frac{2}{3}n_f - \frac{4}{3}n_X T_X\right)$$

$$\beta_1 = \frac{1}{(4\pi)^2}\left[102 - \frac{38}{3}n_f - 20 n_X T_X\left(1 + \frac{C_X}{5}\right)\right]$$

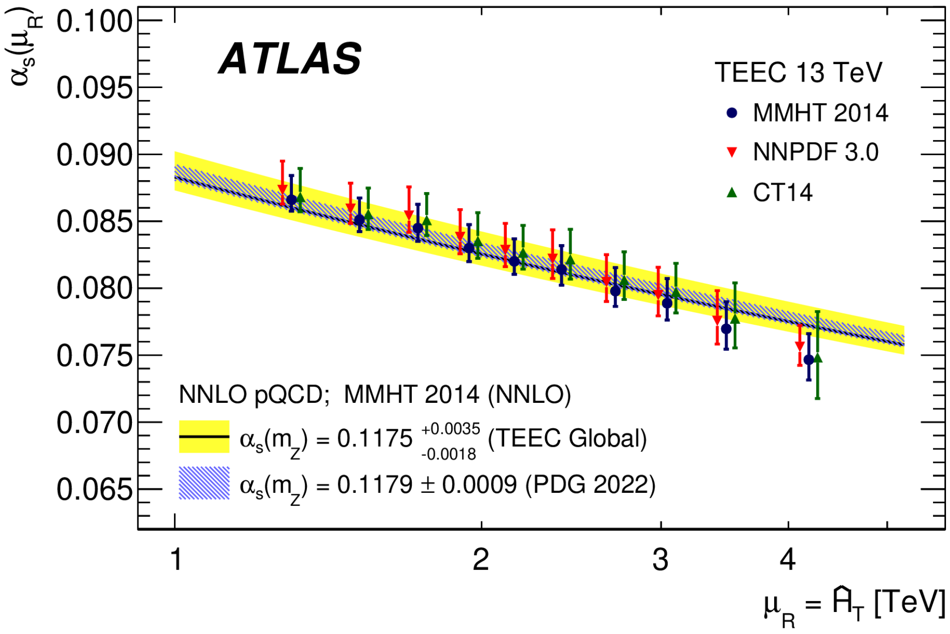
$$\alpha_s(Q) = \frac{1}{\beta_0 \log z}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\log(\log z)}{\log z}\right]; \quad z = \frac{Q^2}{\Lambda^2_{\text{QCD}}}$$



ATLAS TEEC @ 7 TeV data



Or ‘new’ SM dynamics



- Residual PDF effects → very high Q^2 ?
- EW corrections?
- Maybe effect from LC approximation in two-loop ME?

$$\mathcal{R}^{(2)}(\mu_R^2) = 2\,\mathrm{Re}\left[\mathcal{M}^{\dagger(0)}\mathcal{F}^{(2)}\right](\mu_R^2) + \left|\mathcal{F}^{(1)}\right|^2(\mu_R^2)$$

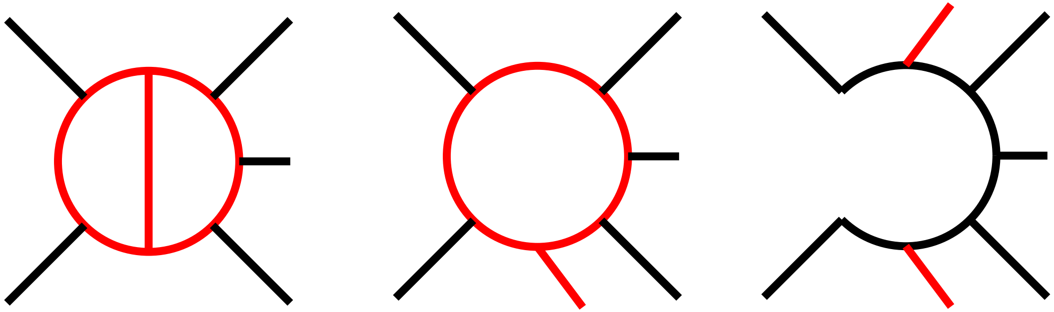
$$\equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^{4} c_i \ln^i\left(\frac{\mu_R^2}{s_{12}}\right)$$

$$\mathcal{R}^{(2)}(s_{12}) \approx \mathcal{R}^{(2)l.c.}(s_{12})$$

- Experimental systematics?
- Resummation?

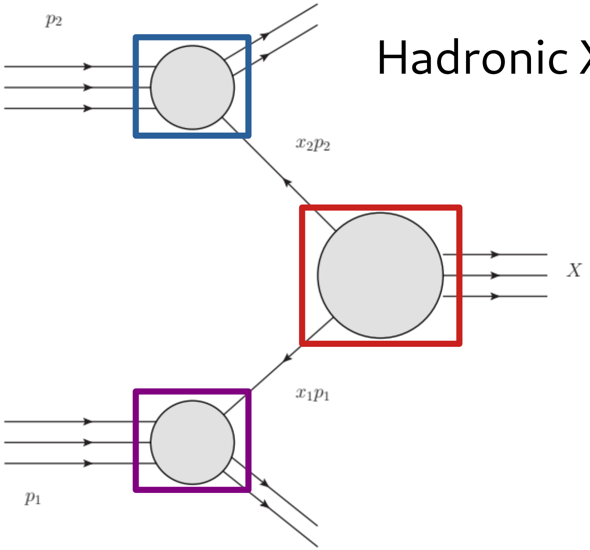
Either case interesting!

NNLO QCD cross sections with the Sector-improved residue subtraction



Rene Poncelet - Cambridge

Hadronic cross section



Hadronic X-section: $\sigma_{h_1 h_2 \to X} = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \phi_{i,h_1}(x_1, \mu_F^2) \phi_{j/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij \to X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$

Parton distribution functions

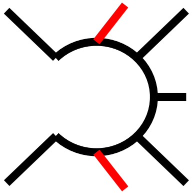
Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab \to X} = \hat{\sigma}^{(0)}_{ab \to X} + \hat{\sigma}^{(1)}_{ab \to X} + \hat{\sigma}^{(2)}_{ab \to X} + \mathcal{O}(\alpha_s^3)$$

The NNLO bit: $\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$

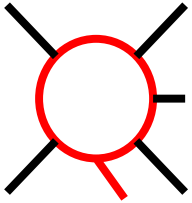
Double real radiation

$$\hat{\sigma}^{\mathrm{RR}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}^{(0)}_{n+2} \middle| \mathcal{M}^{(0)}_{n+2} \right\rangle \mathrm{F}_{n+2}$$



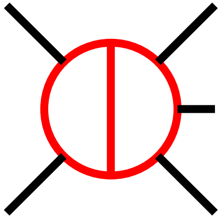
Real/Virtual correction

$$\hat{\sigma}^{\mathrm{RV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(1)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

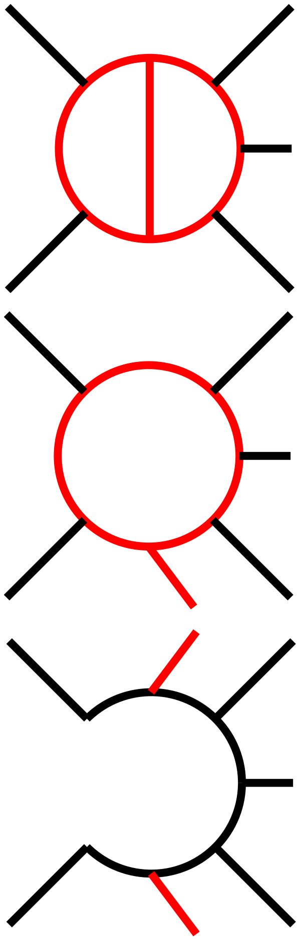


Double virtual corrections

$$\hat{\sigma}^{\mathrm{VV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n} \middle| \mathcal{M}^{(2)}_{n} \right\rangle + \left\langle \mathcal{M}^{(1)}_{n} \middle| \mathcal{M}^{(1)}_{n} \right\rangle \right) \mathrm{F}_{n}$$



NNLO QCD prediction beyond 2 → 2



2 → 3 Two-loop amplitudes:

- (Non-) planar 5 point massless 'pheno ready' [Chawdry'19'20'21,Abreu'20'21,Agarwal'21,Badger'21]
  → triggered by efficient MI representation [Chicherin'20]
- 5 point with one external mass [Abreu'20,Syrrakos'20,Canko'20,Badger'21'22,Chicherin'22]
- For three jet we use the implementation from [Abreu'20'21] checked against NJET

Many leg, IR stable one-loop amplitudes → OpenLoops [Buccioni'19]

Combination with double real radiation

- Various NNLO subtraction schemes are available:
  qT-slicing [Catain'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projetction [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17], Local Analytic [Magnea'18], Sector-improved residue subtraction [Czakon'10-'14,'19]

Partonic cross section beyond LO

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

Contributions with different multiplicities and # convolutions:

$$\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$$

$$\hat{\sigma}^{\mathrm{RR}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\left\langle \mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}^{\mathrm{RV}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re}\left\langle \mathcal{M}^{(0)}_{n+1}\middle|\mathcal{M}^{(1)}_{n+1}\right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\mathrm{VV}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n}\left(2\mathrm{Re}\left\langle \mathcal{M}^{(0)}_{n}\middle|\mathcal{M}^{(2)}_{n}\right\rangle + \left\langle \mathcal{M}^{(1)}_{n}\middle|\mathcal{M}^{(1)}_{n}\right\rangle\right) \mathrm{F}_{n}$$

$$\hat{\sigma}^{\mathrm{C1}}_{ab} = (\text{single convolution})\,\mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\mathrm{C2}}_{ab} = (\text{double convolution})\,\mathrm{F}_{n}$$

Each term separately IR divergent. But sum is:

→ finite

→ regularization scheme independent

Considering CDR ($d = 4 - 2\epsilon$):

→ Laurent expansion:

$$\hat{\sigma}^{C}_{ab} = \sum_{i=-4}^{0} c_i \epsilon^i + \mathcal{O}(\epsilon)$$

Sector decomposition I

Considering working in CDR:

→ Virtuals are usually done in this regularization

→ Real radiation:

→ Very difficult integrals, analytical impractical (except very simple cases)!

→ Numerics not possible, integrals are divergent: ε-poles!

How to extract these poles? → Sector decomposition!

Divide and conquer the phase space:

$$1 = \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right]$$

→

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

Sector decomposition II

Divide and conquer the phase space:

→ Each $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$ has simpler divergences.

appearing as $1/s_{ijk} \quad 1/s_{ik}/s_{jl}$

Soft and collinear (w.r.t parton k,l) of partons i and j

→ Parametrization w.r.t. reference parton:

$$\hat{\eta}_i = \frac{1}{2}(1-\cos\theta_{ir}) \in [0,1] \qquad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$$

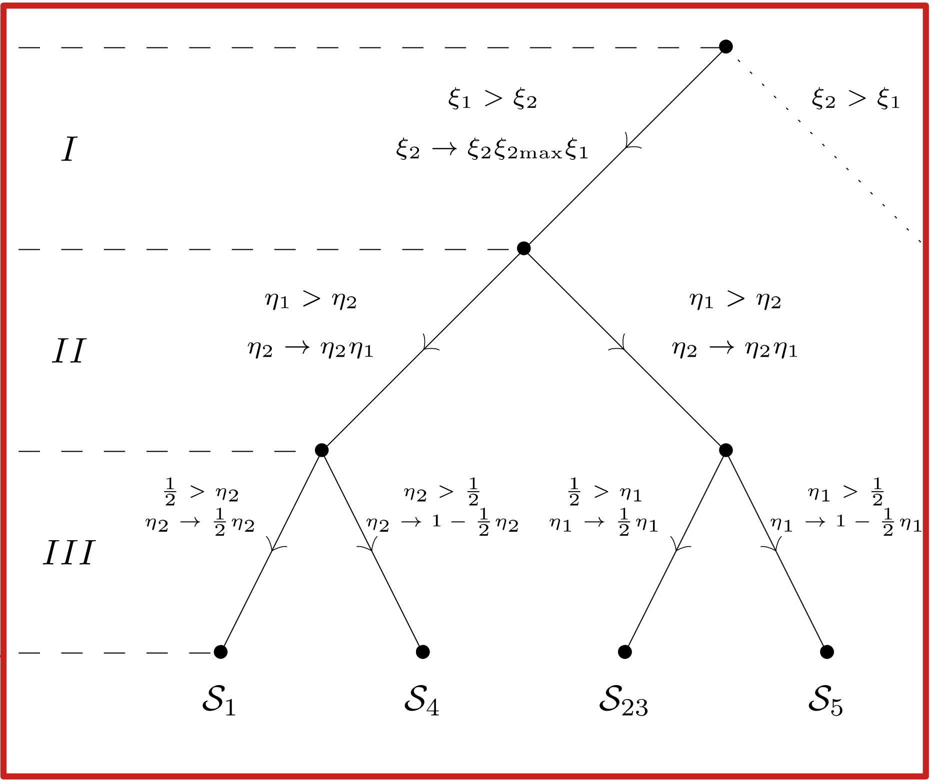
→ Subdivide to factorize divergences

$$s_{u_1 u_2 k} = (p_k + u_1 + u_2)^2 \sim \hat{\eta}_1 u_1^0 + \hat{\eta}_2 u_2^0 + \hat{\eta}_3 u_1^0 u_2^0$$

→ double soft factorization:

$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

→ triple collinear factorization



[Czakon'10,Caola'17]

Sector decomposition III

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\,\mathcal{S}_{kl,m}\left\langle\mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2} = \sum_{\text{sub-sec.}}\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i\,\underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}}\,\mathrm{d}\tilde{\mu}(\{x_i\})\underbrace{\prod x_i^{a_i+1}\left\langle\mathcal{M}_{n+2}|\mathcal{M}_{n+2}\right\rangle}_{\text{regular}}\mathrm{F}_{n+2}$$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 \mathrm{d}x\,\left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x)-f(0)}{x^{1+b\epsilon}}$$

Finite NNLO cross section

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_{n}$$

↓ sector decomposition and master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_{+}}_{\text{reg. + sub.}}$$

$$(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}) \quad (\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}) \quad (\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}) \quad (\sigma_{SU}^{C1}, \sigma_{DU}^{C1}) \quad (\sigma_{DU}^{C2}, \sigma_{FR}^{C2})$$

↓ re-arrangement of terms → 4-dim. formulation [Czakon'14, Czakon'19]

$$\underline{(\sigma_F^{RR})} \quad \underline{(\sigma_F^{RV})} \quad \underline{(\sigma_F^{VV})} \quad \underline{(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1})} \quad \underline{(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2})} \quad \underline{(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2})}$$

separately finite: ε poles cancel

Improved phase space generation

Phase space cut and differential observable introduce

mis-binning : mismatch between kinematics in subtraction terms

→ leads to increased variance of the integrand

→ slow Monte Carlo convergence

New phase space parametrization [Czakon'19]:

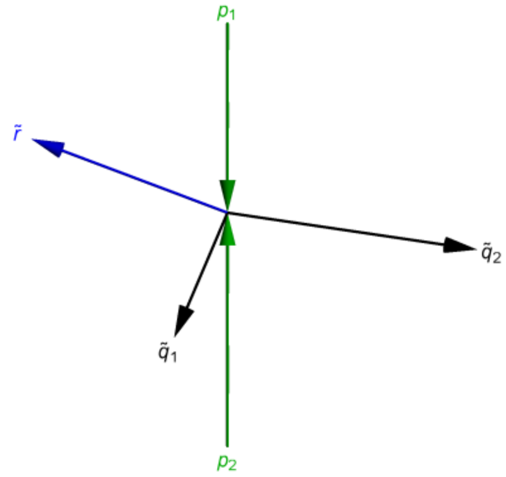
Minimization of # of different subtraction kinematics in each sector

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed : u_i $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,, \;\; \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

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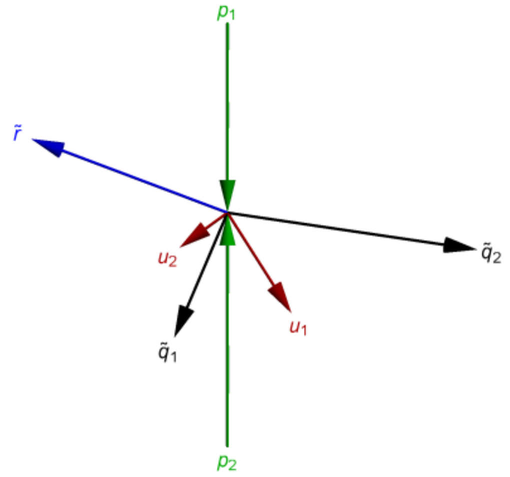

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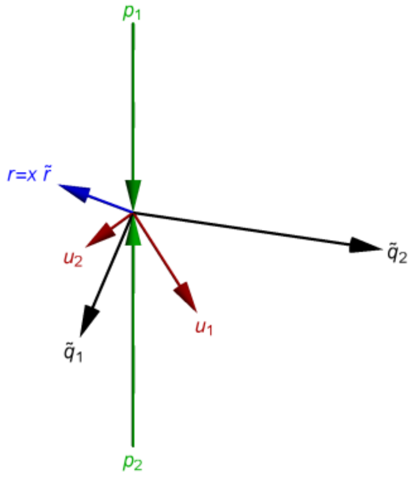

Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i : $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \rightarrow \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,, \quad \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

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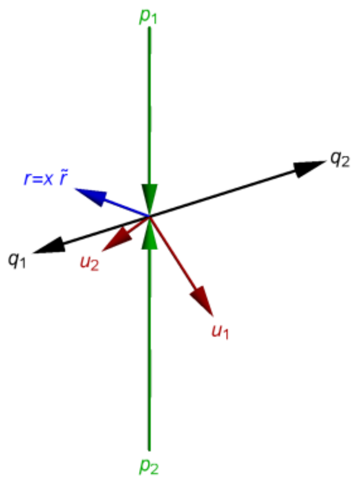

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Main steps:

- Generate Born configuration
- Generate unresolved partons u_i
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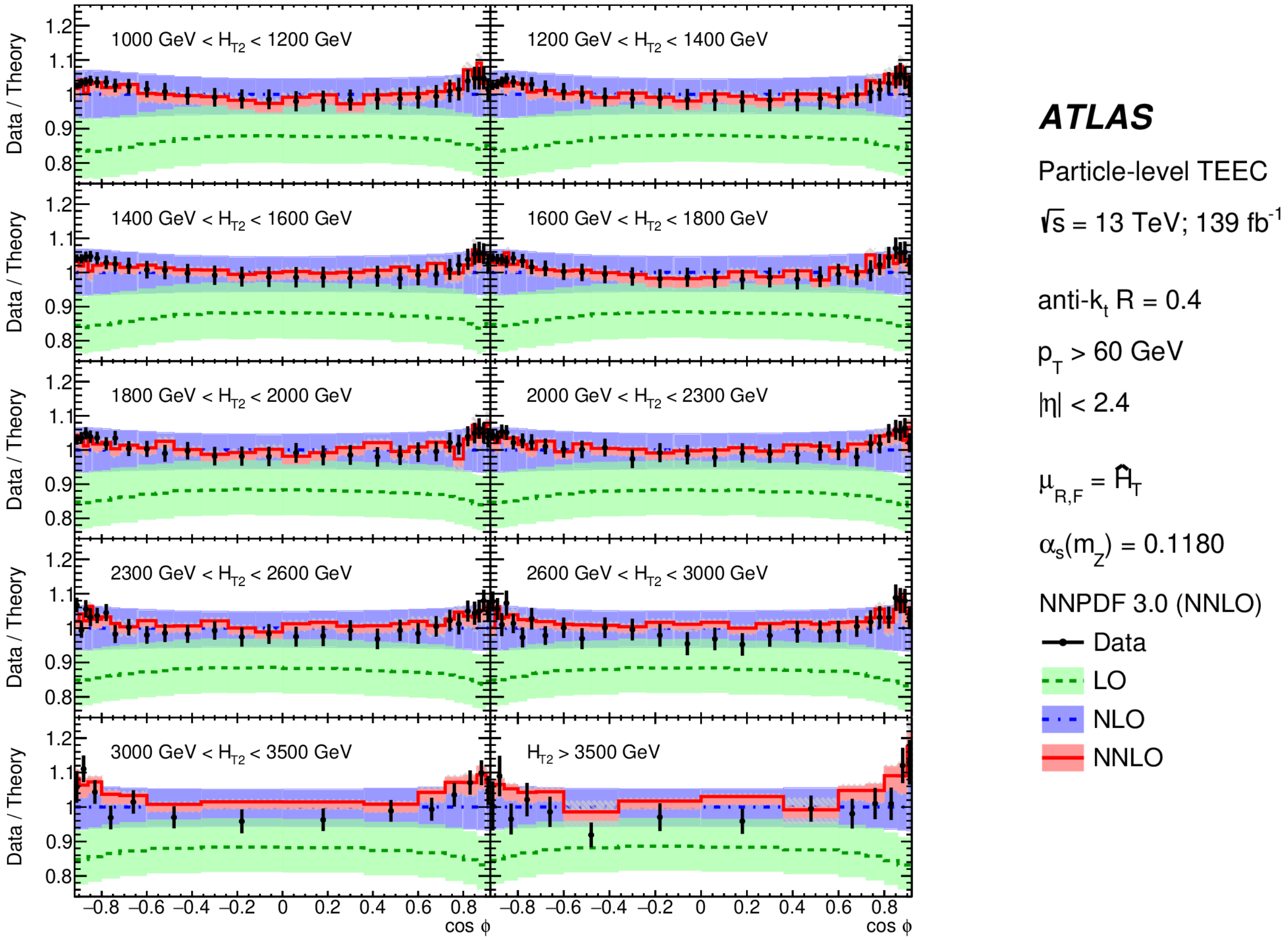
Further technical developments

- Narrow-width-approximation and Double-Pole-Approximation for resonant particles:
  - Top-quark pairs + decays [Czakon'19'20]
  - Polarised vector-bosons [Poncelet'21,Pellen'21'22]
- Automated interfaces to OpenLoops, Recola and Njet
- Implementation of state-of-the-art twoloop matrix-elements:
  - 2→ 2(1) : pp → VV, pp→ Vj, pp → H (j), e+e- → jets, DIS
  - 2 → 3: pp → 3γ, pp → 2γ + j, pp → 3j
- Fragmentation of massless partons into hadrons
  - First application to pp → tt +X → l+l- ν ν~ B + X (NWA) [Czakon'21'22]
- Countless small improvements in terms of organization and efficiency

Closing the loop



The technical developments have been crucial for applications like event shapes @ NNLO (O(10 M) CPUh). Without not feasible!

Summary & Outlook

- Three jet NNLO QCD predictions allow for precision pheno with multi-jet final states
- First predictions for R32 ratios and event shapes
- Extraction of the strong coupling constant from event shapes by ATLAS → will contribute to PDG ave.
- Relatively costly enterprise
  → effective NNLO QCD cross section tools needed
  → optimized STRIPPER subtraction scheme

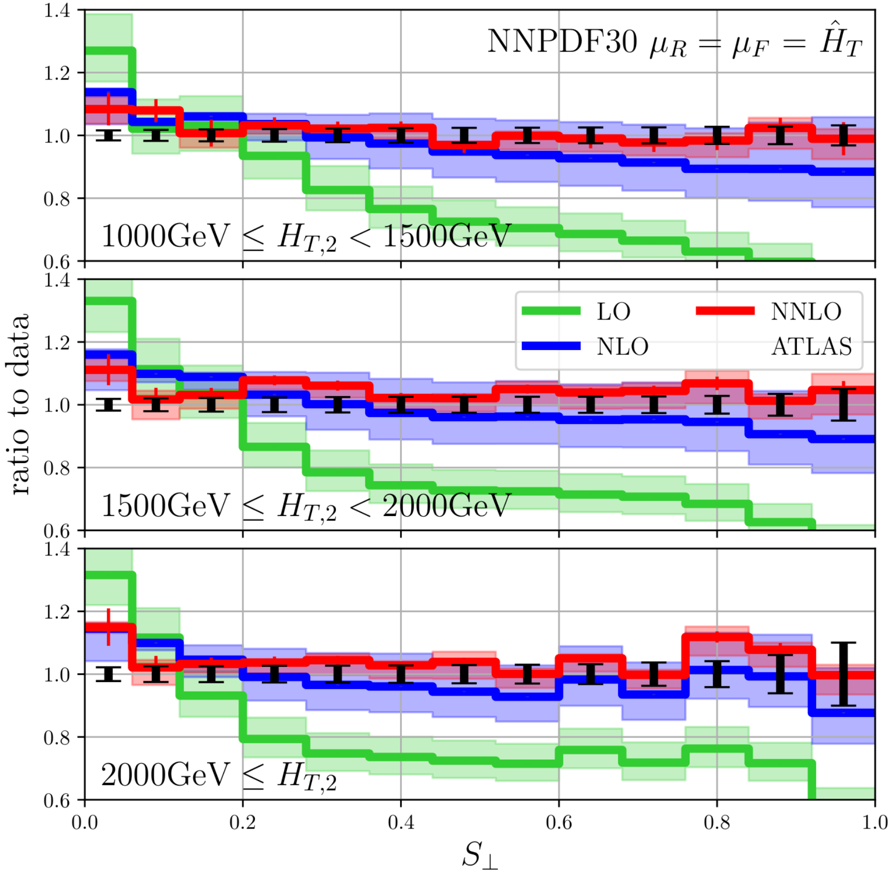
Outlook

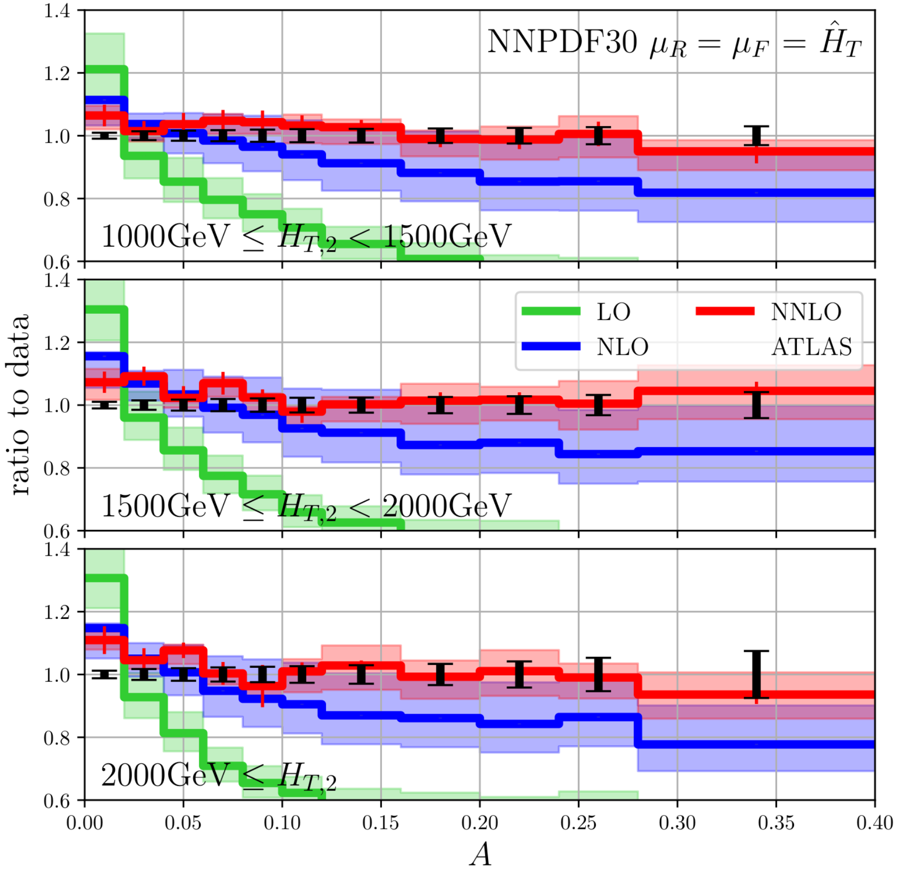
- Many more observables are accessible: azimuthal decorrelation, earth-mover distance based event shapes, …
- Still improvements to be made on subtractions schemes:
  - Better MC integration techniques → ML community has developed a plethora of tools
  - Technical aspects like form of selector function and phase space mappings

    “three factors of 2 are also a order of magnitude” → difference between “doable” and “not doable”!

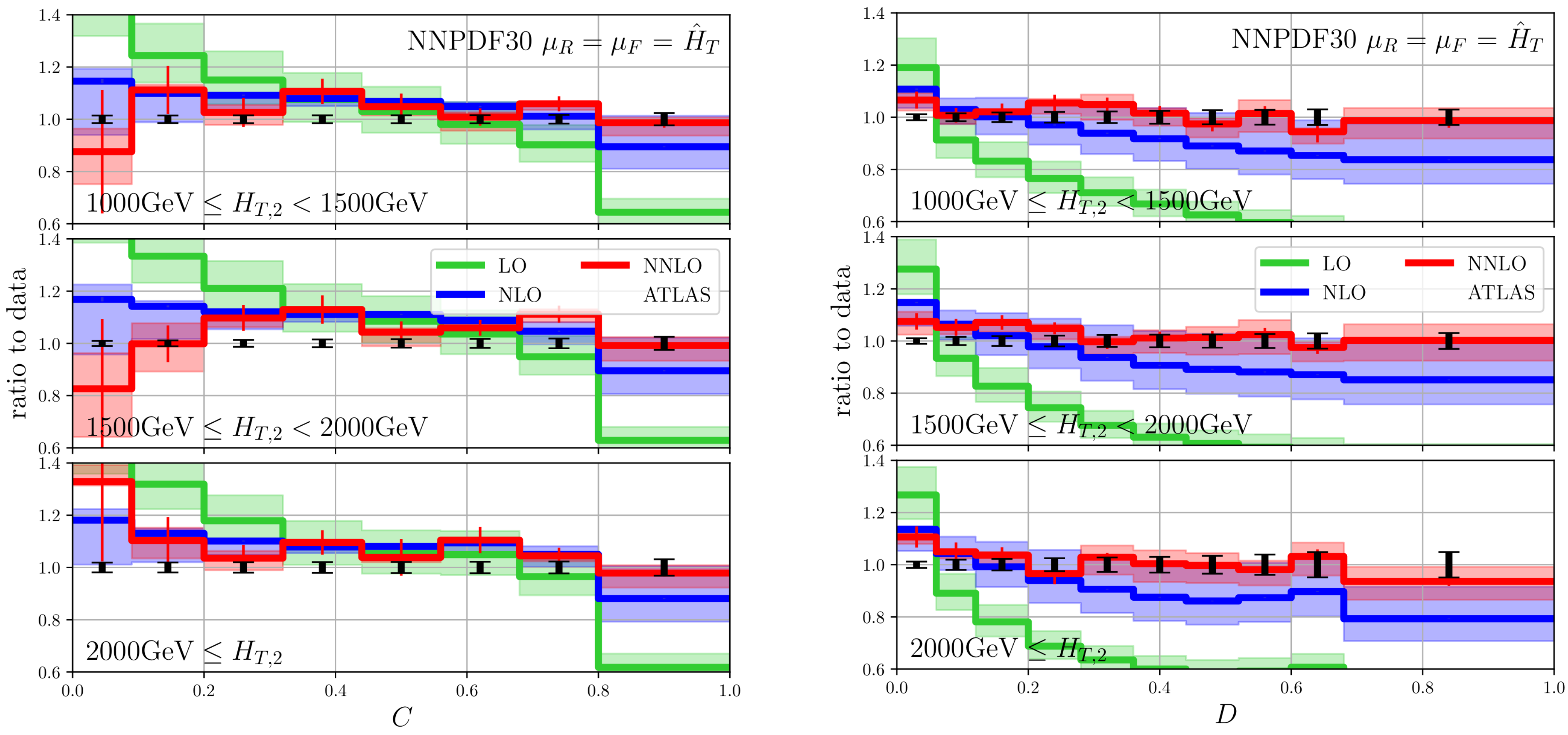
Backup

More event-shapes I





More event-shapes II



Event shapes as MC tuning tool