

# NNLO predictions for three-jet cross sections at the LHC

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in collaboration with Michal Czakon and Alexander Mitov

based on: 1907.12911 and 2106.05331

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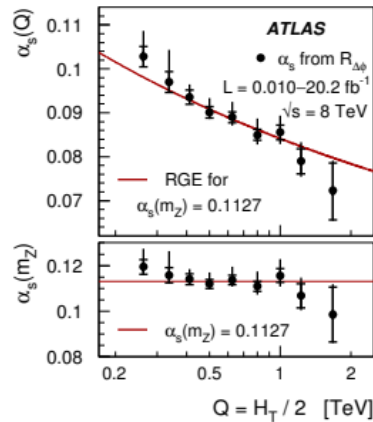
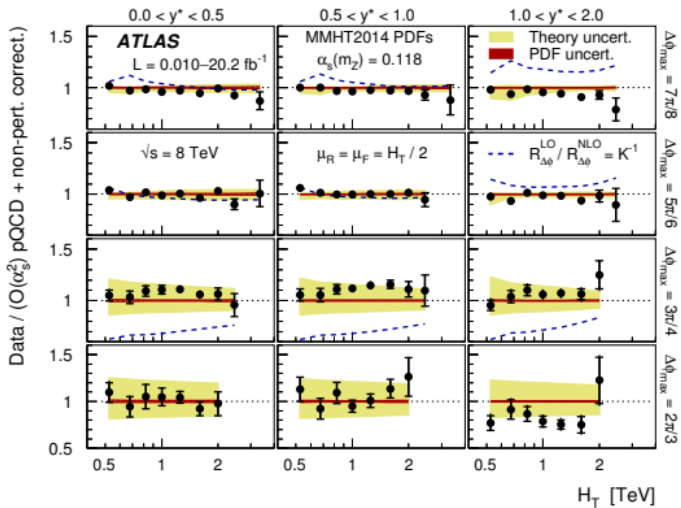
# Jet observables at the LHC

The LHC produces jets abundantly → many phenomenological applications

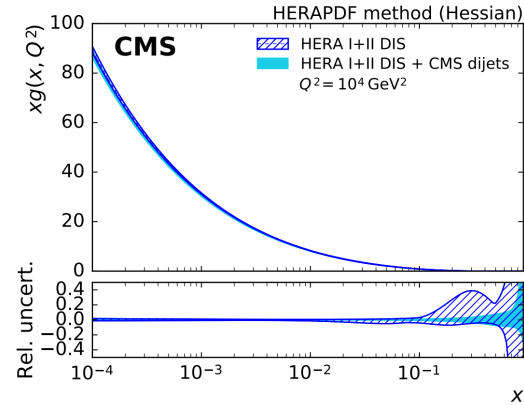
Tests of pQCD,  $\alpha_s$  extraction:  
R32 ratios, event-shapes

PDF determination:  
Single inclusive,  
Multi-differential dijet

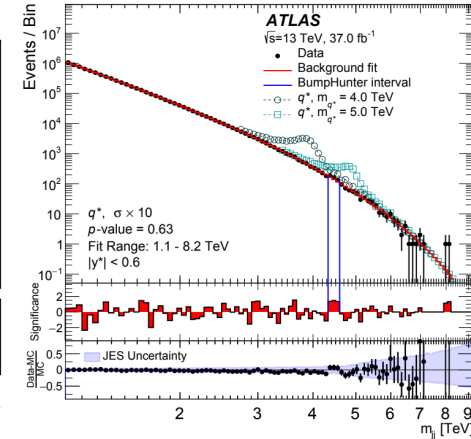
BSM searches:  
dijet mass



[1805.04691]



[1705.02628]



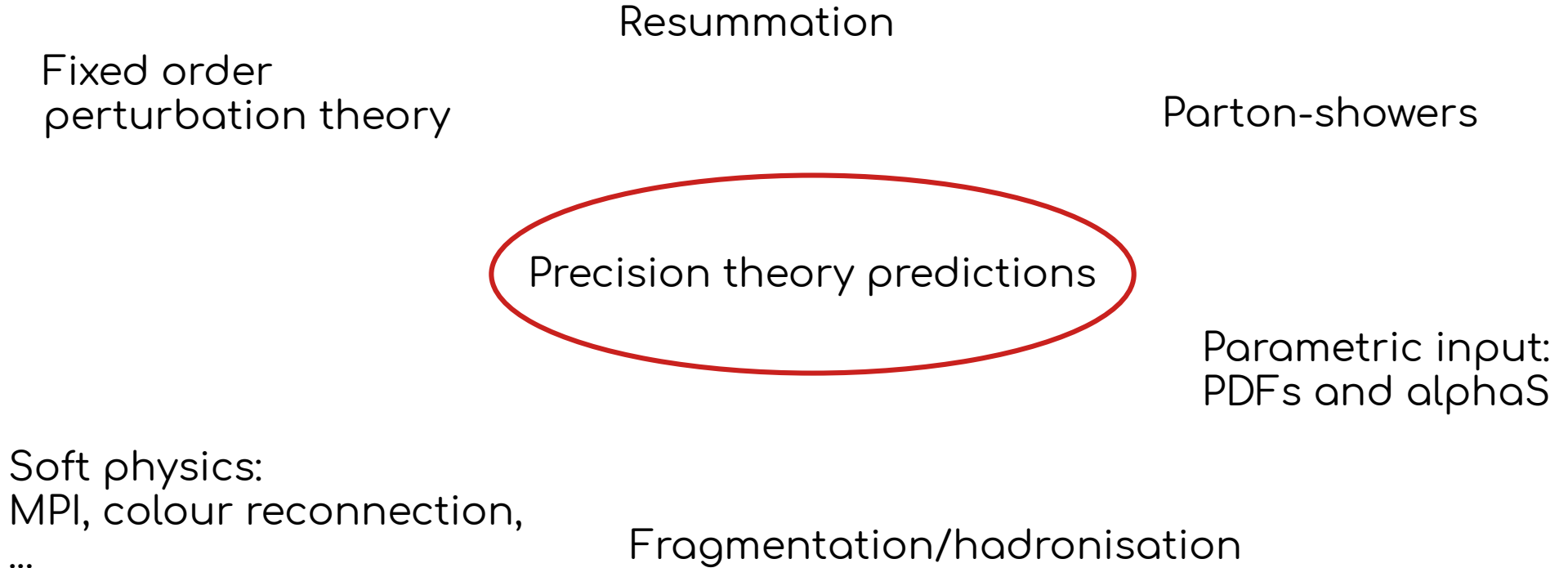
[1703.09127]

Precision theory required!

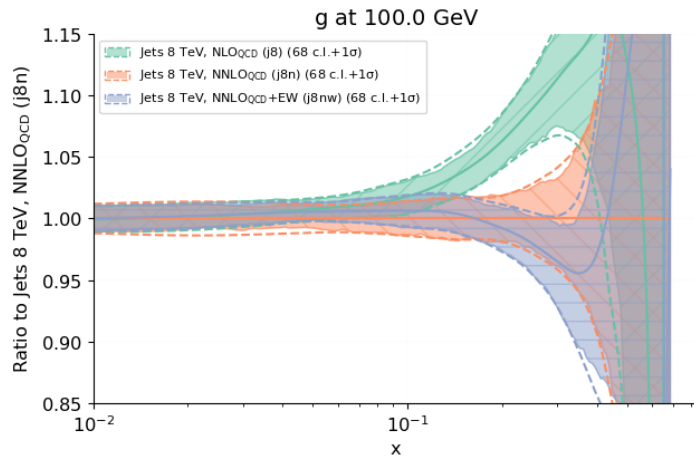
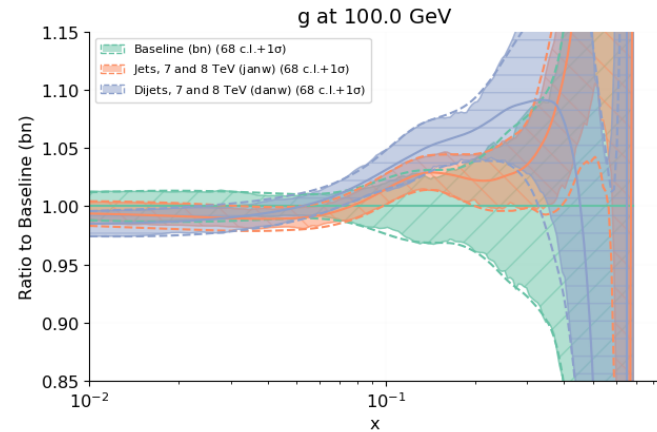
Data driven

# Precision predictions

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# Higher order $\rho$ QCD: PDF fits with jets



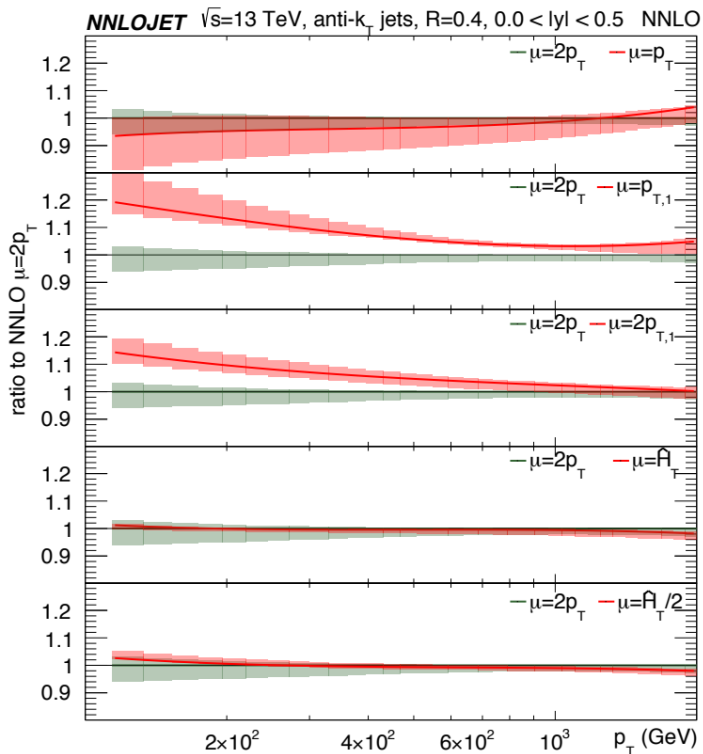
Idea (quite old actually [[Giele'94](#)]):

Here by a collaboration of NNLOJet and NNPDF [[Kholek'20](#)]:

Combine single inclusive and dijet triple differential measurements by ATLAS and CMS to constrain the large gluon- $x$

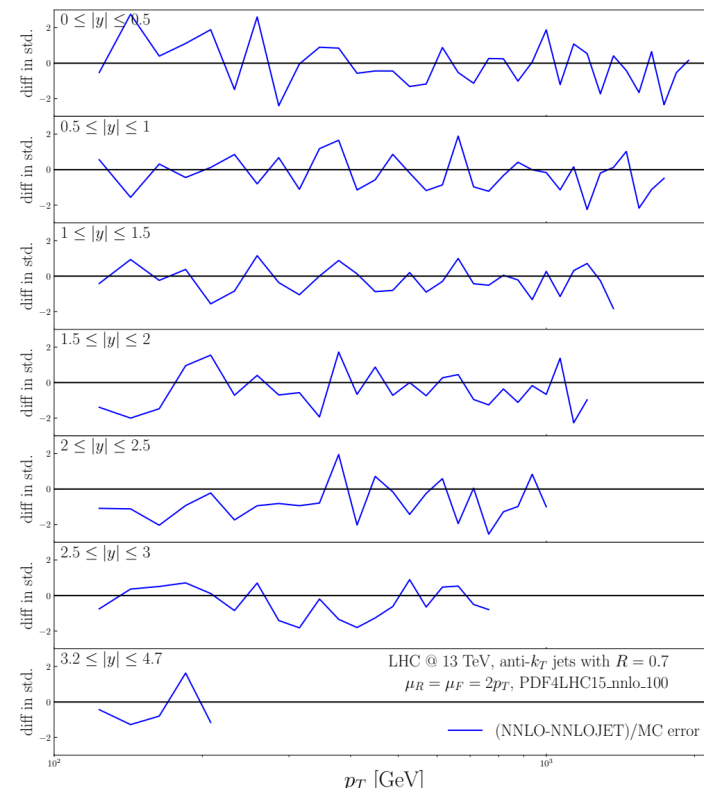
- Reduced uncertainty in large- $x$  gluon PDF
- **NNLO QCD corrections crucial** to obtain consistent results between data sets
- NLO EW [[Dittmaier'12](#)] or full NLO corrections [[Frederix'17](#), [Reyer'19](#)]

# Higher order pQCD: lessons from dijet



Detailed studies of **scale dependence**:  
 Event-based choices vs.  
 single jet choices  
 [Currie'18]

Study of **sub-leading colour effects**  
 in quark channels:  
 smaller than  $O(1\%)$   
 [Czakon'19]

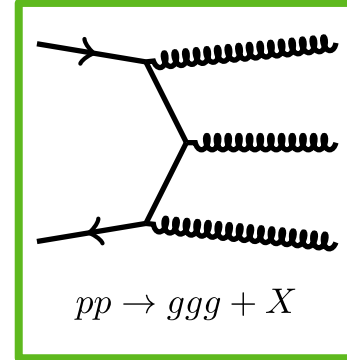


# Three jet production

Advances in perturbative QCD allow to tackle the most complicated  $2 \rightarrow 3$  process

## Bottlenecks:

- Double virtual amplitudes in leading colour approximation [Abreu'21]
- Handling of real radiation:
  - Sector-improved residue subtraction [Czakon'10'14'19] conceptually capable
  - Computationally very challenging!  $\rightarrow O(1M \text{ CPUh})$



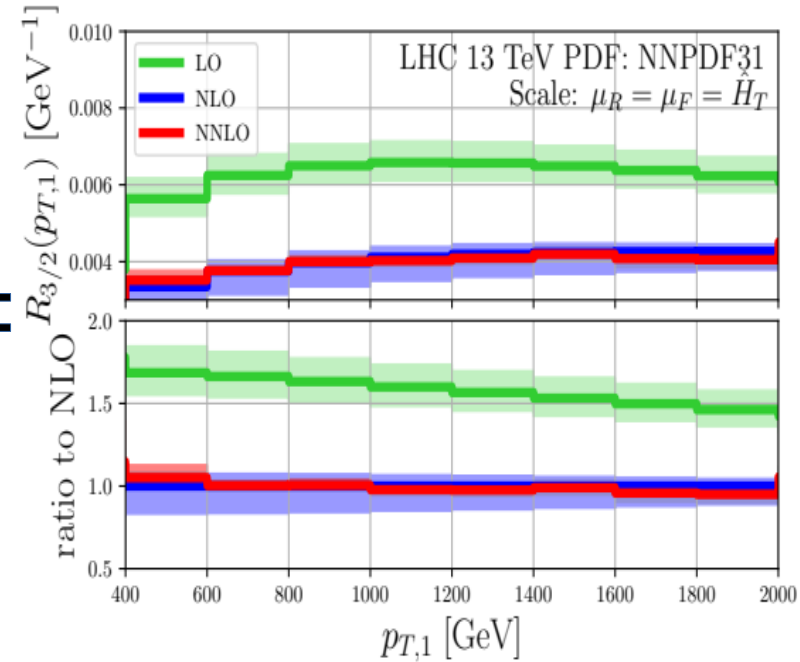
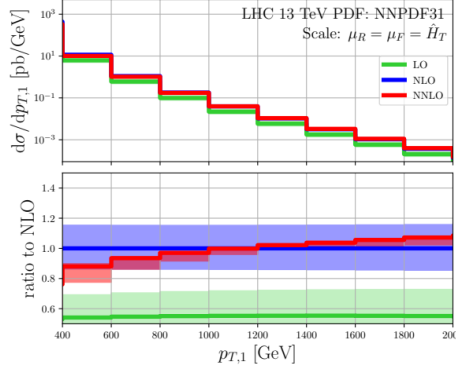
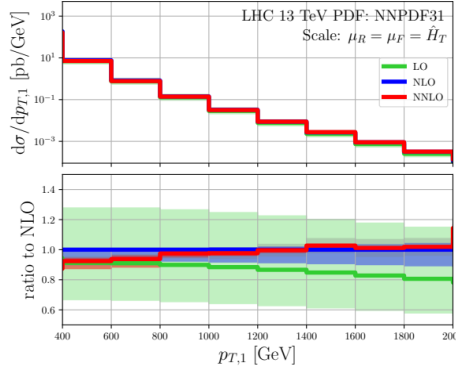
**Only** Approximation made:  $\mathcal{R}^{(2)}(\mu_R^2) = 2 \text{Re} \left[ \mathcal{M}^{\dagger(0)} \mathcal{F}^{(2)} \right] (\mu_R^2) + |\mathcal{F}^{(1)}|^2 (\mu_R^2) \equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^4 c_i \ln^i \left( \frac{\mu_R^2}{s_{12}} \right)$   
 $\rightarrow$  taken from [Abreu'21]  $\mathcal{R}^{(2)}(s_{12}) \approx \mathcal{R}^{(2)l.c.}(s_{12})$

# Three jet production - R32( $p_{T1}$ )

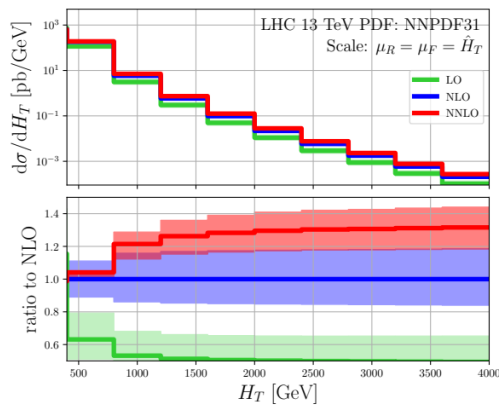
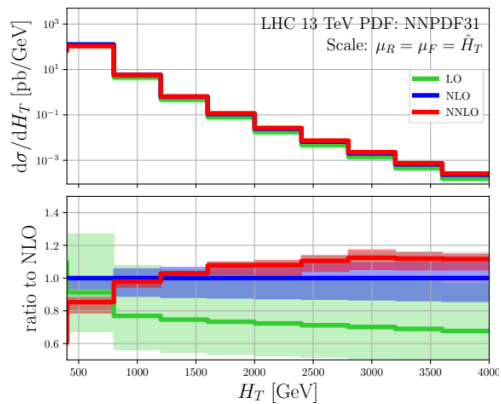
- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets:
  - $p_T(j) > 60$  GeV and  $|y(j)| < 4.4$
  - $H_{T,2} = p_T(j_1) + p_T(j_2) > 250$  GeV
- Scales:

$$\mu_R = \mu_F = \hat{H}_T = \sum_{\text{partons}} p_T$$

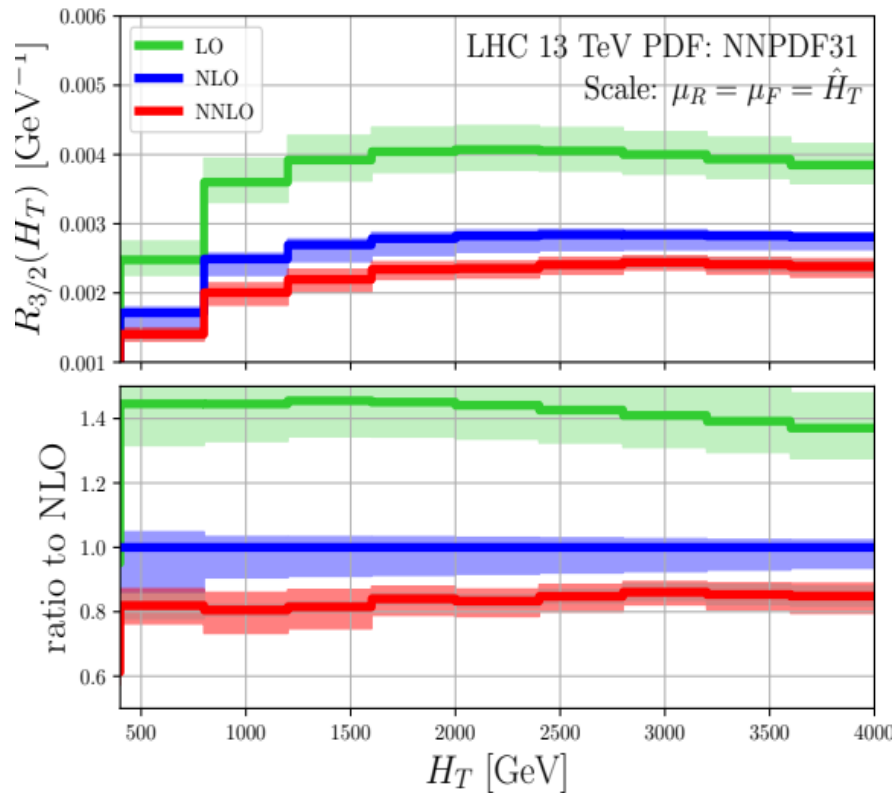
$$R_{3/2}(X, \mu_R, \mu_F) = \frac{d\sigma_3(\mu_R, \mu_F)/dX}{d\sigma_2(\mu_R, \mu_F)/dX}$$



# Three jet production - R32(HT)



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$$H_T = \sum_{\text{jets}} p_T$$

Scale dependence correlated in ratio

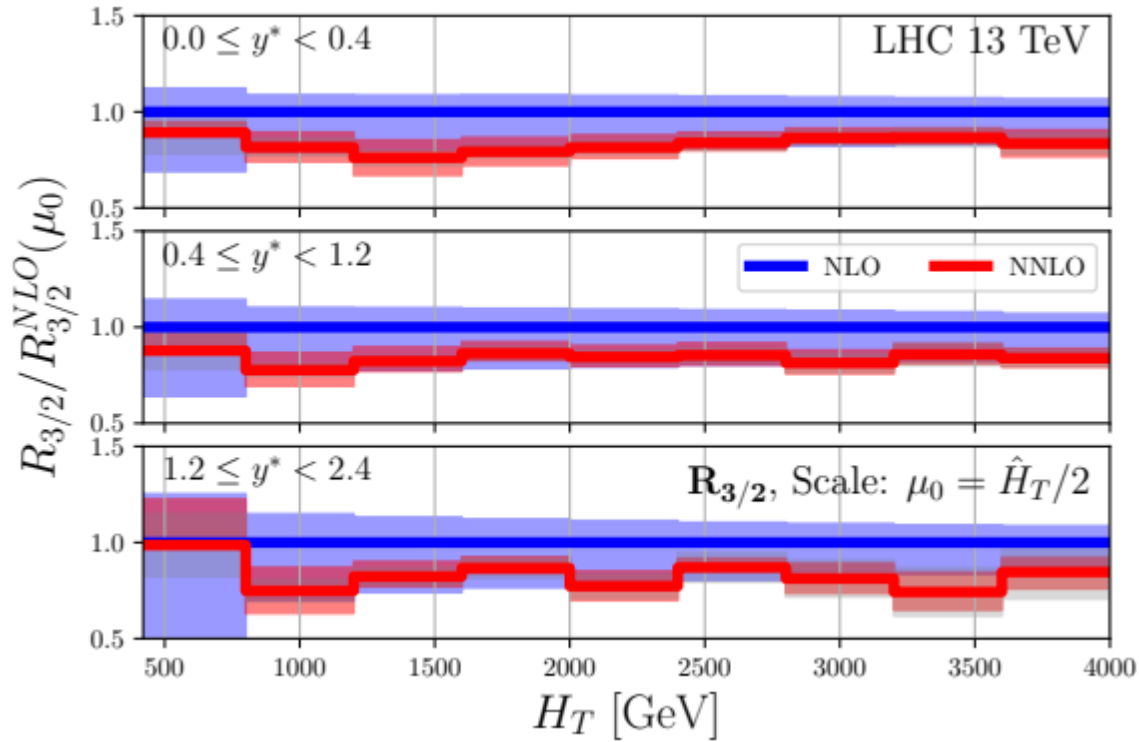
→ reduction of scale dependence

→ flat k-factor

→ scale bands in ratio barely overlap



# Three jet production – R32( $H_T, y^*$ )



Double differential w.r.t.  $y^* = |y(j_1) - y(j_2)|/2$

Different central scale choice:  $\hat{H}_T/2$

# Three jet production – azimuthal decorrelation

Kinematic constraints on the azimuthal separation between the two leading jets ( $\phi_{12}$ )

$\phi_{12}$  sensitive to the jet multiplicity:

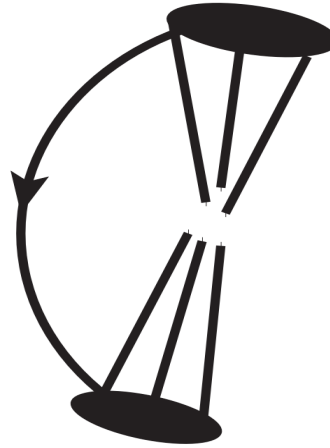
2j:  $\phi_{12} = \pi$

3j:  $\phi_{12} > 2/3\pi$

4j: unconstrained

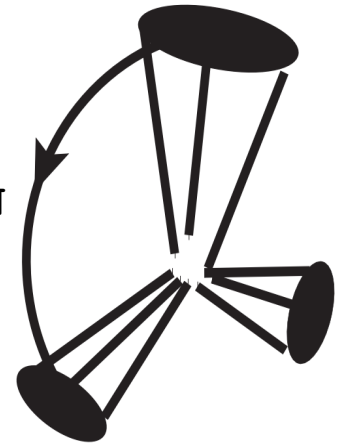
Dijet:

$\phi_{12} = \pi$



Trijet:

$\phi_{12} > 2/3\pi$



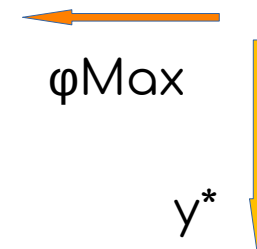
Study of the ratio

$$R_{32}(HT, y^*, \phi_{\text{Max}}) = \frac{d\sigma_3(\phi < \phi_{\text{Max}})/dHT/dy^*}{(d\sigma_2/dHT/dy^*)}$$

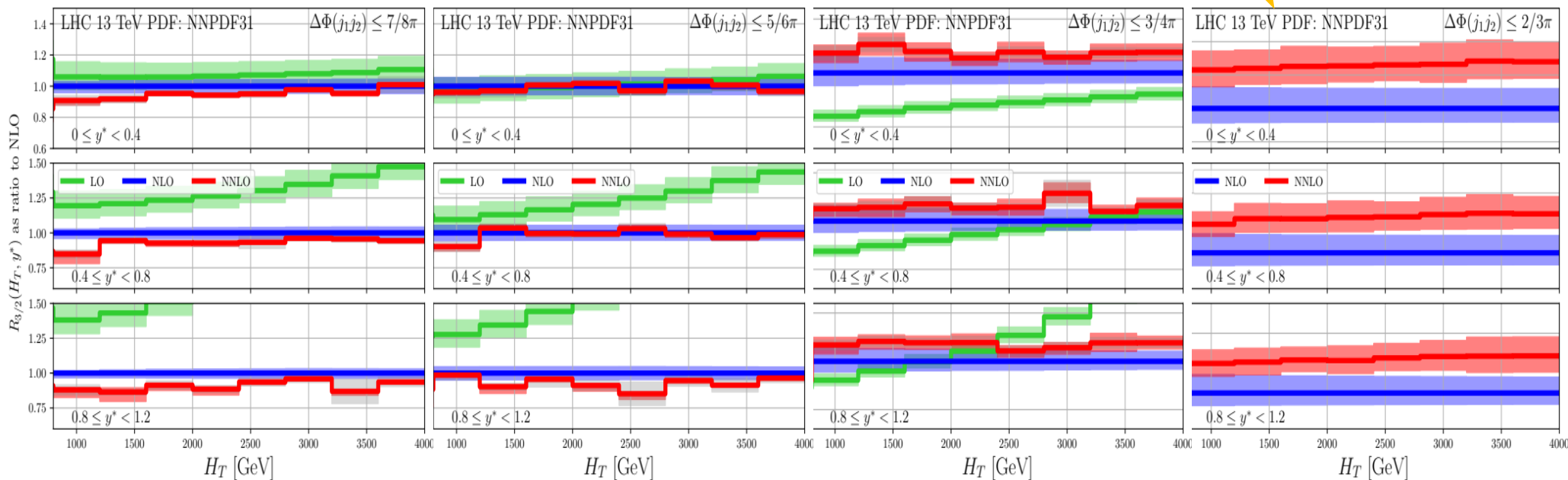
With  $y^* = |y_1 - y_2|/2$

# Three jet production – R32(HT,y\*,φMax)

NNLO/NLO K-factor smaller than NLO/LO  
Scale dependence is reduced

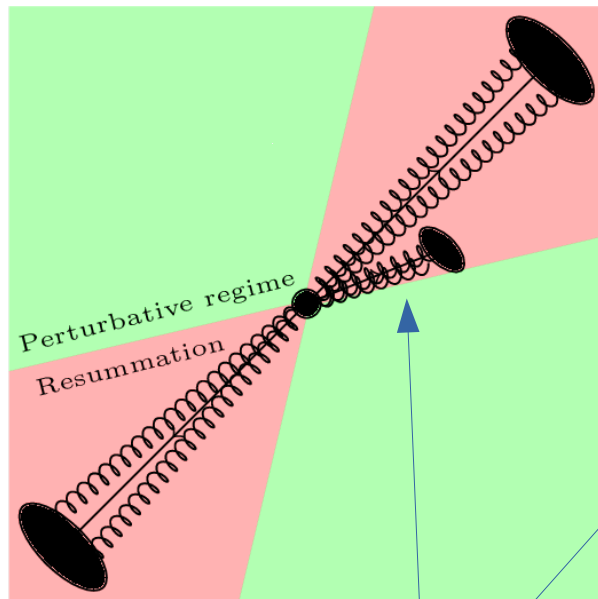


NLO 4-jet

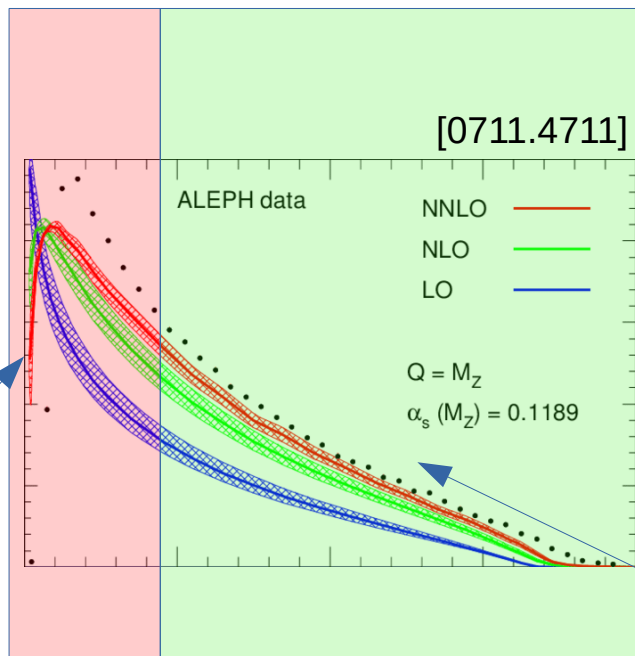


# Event-shapes regimes

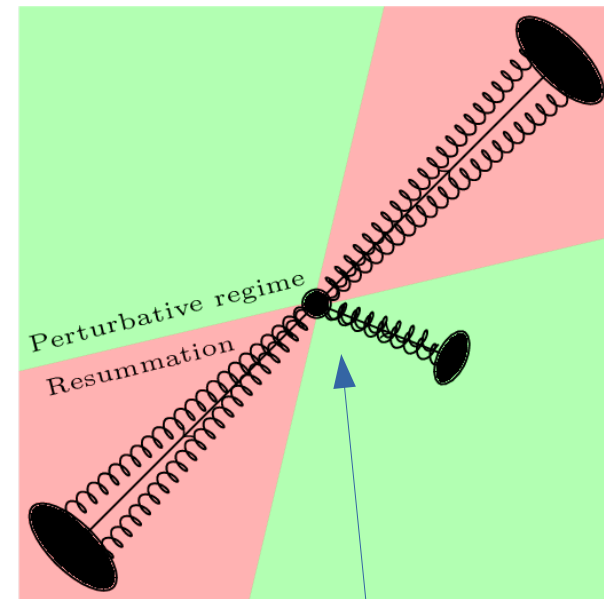
Typically event-shapes measure departure from N hard jet case



Anisotropic, 2-prong like  
Sensitivity to resummation,  
non-perturbative effects



Example: 1-Thrust at LEP

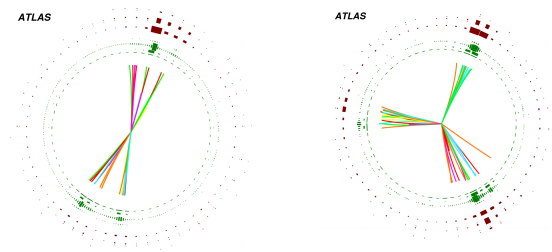


Isotropic, multi-jet  
Sensitive to hard  
matrix elements

# Event-shapes at the LHC

Event-shapes are measured using multi-jet events  
 → three jet is the leading contribution

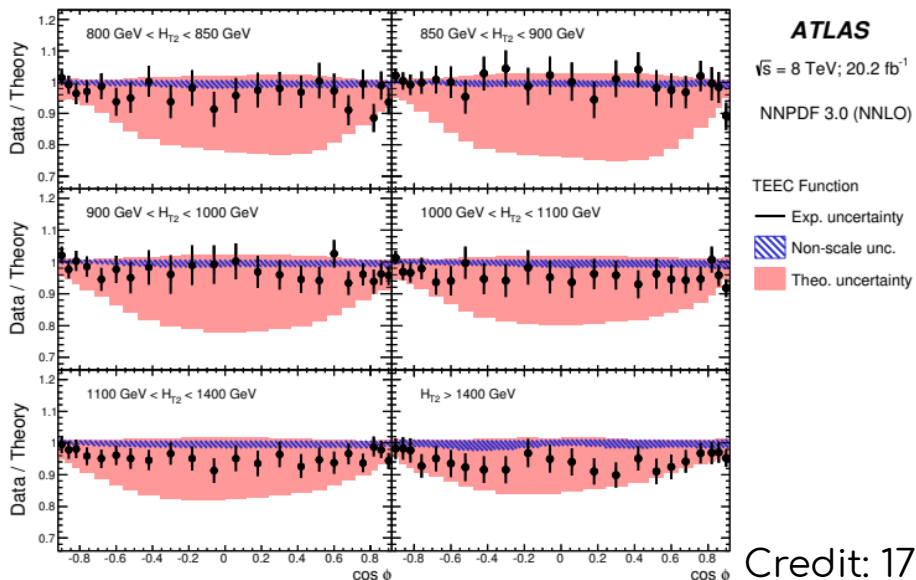
Credit: ATLAS 2007.12600



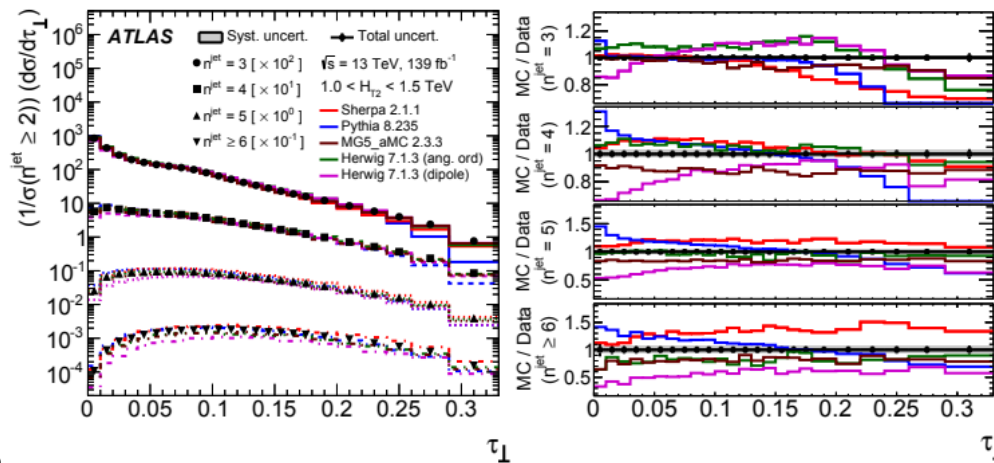
TEEC (Transverse Energy-Energy Correlation)

$$\frac{1}{\sigma} \frac{d\Sigma}{d \cos \phi} = \frac{1}{N} \sum_{A=1}^N \sum_{ij} \frac{E_{\perp,i}^A E_{\perp,j}^A}{\left(\sum_k E_{T,k}^A\right)^2} \delta(\cos \phi - \cos \phi_{ij})$$

Transverse Thrust:  $\tau_T = 1 - \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}|}{\sum |\vec{p}_{T,i}|}$



Credit: 1707.02562



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# Summary and Outlook

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## Jet rates with the sector-improved residue subtraction framework

- Full NNLO QCD predictions for di-jet production available
  - scale choice important
  - sub-leading colour contributions small
- Three jets @ the LHC:
  - First predictions available with approximate two-loop contribution!
    - improved scale dependence and stabilized K-factors
    - $p_T$  spectra, HT, double differential
  - Real radiation for  $2 \rightarrow 3$  can be handled.  
But efficiency is a concern and needs some attention!

Many interesting applications ahead! → Event-shapes  
Stay tuned!

Thank you for your attention!