NNLO QCD corrections for three-jet production

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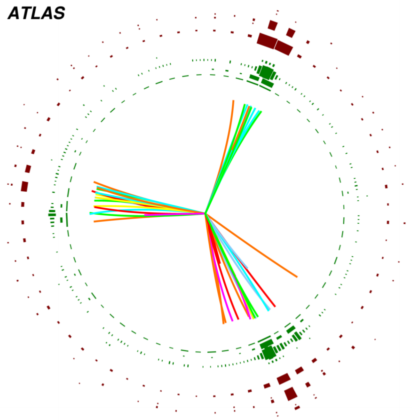
Multi-jet observables at the LHC

Multi-jet final states:

- Tests of pQCD at high energy
- Tests of MC modelling of LHC events
- Search for new physics



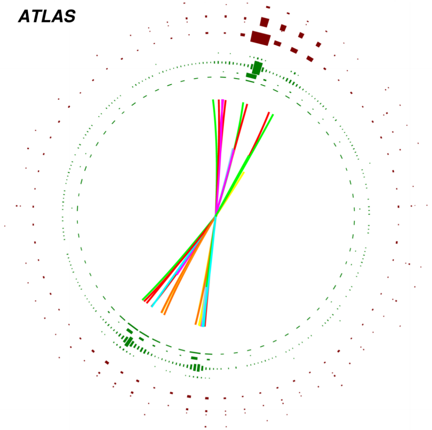
Study of perturbative QCD:

- R32 ratios

$$R_{3/2}(X, \mu_R, \mu_F) = \frac{\mathrm{d}\sigma_3(\mu_R, \mu_F)/\mathrm{d}X}{\mathrm{d}\sigma_2(\mu_R, \mu_F)/\mathrm{d}X} \sim \alpha_s$$

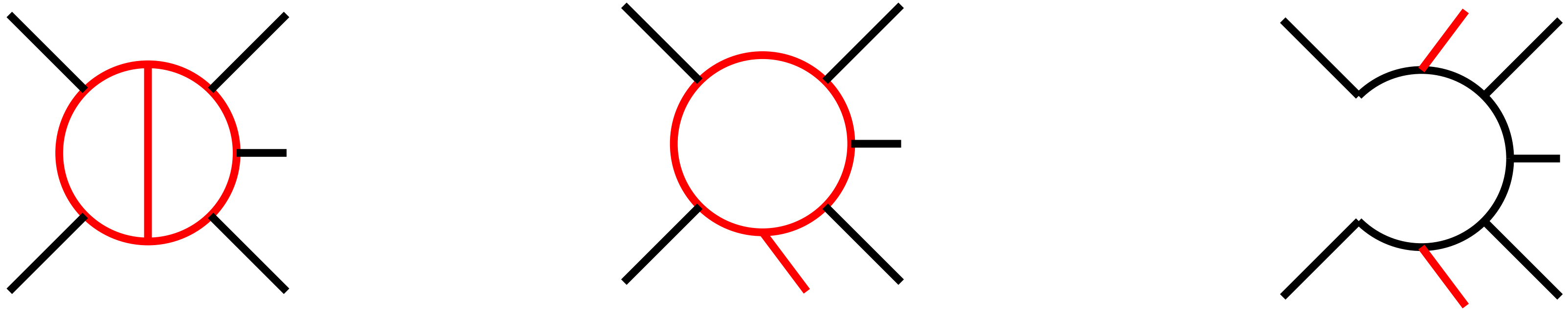
→ Extraction of the strong coupling constant

- Transverse Energy-Energy Correlator
- Event shapes



Credits: [ATLAS:2007.12600]

NNLO QCD prediction beyond 2 → 2



2 → 3 Two-loop amplitudes:

- Advances in amplitude techniques:

  IBPs, amplitude reconstruction and master integrals
- (Non-) planar 5 point massless amplitudes [Chawdry'19'20'21,Abreu'20'21,Agarwal'21, Badger'21]

  → triggered by efficient MI representation [Chicherin'20]

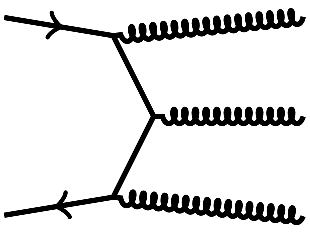
Cross-sections → Combination with real radiation

- Various NNLO subtraction schemes available:

  qT-slicing [Catain'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projetction [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17], Sector-improved residue subtraction [Czakon'10-'14,'19]

Three-jet production

- Sector-improved residue subtraction [Czakon'10'14'19]
  - Efficient c++ implementation → STRIPPER
  - Highly automated to deal with enormous amount of channels in three-jet production → O(1k) sectors →O(1M) individual MC integrals
  - Still computationally very challenging! → O(1M CPUh)
- Many-leg, IR stable one-loop amplitudes → OpenLoops [Buccioni'19]
- Double virtual amplitudes in leading-colour approximation [Abreu'21]
  - Sub-leading colour corrections expected to be small
  - Analytical expressions challenging
  - Fast numerical evaluation → very small contribution to computational cost



Only Approximation made:

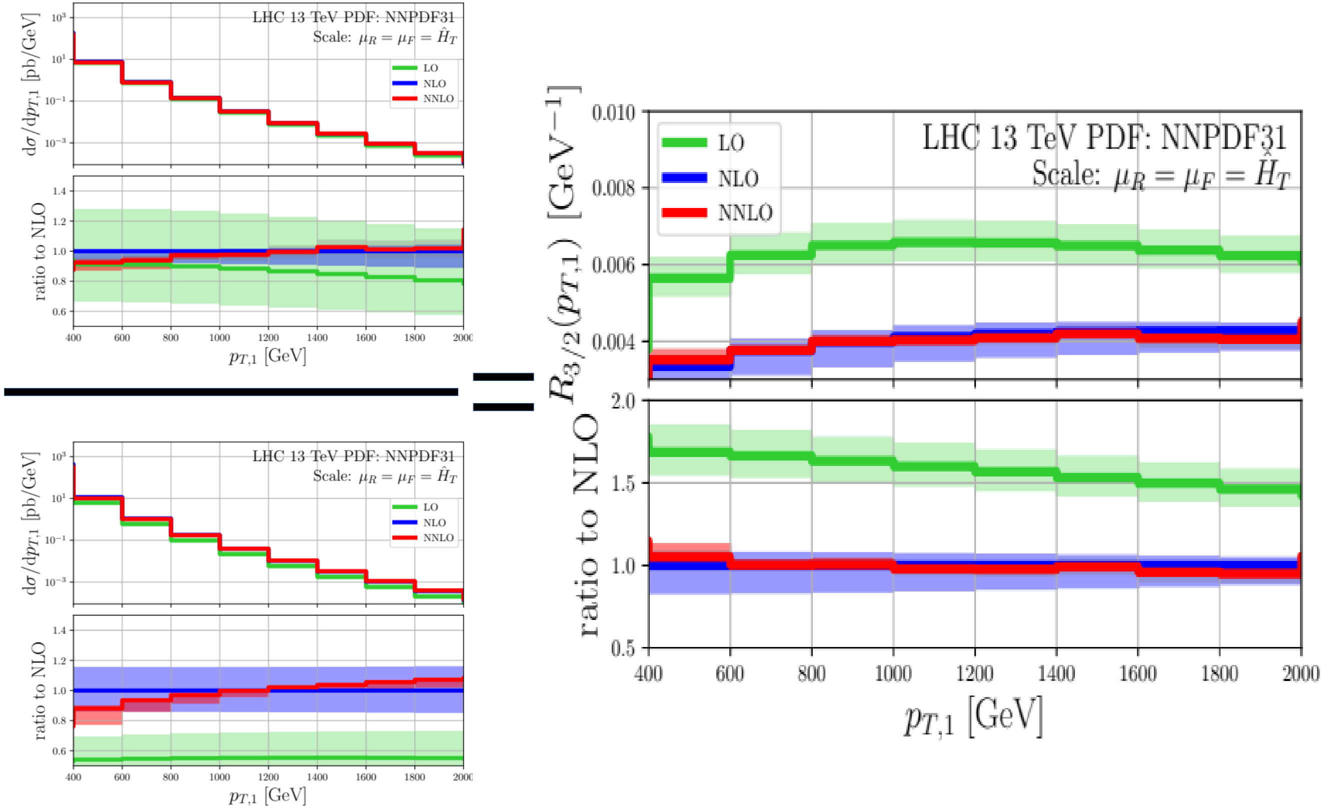
$$\mathcal{R}^{(2)}(\mu_R^2) = 2\,\mathrm{Re}\left[\mathcal{M}^{\dagger(0)}\mathcal{F}^{(2)}\right](\mu_R^2) + \left|\mathcal{F}^{(1)}\right|^2(\mu_R^2) \equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^{4} c_i \ln^i\left(\frac{\mu_R^2}{s_{12}}\right)$$

$$\mathcal{R}^{(2)}(s_{12}) \approx \mathcal{R}^{(2)l.c.}(s_{12})$$

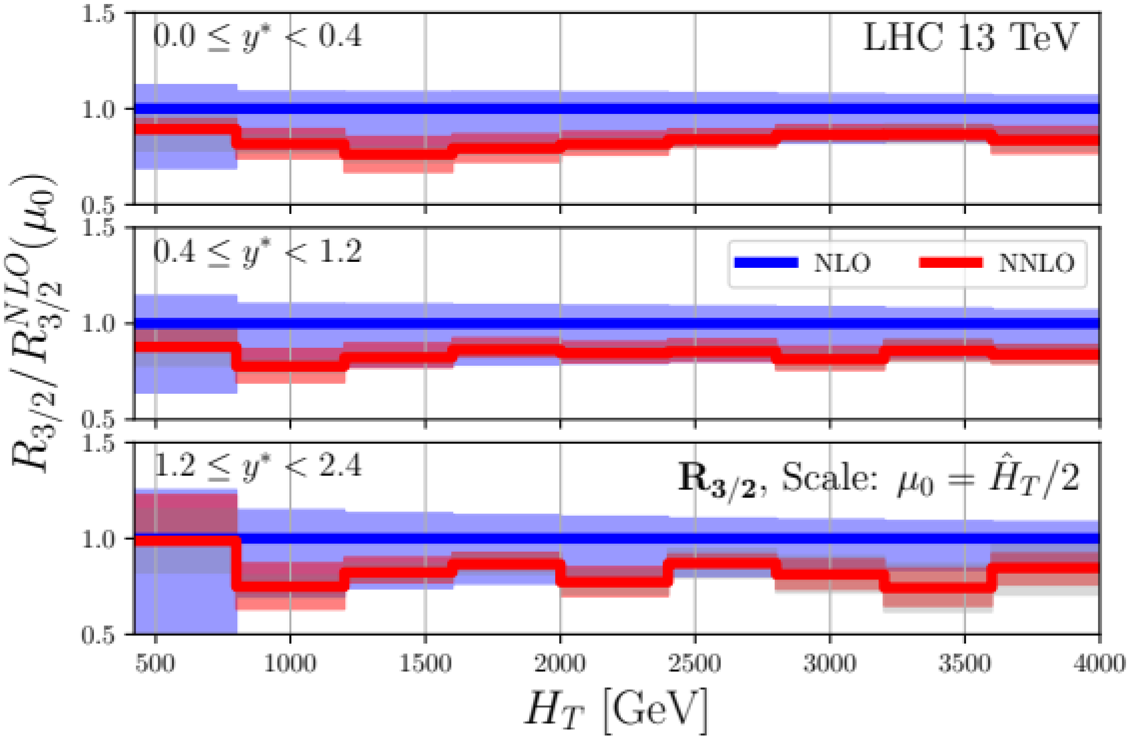
Three-jet production - R32(pT1)

- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets:
  - $p_T(j) > 60$ GeV and $|y(j)| < 4.4$
  - $H_{T,2} = p_T(j_1) + p_T(j_2) > 250$ GeV
- Scales:

$$\mu_R = \mu_F = \hat{H}_T = \sum_{\text{partons}} p_T$$



Three-jet production – R32(HT,y*)



Double differential w.r.t. $H_T = \sum_{\text{jets}} p_T$ and $y^* = |y(j_1) - y(j_2)|/2$

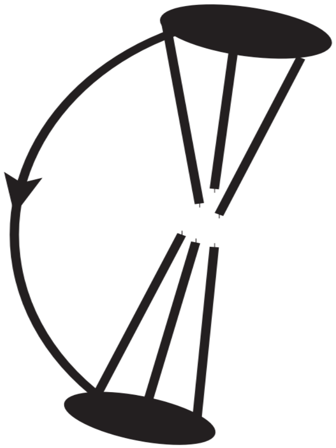
Central scale choice: $\hat{H}_T/2$

Three-jet production – azimuthal decorrelation

Kinematic constraints on the azimuthal separation between the two leading jets (ϕ_{12})

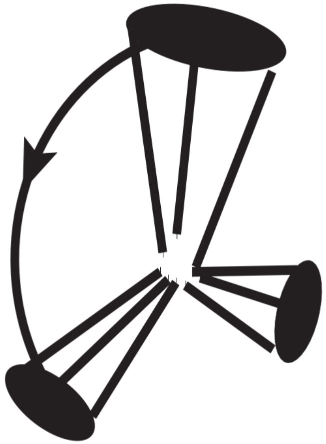
Dijet:

$\phi_{12} = \pi$



Trijet:

$\phi_{12} > \frac{2\pi}{3}$



φ12 sensitive to the jet multiplicity:

2j: $\phi_{12} = \pi$

3j: $\phi_{12} > \frac{2\pi}{3}$

4j: unconstrained

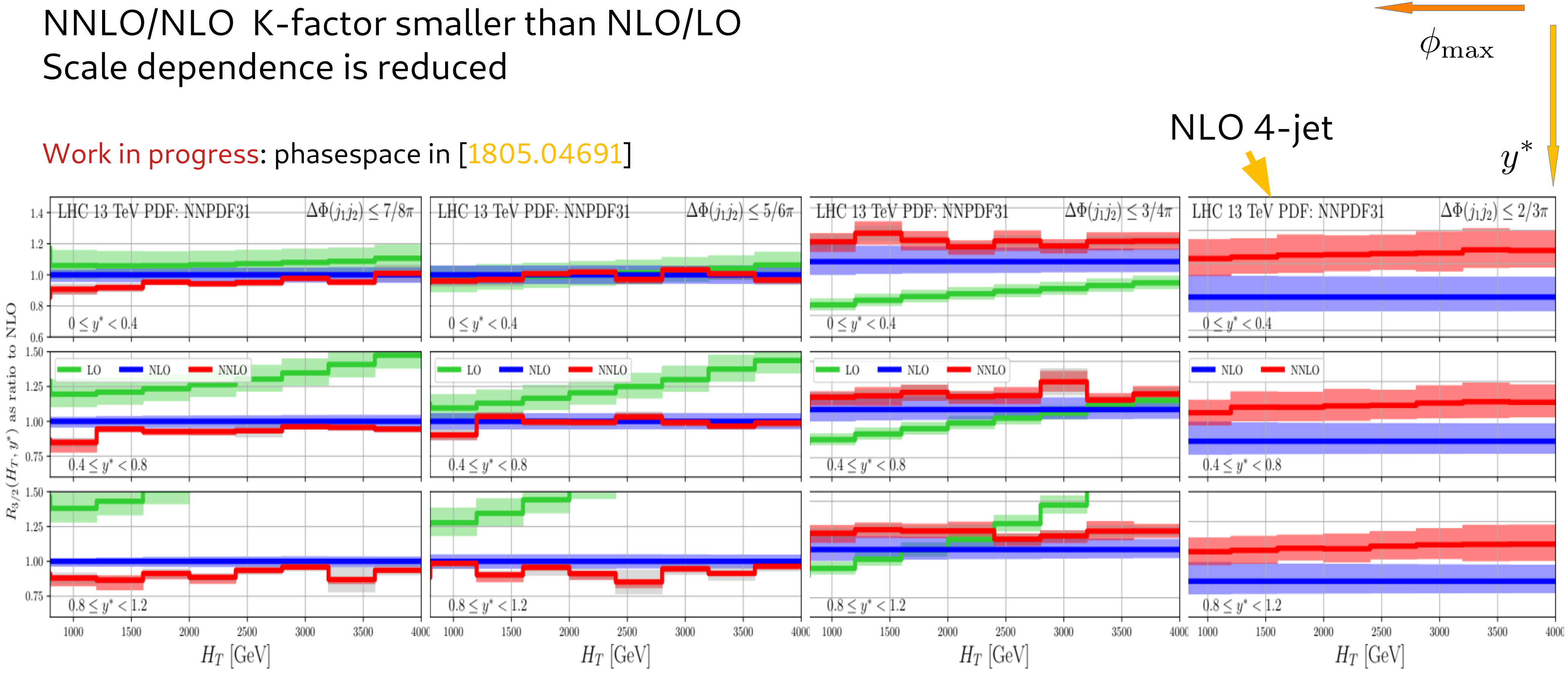
Study of the ratio:

$$R_{32}(H_T, y^*, \phi_{\max}) = \frac{\mathrm{d}\sigma_3(H_T, y^*, \phi_{12} < \phi_{\max})}{\mathrm{d}\sigma_2(H_T, y^*)}$$

Three-jet production - azimuthal decorrelation

NNLO/NLO K-factor smaller than NLO/LO
Scale dependence is reduced

Work in progress: phasespace in [1805.04691]



Outlook: Extraction of the strong coupling constant from multi-jet events at the LHC

→ Transverse Energy-Energy Correlator TEEC

→ Event shapes

Transverse Energy-Energy Correlator @ LHC

TEEC: Transverse Energy-Energy Correlation

$$\frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi} = \frac{1}{N}\sum_{A=1}^{N}\sum_{ij}\frac{E^A_{\perp,i}E^A_{\perp,j}}{\left(\sum_k E^A_{T,k}\right)^2}\delta(\cos\phi - \cos\phi_{ij})$$

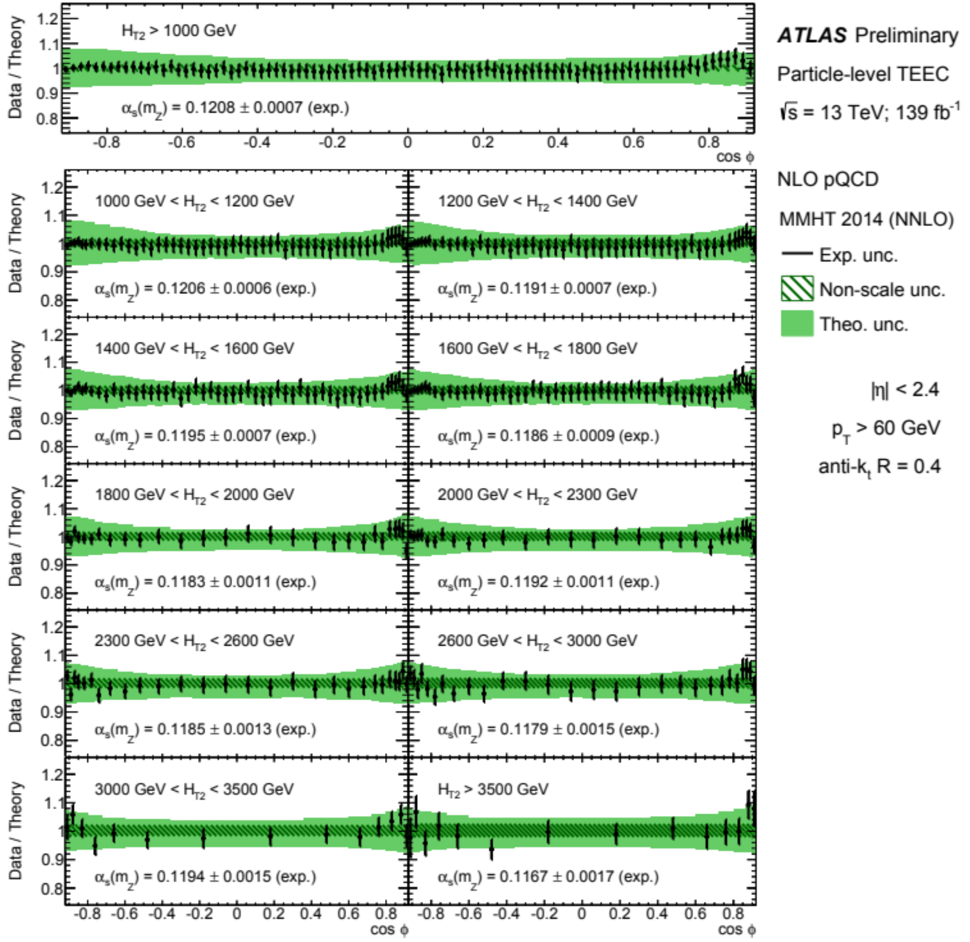
ATLAS measurement of the TEEC and ATEEC:

- @ 8 TeV [ATLAS:1707.02562]
- @ 13 TeV [ATLAS-CONF-2020-025]

TEEC in HT2 bins:

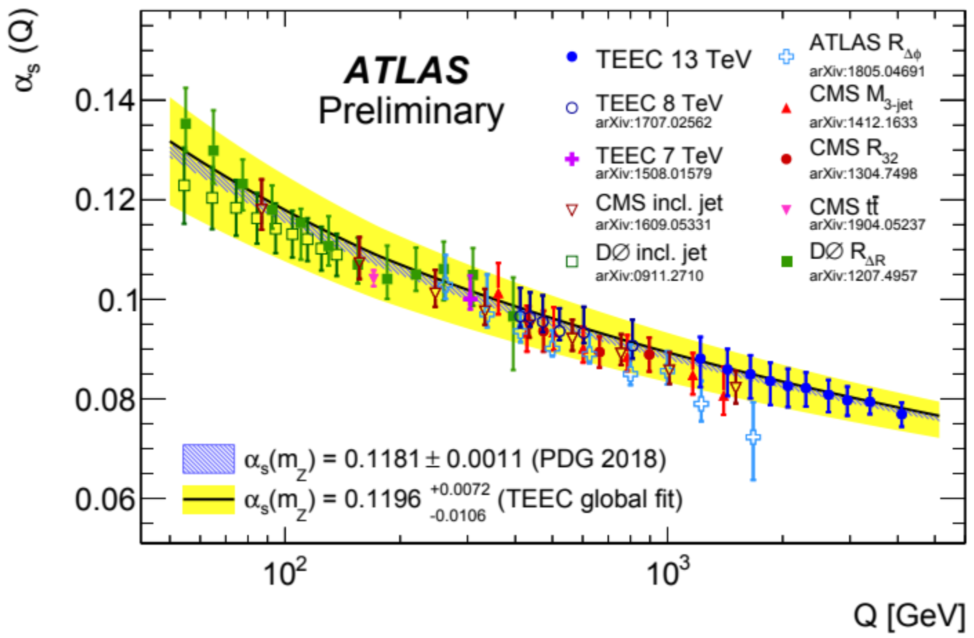
→ from 1000 GeV to 3500 GeV and above

→ sensitivity to different energy scales



Transverse Energy-Energy Correlator @ LHC

Extraction of alphas in different HT bins → test of SM running



$\langle Q \rangle$ [GeV]	$\alpha_s(m_Z)$ value (MMHT 2014)
Global	0.1195 ± 0.0002 (stat.) ± 0.0006 (syst.) $^{+0.0084}_{-0.0106}$ (scale) ± 0.0009 (PDF) ± 0.0003 (NP)
Inclusive	0.1198 ± 0.0002 (stat.) ± 0.0006 (syst.) $^{+0.0078}_{-0.0095}$ (scale) ± 0.0010 (PDF) ± 0.0002 (NP)
1219	0.1202 ± 0.0003 (stat.) ± 0.0006 (syst.) $^{+0.0079}_{-0.0098}$ (scale) ± 0.0010 (PDF) ± 0.0002 (NP)
1434	0.1184 ± 0.0003 (stat.) ± 0.0007 (syst.) $^{+0.0078}_{-0.0098}$ (scale) ± 0.0011 (PDF) ± 0.0002 (NP)
1647	0.1188 ± 0.0004 (stat.) ± 0.0007 (syst.) $^{+0.0073}_{-0.0087}$ (scale) ± 0.0012 (PDF) ± 0.0001 (NP)
1856	0.1177 ± 0.0006 (stat.) ± 0.0008 (syst.) $^{+0.0072}_{-0.0083}$ (scale) ± 0.0013 (PDF) ± 0.0006 (NP)
2064	0.1174 ± 0.0008 (stat.) ± 0.0009 (syst.) $^{+0.0069}_{-0.0078}$ (scale) ± 0.0013 (PDF) ± 0.0007 (NP)
2300	0.1185 ± 0.0009 (stat.) ± 0.0010 (syst.) $^{+0.0063}_{-0.0067}$ (scale) ± 0.0014 (PDF) ± 0.0005 (NP)
2636	0.1166 ± 0.0016 (stat.) ± 0.0012 (syst.) $^{+0.0062}_{-0.0066}$ (scale) ± 0.0015 (PDF) ± 0.0000 (NP)
2952	0.1141 ± 0.0029 (stat.) ± 0.0013 (syst.) $^{+0.0062}_{-0.0069}$ (scale) ± 0.0018 (PDF) ± 0.0003 (NP)
3383	0.1164 ± 0.0043 (stat.) ± 0.0015 (syst.) $^{+0.0050}_{-0.0044}$ (scale) ± 0.0017 (PDF) ± 0.0001 (NP)
4095	0.1029 ± 0.0163 (stat.) ± 0.0014 (syst.) $^{+0.0066}_{-0.0012}$ (scale) ± 0.0010 (PDF) ± 0.0003 (NP)

FO scale uncertainty limiting factor!

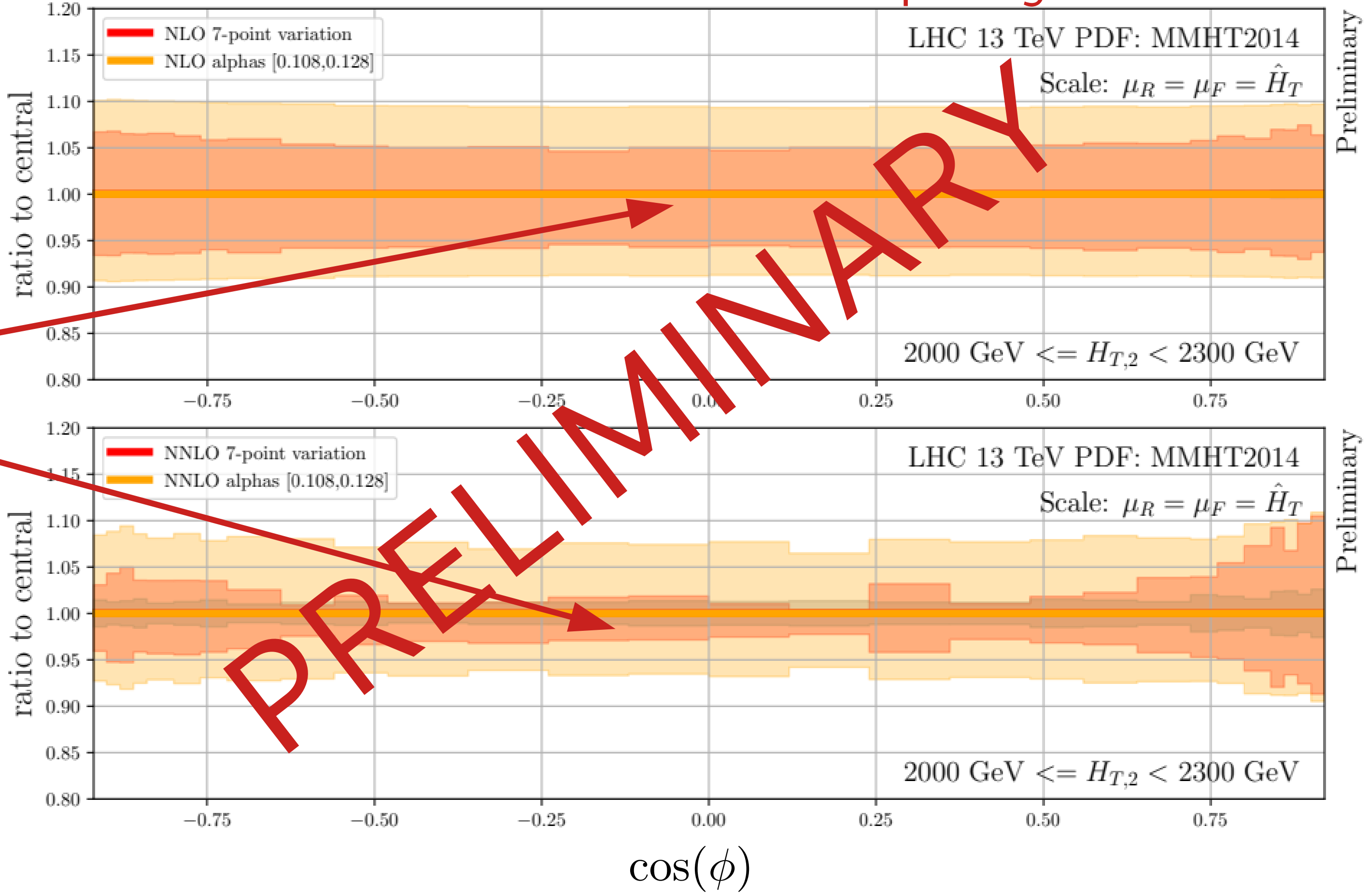
NNLO QCD corrections to TEEC @ LHC

Massive thanks to Manuel Alvarez and Javier Llorente for computing!

NLO

Reduction in scale dependence by factor 2-3

NNLO



Event shapes at the LHC

ATLAS measurement of event shapes @ 13 TeV using multi-jet events (139fb-1) in HT2 bins and high pT jets (> 100 GeV): [ATLAS:2007.12600]

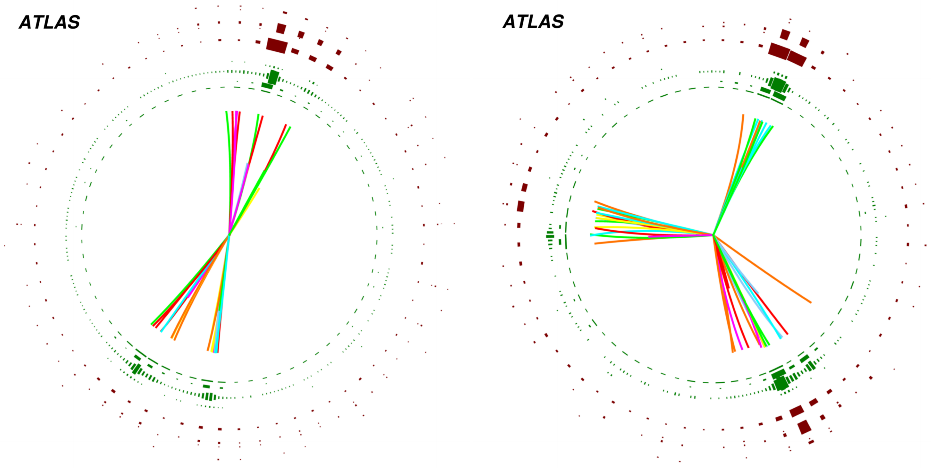
Transverse Thrust: $\tau_T = 1 - \frac{\sum_i^{\text{jets}} |\vec{p}_{T,i} \cdot \hat{n}|}{\sum_i^{\text{jets}} |\vec{p}_{T,i}|}$

Thrust Minor: $T_m = \frac{\sum_i^{\text{jets}} |\vec{p}_{T,i} \times \hat{n}|}{\sum_i^{\text{jets}} |\vec{p}_{T,i}|}$

More quantities based on eigenvalues of (transverse) linearised sphericity tensor:

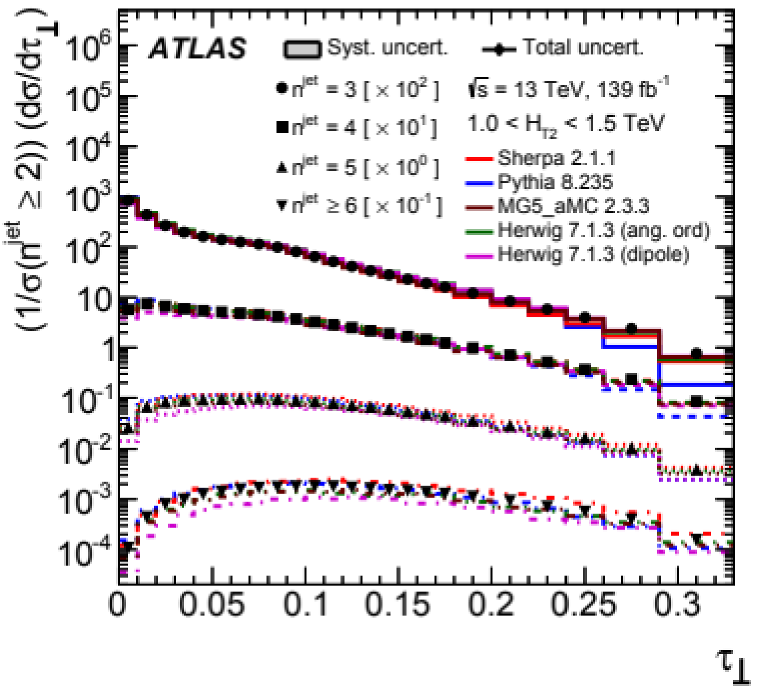
$$\mathcal{M}_{xyz} = \frac{1}{\sum_i^{\text{jets}} |\vec{p}_i|} \sum_i^{\text{jets}} \frac{1}{|\vec{p}_i|} \begin{pmatrix} p_{x,i}^2 & p_{x,i}p_{y,i} & p_{x,i}p_{z,i} \\ p_{y,i}p_{x,i} & p_{y,i}^2 & p_{y,i}p_{z,i} \\ p_{z,i}p_{x,i} & p_{z,i}p_{y,i} & p_{z,i}^2 \end{pmatrix}$$

Back-to-Back Spherical

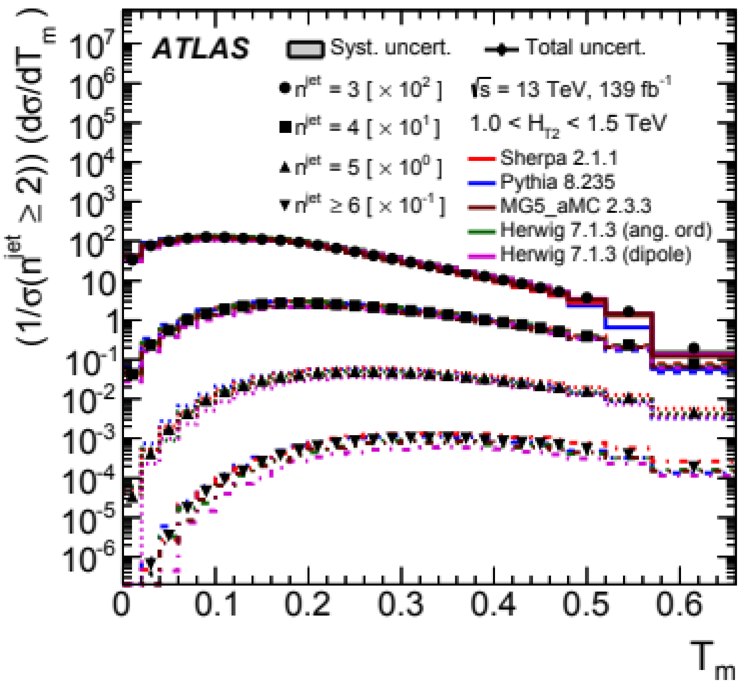


Event shapes at the LHC

Transverse thrust:



Transverse thrust minor:

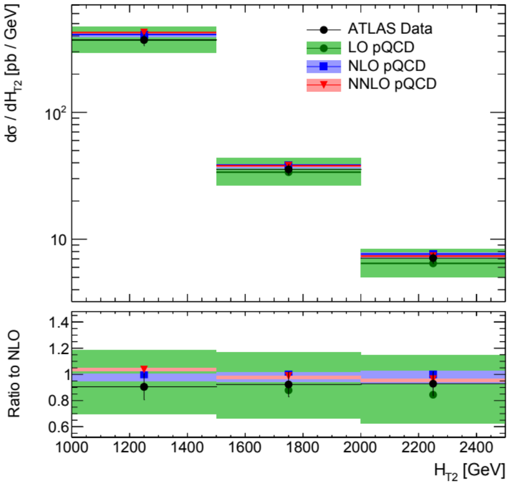


[ATLAS:2007.12600]

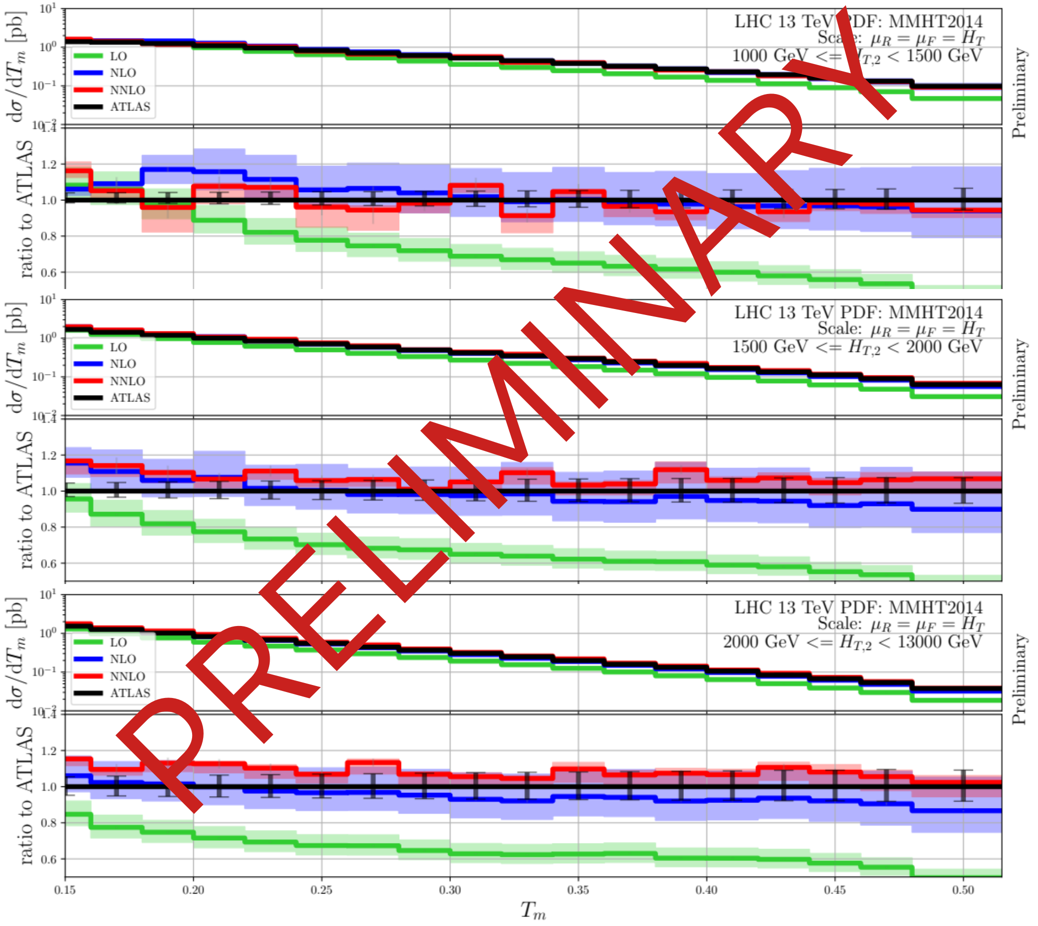
NNLO QCD corrections to event shapes

Comparison of public data from HEPdata

HT 2 denominator:



Credits:
Javier Llorente!



Example Thrust-Minor:

- Beautiful perturbative convergence
- Significant reduction of perturbative corrections

Summary and Outlook

NNLO predictions with the sector-improved residue subtraction framework

- First computations of 2 → 3 processes: 3 photon, 2 photon+jet and three-jet production
- Three jets @ the LHC:
  - R32 ratios → reduction of scale uncertainties, stabilization of K-factors
  - alphaS extractions from
    - R32 ratios
    - Event-shapes
    - TEEC

Many interesting applications ahead!
Stay tuned!

Thank you for your attention!

Backup

State of NNLO QCD at the LHC

NNLO QCD completed for 2 → 1, 2 → 2 SM processes:

- Colour singlet production: pp → H, pp → VV (available in MATRX [Grazzini'17], MCFM [Boughezal'16])
- Massive quark production: pp → ttbar (+decays) [Czakon'15,19], pp → bbar [Kallweit'20], single top [Campbell'17]
- Vector plus jet: pp → V+jet, pp → A + X, flavoured jets: pp → Z+b-jets, V+c-jets [NNLOJet '16-'20,Boughezal'15, Czakon'20]
- Di-jets: pp → j + X, pp → jj + X [NNLOJet '16-'20, Czakon,'19]

Recently first steps in the realm of 2 → 3 processes:

- Three photons [Chawdhry'19, Kallweit '20]
- Diphoton plus jet [Chawdhry'21], gg-induced [Badger'21]
- Three jets [Czakon,Mitov,Poncelet'21]

Beyond fixed-order QCD:

- Dedicated resummation calculations for specific observables
- First NNLO + PS appear for colour singlet and ttbar: MiNNLOPS with MATRIX [Monni '20]
- Identified hadron production: B-hadrons in ttbar production [Czakon'21]
- Photon fragmentation [Gehrmann'21]