

NNLO QCD corrections to $W+2$ b-jet production

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in collaboration with Bayu Hartanto, Andrei Popescu, Simone Zoia
based on: [2102.02516], [2205.01687] and [2209.03280]

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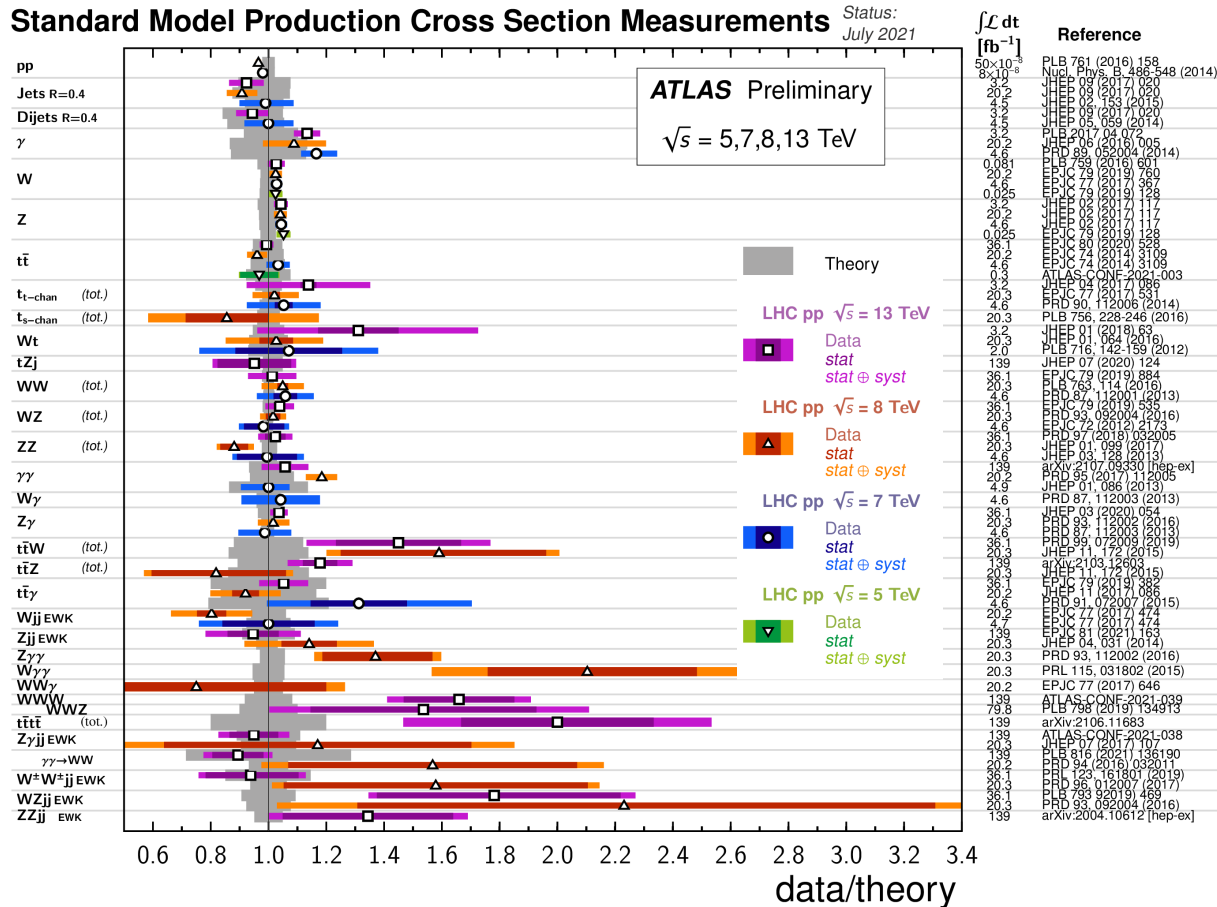
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Outline

- Introduction
- Sector-improved residue subtraction
- Two-loop five-point amplitude for $ud \rightarrow W (\rightarrow lv) b \bar{b}$
- $W+2$ b-jet production @ LHC
 - Phenomenology and flavour jet definitions
- Summary and Outlook

SM measurements at the LHC

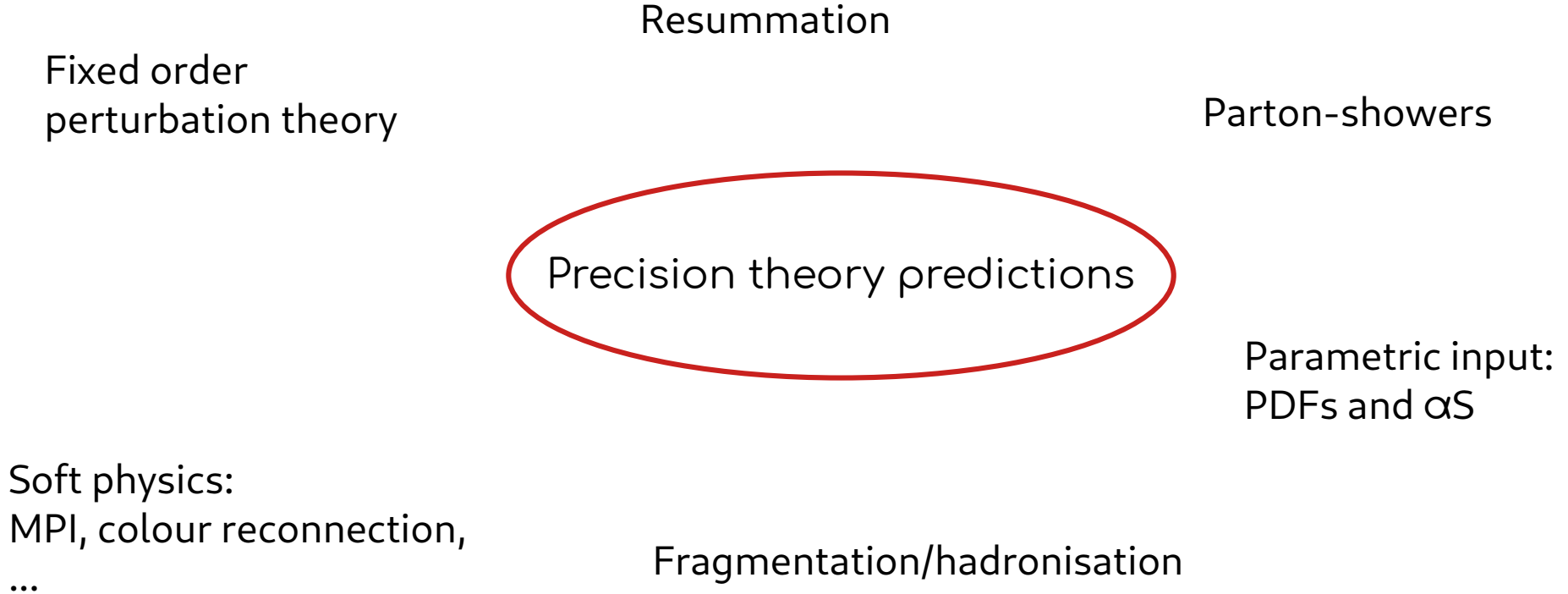


New physics around the corner?

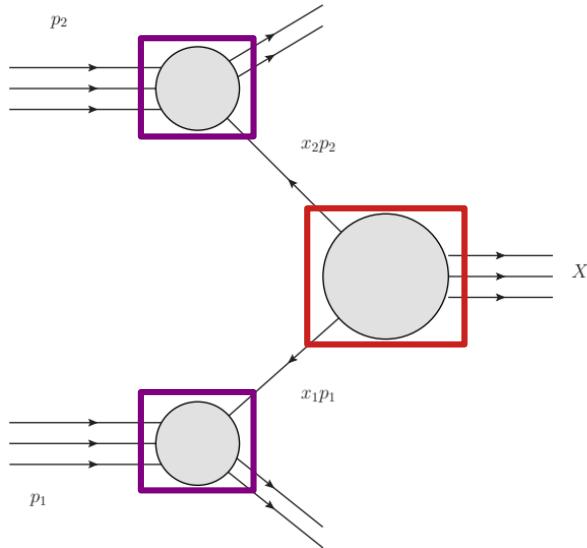
Precise measurements
 <->
 Precise theory
 Win-Win situation

- improved SM understanding
- search for indirect NP signals

Precision predictions



Perturbative QCD



Hadronic cross section:

$$\sigma_{h_1 h_2 \rightarrow X} = \sum_{ij} \int_0^1 \int_0^1 dx_1 dx_2 \underbrace{\phi_{i,h_1}(x_1, \mu_F^2)}_{\text{PDF}} \underbrace{\phi_{j/h_2}(x_2, \mu_F^2)}_{\text{PDF}} \underbrace{\hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)}_{\text{Partonic CS}}$$

Parton distribution functions: $\delta \sim 1-3\%$

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \hat{\sigma}_{ab \rightarrow X}^{(1)} + \hat{\sigma}_{ab \rightarrow X}^{(2)} + \mathcal{O}(\alpha_s^3)$$

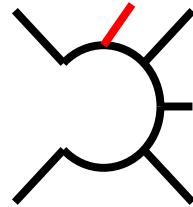
Typical uncertainties from scale variations: $\delta \text{LO} \sim 20-30\%$ $\delta \text{NLO} \sim 5-20\%$ $\delta \text{NNLO} \sim 1-5\%$

Next-to-leading order case

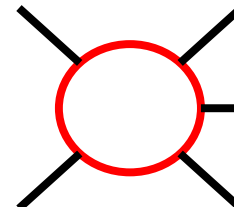
Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \hat{\sigma}_{ab \rightarrow X}^{(1)} + \hat{\sigma}_{ab \rightarrow X}^{(2)} + \mathcal{O}(\alpha_s^3)$$

Real corrections:



Virtual corrections:



$$\hat{\sigma}_{ab}^{(1)} = \hat{\sigma}_{ab}^{\text{R}} + \hat{\sigma}_{ab}^{\text{V}} + \hat{\sigma}_{ab}^{\text{C}}$$



Each term separately infrared (IR) divergent.

→ KLN theorem: sum is finite for sufficiently inclusive observables and regularization scheme independent

$$\hat{\sigma}_{ab}^{\text{R}} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1}$$

$$\hat{\sigma}_{ab}^{\text{V}} = \frac{1}{2\hat{s}} \int d\Phi_n 2\text{Re} \langle \mathcal{M}_n^{(0)} | \mathcal{M}_n^{(1)} \rangle F_n$$

$$\hat{\sigma}_{ab}^{\text{C}} = (\text{single convolution}) F_n$$

Slicing and Subtraction

Central idea: Divergences arise from IR limits \rightarrow Factorization!

Slicing:

$$\hat{\sigma}_{ab}^R = \frac{1}{2\hat{s}} \int_{\delta(\Phi) \geq \delta_c} d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1} + \frac{1}{2\hat{s}} \int_{\delta(\Phi) < \delta_c} d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1}$$

$$\approx \frac{1}{2\hat{s}} \int_{\delta(\Phi) \geq \delta_c} d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1} + \frac{1}{2\hat{s}} \int d\Phi_n \tilde{M}(\delta_c) F_n + \mathcal{O}(\delta_c)$$

Subtraction:

$$\hat{\sigma}_{ab}^R = \frac{1}{2\hat{s}} \int \left(d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1} - d\tilde{\Phi}_{n+1} \mathcal{S}F_n \right) + \frac{1}{2\hat{s}} \int d\tilde{\Phi}_{n+1} \mathcal{S}F_n$$

$$\frac{1}{2\hat{s}} \int d\tilde{\Phi}_{n+1} \mathcal{S}F_n = \frac{1}{2\hat{s}} \int d\Phi_n d\Phi_1 \mathcal{S}F_n$$

$\dots + \hat{\sigma}_{ab}^V = \text{finite}$

Slicing:

- Conceptually simple
- Recycling of lower computations
- Non-local cancellations
 \rightarrow computationally expensive

Subtraction:

- Conceptually more difficult
- Local subtraction \rightarrow efficient
- Better numerical stability

Slicing and Subtraction

Central idea: Divergences arise from IR limits \rightarrow Factorization!

Slicing:

$$\begin{aligned}\hat{\sigma}_{ab}^R &= \frac{1}{2\hat{s}} \int_{\delta(\Phi) \geq \delta_c} d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1} + \frac{1}{2\hat{s}} \int_{\delta(\Phi) < \delta_c} d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1} \\ &\approx \frac{1}{2\hat{s}} \int_{\delta(\Phi) \geq \delta_c} d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1} + \frac{1}{2\hat{s}} \int d\Phi_n \tilde{M}(\delta_c) F_n + \mathcal{O}(\delta_c)\end{aligned}$$

Subtraction:

$$\hat{\sigma}_{ab}^R = \frac{1}{2\hat{s}} \int \left(d\Phi_{n+1} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle F_{n+1} - d\tilde{\Phi}_{n+1} \mathcal{S}F_n \right) + \frac{1}{2\hat{s}} \int d\tilde{\Phi}_{n+1} \mathcal{S}F_n$$

$$\frac{1}{2\hat{s}} \int d\tilde{\Phi}_{n+1} \mathcal{S}F_n = \frac{1}{2\hat{s}} \int d\Phi_n d\Phi_1 \mathcal{S}F_n$$

... + $\hat{\sigma}_{ab}^V$ = finite

Slicing:

qT-slicing [Catain'07],
N-jettiness slicing [Gaunt'15/Boughezal'15]

Subtraction:

Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15],
Projection [Cacciari'15], Geometric [Herzog'18],
Unsubtraction [Aguilera-Verdugo'19],
Nested collinear [Caola'17],
Sector-improved residue subtraction [Czakon'10-'14]

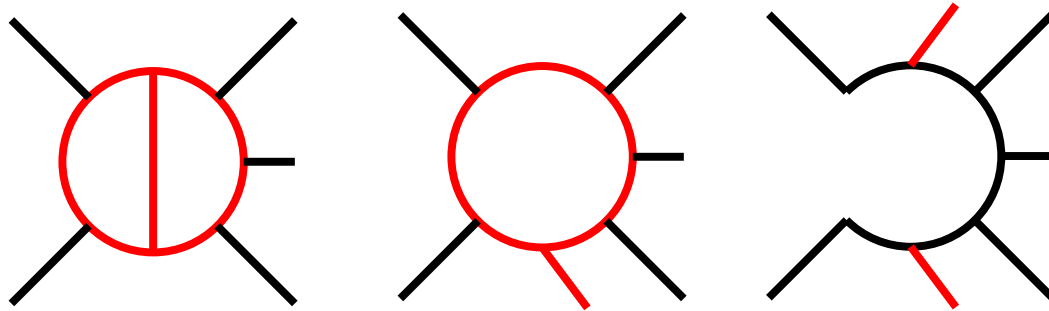
Partonic cross section beyond NLO

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \hat{\sigma}_{ab \rightarrow X}^{(1)} + \hat{\sigma}_{ab \rightarrow X}^{(2)} + \mathcal{O}(\alpha_s^3)$$

Contributions with different multiplicities and # convolutions:

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\text{VV}} + \hat{\sigma}_{ab}^{\text{RV}} + \hat{\sigma}_{ab}^{\text{RR}} + \hat{\sigma}_{ab}^{\text{C2}} + \hat{\sigma}_{ab}^{\text{C1}}$$



$$\hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2}$$

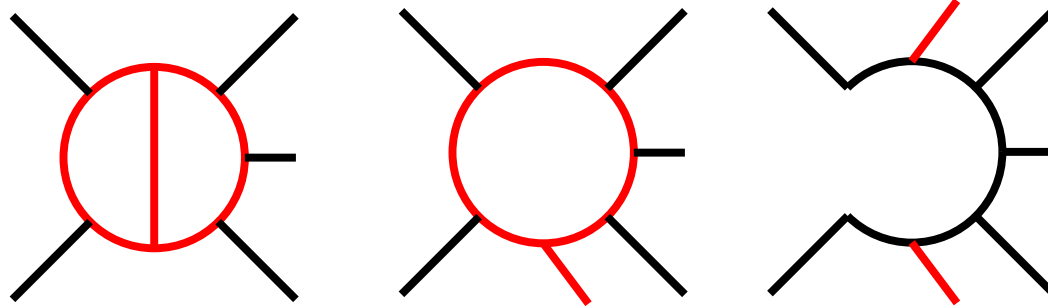
$$\hat{\sigma}_{ab}^{\text{RV}} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(1)} \rangle F_{n+1}$$

$$\hat{\sigma}_{ab}^{\text{VV}} = \frac{1}{2\hat{s}} \int d\Phi_n \left(2\text{Re} \langle \mathcal{M}_n^{(0)} | \mathcal{M}_n^{(2)} \rangle + \langle \mathcal{M}_n^{(1)} | \mathcal{M}_n^{(1)} \rangle \right) F_n$$

$$\hat{\sigma}_{ab}^{\text{C1}} = (\text{single convolution}) F_{n+1}$$

$$\hat{\sigma}_{ab}^{\text{C2}} = (\text{double convolution}) F_n$$

Sector-improved residue subtraction



Sector decomposition I

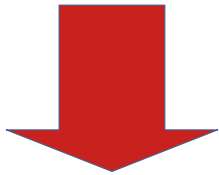
Considering working in CDR:

→ Virtuals are usually done in this regularization: $\hat{\sigma}_{ab}^{VV} = \sum_{i=-4}^0 c_i \epsilon^i + \mathcal{O}(\epsilon)$

→ Can we write the real radiation as such expansion?

→ Difficult integrals, analytical impractical (except very simple observables)!

→ Numerics not possible, integrals are divergent → ϵ -poles!



How to extract these poles? → Sector decomposition!

Divide and conquer the phase space:

$$1 = \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \quad \longrightarrow \quad \hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle_{\text{F}_{n+2}}$$

Sector decomposition II

Divide and conquer the phase space:

- Each $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$ has simpler divergences.
(Soft and collinear (w.r.t parton k,l) of partons i and j)
Parametrization w.r.t. reference parton
(makes divergences explicit:)

$$\hat{\eta}_i = \frac{1}{2}(1 - \cos \theta_{ir}) \in [0, 1] \quad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0, 1]$$

- Example: Splitting function $\sim \frac{1}{s_{r1}} P(z)$

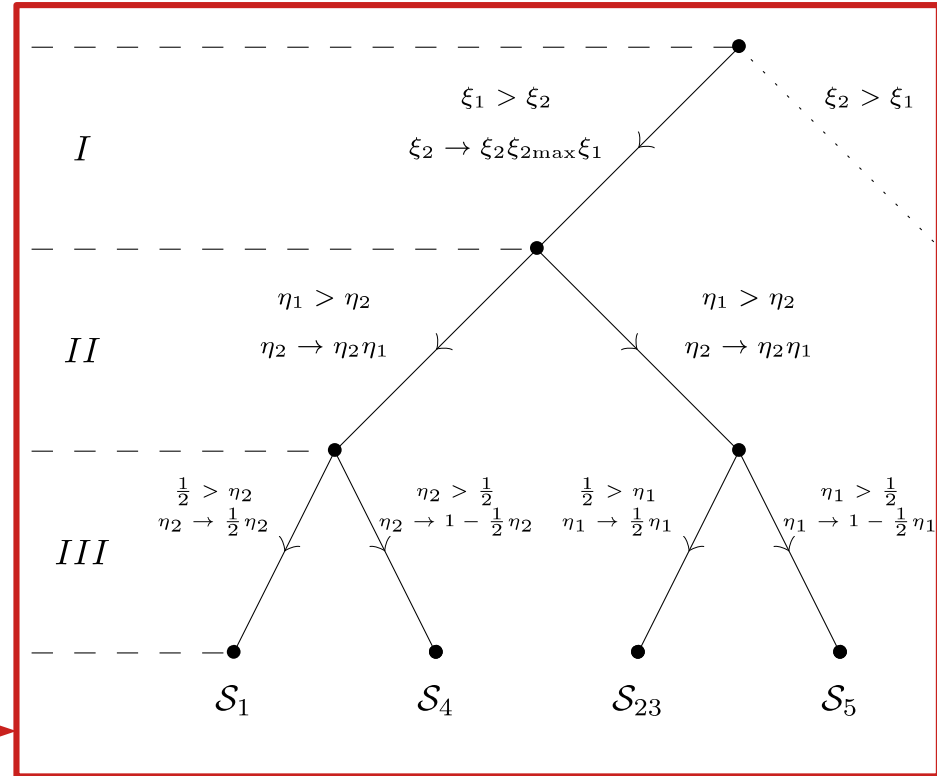
$$s_{r1} = (p_r + p_1)^2 = 2p_r^0 u_{\max}^1 \xi_1 \eta_1$$

$$(p_r + u_1 + u_2)^2 = 2p_r^0 (\xi_1 \eta_1 u_{\max}^1 + \xi_2 \eta_2 u_{\max}^2 + \xi_1 \xi_2 \frac{u_{\max}^1 u_{\max}^2}{p_r^0} \angle(u_1, u_2))$$

- Subdivide to factorize divergences

→ double soft factorization: $\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$

→ triple collinear factorization



[Czakon'10, Caola'17]

Sector decomposition III

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}} \int d\Phi_{n+2} \mathcal{S}_{kl,m} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2} = \sum_{\text{sub-sec.}} \int d\Phi_n \prod dx_i \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}} d\tilde{\mu}(\{x_i\}) \underbrace{\prod x_i^{a_i+1} \langle \mathcal{M}_{n+2} | \mathcal{M}_{n+2} \rangle}_{\text{regular}} F_{n+2}$$

$x_i \in \{\eta_1, \xi_1, \eta_2, \xi_2\}$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{[x^{-1-b\epsilon}]_+}_{\text{reg. + sub.}} \quad \int_0^1 dx [x^{-1-b\epsilon}]_+ f(x) = \int_0^1 \frac{f(x) - f(0)}{x^{1+b\epsilon}}$$

Finite NNLO cross section

$$\hat{\sigma}_{ab}^{RR} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2}$$

$$\hat{\sigma}_{ab}^{C1} = (\text{single convolution}) F_{n+1}$$

$$\hat{\sigma}_{ab}^{RV} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(1)} \rangle F_{n+1}$$

$$\hat{\sigma}_{ab}^{C2} = (\text{double convolution}) F_n$$

$$\hat{\sigma}_{ab}^{VV} = \frac{1}{2\hat{s}} \int d\Phi_n \left(2\text{Re} \langle \mathcal{M}_n^{(0)} | \mathcal{M}_n^{(2)} \rangle + \langle \mathcal{M}_n^{(1)} | \mathcal{M}_n^{(1)} \rangle \right) F_n$$



sector decomposition and master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{[x^{-1-b\epsilon}]_+}_{\text{reg. + sub.}}$$

$$(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}) \quad (\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}) \quad (\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}) \quad (\sigma_{SU}^{C1}, \sigma_{DU}^{C1}) \quad (\sigma_{DU}^{C2}, \sigma_{FR}^{C2})$$



re-arrangement of terms \rightarrow 4-dim. formulation [Czakon'14, Czakon'19]

$$\underline{(\sigma_F^{RR})} \quad \underline{(\sigma_F^{RV})} \quad \underline{(\sigma_F^{VV})} \quad \underline{(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1})} \quad \underline{(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2})} \quad \underline{(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2})}$$

separately finite: ϵ poles cancel

Two-loop five-point amplitude

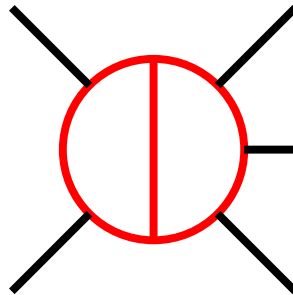
Massless:

[Chawdry'19'20'21] $(3A+2j, 2A+3j)$

[Abreu'20'21] $(3A+2j, 5j)$

[Agarwal'21] $(2A+3j)$

[Badger'21'] $(5j, gggAA)$



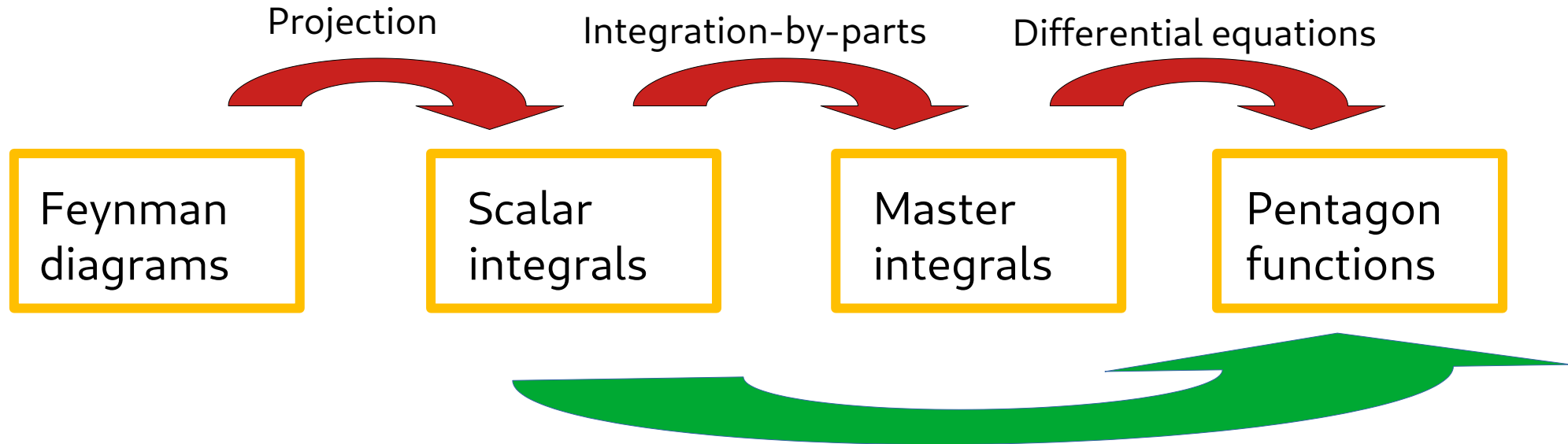
1 external mass:

[Abreu'21] $(W+4j)$

[Badger'21'22] $(Hqqgg, W4q, WAjjj)$

Overview

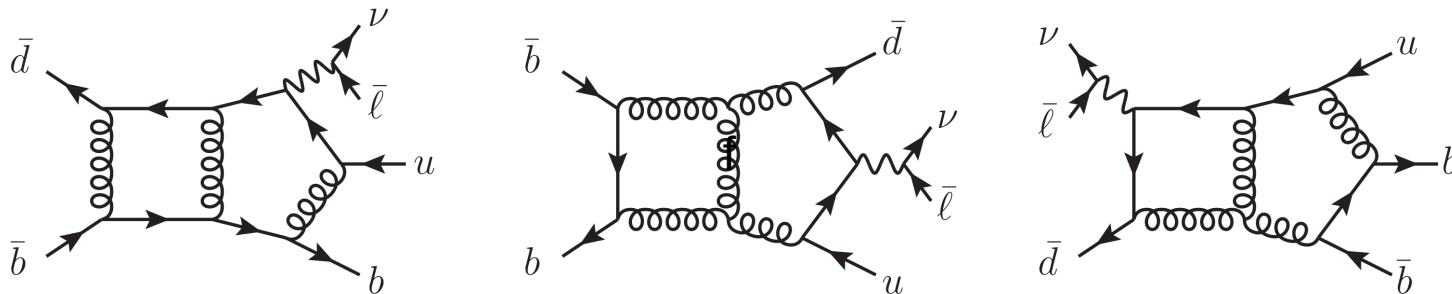
Old school approach:



Automated framework using finite fields
to avoid expression swell based on
FiniteFlow [[Peraro'19](#)]

Projection to scalar integrals

Generate diagrams (contributing to leading-colour) with QGRAF



Factorizing decay: $A_6^{(L)} = A_5^{(L)\mu} D_\mu P$

$$M_6^{2(L)} = \sum_{\text{spin}} A_6^{(0)*} A_6^{(L)} = M^{(L)\mu\nu} D_{\mu\nu} |P|^2$$

Projection on scalar functions (FORM+Mathematica):
 \rightarrow anti-commuting γ_5 + Larin prescription

$$M_5^{(L)} = \sum_{i=1}^{16} a_i^{(L)} v_i^{\mu\nu}$$



$$a_i^{(L)} = a_i^{(L),\text{even}} + \text{tr}_5 a_i^{(L),\text{odd}}$$

$$a_i^{(L),p} = \sum_j c_{j,i}(\{p\}, \epsilon) \mathcal{I}(\{p\}, \epsilon)$$

Integration-By-Parts reduction

$$a_i^{(L),p} = \sum_i c_{j,i}(\{p\}, \epsilon) \mathcal{I}(\{p\}, \epsilon)$$

Prohibitively large number of integrals

$$\mathcal{I}_i(\{p\}, \epsilon) \equiv \mathcal{I}(\vec{n}_i, \{p\}, \epsilon) = \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\}, \{k\})$$

Integration-By-Parts identities connect different integrals \rightarrow system of equations
 \rightarrow only a small number of independent "master" integrals

$$0 = \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} l^\mu \frac{\partial}{\partial l^\mu} \prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\}, \{k\}) \quad \text{with } l \in \{p\} \cap \{k\}$$

LiteRed (+ Finite Fields)



$$a_i^{(L),p} = \sum_i d_{j,i}(\{p\}, \epsilon) \text{MI}(\{p\}, \epsilon)$$

Master integrals & finite remainder

Differential Equations: $d\vec{M}\mathbb{I} = dA(\{p\}, \epsilon)\vec{M}\mathbb{I}$

[Remiddi, 97]
[Gehrmann, Remiddi, 99]
[Henn, 13]

Canonical basis: $d\vec{M}\mathbb{I} = \epsilon d\tilde{A}(\{p\})\vec{M}\mathbb{I}$

Simple iterative solution



$$M\mathbb{I}_i = \sum_w \epsilon^w \tilde{M}\mathbb{I}_i^w \quad \text{with} \quad \tilde{M}\mathbb{I}_i^w = \sum_j c_{i,j} m_j$$

Chen-iterated integrals
"Pentagon"-functions

[Chicherin, Sotnikov, 20]
[Chicherin, Sotnikov, Zoia, 21]

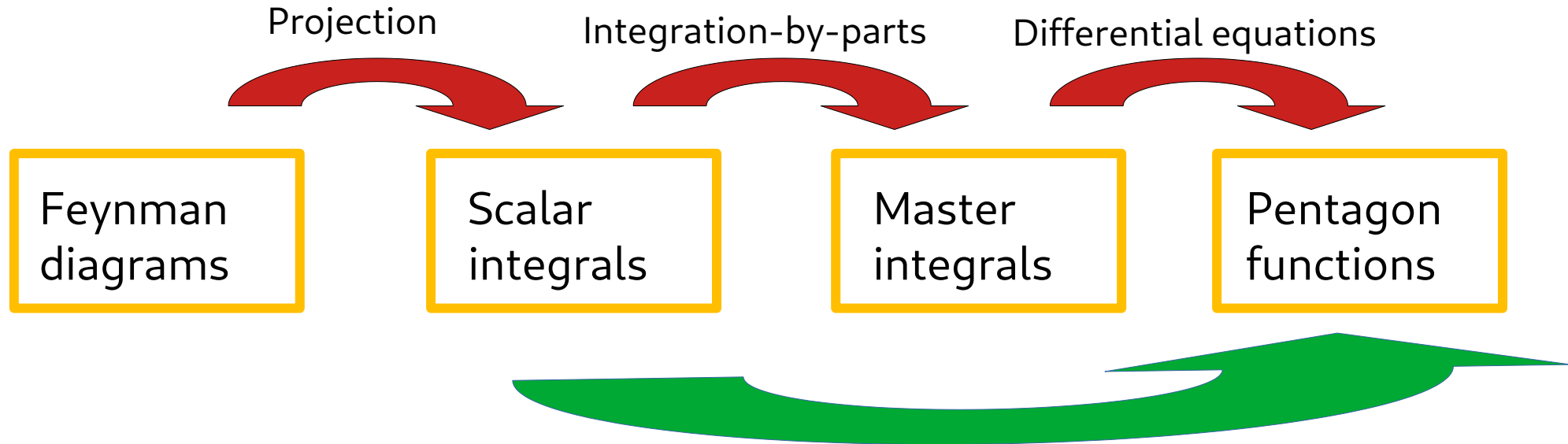
Putting everything together (and removing of IR poles):

$$f_i^{(L),p} = a_i^{(L),p} - \text{poles}$$

$$f_i^{(L),p} = \sum_j c_{i,j}(\{p\}) m_j + \mathcal{O}(\epsilon)$$

Overview

Old school approach:

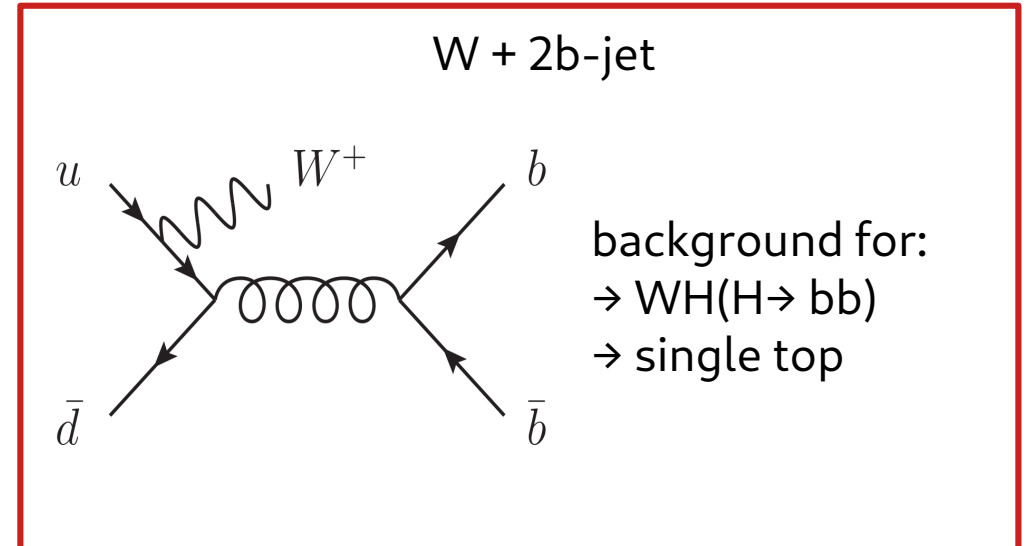
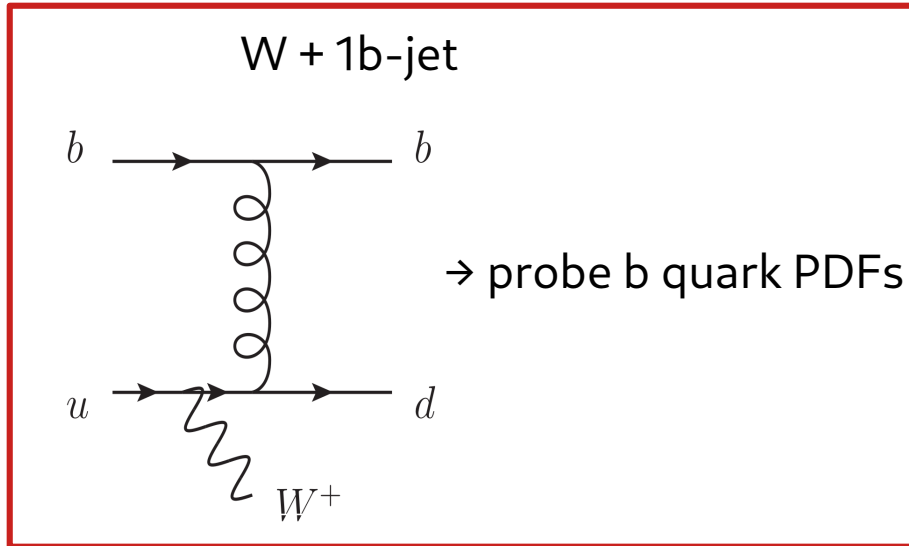


Automated framework using finite fields
to avoid expression swell based on
FiniteFlow [[Peraro'19](#)]

W+2 b-jet production @ LHC

W + b - jets

Motivation: → testing perturbative QCD: large NLO QCD corrections, 4FS vs. 5 FS
→ modelling of flavoured jets



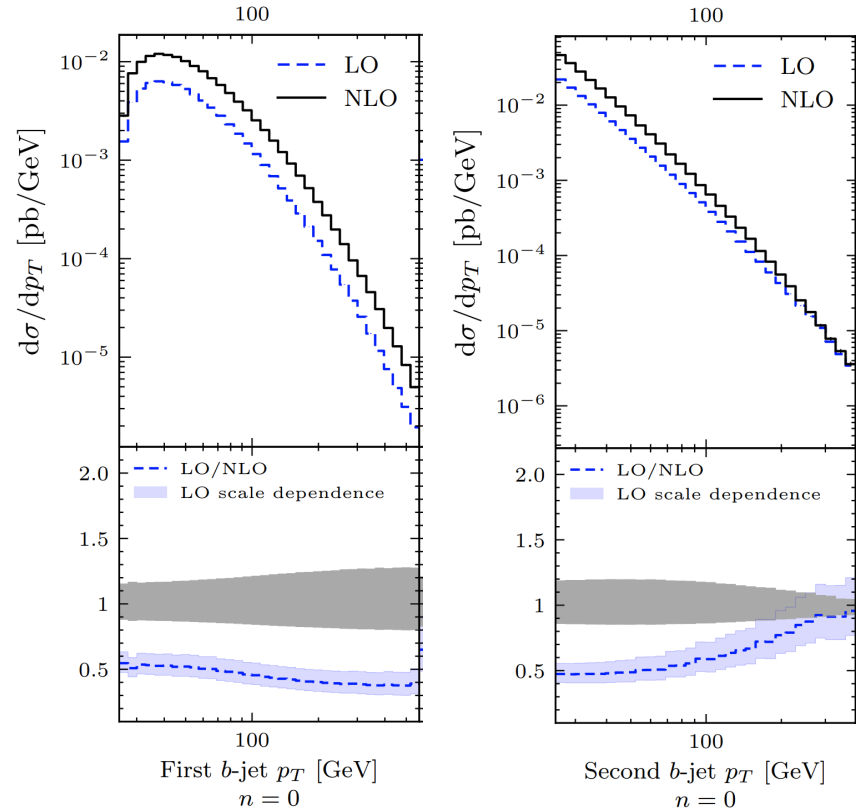
Experiment: [D0,1210.0627,0410062] [ATLAS,1109.1470,1302.2929][CMS,1312.6608,1608.07561]

Theory W+1 b-jet: [Campbell et al,0611348,0809.3003][Caola et.al.,1107.3714]

Theory W+2 b-jet: mb=0 [Ellis et al,9810489] onshell W: [Cordero et al,0606102] W(lv)bb: [Campbell et al,1011.6647]
NLO+PS: [Oleari et al,1105.4488][Frederix et al,1110.5502] W(lv)bb: [Luisoni et al,1502.01213]
W(lv)bb+≤3j: [Anger et al, 1712.05721]

NLO QCD corrections

[Anger et al, 1712.05721]



- Large NLO QCD corrections + scale dependence
 - Opening of qg -channel
- ↓
- Computation of NNLO QCD corrections
 - Amplitudes:
 - Born: AvH library [Bury'15]
 - Oneloop: OpenLoops2 [Buccioni'19]
 - Twoloop [Bager'21, Hartanto'22]
 - Subtraction → Stripper [Czakon'10'14'19]

Setup

NNLO QCD corrections to $Wb\bar{b}$ production at the LHC
Hartanto, Poncelet, Popescu, Zoia 2205.01687

- LHC @ 8 TeV in 5 FS, NNPDF31, scale: $H_T = E_T(l\nu) + p_T(b1) + p_T(b2)$
- Phasespace definition to model **[CMS, 1608.07561]**:
 $p_T(l) \geq 30 \text{ GeV}$ $|y(l)| < 2.1$ $p_T(j) \geq 25 \text{ GeV}$, $|y(j)| < 2.4$
- Inclusive (at least 2 b-jets) and exclusive (exactly 2 b-jets, no other jets) jet phase spaces (defined by the flavour-kT jet algorithm **[Banfi'06]**)

- Inclusive:

~ +20% corrections

~ 7% scale dependence

- Exclusive:

~ + 6% corrections

~ 2.5% scale dependence (7-pt)

Compare decorrelated model: **[Steward'12]**

~ 11% scale dependence

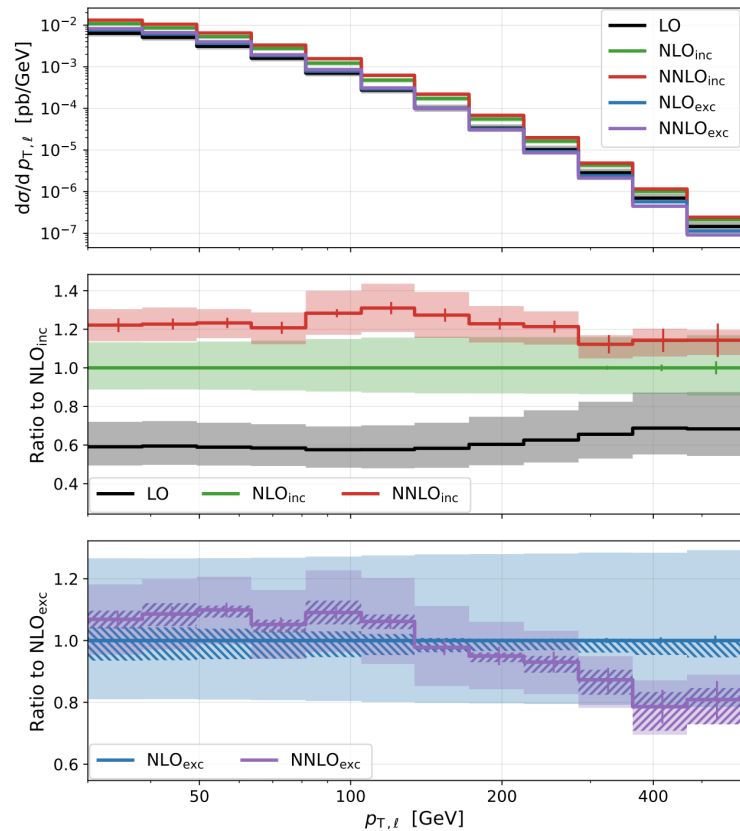
	inclusive [fb]	\mathcal{K}_{inc}	exclusive [fb]	\mathcal{K}_{exc}
σ_{LO}	$213.2(1)^{+21.4\%}_{-16.1\%}$	-	$213.2(1)^{+21.4\%}_{-16.1\%}$	-
σ_{NLO}	$362.0(6)^{+13.7\%}_{-11.4\%}$	1.7	$249.8(4)^{+3.9(+27)\%}_{-6.0(-19)\%}$	1.17
σ_{NNLO}	$445(5)^{+6.7\%}_{-7.0\%}$	1.23	$267(3)^{+1.8(+11)\%}_{-2.5(-11)\%}$	1.067

$$\sigma_{Wb\bar{b},\text{excl.}} = \sigma_{Wb\bar{b},\text{incl.}} - \sigma_{Wb\bar{b}j,\text{incl.}}$$

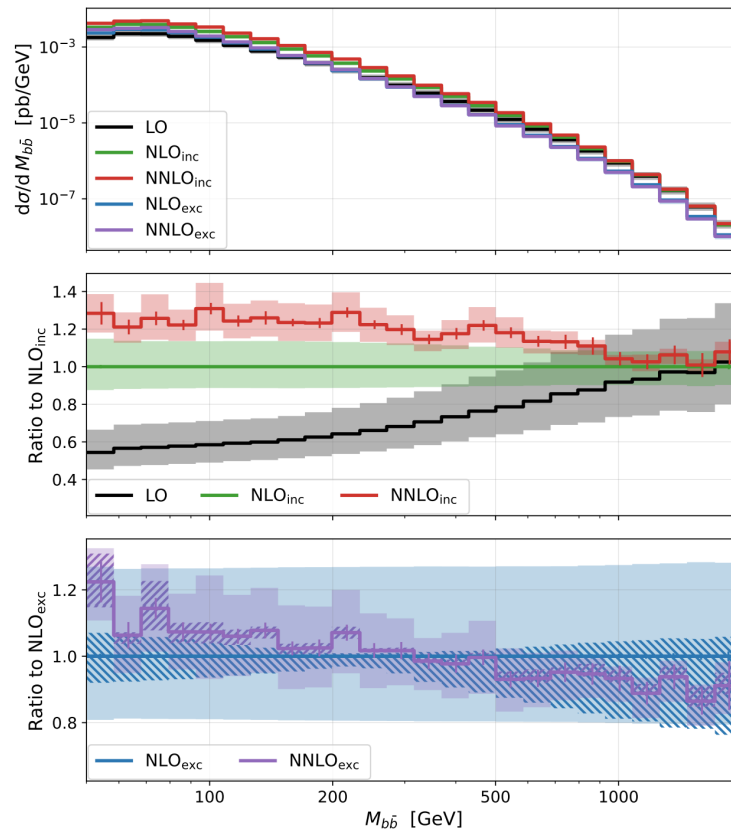
$$\Delta\sigma_{Wb\bar{b},\text{excl.}} = \sqrt{(\Delta\sigma_{Wb\bar{b},\text{incl.}})^2 + (\Delta\sigma_{Wb\bar{b}j,\text{incl.}})^2}$$

Differential cross sections

Transverse momentum of lepton



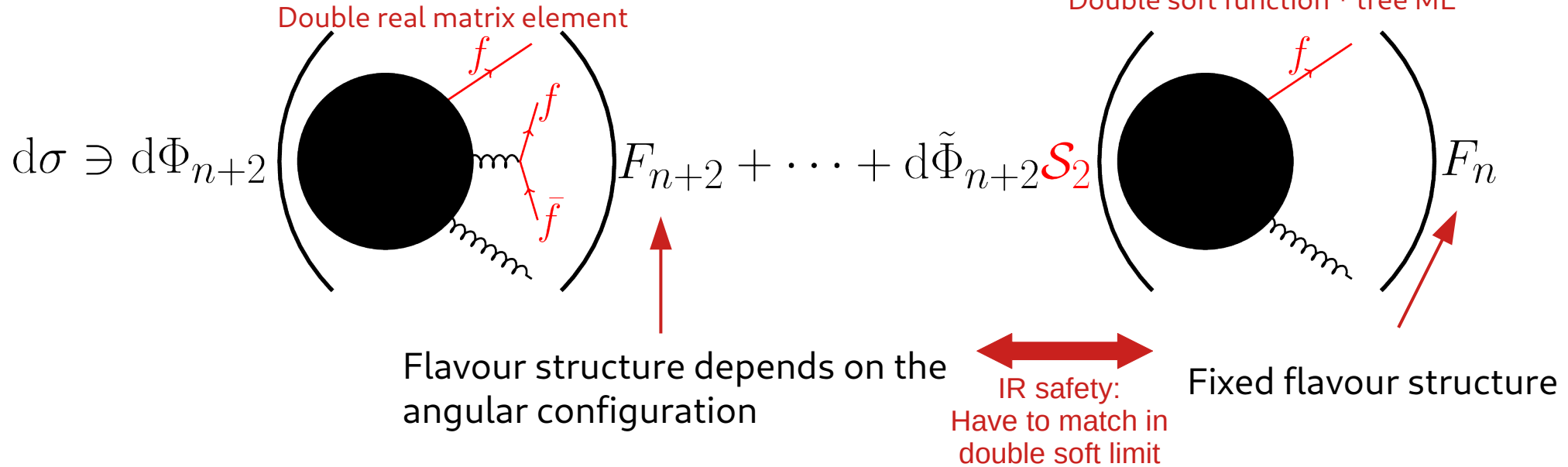
Invariant mass b-jet pair



Fixed order flavoured jets beyond NLO

What is the problem with FO flavoured jets?

Example NNLO: double real radiation and subtraction



- If $F(n+2)$ does not treat the flavour pair appropriately:
→ double soft singularity not subtracted
- **Implies correlated treatment of kinematics and flavour information**

Solution: Modified jet algorithms

→ Implies correlated treatment of kinematics and flavour information

Standard kT algorithm:

Pair distance:

$$d_{ij} = \min(k_{T,i}^2, k_{T,j}^2) R_{ij}^2$$

$$R_{ij}^2 = (\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2) / R^2$$

“Beam” distance for determination condition:

$$d_i = k_{T,i}^2$$

Flavour kT algorithm:

Infrared safe definition of jet flavor,
Banfi, Salam, Zanderighi hep-ph/0601139

Pair distance:

$$d_{ij} = R_{ij}^2 \begin{cases} \max(k_{T,i}, k_{T,j})^\alpha \min(k_{T,i}, k_{T,j})^{2-\alpha} & \text{softer of } i, j \text{ is flavoured} \\ \min(k_{T,i}, k_{T,j})^\alpha & \text{else} \end{cases}$$

Beam distance:

$$d_{i,B} = \begin{cases} \max(k_{T,i}, k_{T,B}(y_i))^\alpha \min(k_{T,i}, k_{T,B}(y_i))^{2-\alpha} & i \text{ is flavoured} \\ \min(k_{T,i}, k_{T,B}(y_i))^\alpha & \text{else} \end{cases}$$

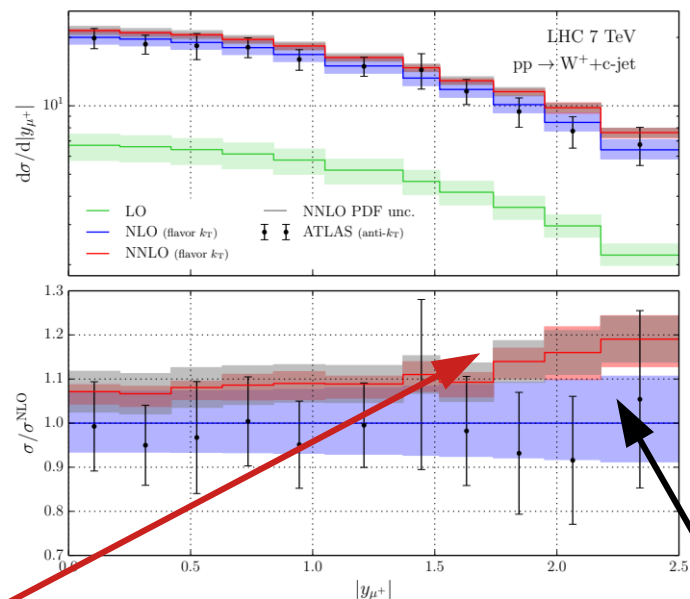
$$d_B(\eta) = \sum_i k_{T,i} (\theta(\eta_i - \eta) + \theta(\eta - \eta_i)) e^{\eta_i - \eta}$$

$$d_{\bar{B}}(\eta) = \sum_i k_{T,i} (\theta(\eta - \eta_i) + \theta(\eta_i - \eta)) e^{\eta - \eta_i}$$

Problem solved, isn't it?

Example: $W+c$ -jet at NNLO QCD with flavour-kT

NNLO QCD predictions for $W+c$ -jet production at the LHC
Czakon, Mitov, Pellen, Poncelet 2011.01011

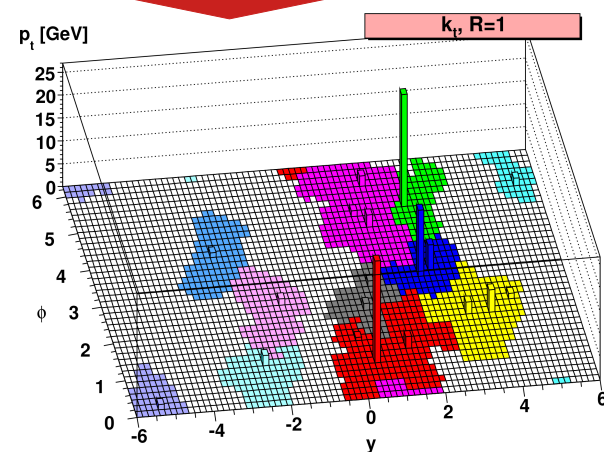
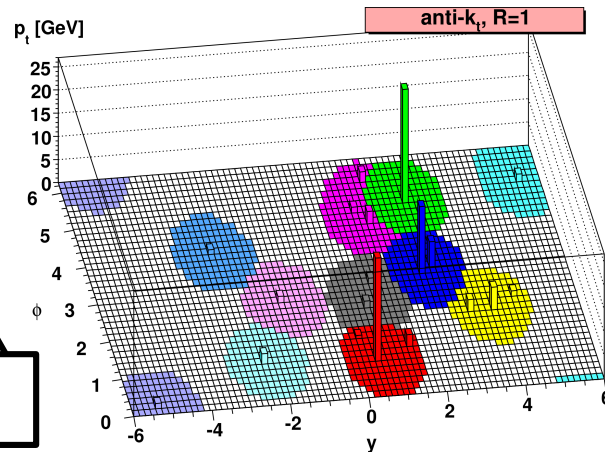


A proper comparison would require to **unfold experimental data**
→ (flavour-) kT and anti-kT cluster partonic jets differently → Non-trivial procedure.

Towards Jetography
Salam 0906.1833

NNLO QCD with flavour kT

ATLAS data with standard anti-kT



Old problem, new approaches

Renewed interest:

- Anti-kT + flv.-kT flavour matching:

QCD-aware partonic jet clustering for truth-jet flavour labelling Buckley, Pollard 1507.00508

Practical Jet Flavour Through NNLO

Caletti, Larkoski, Marzani, Reichelt 2205.01109

A dress of flavour to suit any jet

Gauld, Huss, Stagnitto 2208.11138

- Fixed-order fragmentation:

B-hadron production in NNLO QCD: application to LHC ttbar events with leptonic decays, Czakon, Generet, Mitov and Poncelet, 2102.08267

A Fragmentation Approach to Jet Flavor

Caletti, Larkoski, Marzani, Reichelt 2205.01117

- Modified anti-kT algorithm:

Infrared-safe flavoured anti-kT jets, Czakon, Mitov, Poncelet 2205.11879

Proposed modification:

A soft term designed to modify the distance of flavoured pairs.

$$d_{ij}^{(F)} = d_{ij} \begin{cases} \mathcal{S}_{ij} & i,j \text{ is flavoured pair} \\ 1 & \text{else} \end{cases}$$

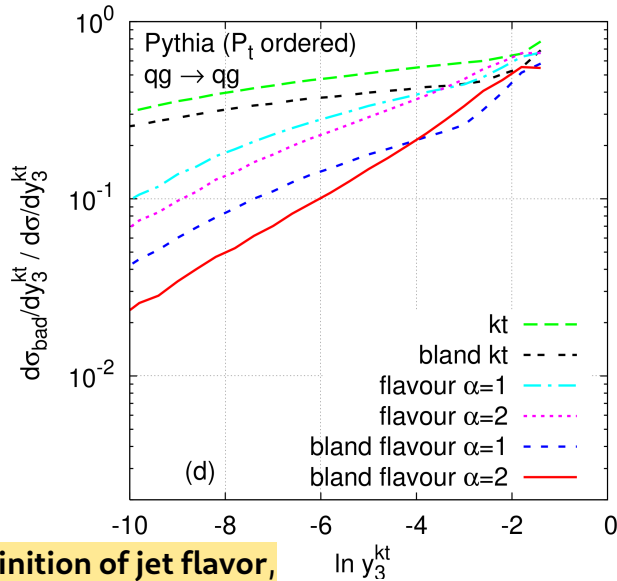
$$\mathcal{S}_{ij} = 1 - \theta(1 - x) \cos\left(\frac{\pi}{2}x\right) \quad \text{with} \quad x = \frac{k_{T,i}^2 + k_{T,j}^2}{2ak_{T,\max}^2}$$

Tests of IR safety with parton showers

Dress tree-level di-jet events (definite flavour structure: "qq", "qg" or "gg") with radiation and study jet flavour (q or g) as function of kinematics.

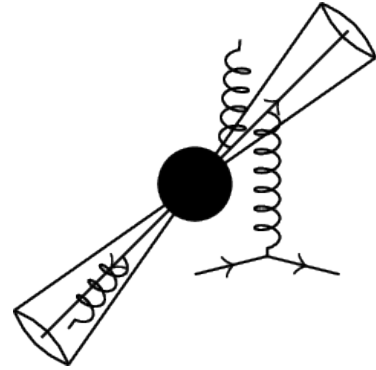
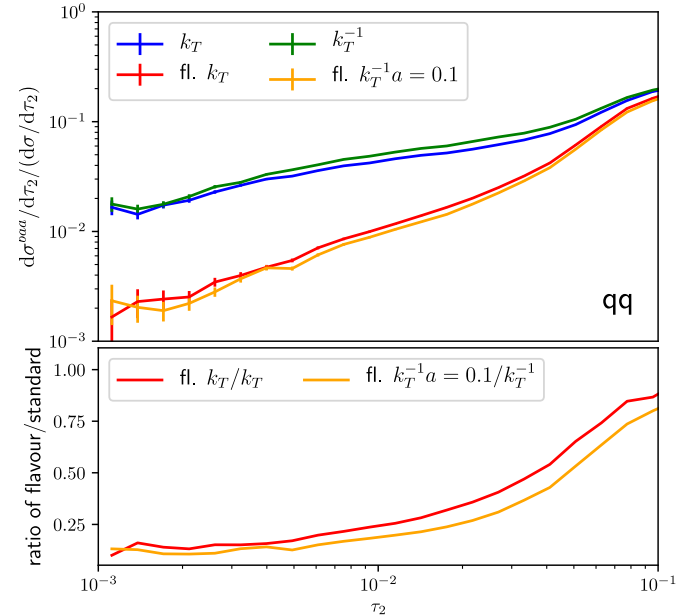
In the di-jet limit the flavour needs to correspond to tree level flavours
 → misidentification rate needs to vanish in di-jet back-to-back limit

Flavour k_T vs. k_T :



Infrared safe definition of jet flavor,
 Banfi, Salam, Zanderighi hep-ph/0601139

Flavour anti- k_T :



Tests of IR safety with NNLO FO computations

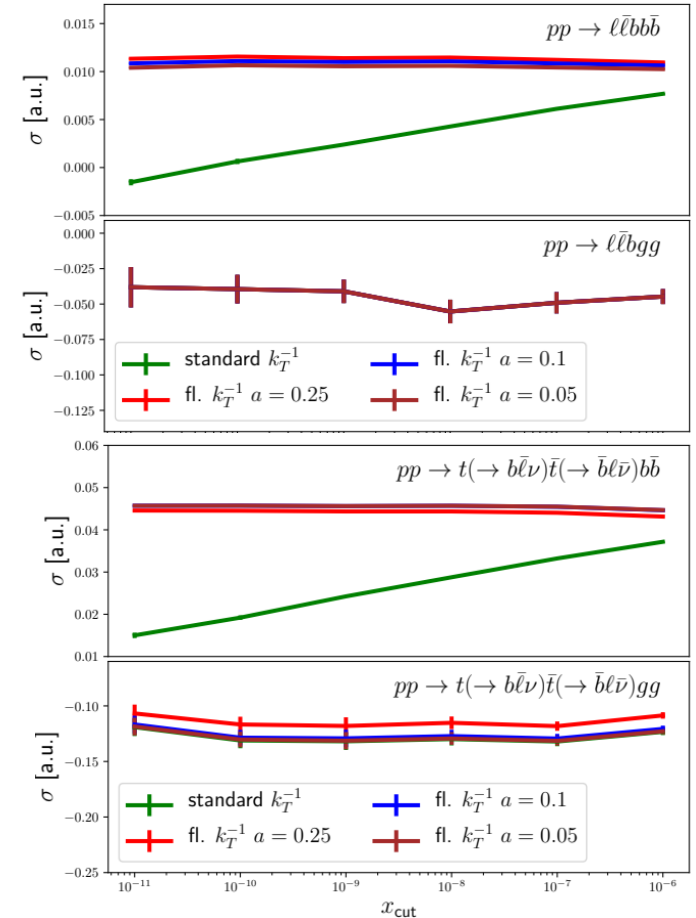
IR sensitivity of jet cross sections on
(technical) IR regulating parameter x

$$d\sigma \ni d\Phi_{n+2} \left(\text{Diagram 1} \right) F_{n+2} + \dots + d\tilde{\Phi}_{n+2} \mathcal{S}_2 \left(\text{Diagram 2} \right) F_n \theta(x - x_{\text{cut}})$$

In the limit $x_{\text{cut}} \rightarrow 0$:

IR safe jet flavour \rightarrow no dependence on x_{cut}

IR non-safe jet flavour \rightarrow logarithmic divergent



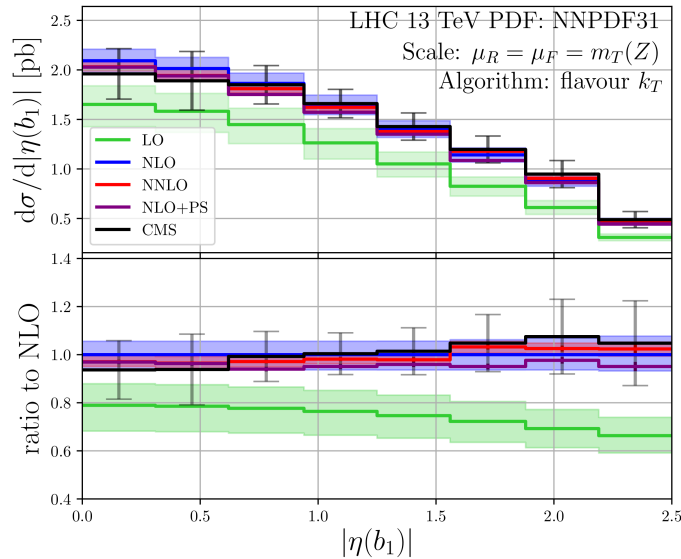
Z+b-jet Phenomenology: Tunable parameter

Benchmark process: $pp \rightarrow Z(\ell\ell) + b\text{-jet}$

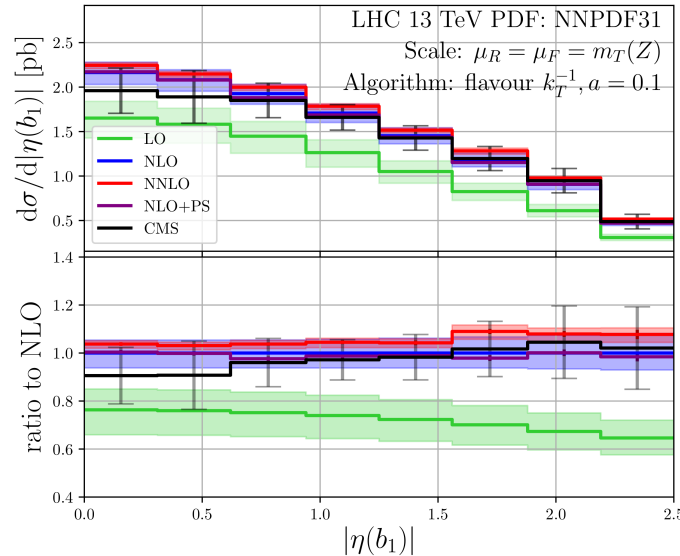
Tunable parameter a :

- Limit $a \rightarrow 0 \Leftrightarrow$ original anti-kT (IR unsafe)
- Large $a \Leftrightarrow$ large modification of cluster sequence

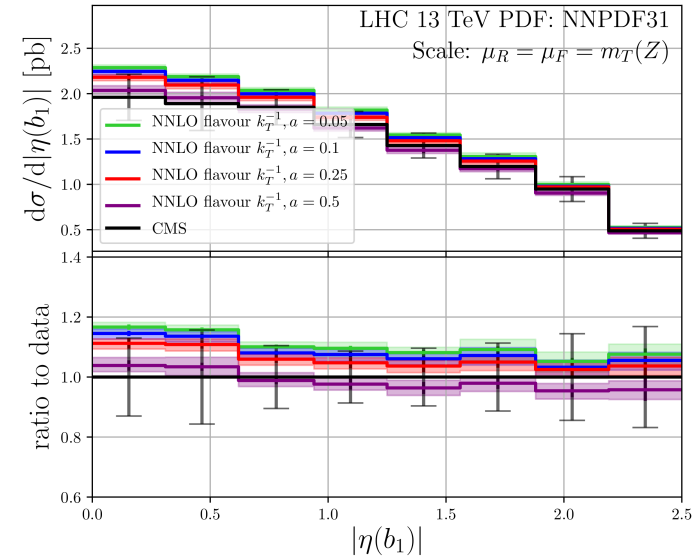
Flavour kT:



Flavour anti-kT: $a = 0.1$

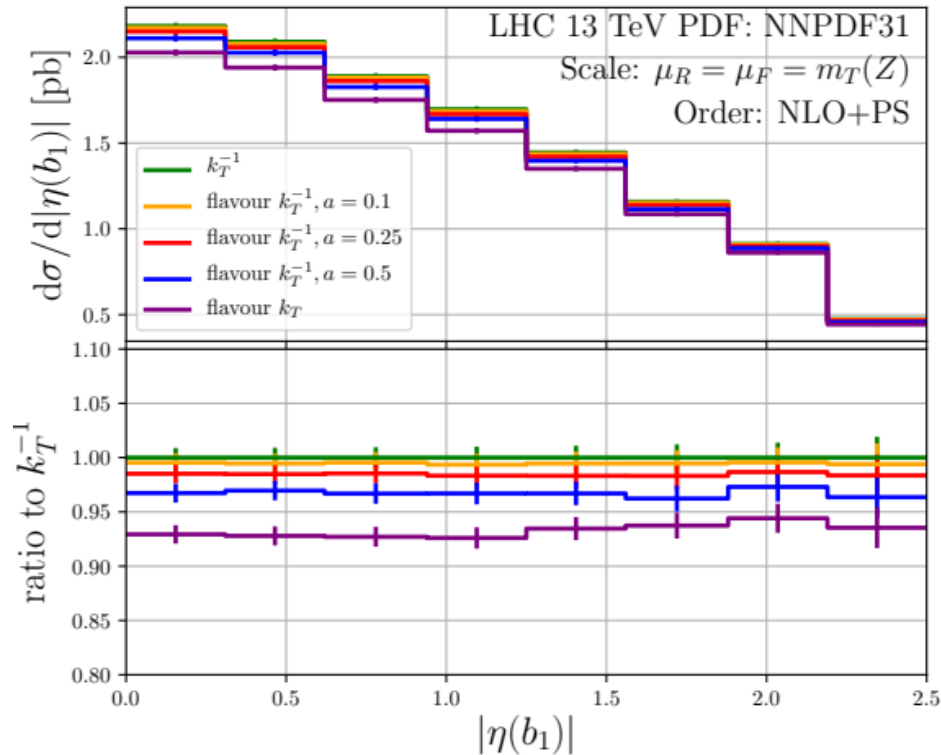


Comparison of different parameter a to data:



Z+b-jet Phenomenology: Tunable parameter II

What happens in the presence of many flavoured partons? \rightarrow NLO PS



Tunable parameter a:

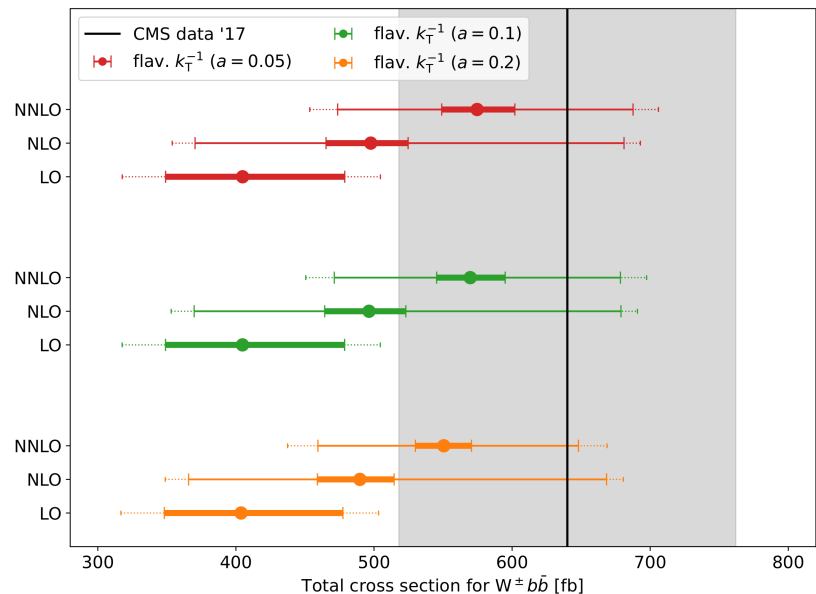
- Small a: Flavour anti-kT results are more similar to standard anti-kT \rightarrow **small unfolding factors**
- Larger a: Larger modification of clustering

Good FO perturbative convergence +
Small difference to standard anti-kT
 \rightarrow $a \sim 0.1$ is a good candidate

W+2 bjets: flavour anti-kT

Flavour anti-kT algorithm applied to Wbb production at the LHC

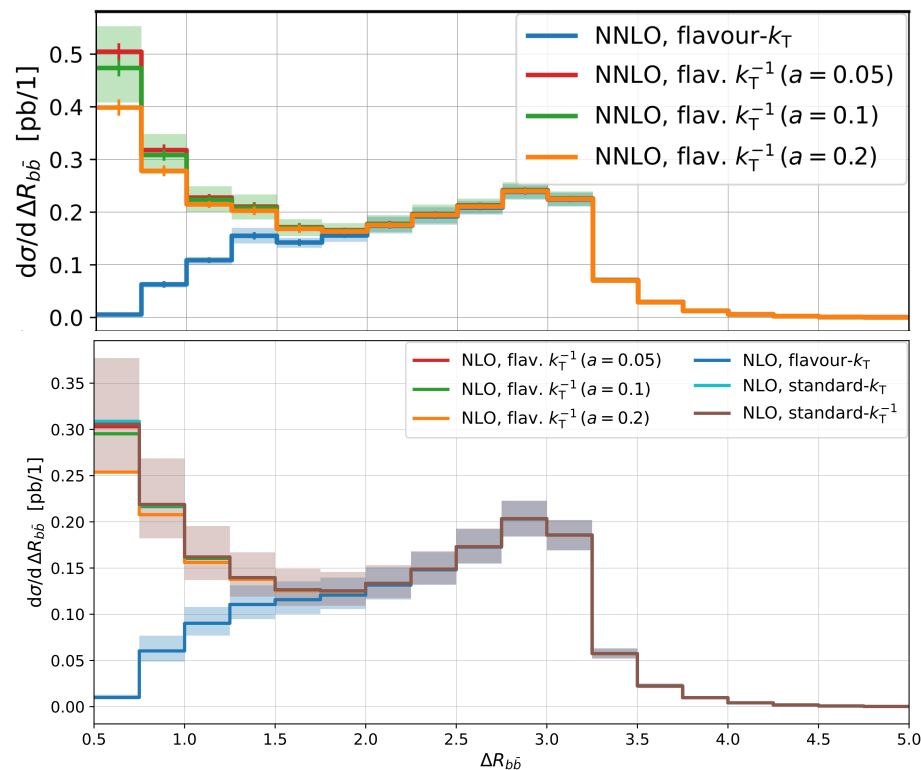
Hartanto, Poncelet, Popescu, Zoia 2209.03280



Comparison to data

Measurement of the production cross section of a W boson in association with two b jets in pp collisions at $\sqrt{s} = 8$ TeV, CMS 1608.07561

(assumes small unfolding corrections → wip)



Significant differences between kT and anti-kT
In small DeltaR(bb) region? Beam-function?!

Summary & Outlook

Summary & Outlook

Summary

- Sector-improved residue subtraction scheme
- Two-loop five-point amplitudes with external mass
- NNLO QCD corrections to $W+2b$ -jet production at the LHC
- Flavour sensitive jet-algorithms

Outlook

- Application of Stripper to further 5-point signatures
- Working towards non-planar contributions (also for 1 ext. mass)
→ See Abreu's summary at (HP)²
- Flavour-tagging
→ more studies and comparisons between different algorithms needed

Backup

Improved phase space generation

Phase space cut and differential observable introduce

mis-binning: mismatch between kinematics in subtraction terms

→ leads to increased variance of the integrand

→ slow Monte Carlo convergence

New phase space parametrization [Czakon'19]:

Minimization of # of different subtraction kinematics in each sector

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

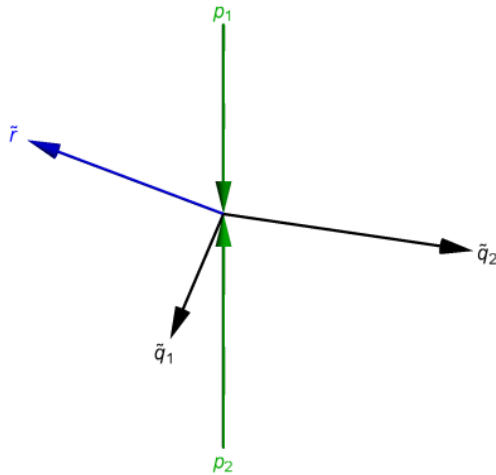
Mapping from $n+2$ to n particle phase space: $\{P, r_j, u_k\} \rightarrow \{\tilde{P}, \tilde{r}_j\}$

Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i : $\{\tilde{P}, \tilde{r}_j, u_k\} \rightarrow \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2, \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration



Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

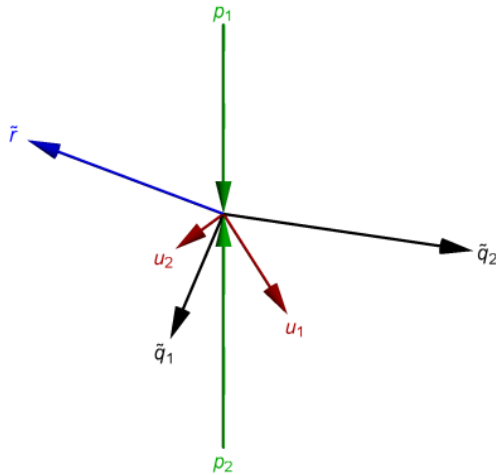
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Improved phase space generation

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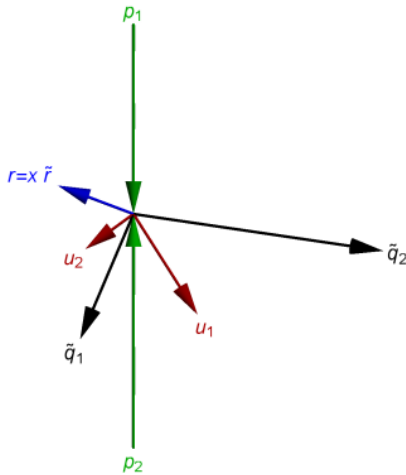
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Improved phase space generation

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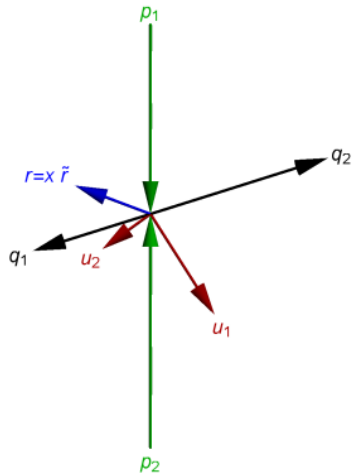
Mapping from $n+2$ to n particle phase space: $\{P, r_j, u_k\} \rightarrow \{\tilde{P}, \tilde{r}_j\}$

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- Invertible for fixed u_i : $\{\tilde{P}, \tilde{r}_j, u_k\} \rightarrow \{P, r_j, u_k\}$
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Main steps:

- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration



Further technical developments

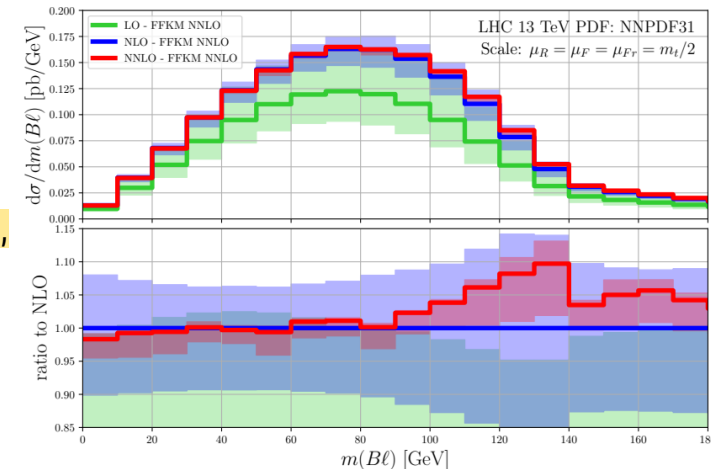
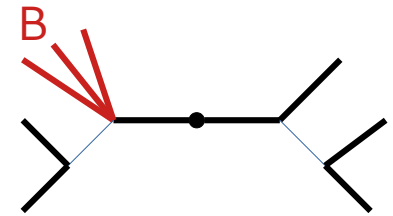
- Narrow-width-approximation and Double-Pole-Approximation for resonant particles:
 - Top-quark pairs + decays [Czakon'19'20]
 - W+W- polarization [Poncelet'21]
- Automated interfaces to OpenLoops, Recola and Njet
- Implementation of state-of-the-art twoloop matrix-elements:
 - $2 \rightarrow 2(1)$: $pp \rightarrow VV$, $pp \rightarrow Vj$, $pp \rightarrow H(j)$, $e+e- \rightarrow \text{jets}$, DIS
 - $2 \rightarrow 3$: $Pp \rightarrow 3\gamma$, $pp \rightarrow 2\gamma + j$, $pp \rightarrow 3j$
- Fragmentation of massless partons into hadrons
 - First application to $pp \rightarrow tt + X \rightarrow l+l- \nu \bar{\nu} B + X$ (NWA) [Czakon'21]
- Countless small improvements in terms of organization and efficiency

Flavour tagging and fixed order fragmentation

- Fixed order QCD predictions with a final state hadron
- Partonic computation + transition of parton to hadron (collinear fragmentation of massless partons)
- Non-perturbative fragmentation function (similar to PDFs): Probability to find a hadron with a fraction x of a parton
- Advantage is that the hadrons momentum is measurable
→ usage as b-tag?
- Implementation in the STRIPPER framework through NNLO QCD:

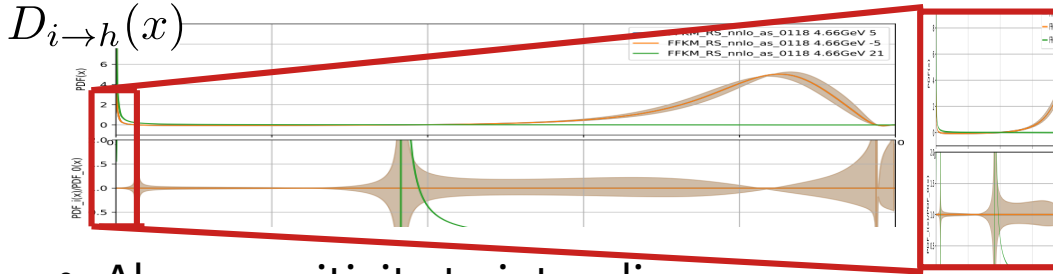
B-hadron production in NNLO QCD: application to LHC $t\bar{t}$ events with leptonic decays,
Czakon, Generet, Mitov and Poncelet, 2102.08267

$$pp \rightarrow t\bar{t} \rightarrow B\ell\bar{\ell}\nu\bar{\nu}b + X$$



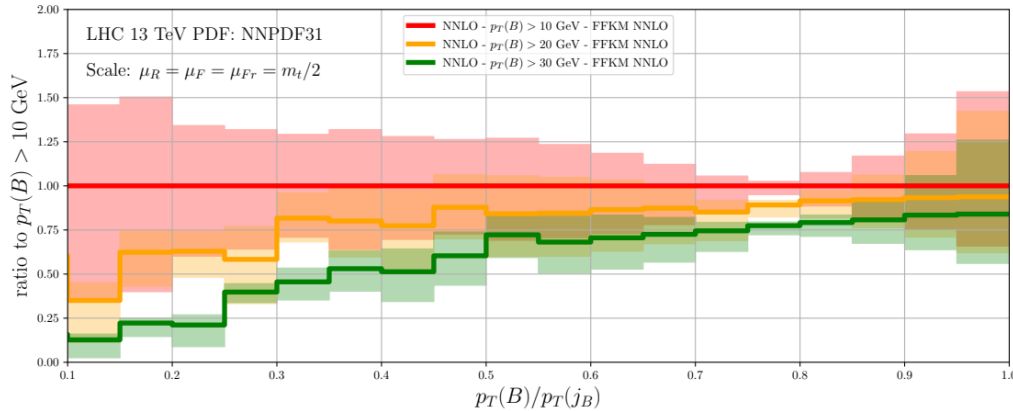
Subtleties

- $p_T(B)$ requirement necessary since NNLO fragmentation function divergent for $x \rightarrow 0$ due to $g \rightarrow b\bar{b}$ splitting:



- Also: sensitivity to jet radius

→ Usage as b-tag needs tuning



Jet radius variation $R = 0.8, 0.6, 0.4$

