Status of (N)NNLO calculations

Rene Poncelet

LEVERHULME TRUST

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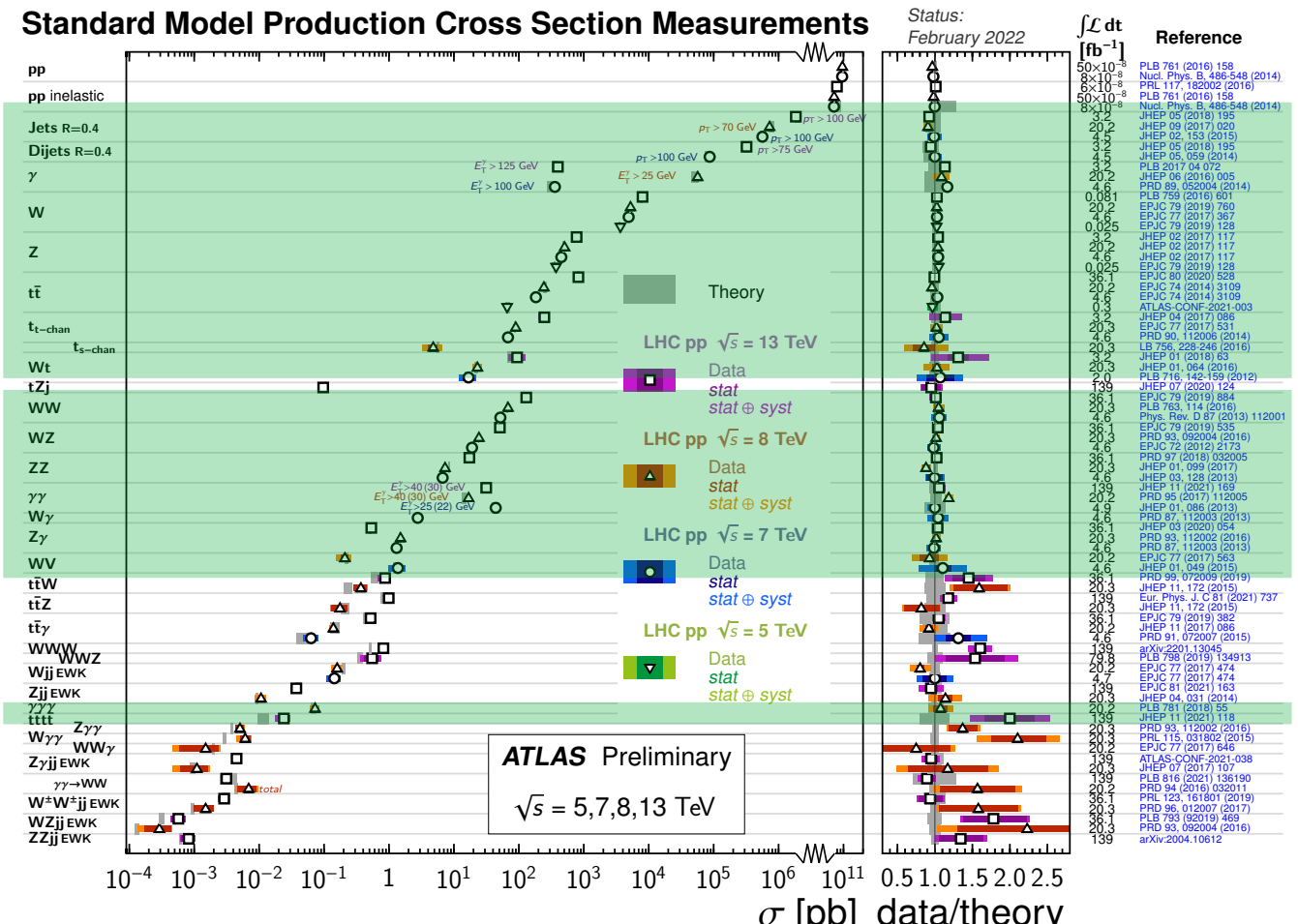
NNLO QCD coverage



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N3LO computations

2→3 NNLO QCD

NNLO QCD + PS

Fragmentation

Mixed EW-QCD

Differential N3LO QCD for Colour Singlets

First N3LO predictions become available

- Based on
  → Analytical integration (inclusive)
  → Projection-to-Born (differential)
  → qT-slicing+resummation (differential)
  Processes:
  - pp → H (+ diphoton decay with fiducial cuts)
  - pp → W/Z/A (+leptonic decays)

Higgs Boson Gluon-Fusion Production in QCD at Three Loops,
Anastasiou, Duhr, Dulat, Herzog and Mistlberger, 1503.06056
Charged current Drell-Yan production at N3LO,
Duhr, Dulat and Mistlberger, 2007.13313
Fully Differential Higgs Boson Production to Third Order in QCD,
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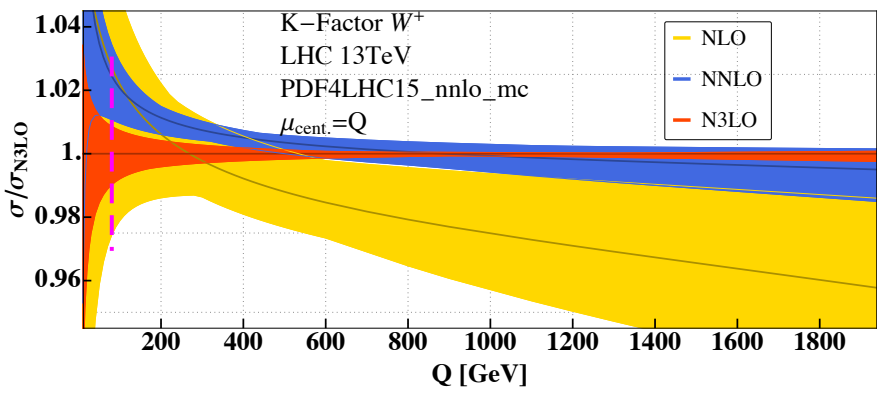
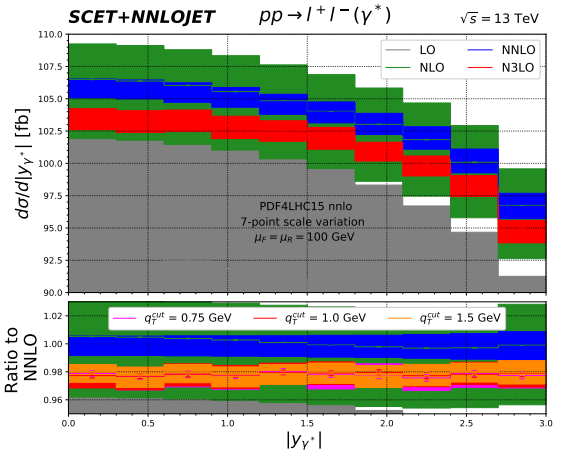
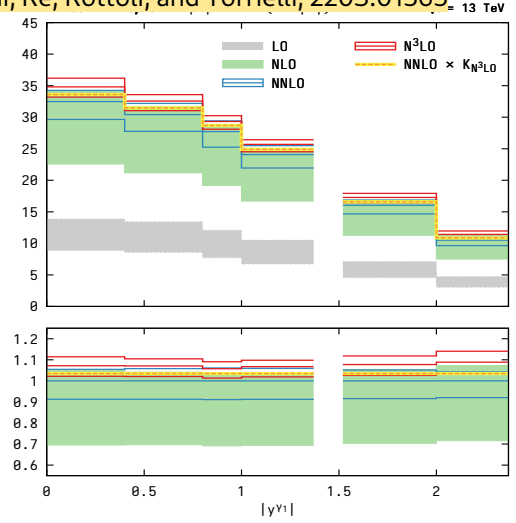


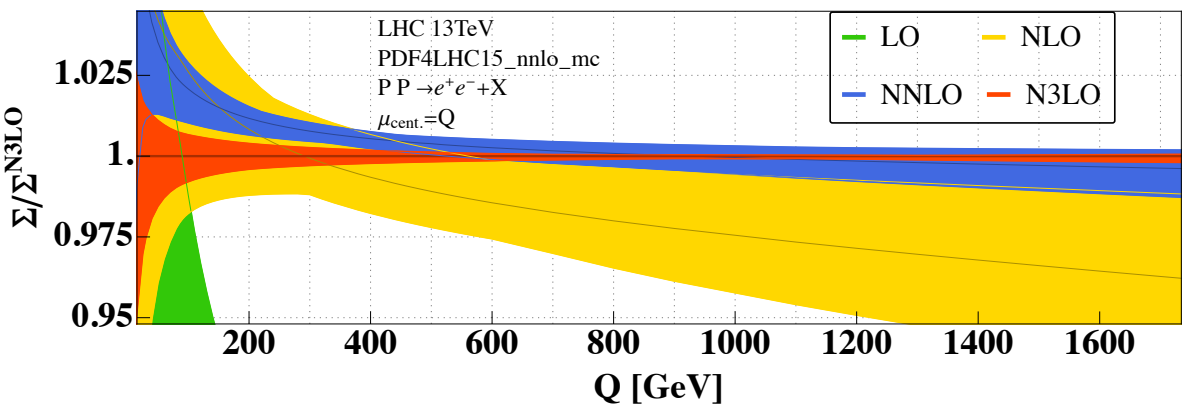


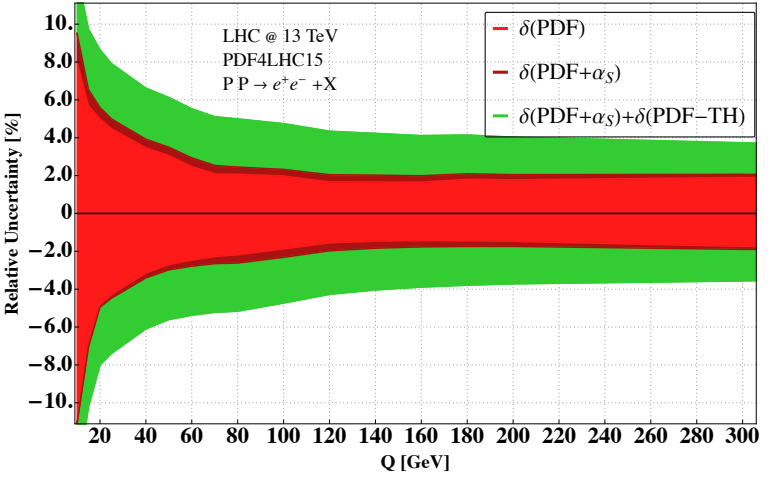


More loops: N3LO QCD DrellYan II

- Tiny scale dependence with respect to NNLO, in particular for large Q
- No overlap of scale bands for 50 < Q < 400
- Maybe a PDF effect? No N3LO PDFs available

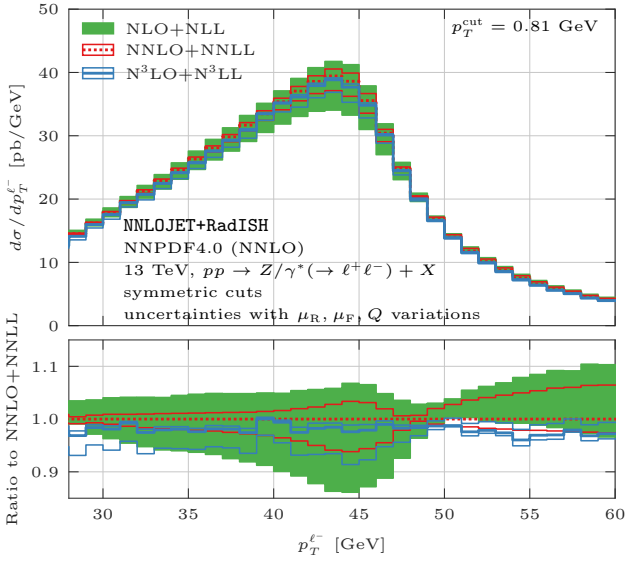
  → Estimated effect relatively large





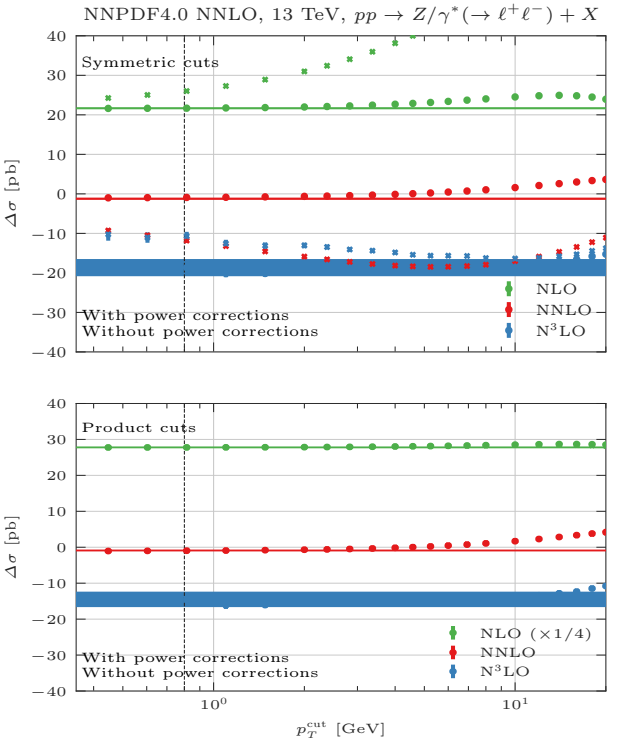
Fiducial predictions at N3LO

Third order fiducial predictions for Drell-Yan at the LHC,
Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, and Torrielli, 2203.01565



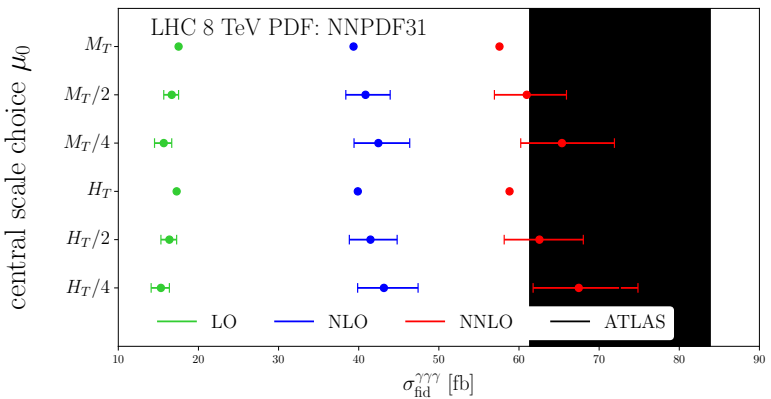
- N3LO + N3LL fiducial predictions
- Power corrections crucial for symmetric cuts → Rottoli's talk
- Symmetric vs. product cuts

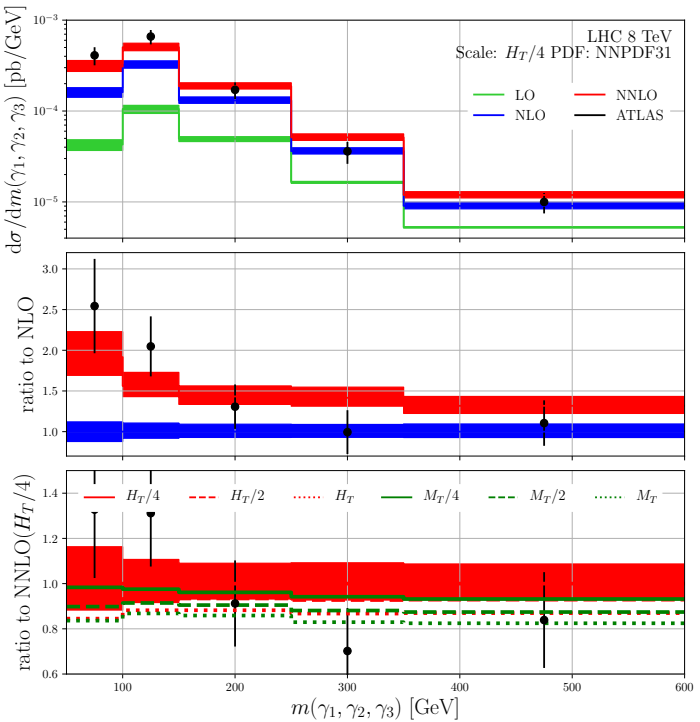
Cuts for two-body decays at colliders
Gavin P. Salam, Emma Slade, 2106.08329



Three photon production

- First NNLO QCD 2 → 3 cross sections:
  NNLO QCD corrections to three-photon production at the LHC, Chawdhry, Czakon, Mitov and Poncelet, 1911.00479
  Triphoton production at hadron colliders in NNLO QCD, Kallweit, Sotnikov and Wiesemann, 2010.04681
- Simplest among the 2→3 massless cases: colour singlet
- Approximation in two-loop virtuals: only planar diagrams
  → overall small contribution
- Large NNLO/NLO K-factors
- NNLO QCD corrections essential for theory/data comparison
  Here: ATLAS





Diphoton plus jet production

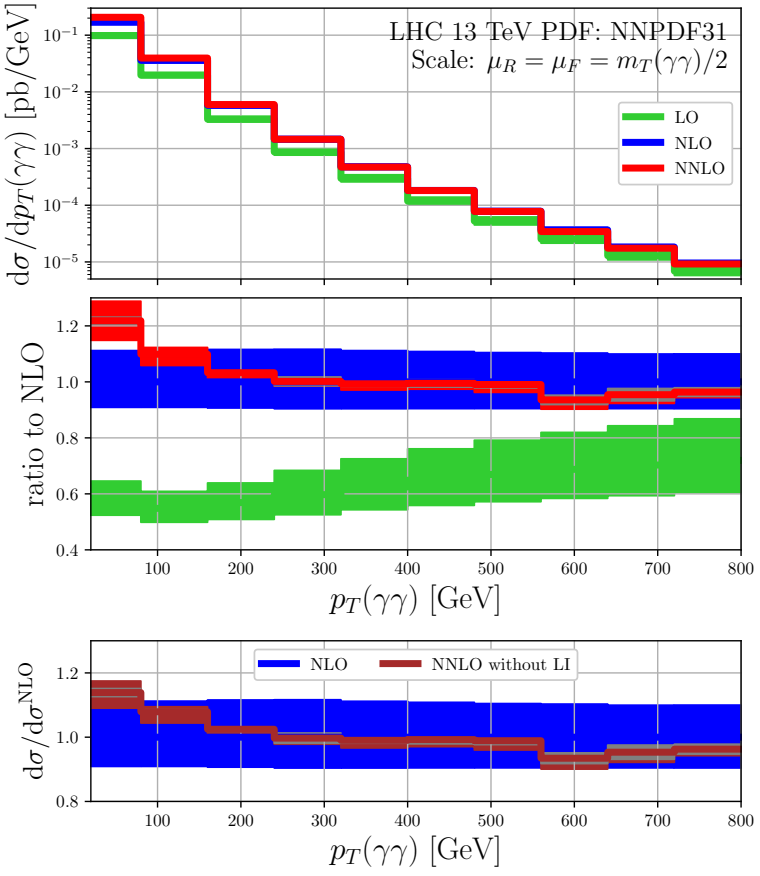
Photon pair production @ LHC is of particular interest:

- Main background to cleanest Higgs decay channel
- Large NNLO QCD corrections!
  Perturbative convergence @ N3LO?
- Diphoton plus jet @ NNLO QCD (pT(AA) → 0 limit)
- pT(γγ) spectrum itself interesting for Higgs → γγ

First NNLO QCD for pp → AAj

NNLO QCD corrections to diphoton production with an additional jet at the LHC, Chawdhry, Czakon, Mitov and Poncelet, 2105.06940

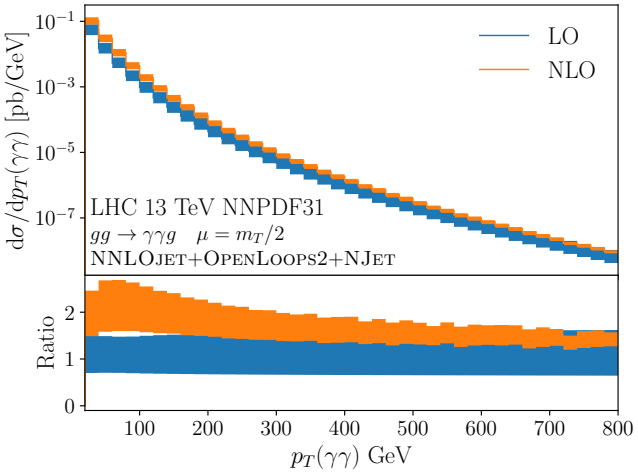
- Beautiful perturbative convergence
- Scale dependence: NLO: ~10% NNLO: ~1-2%
- Low pT region:
  - ? Resummation for $p_T(\gamma\gamma)/m(\gamma\gamma) \ll 1$
  - Strong effect from the loop induced!



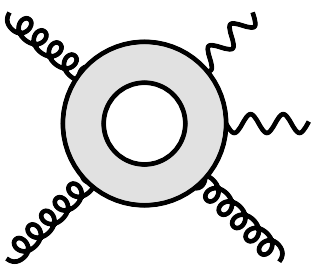
Diphoton plus jet – gg fusion

Next-to-leading order QCD corrections to diphoton-plus-jet production through gluon fusion at the LHC, Badger, Gehrmann, Marcoli and Moodie, 2109.12003

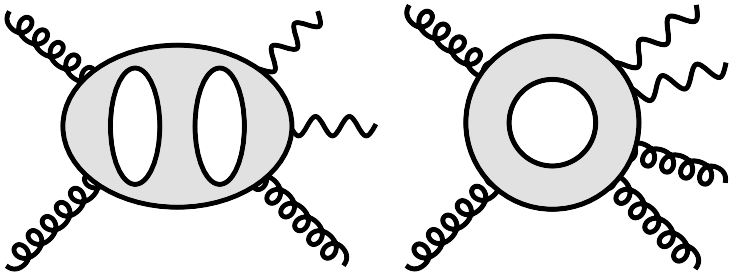
- NLO QCD to gg → yyg (formally N3LO for pp→yyj)
- Challenging double virtual matrix element
- Large corrections of up to 100% → relate to 5% in full pp → yyj
- Reduction of scale dependence at high transverse momentum

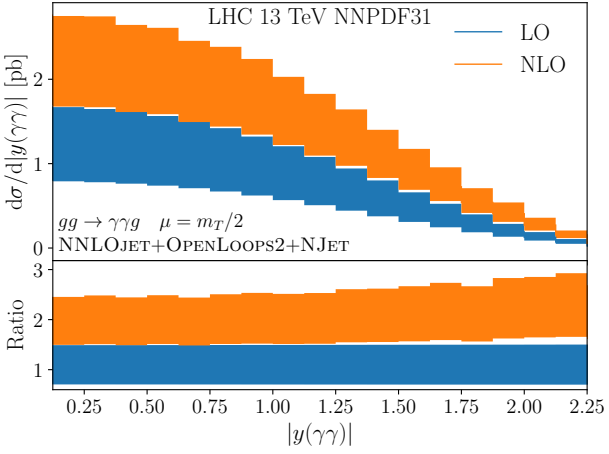


LO:



NLO:





Three-jet production

Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC, Czakon, Mitov and Poncelet, 2106.05331

Computational challenges:

- Sector-improved residue subtraction for real radiation
  - Efficient c++ implementation → STRIPPER
  - Highly automated to deal with enormous amount of channels in three-jet production
    → O(1k) sectors →O(1M) individual MC integrals
- Many-leg, IR stable one-loop amplitudes → OpenLoops 2
- Double virtual amplitudes in leading-colour approximation
  - Sub-leading colour corrections expected to be small
  - Analytical expressions challenging
  - Fast numerical evaluation → very small contribution to computational cost
- The pure gluonic process evaluated within the NNLOJet framework:

A novel subtraction scheme for double-real radiation at NNLO,
Czakon, 1005.0274
Four-dimensional formulation of the sector-improved residue subtraction scheme, Czakon and Heymes, 1408.2500
Single-jet inclusive rates with exact color at O(as^4)
Czakon, van Hameren, Mitov and Poncelet, 1907.12911
OpenLoops 2, Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller, 1907.13071

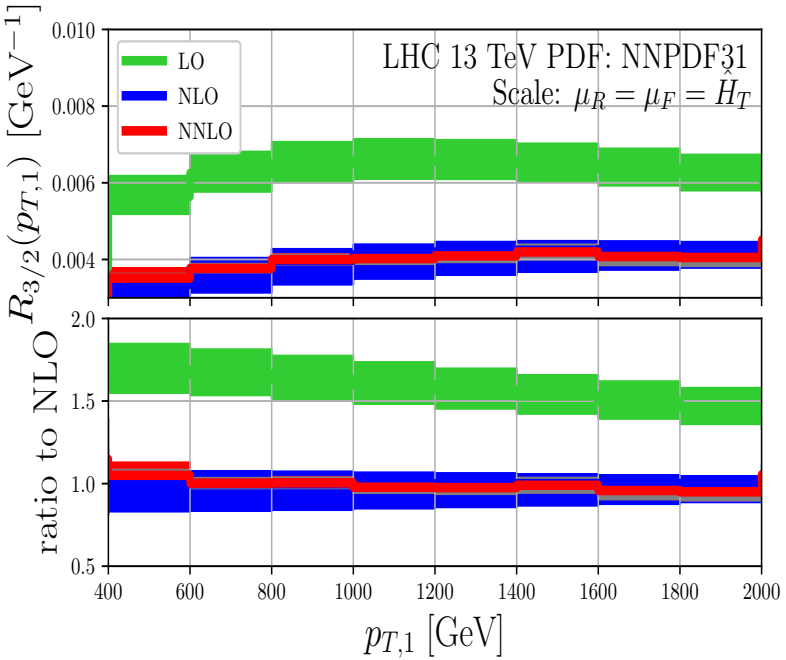
Leading-color two-loop QCD corrections for three-jet production at hadron colliders,
Abreu, Cordero, Ita, Klinkert, Page, Sotnikov, 2110.07541

Automation of antenna subtraction in colour space: gluonic processes,
Chen, Gehrmann, Glover, Huss and Marcoli, 2203.13531

Three-jet production - R32

- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets:
  - $p_T(j) > 60\ \text{GeV}$ and $|y(j)| < 4.4$
  - $H_{T,2} = p_T(j_1) + p_T(j_2) > 250\ \text{GeV}$
- Scales: $\mu_R = \mu_F = \hat{H}_T = \sum_{\text{partons}} p_T$

$$R_{3/2}(X, \mu_R, \mu_F) = \frac{\mathrm{d}\sigma_3(\mu_R, \mu_F)/\mathrm{d}X}{\mathrm{d}\sigma_2(\mu_R, \mu_F)/\mathrm{d}X} \sim \alpha_s$$



Interesting phenomenological applications:

- Extraction of alphaS, tests of SM running and tests of QCD matrix elements
  R32, event-shapes, TEEC, azimuthal decorrelation

NNLO QCD meets parton showers

NNLO QCD + PS available (MiNNLO_PS, Geneva + Pythia)

- pp →H
- pp →W/Z
- pp → VH
- pp →WW/ZZ/Zy
- pp →ttbar (+PS decays
  → Wiesemann's talk

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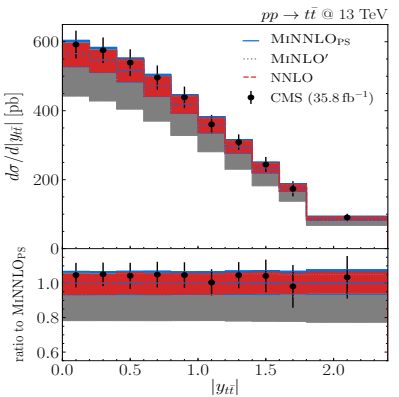
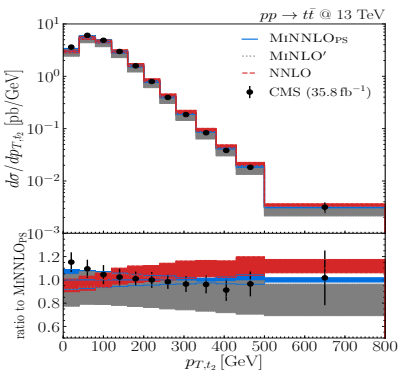
NNLO+PS with MiNNLO PS: status and prospects, Buonocore et al., 2203.07240
NNLO event gen … field theory, Haisch, Scott, Wiesemann, Zanderighi, Zanoli, 2204.00663

Matching to LL accuracy → Prestel's talk

- keeping NNLO accuracy for inclusive observables
- but lifting fixed order kinematic constraints
  → example sub-leading top-quark pT



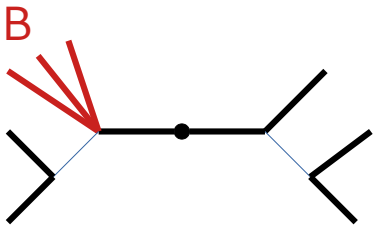


Fixed-order Fragmentation

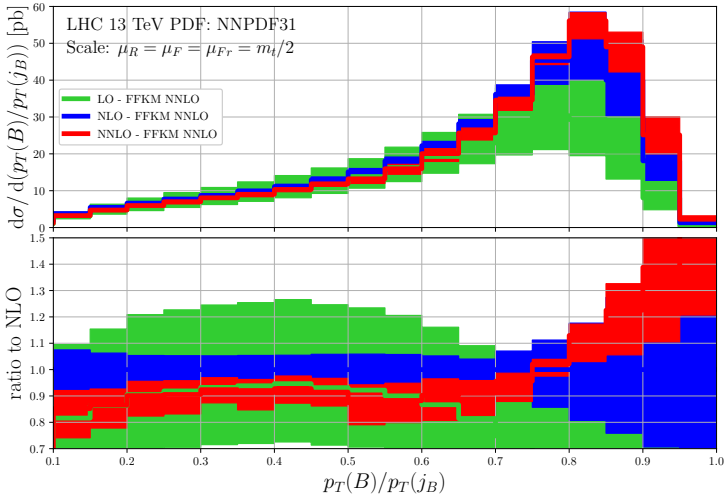
- Fixed order QCD predictions with a final state hadron
- Considering partonic computation + transition of parton to hadron
  (collinear fragmentation of massless partons)
- Advantage is that the hadrons momentum is measurable while the quark's is not
- Fragmentation function (similar to PDFs)
  Probability to find a hadron with a fraction x of the quarks momentum: $D_{i\to h}(x)$
- No Parton-shower needed
- Implementation in the STRIPPER framework through NNLO QCD → Czakon's talk
  B-hadron production in NNLO QCD: application to LHC ttbar events with leptonic decays,
  Czakon, Generet, Mitov and Poncelet, 2102.08267
- Photon fragmentation in NNLOJet → Höfer's talk
  Photon Fragmentation in the Antenna Subtraction Formalism,
  Gehrmann and Schürmann, 2201.06982

B-hadrons in ttbar production

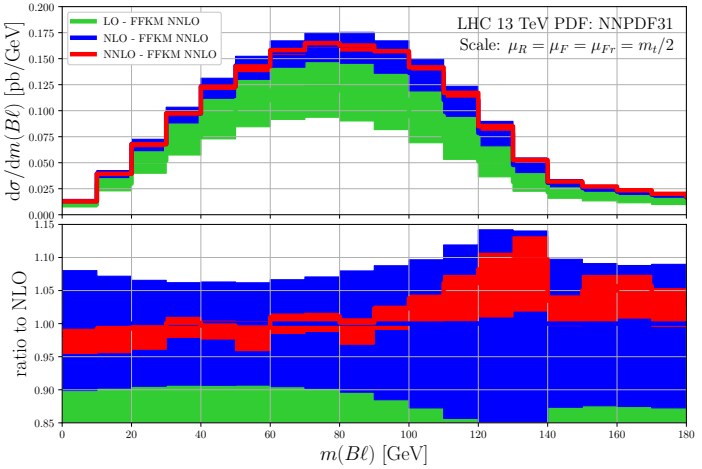
$$pp \to t\bar{t} \to B\ell\bar{\ell}\nu\bar{\nu}b + X$$

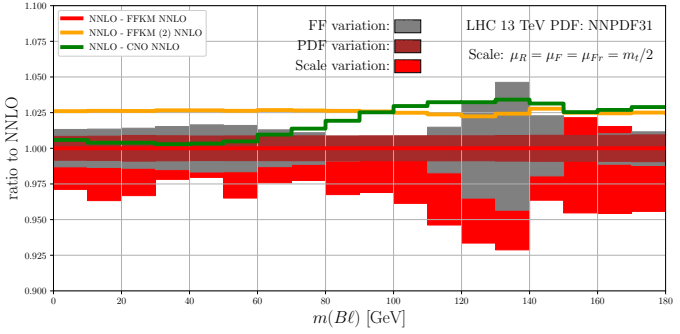


pT(B)/pT(jB): sensitive to B-hadron fraction x



m(lB): sensitive to top-quark mass





Further interesting topics

- Mixed QCD-EW → Freitas', Vicini's, Bonetti's talk
- Flavoured jets → useful for PDF fits BUT: → Czakon's talk
  - Anti-kT is IR unsafe at NNLO QCD
  - Ways around by using the flavour kT algorithm
  - But: Measurements are performed with anti-kT algorithm
    → additional unfolding necessary
    → makes applications to PDF fits difficult
    → Room for improvement → flavour sensitive anti-kT algorithm
- New FastNLO tables @ NNLO by the NNLOJet group for DIS, single inclusive jets, pp → Zj
- SMEFT meets NNLO+PS
  NNLO event generation for pp → Zh → l+l- bbar production in the SM effective field theory,
  Haisch, Scott, Wiesemann, Zanderighi, Zanoli, 2204.00663

What's next?

N3LO:

- Techniques based on qT slicing+resummation do work for any colour singlet process
- Limitations:
  - Computational complexity:
    - Numerical stability of two-loop amplitudes
    - Stability of NNLO subtractions in qT → 0 limit
    - Power corrections in fiducial phase spaces
  - Three loop amplitudes pp → S
  - Numerical stable two-loop amplitudes: 2→3 amplitudes

pp → AA+X

Diphoton Amplitudes in Three-Loop Quantum Chromodynamics,
Caola, Von Manteuffel and Tancredi, 2011.13946
Two-Loop Helicity Amplitudes for Diphoton Plus Jet Production in Full Color,
Agarwal, Buccioni, von Manteuffel and Tancredi, 2105.04585

pp → WA+X ?

Two-loop leading colour helicity amplitudes for WAj production at the LHC,
Badger, Hartanto, Krys and Zoia, 2105.04585

What's next?

NNLO QCD:

- New processes → what two-loop amplitudes are or will be available soon?
  - 2 → 3 massless: pp → AAA (LC), pp → AAj (FC), pp → jjj (LC) , pp → Ajj ?
    → All ingredients available for the complete set → at most technical challenges
  - 2 → 3 one-mass:
    - Progress on master integrals:
      Analytic representation of all planar two-loop five-point Master Integrals with one off-shell leg, Canko, Papadopoulos, Syrrakos, 2009.13917
      Pentagon functions for one-mass planar scattering amplitudes, Chicherin, Sotnikov and Zoia, 2110.10111
      Two-loop hexa-box integrals for non-planar five-point one-mass processes, Abreu, Ita, Page and Tschernow, 2107.14180
    - pp → Wjj (planar/LC):
      Two-Loop QCD Corrections to Wbb Production at Hadron Colliders, Badger, Hartanto and Zoia, 2102.02516
      Leading-Color Two-Loop Amplitudes for Four Partons and a W Boson in QCD, Abreu, Cordero, Ita,Klinkert, Page, Sotnikov, 2110.07541
  - 2 → 3 two-mass ???
- Matching to parton showers with final state jets:
  Towards NNLO+PS Matching with Sector Showers, Campbell, Höche, Li, Preuss and Skands, 2108.07133

Summary

- NNLO QCD covers more or less all 2 → 1 and 2 → 2 processes
- First 2 → 3 processes become available: pp → 3y, pp → yyj, pp→jjj
  - Double virtual amplitudes for 2 → 3 one-mass processes start to appear but virtual amplitudes are the main bottleneck for extending the portfolio → automation of numerical two-loop amplitudes?
- Drell-Yan type processes at N3LO
  - Fully differential → fiducial phase spaces
  - Need for N3LO PDFs
  - In foreseeable future: pp → yy ? → many technical challenges
- NNLO QCD + parton showers: pp → S, pp → ttbar
- Fragmentation at NNLO QCD
- And many other exciting applications!