Jets at the LHC: a fixed order perspective

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Outline

→ Jet measurements at the LHC

→ Three jet observables at NNLO QCD

R32 ratios
Event-shapes

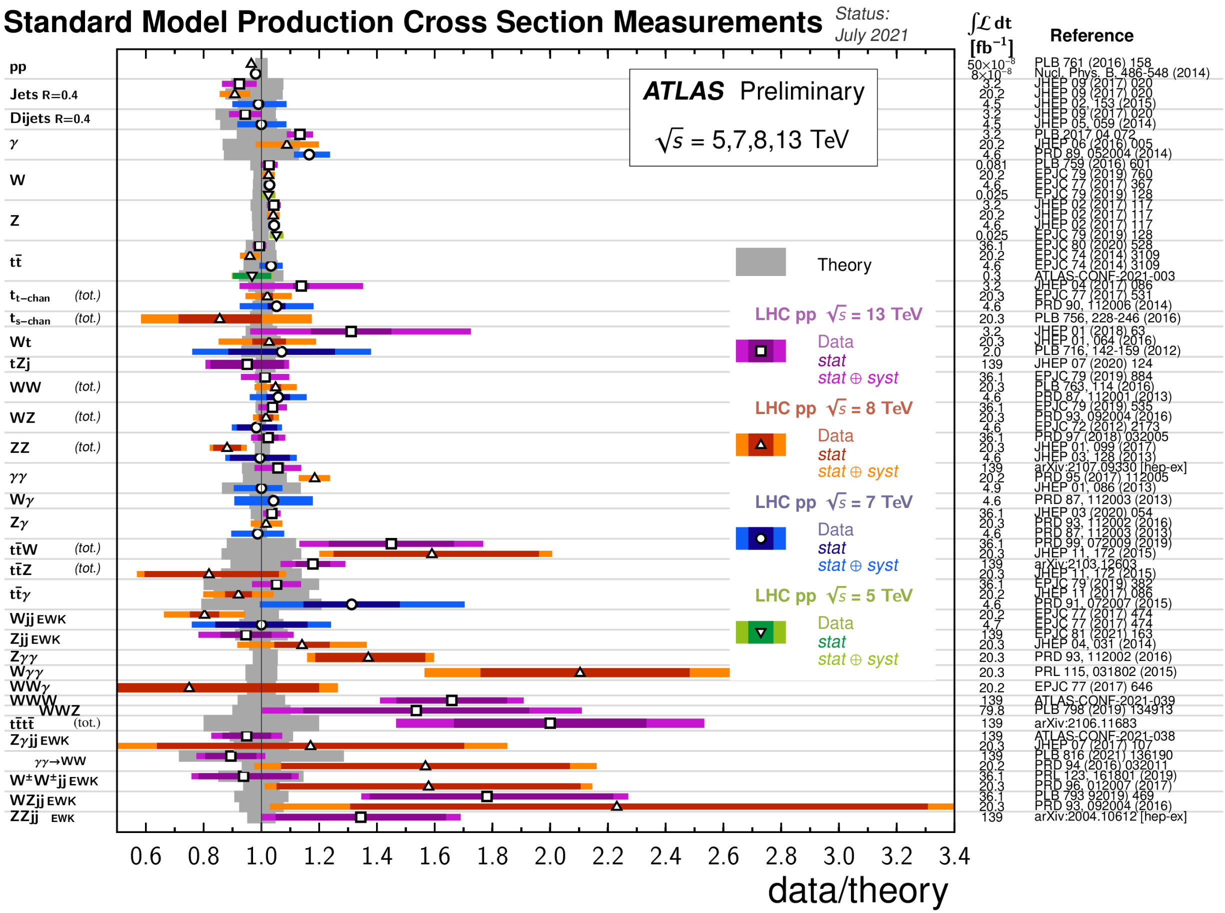
→ Flavoured jets

Infrared safe definition of jet flavour?
→ New proposal for a flavour safe algorithm.

→ Wrap-up and outlook

Jet measurements at the LHC

SM measurements at the LHC

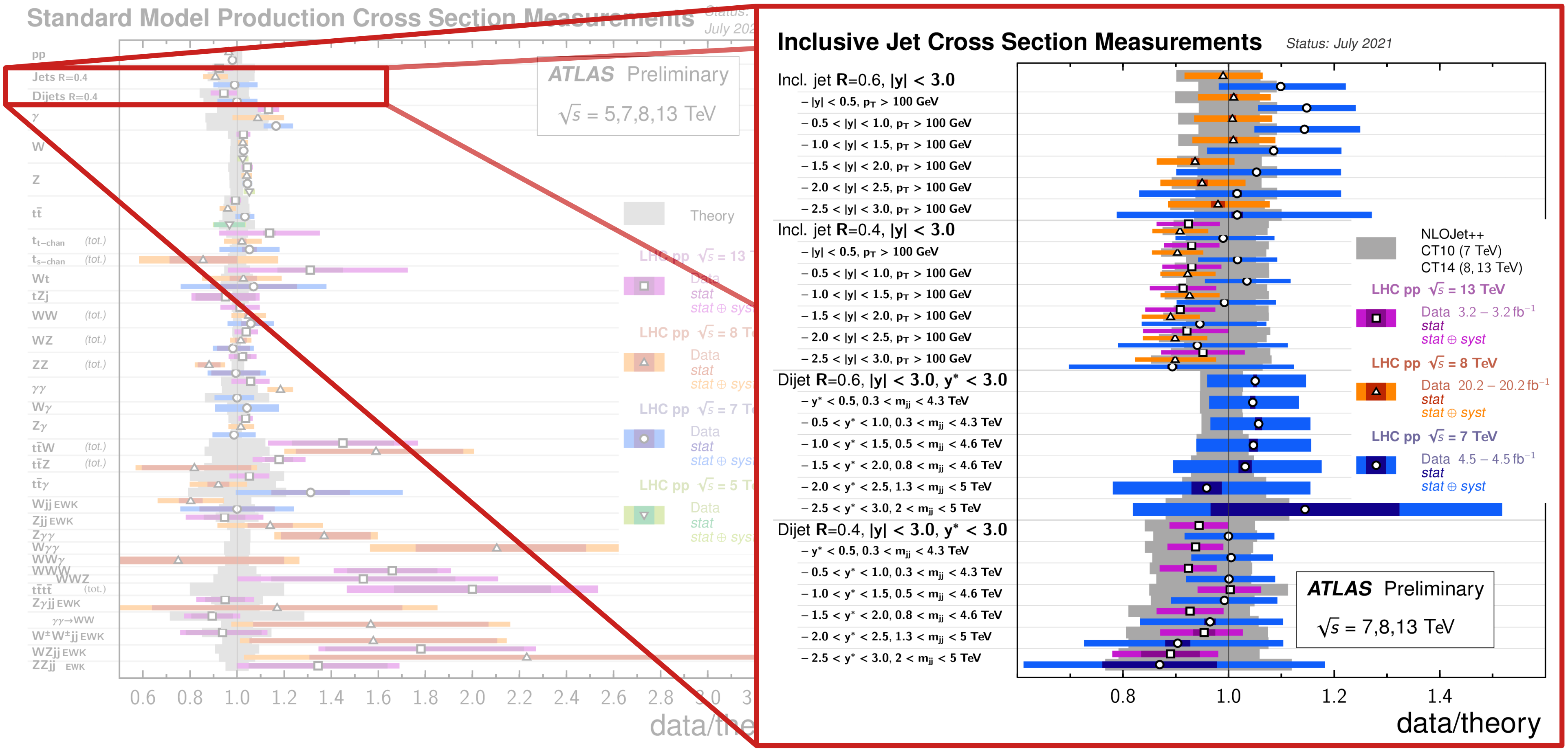


New physics
around the corner?

Precise measurements
<->
Precise theory

Win-Win situation
- improved SM understanding
- possible indirect BSM signals

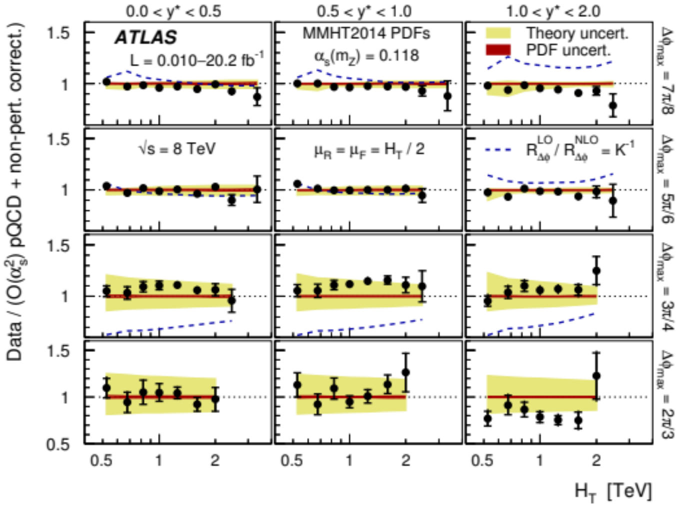
SM measurements at the LHC

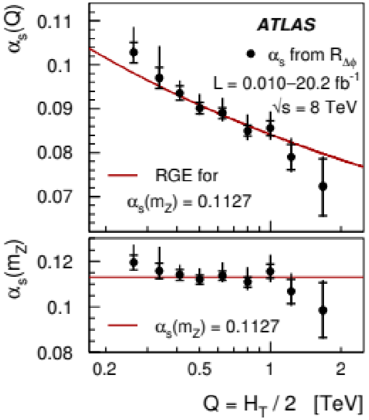


Jet observables at the LHC

The LHC produces jets abundantly → many phenomenological applications

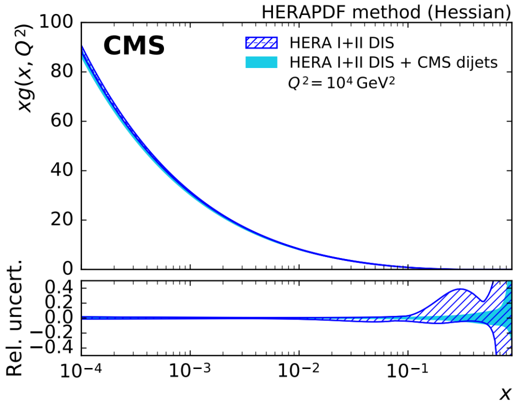
Tests of pQCD, α_s extraction:
R32 ratios, event-shapes





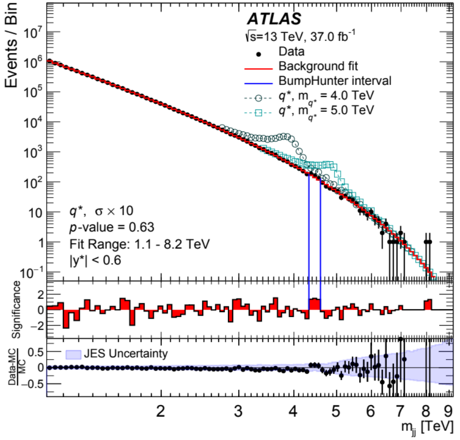
[1805.04691]

PDF determination:
Single inclusive,
Multi-differential dijet



[1705.02628]

BSM searches:
dijet mass



[1703.09127]

Precision theory required!

Data driven

Precision predictions

Resummation

Fixed order
perturbation theory

Parton-showers

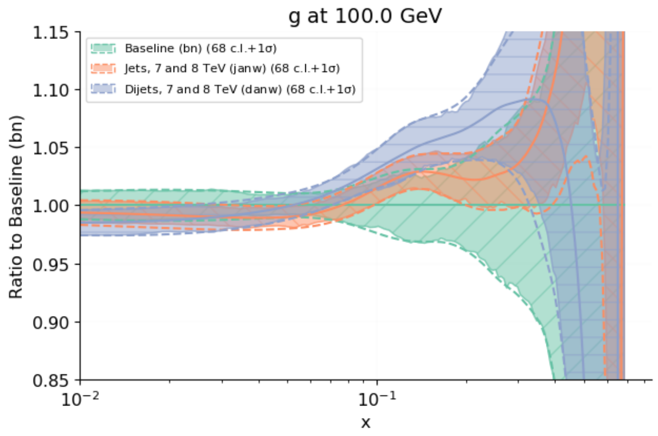
Precision theory predictions

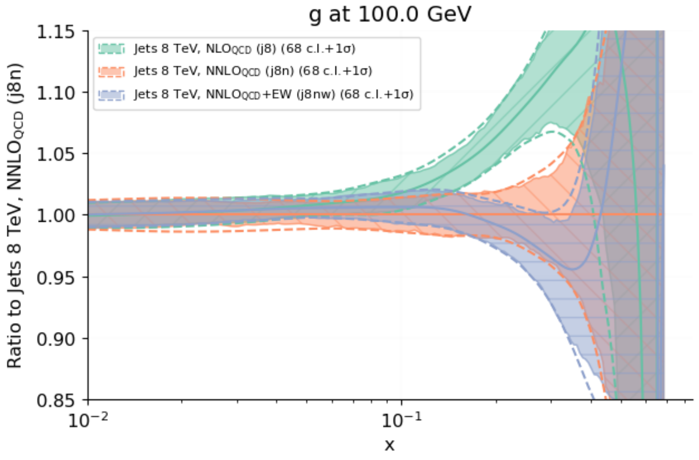
Parametric input:
PDFs and alphaS

Soft physics:
MPI, color reconnection,
...

Fragmentation/hadronisation

Example: PDF fits with jets





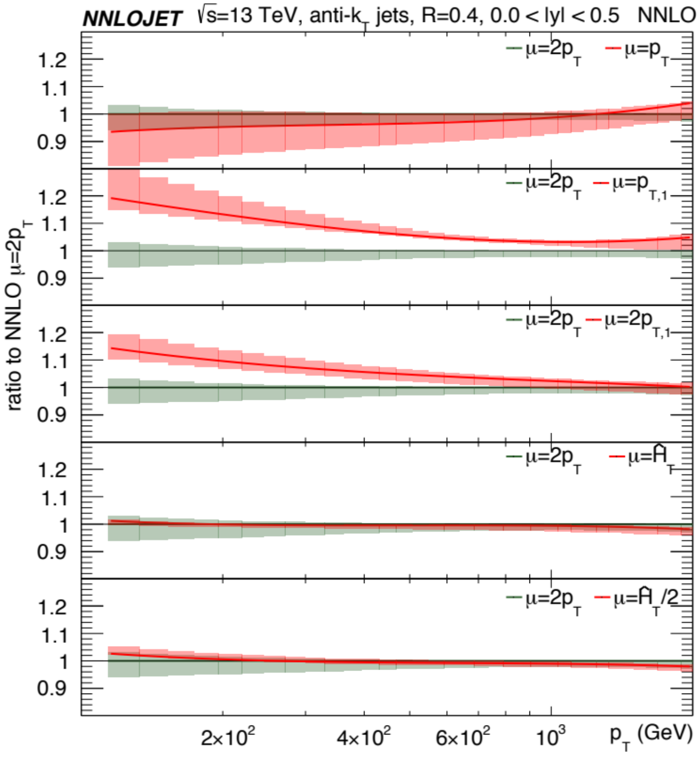
Idea (quite old actually [Giele'94]):

Combine single inclusive and dijet triple differential measurements by ATLAS and CMS to constrain the large gluon-x

Here by a collaboration of NNLOJet and NNPDF [Khalek'20]:

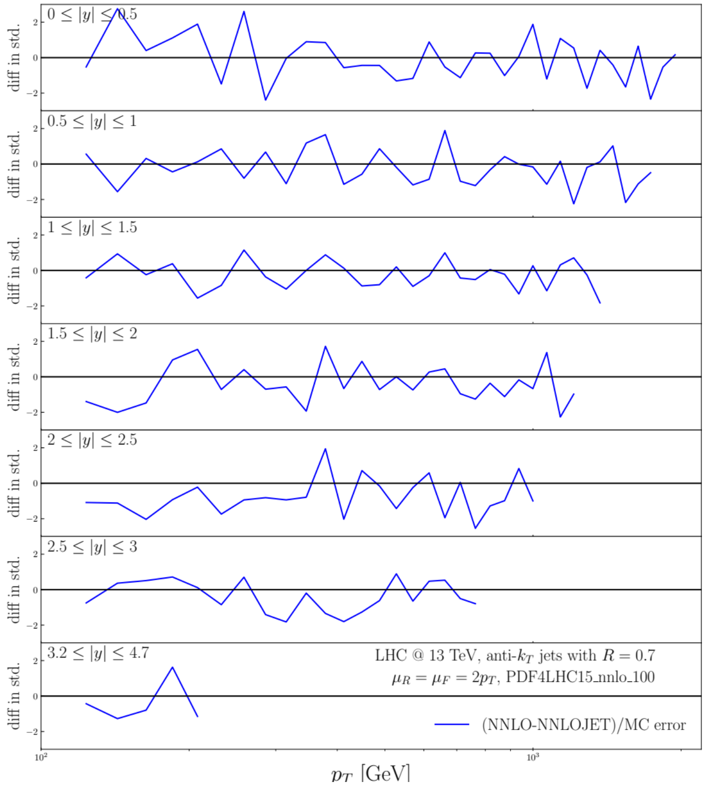
- Reduced uncertainty in large-x gluon PDF
- NNLO QCD corrections crucial to obtain consistent results between data sets
- NLO EW[Dittmaier'12] or full NLO corrections [Frederix'17,Reyer'19]

Control over theory uncertainties



Detailed studies of scale dependence: Event-based choices vs. Single jet choices [Currie'18]

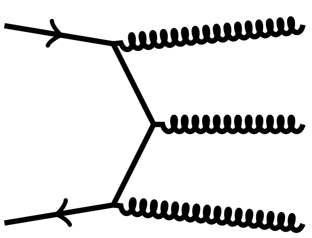
Study of sub-leading colour effects in quark channels: Smaller than O(1%) [Czakon'19]



Three jet production @ NNLO QCD

Three jet production

Advances in perturbative QCD allow to tackle the most complicated 2→3 process


$pp \to ggg + X$

Bottlenecks:

- Double virtual amplitudes: recently published in leading colour approximation [Abreu'21]
- Handling of real radiation:
  - Sector-improved residue subtraction [Czakon'10'14'19] conceptually capable
  - Computationally very challenging! → O(1M CPUh)

Only Approximation made: $\mathcal{R}^{(2)}(\mu_R^2) = 2\,\mathrm{Re}\left[\mathcal{M}^{\dagger(0)}\mathcal{F}^{(2)}\right](\mu_R^2) + \left|\mathcal{F}^{(1)}\right|^2(\mu_R^2) \equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^{4} c_i \ln^i\left(\frac{\mu_R^2}{s_{12}}\right)$

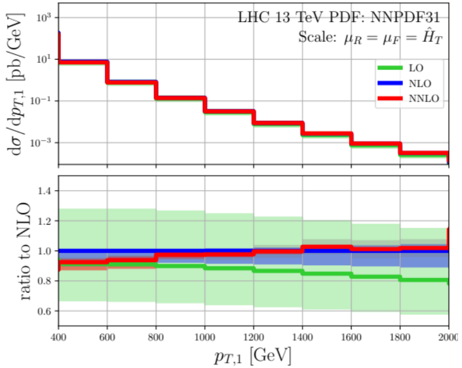
→ taken from [Abreu'21] $\mathcal{R}^{(2)}(s_{12}) \approx \mathcal{R}^{(2)l.c.}(s_{12})$

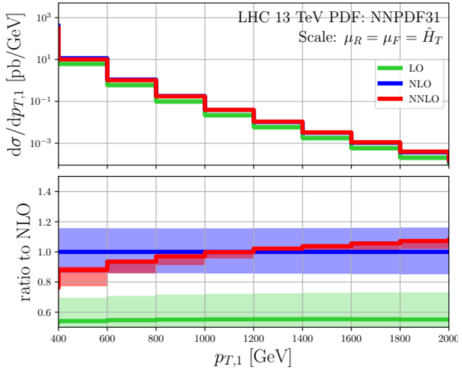
Three jet production - R32(pT1)

- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets:
  - $p_T(j) > 60$ GeV and $|y(j)| < 4.4$
  - $H_{T,2} = p_T(j_1) + p_T(j_2) > 250$ GeV
- Scales:

$$\mu_R = \mu_F = \hat{H}_T = \sum_{\text{partons}} p_T$$

$$R_{3/2}(X, \mu_R, \mu_F) = \frac{\mathrm{d}\sigma_3(\mu_R, \mu_F)/\mathrm{d}X}{\mathrm{d}\sigma_2(\mu_R, \mu_F)/\mathrm{d}X}$$





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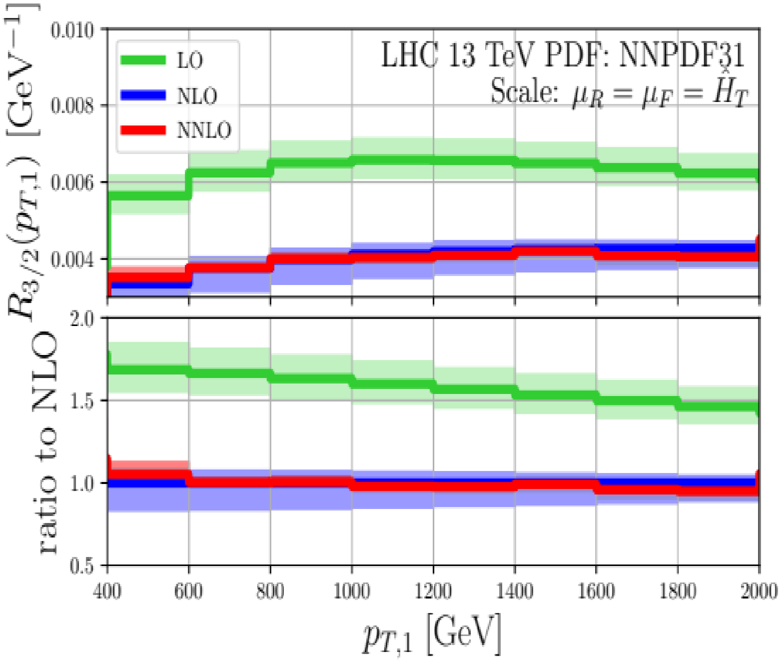
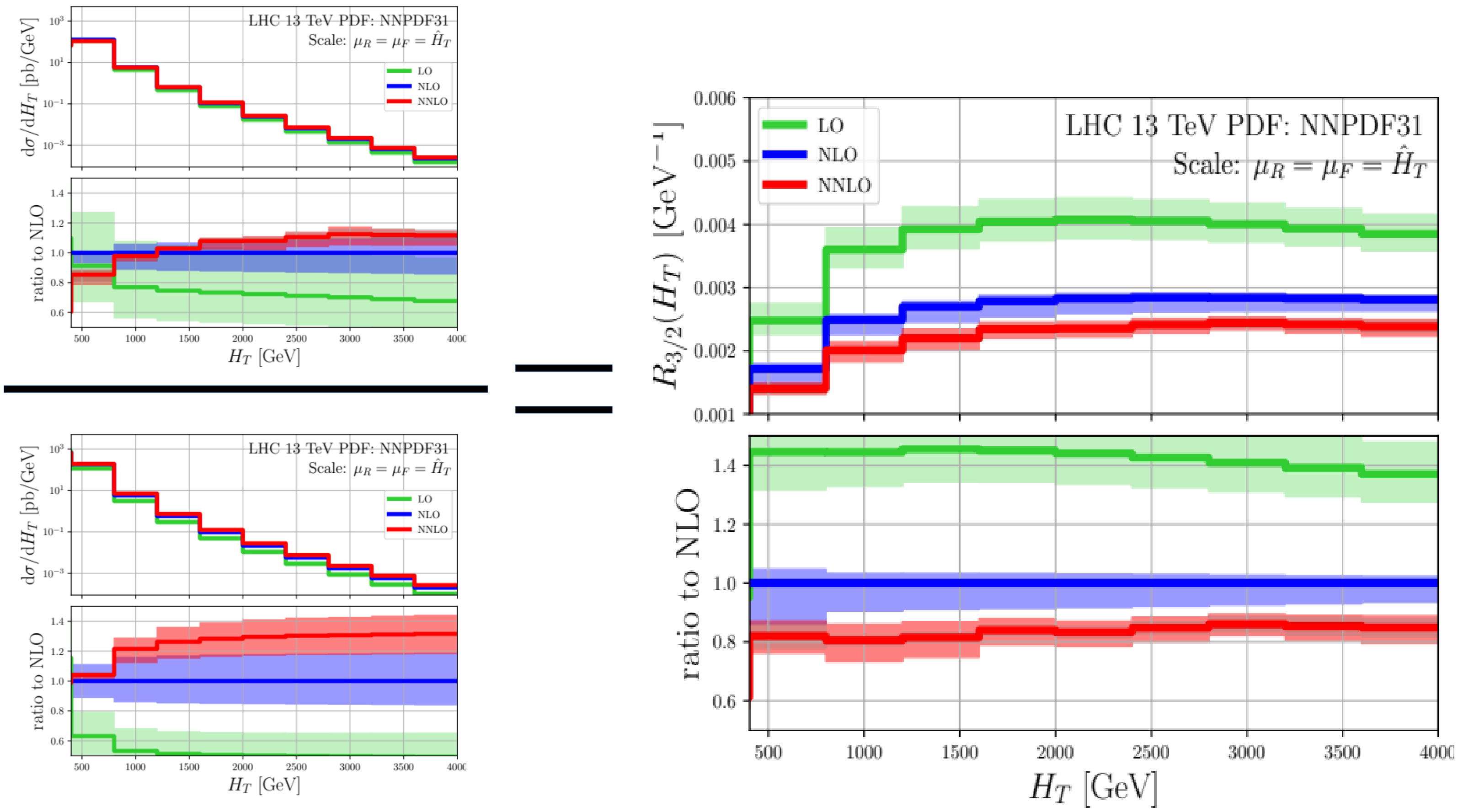


Three jet production - R32(HT)



$$H_T = \sum_{\text{jets}} p_T$$

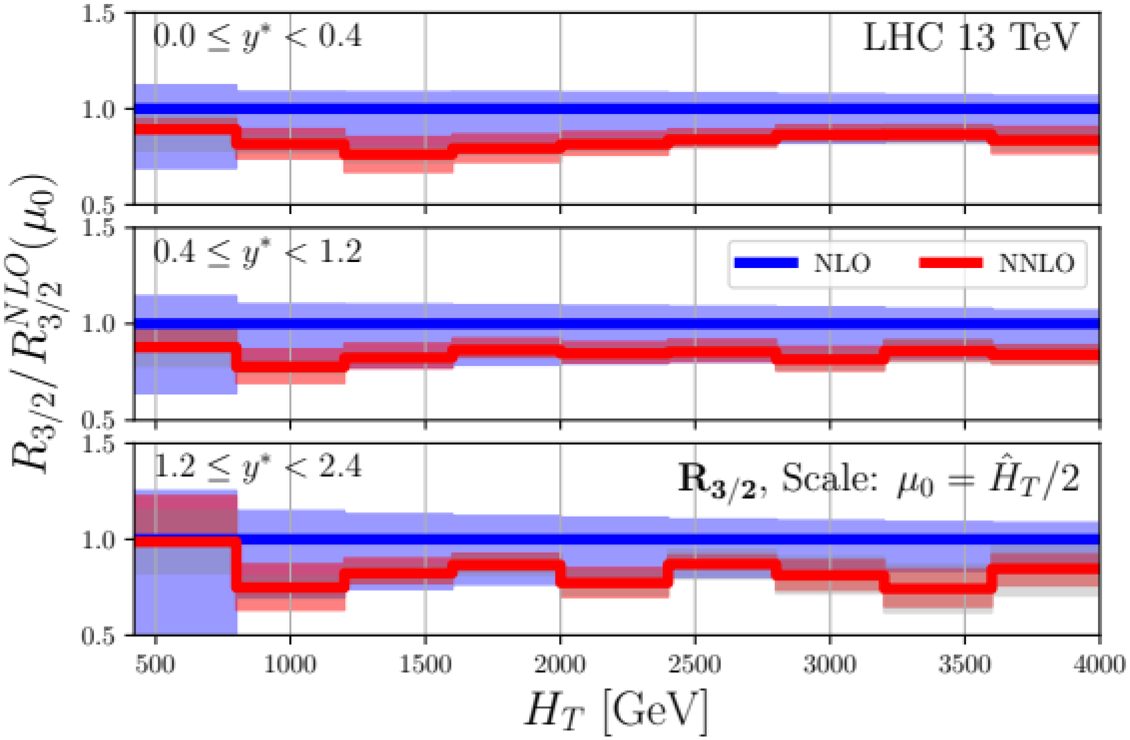
Scale dependence correlated in ratio

→ reduction of scale dependence

→ flat k-factor

→ scale bands in ratio barely overlap

Three jet production – R32(HT,y*)



Double differential w.r.t. $y^* = |y(j_1) - y(j_2)|/2$

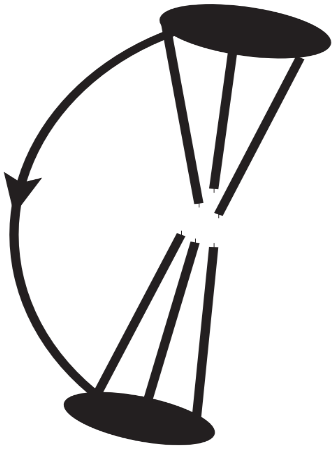
Different central scale choice: $\hat{H}_T/2$

Three jet production – azimuthal decorrelation

Kinematic constraints on the azimuthal separation between the two leading jets ($\varphi12$)

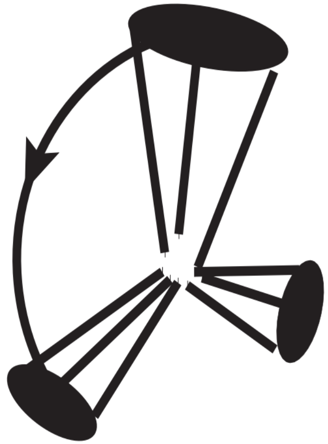
Dijet:

$\varphi12 = \pi$



Trijet:

$\varphi12 > 2/3\pi$



$\varphi12$ sensitive to the jet multiplicity:

2j: $\varphi12 = \pi$

3j: $\varphi12 > 2/3\pi$

4j: unconstrained

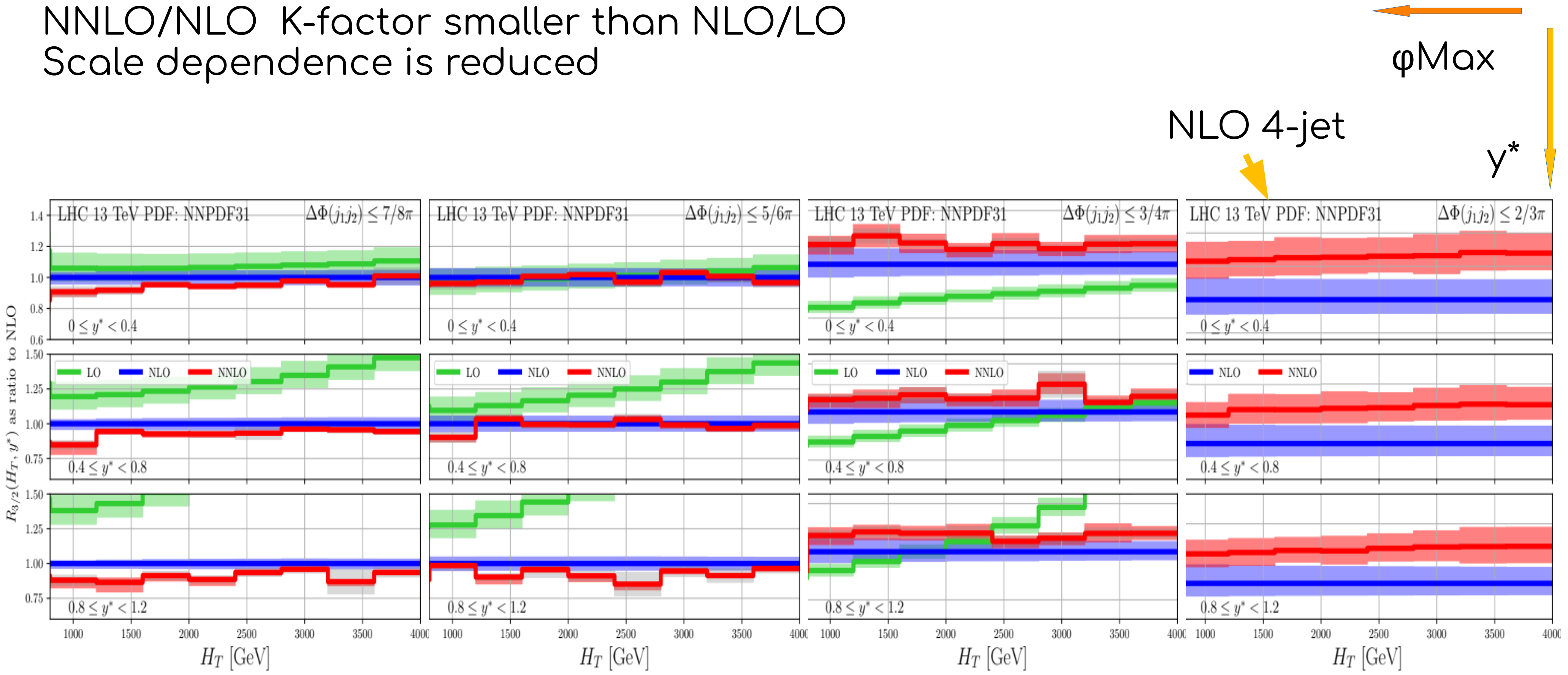
Study of the ratio

$$R32(HT,y^*,\varphi Max) = (d\sigma3(\varphi<\varphi Max)/dHT/dy^*)/(d\sigma2/dHT/dy^*)$$

With $y^* = |y1-y2|/2$

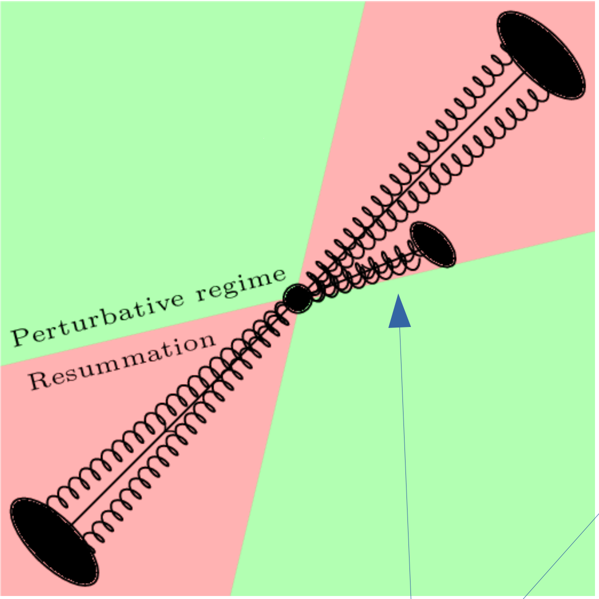
Three jet production – R32(HT,y*,φMax)

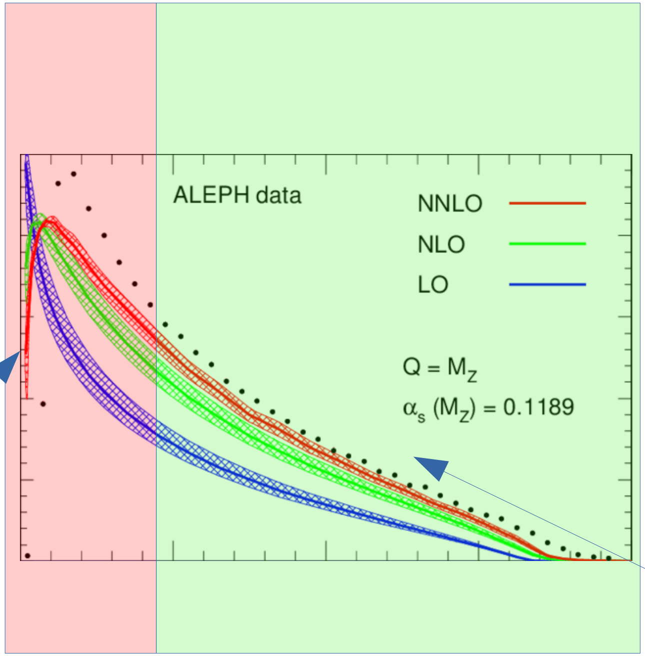
NNLO/NLO K-factor smaller than NLO/LO
Scale dependence is reduced

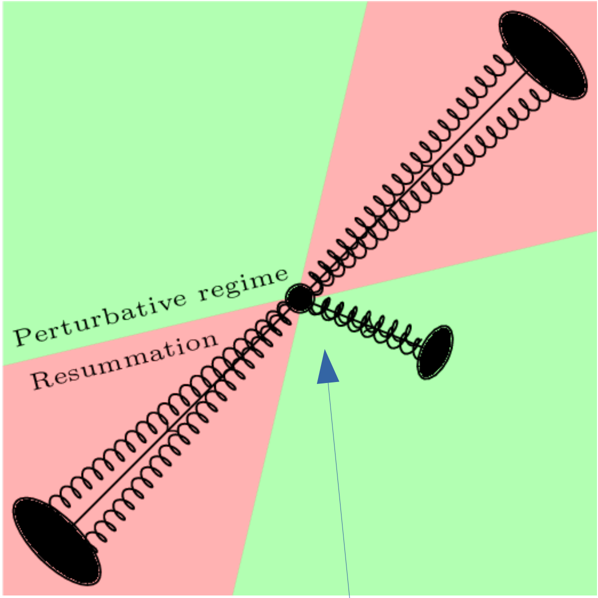


Event shapes regimes

Typically event shapes measure departure from N hard jet case







Anisotropic, 2-prong like

Sensitivity to resummation

Example: 1-Thrust at LEP

Isotropic, multi-jet

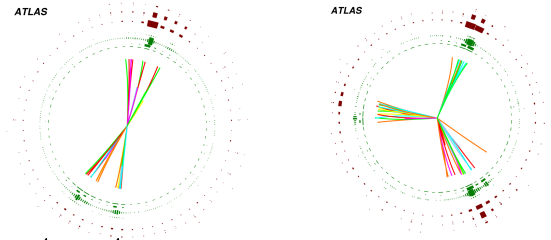
Sensitive to hard matrix elements

Event shapes at the LHC

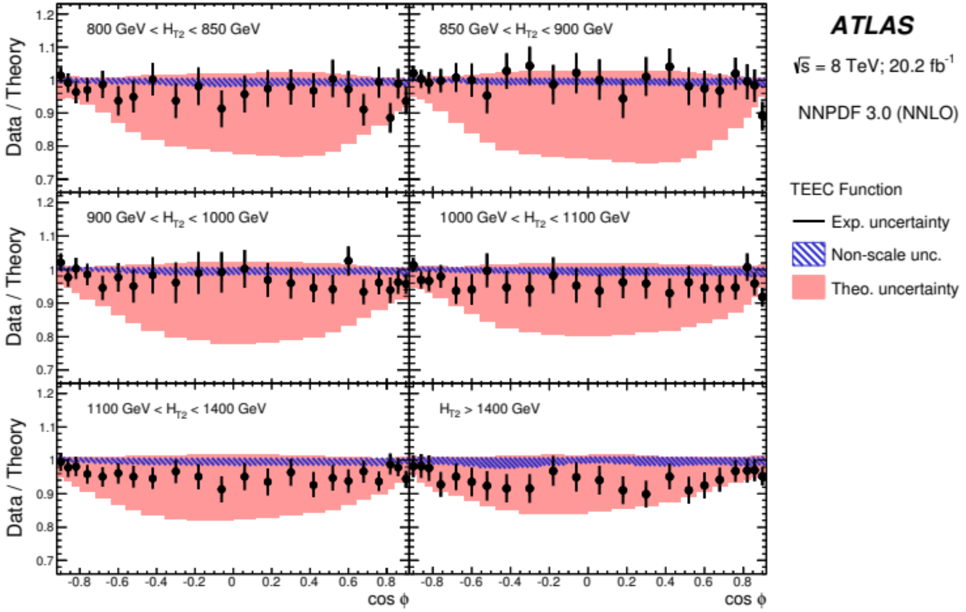
Event-shapes are measured using multi-jet events
→ three jet is often the leading contribution

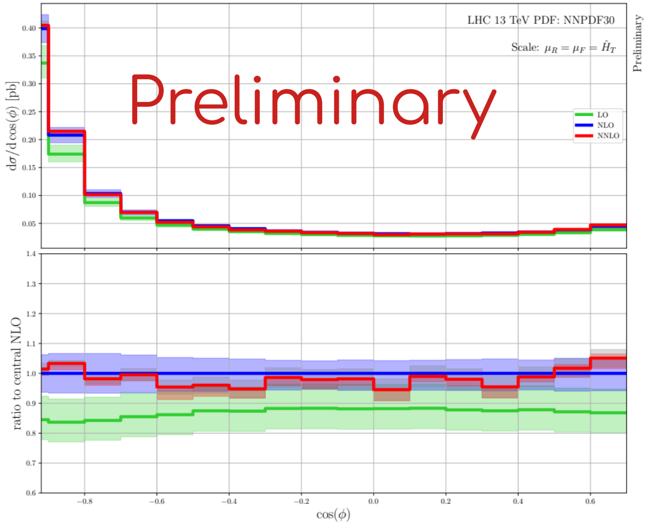
Example: TEEC (Transverse Energy-Energy Correlation)

Credit: ATLAS 2007.12600



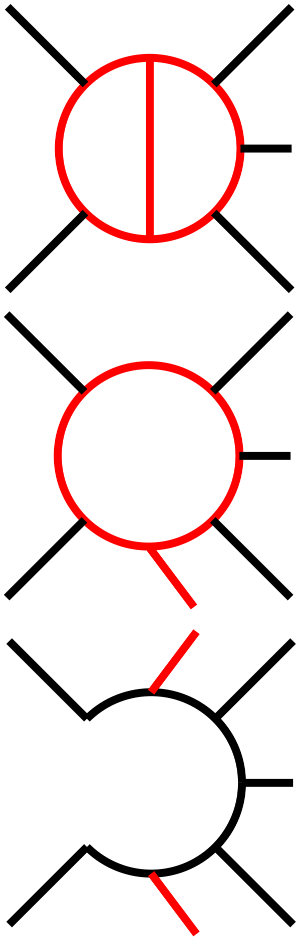
$$\frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi} = \frac{1}{N}\sum_{A=1}^{N}\sum_{ij}\frac{E^A_{\perp,i}E^A_{\perp,j}}{\left(\sum_k E^A_{T,k}\right)^2}\delta(\cos\phi - \cos\phi_{ij})$$





Technical aspects (~10mins)

NNLO QCD prediction beyond 2 → 2



2 → 3 Two-loop amplitudes:

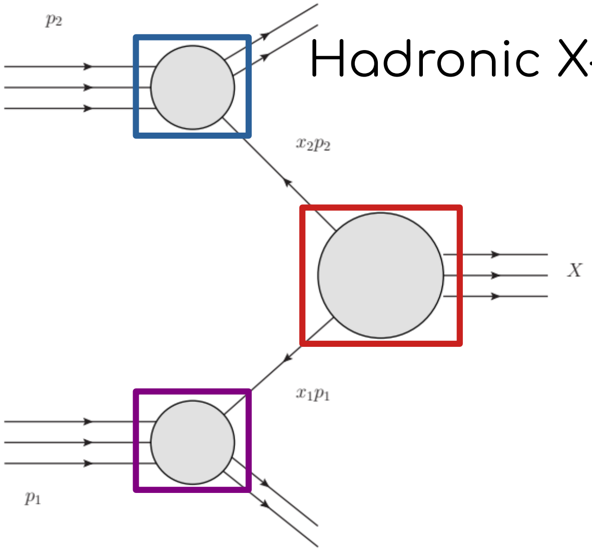
- (Non-) planar 5 point massless 'pheno ready' [Chawdry'19'20'21,Abreu'20'21,Agarwal'21,Badger'21]
  fast progress in the last year
  → triggered by efficient MI representation [Chicherin'20]
- 5 point with one external mass [Abreu'20,Syrrakos'20,Canko'20,Badger'21]

Many leg, IR stable one-loop amplitudes → OpenLoops [Buccioni'19]

Cross sections → Combination with real radiation

- Various NNLO subtraction schemes are available:
  qT-slicing [Catain'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projetction [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17], Sector-improved residue subtraction [Czakon'10-'14,'19]

Hadronic cross section



Hadronic X-section: $\sigma_{h_1h_2\to X} = \sum_{ij}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\phi_{i,h_1}(x_1,\mu_F^2)\phi_{j/h_2}(x_2,\mu_F^2)\hat{\sigma}_{ij\to X}(\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)$

Parton distribution functions

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

The NNLO bit: $\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$

Double real radiation	Real/Virtual correction	Double virtual corrections
$\hat{\sigma}^{\mathrm{RR}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\left\langle\mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2}$	$\hat{\sigma}^{\mathrm{RV}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re}\left\langle\mathcal{M}^{(0)}_{n+1}\middle|\mathcal{M}^{(1)}_{n+1}\right\rangle \mathrm{F}_{n+1}$	$\hat{\sigma}^{\mathrm{VV}}_{ab} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n}\left(2\mathrm{Re}\left\langle\mathcal{M}^{(0)}_{n}\middle|\mathcal{M}^{(2)}_{n}\right\rangle + \left\langle\mathcal{M}^{(1)}_{n}\middle|\mathcal{M}^{(1)}_{n}\right\rangle\right) \mathrm{F}_{n}$

Each term separately IR divergent. But sum is:
finite and regularization scheme independent

Considering CDR ($d = 4 - 2\epsilon$):

$$\hat{\sigma}^{C}_{ab} = \sum_{i=-4}^{0} c_i\epsilon^i + \mathcal{O}(\epsilon)$$

Sector decomposition I

Considering working in CDR:

→ Virtuals are usually done in this regularization

→ Real radiation:

→ Very difficult integrals, analytical impractical (except very simple cases)!

→ Numerics not possible, integrals are divergent: ε-poles!

How to extract these poles? → Sector decomposition!

Divide and conquer the phase space:

$$1 = \sum_{i,j}\left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l}\mathcal{S}_{i,k;j,l}\right] \rightarrow \hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\sum_{i,j}\left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l}\mathcal{S}_{i,k;j,l}\right]\left\langle \mathcal{M}_{n+2}^{(0)}\middle|\mathcal{M}_{n+2}^{(0)}\right\rangle \mathrm{F}_{n+2}$$

Sector decomposition II

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\,\mathcal{S}_{kl,m}\left\langle \mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2} = \sum_{\text{sub-sec.}}\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i\,\underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}}\,\mathrm{d}\tilde{\mu}(\{x_i\})\underbrace{\prod x_i^{a_i+1}\left\langle \mathcal{M}_{n+2}|\mathcal{M}_{n+2}\right\rangle}_{\text{regular}}\mathrm{F}_{n+2}$$

Regularization of divergences: $x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$ $\qquad \int_0^1 \mathrm{d}x\,\left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x)-f(0)}{x^{1+b\epsilon}}$

↓ Numerical integrable in d = 4 – 2ε dimensions

$$\left(\sigma^{RR}_F, \sigma^{RR}_{SU}, \sigma^{RR}_{DU}\right)\quad \left(\sigma^{RV}_F, \sigma^{RV}_{SU}, \sigma^{RV}_{DU}\right)\quad \left(\sigma^{VV}_F, \sigma^{VV}_{DU}, \sigma^{VV}_{FR}\right)\quad \left(\sigma^{C1}_{SU}, \sigma^{C1}_{DU}\right)\quad \left(\sigma^{C2}_{DU}, \sigma^{C2}_{FR}\right)$$

↓ re-arrangement of terms → 4-dim. formulation [Czakon'14,Czakon'19]

$$\underline{\left(\sigma^{RR}_F\right)}\quad \underline{\left(\sigma^{RV}_F\right)}\quad \underline{\left(\sigma^{VV}_F\right)}\quad \underline{\left(\sigma^{RR}_{SU}, \sigma^{RV}_{SU}, \sigma^{C1}_{SU}\right)}\quad \underline{\left(\sigma^{RR}_{DU}, \sigma^{RV}_{DU}, \sigma^{VV}_{DU}, \sigma^{C1}_{DU}, \sigma^{C2}_{DU}\right)}\quad \underline{\left(\sigma^{RV}_{FR}, \sigma^{VV}_{FR}, \sigma^{C2}_{FR}\right)}$$

separately finite: ε poles cancel

Five-point amplitudes - Overview

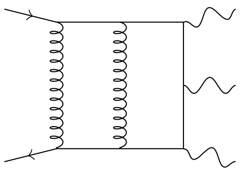
The all massless case:

- $pp \to jjj$
  - Euclidean results: insights in rational structure of amplitudes [Abreu'19]
  - Physical phase space [Abreu'21]:
    - based on 'pentagon-functions' by Chicherin and Sotnikov [Chicherin'20]
    - efficient evaluation times (~1sec)→ 'pheno-ready'
- $pp \to \gamma\gamma\gamma$
  - First, squared matrix elements with 'pentagon-functions' by [Gehrmann'18]. Very slow, however usable for pheno application [Chawdhry'19].
  - Helicity amplitudes with new 'pentagon-functions' [Abreu'20,Chawdhry'20]
- $pp \to \gamma\gamma j$
  - Squared matrix element in planar limit [Agarwal'21]
  - Helicity amplitudes in planar limit [Chawdhry'21]
  - Both in full glory [Agarwal'21] + gg induced [Badger'21]
- $pp \to \gamma jj$ ← untouched territory so far...

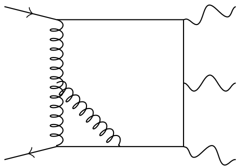
Planar five-point amplitudes

$q\bar{q} \to \gamma\gamma\gamma$

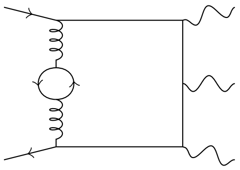
- 3 independent helicities
- QED x QCD → leading color ≠ planar



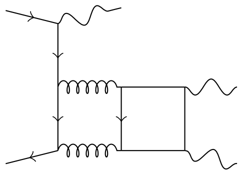
$\sim C_F^2$



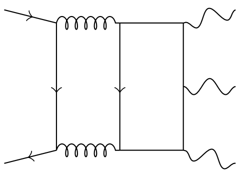
$\sim C_F C_A$



$\sim C_F n_f$



$\sim C_F \sum_{q'} Q_q'^2$

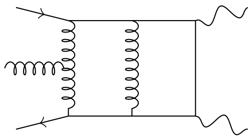


$\sim C_F \sum_{q'} Q_q'^3$

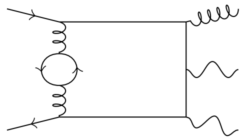
$$\mathcal{F}^{(2)}(q\bar{q} \to \gamma\gamma\gamma)\Big|_{\text{planar}} = Q_q^3 N_c^2 \left(\mathcal{F}^{C_F^2} + 2\mathcal{F}^{C_F C_A} \right) + Q_q^3 C_F n_f \mathcal{F}^{C_F n_f}$$

$q\bar{q} \to g\gamma\gamma \quad qg \to q\gamma\gamma$

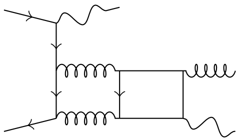
- Kinematics: $\{s_{ij}\} = \{s_{12}, s_{23}, s_{34}, s_{45}, s_{51}\}$
  $\text{tr}_5 = 4i\epsilon(p_1, p_2, p_3, p_4)$



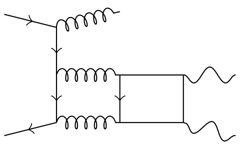
$\sim \mathcal{F}^{(2),Q_q^2,N_c^2}$



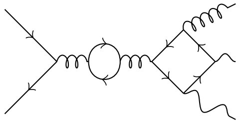
$\sim \mathcal{F}^{(2),Q_q^2,n_f}$



$\sim \mathcal{F}^{(2),Q_q Q_{q'} N_c}$



$\sim \mathcal{F}^{(2),Q_{q'}^2 N_c}$



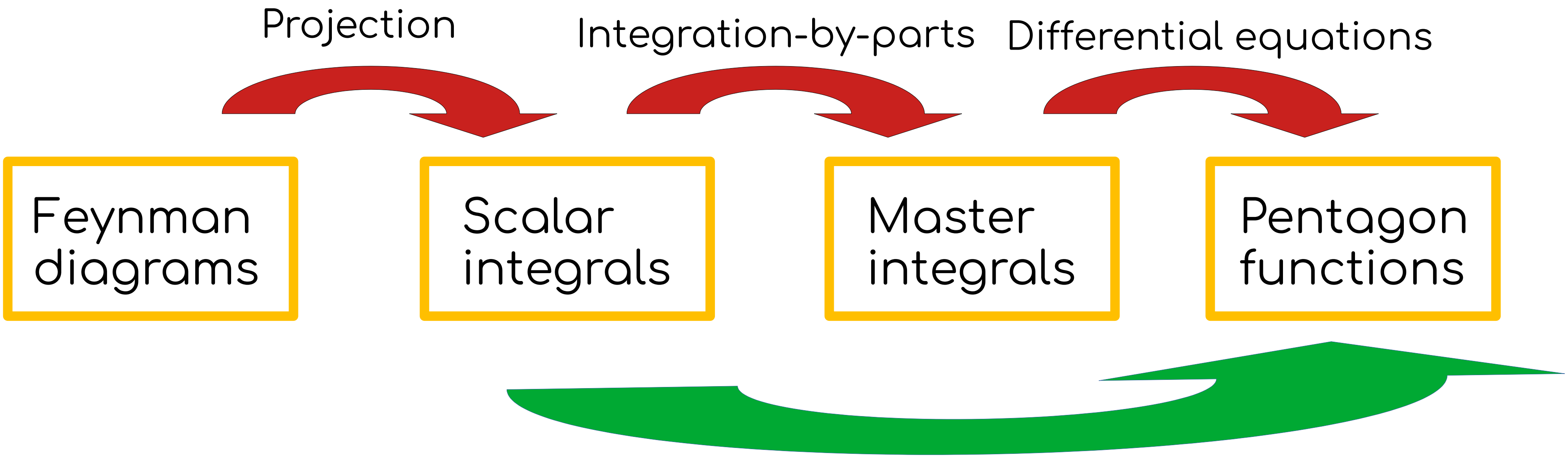
$\sim \mathcal{F}^{(2),Q_{q'}^2 n_f}$

$$\mathcal{F}^{(2)}(q\bar{q} \to \gamma\gamma g)\Big|_{\text{planar}} = Q_q^2 N_c^2 \left(\mathcal{F}^{(2),Q_q^2,N_c^2} + \frac{n_f}{N_c} \mathcal{F}^{(2),Q_q^2,n_f} \right) + Q_{l,2} n_f \mathcal{F}^{(2),Q_{q'}^2,n_f}$$

= non-planar diagrams at LC

Our framework

Old school approach:



Automated framework using finite fields to avoid expression swell based on Firefly [Klappert'19'20]

Projection

Projection to helicity amplitudes based on [Chen '19]

Spin structure of $q\bar{q} \to \gamma\gamma\gamma$ and $q\bar{q} \to g\gamma\gamma$:
$$\mathcal{M}^{\bar{h}} = \epsilon^{*\mu}_{3,h_3}\epsilon^{*\nu}_{4,h_4}\epsilon^{*\rho}_{5,h_5}\bar{v}(h_2)\Gamma_{\mu\nu\rho}u(h_1)$$

Explicit representation of polarization vectors in terms of momenta (d=4):

$$\epsilon^{\mu}_{i,h} = \frac{1}{\sqrt{2}}(\epsilon^{\mu}_{i,X} + hi\epsilon^{\mu}_{i,Y})$$

Ansatz:

$$\epsilon^{\mu}_{i,X} = c^{X}_{i,1}p^{\mu}_{1} + c^{X}_{i,2}p^{\mu}_{2} + c^{X}_{i,3}p^{\mu}_{i}$$
$$\Rightarrow \epsilon^{\mu}_{i,Y} = \mathcal{N}_{i,Y}\epsilon^{\mu}{}_{\nu\rho\sigma}q^{\nu}p^{\rho}_{i}\epsilon^{\sigma}_{i,X}$$

Constraints:

$$(\epsilon_{i,X})^2 = -1, \quad \epsilon_{i,X}\cdot q = 0, \quad \epsilon_{i,X}\cdot p_i = 0$$

Spinors expressed through trace:

$$\mathcal{M} = \bar{v}(p_2,h_2)\Gamma u(p_1,h_1) = \mathrm{Tr}\Big\{(u\otimes\bar{v})\,\Gamma\Big\}$$
$$(u\otimes\bar{v})_{\alpha\beta} = \frac{\bar{u}Nv}{\bar{u}Nv}(u\otimes\bar{v})_{\alpha\beta} = \frac{1}{\mathcal{N}}[(u\otimes\bar{u})N(v\otimes\bar{v})]_{\alpha\beta}$$

Application to Feynman diagrams → scalar expression: $\mathcal{M} = \sum c(\{s_{ij}\},\mathrm{tr}_5,d)I(\{s_{ij}\},d)$

Note: bare amplitudes are scheme-dependent, finite remainders are not

Amplitudes! Assemble!

Analytically derived IBP tables [Chawdhry'18]:

$$I(\{s_{ij}\},d) = \sum \tilde{c}(\{s_{ij}\},d)\mathrm{UT}(\{s_{ij}\},d)$$

Master Integrals:
(pentagon-functions)

$$\mathrm{UT}(\{s_{ij}\},d) = \sum_{i=0}^{4} \left(\vec{c_i}\cdot\vec{t_i}\right)\epsilon^i$$

All bits known analytically, but adding them up is cumbersome...

Using the increasingly adapted finite field approach (using Firefly):

→ evaluating all components in finite field points

→ doing the sums

→ reconstruct the finite remainder amplitude:

$$\mathcal{R}^{(\ell),i,c} = \sum_e \boxed{r_e^{(\ell),i,c}}\,\boxed{t_e}$$

$\boxed{t_e}$: Combinations of transcendental functions

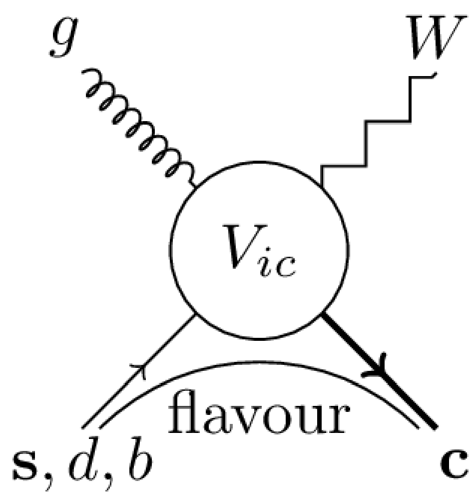
$\boxed{r_e^{(\ell),i,c}}$: rational in s_{ij} and linear in tr_5

→ Exploiting Q-linear relations among rationals:

$q\bar{q}\to g\gamma\gamma$	# tot./ # ind.
$\mathcal{R}^{+----,(2),Q_q^2,N_c^2}$	96 / 33
$\mathcal{R}^{+----,(2),Q_q^2,n_f}$	48 / 22
$\mathcal{R}^{+----,(2),Q_{q'}^2,n_f}$	6 / 2
$\mathcal{R}^{+-+--,(2),Q_q^2,N_c^2}$	7266 / 66
$\mathcal{R}^{+-+--,(2),Q_q^2,n_f}$	504 / 27
$\mathcal{R}^{+-+--,(2),Q_{q'}^2,n_f}$	58 / 8
$\mathcal{R}^{+--+-,(2),Q_q^2,N_c^2}$	7252 / 101
$\mathcal{R}^{+--+-,(2),Q_q^2,n_f}$	736 / 59
$\mathcal{R}^{+--+-,(2),Q_{q'}^2,n_f}$	58 / 8

Flavoured jets

Example: W+c-jet



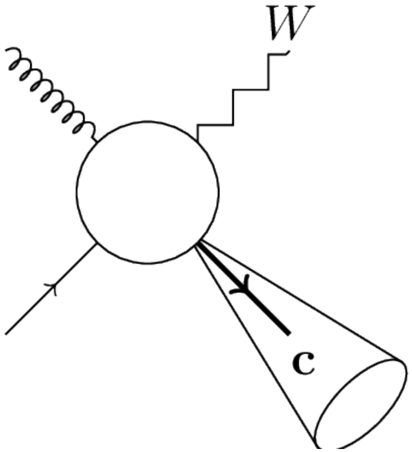
Vsc > Vdc >> Vbc
→ Sensitivity to strange PDF
Use measurement for:
→ Reduction of PDF uncertainties
→ Shed light on ssbar asymmetry

Idea is simple:

Identify final state c-quarks to access s-quark PDFs.

But:
- Non-diagonal CKM contributions reduce sensitivity
- Theoretical treatment for PDF fits:
  - Large NLO corrections: g → c cbar
  - Massive c:
    - Resummation of mass logs at high pT
    - Higher order predictions?
  - Massless c:
    - Appropriate for high pT
    - NNLO QCD available
    - Jet definition?

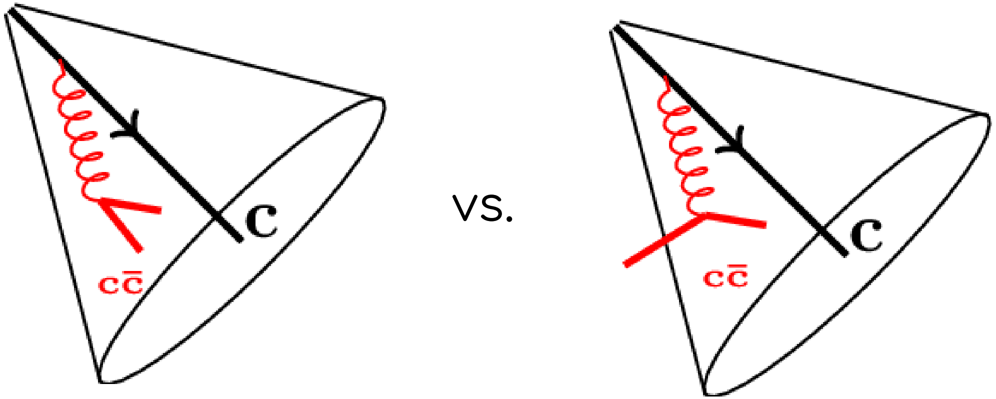
W+c-jet: IR safe jet flavour



High energy c-quark will:
- Radiate QCD partons
  → eventually form a jet
- After hadronization/fragmentation:
  Identification through heavy mesons

For PDF fits we need a fixed order prediction.

Well know problem at NNLO QCD [Banfi'06]:



Wide angle soft flavour pairs lead to IR unsafe jet clustering

Solution: Modified jet algorithms

Standard kT algorithm [Ellis'93]:

Pair distance: $$d_{ij} = \min(k_{T,i}^2, k_{T,j}^2) R_{ij}^2$$
$$R_{ij}^2 = (\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2)/R^2$$

Beam distance: $$d_i = k_{T,i}^2$$

Flavour kT algorithm [Banfi'06]:

Pair distance:

$$d_{ij} = R_{ij}^2 \begin{cases} \max(k_{T,i}, k_{T,j})^\alpha \min(k_{T,i}, k_{T,j})^{2-\alpha} & \text{softer of i,j is flavoured} \\ \min(k_{T,i}, k_{T,j})^\alpha & \text{else} \end{cases}$$

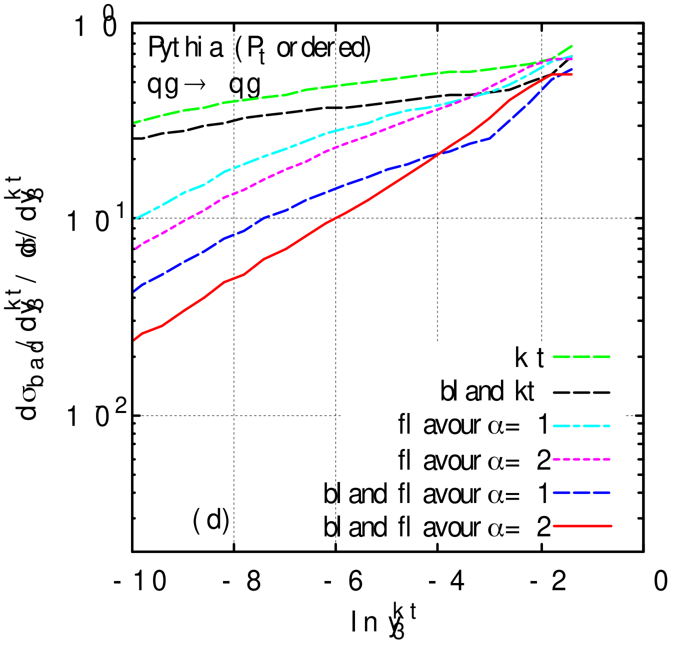
Beam distance:

$$d_{i,B} = \begin{cases} \max(k_{T,i}, k_{T,B}(y_i))^\alpha \min(k_{T,i}, k_{T,B}(y_i))^{2-\alpha} & \text{i is flavoured} \\ \min(k_{T,i}, k_{T,B}(y_i))^\alpha & \text{else} \end{cases}$$

$$d_B(\eta) = \sum_i k_{T,i}(\theta(\eta_i - \eta) + \theta(\eta - \eta_i) e^{\eta_i - \eta}$$

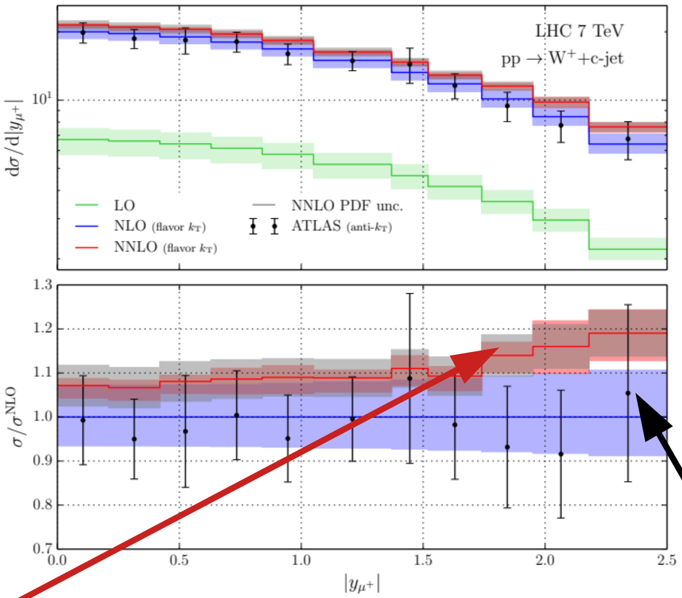
$$d_{\bar{B}}(\eta) = \sum_i k_{T,i}(\theta(\eta - \eta_i) + \theta(\eta_i - \eta) e^{\eta - \eta_i}$$

Numerical check in 2jet events: Misidentification rate as a function of y3kt



Problem solved, isn't it?

W+c-jet at NNLO QCD with flavour-kT [Czakon'20]

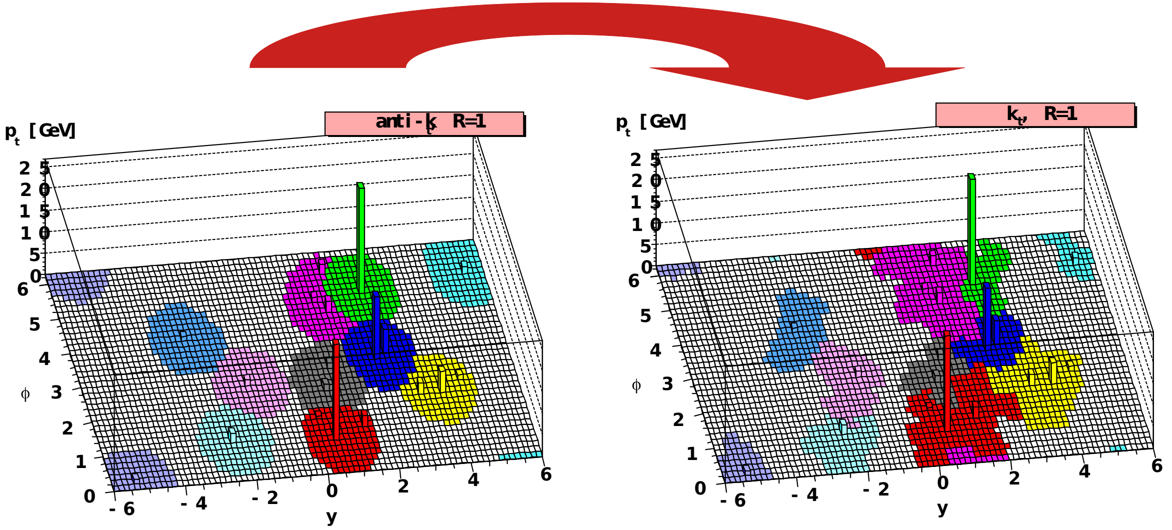


NNLO QCD with flavour kT

ATLAS data with standard anti-kT

A proper comparison would require to unfold experimental data

→ (flavour-) kT and anti-kT cluster partonic jets differently → Non-trivial procedure.



What about flavour anti-kT?

Anti-kT: $d_{ij} = \min(k_{T,i}^{-2}, k_{T,j}^{-2})R_{ij}^2 \qquad d_i = k_{T,i}^{-2}$

The energy ordering in anti-kT prevents correct recombination of flavoured pairs in the double soft limit.

Proposed modification:

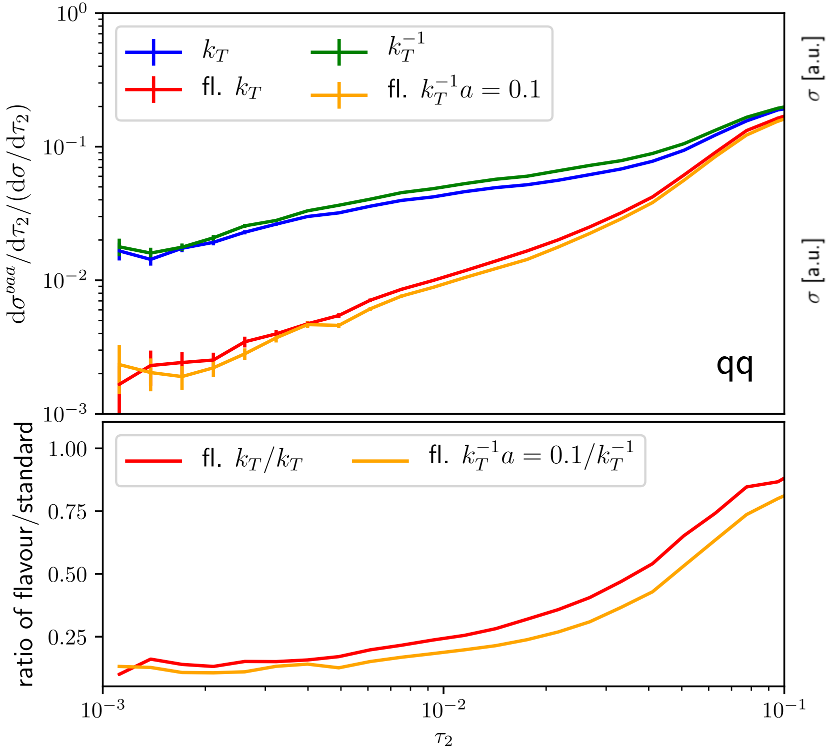
A soft term designed to modify the distance of flavoured pairs.

$$d_{i,j}^{(F)} = d_{i,j} \begin{cases} \mathcal{S}_{ij} & \text{i,j is flavoured pair} \\ 1 & \text{else} \end{cases}$$

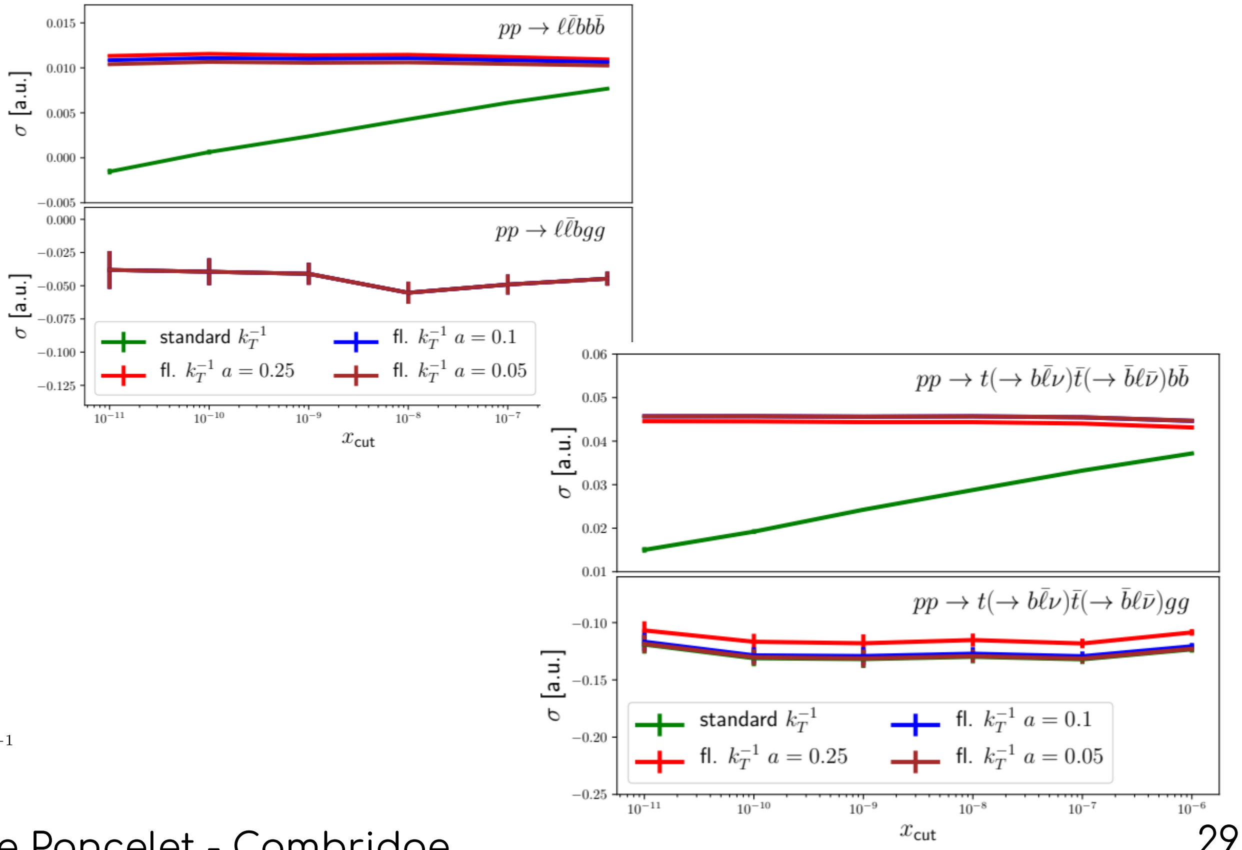
$$\mathcal{S}_{ij} = 1 - \theta(1-x)\cos\left(\frac{\pi}{2}x\right) \quad \text{with} \quad x = \frac{k_{T,i}^2 + k_{T,j}^2}{2ak_{T,\max}^2}$$

IR safety of flavoured Anti-kT

Misidentification rate as function of two jetti-ness



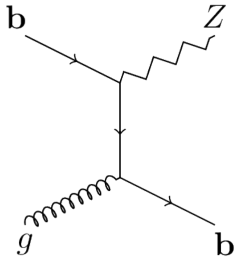
IR sensitivity of jet cross sections:



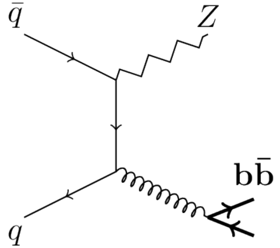
Phenomenology: Z+b-jet

Benchmark process: pp → Z(ll) + b-jet

5fs:



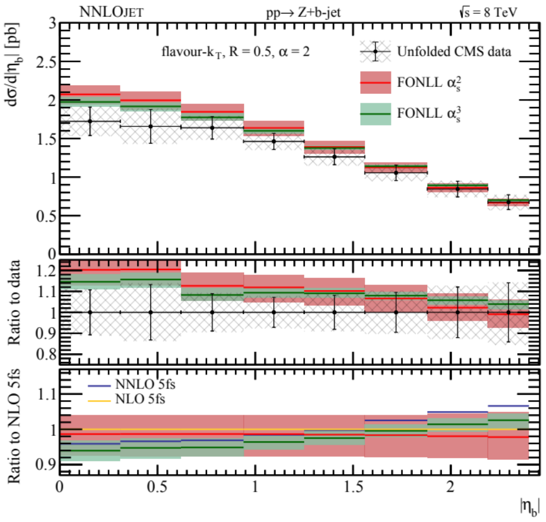
4fs:

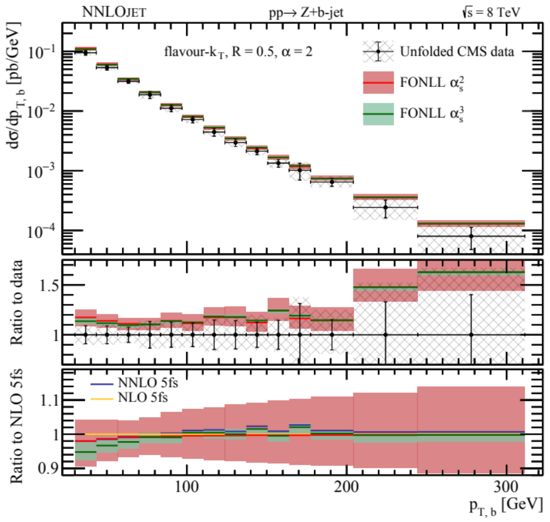


Well studied up to $\mathcal{O}(\alpha_s^3)$ [Gauld'20]:

- Defined with flavour-kT algorithm
- Unfolding of experimental data (RooUnfold,bin-by-bin unfolding)
- Matching between four- and five-flavour schemes (FONLL) [Gauld'21]

$$\mathrm{d}\sigma^{\mathrm{FONLL}} = \mathrm{d}\sigma^{5\mathrm{fs}} + (\mathrm{d}\sigma^{4\mathrm{fs}}_{m_b} - \mathrm{d}\sigma^{4\mathrm{fs}}_{m_b \to 0})$$





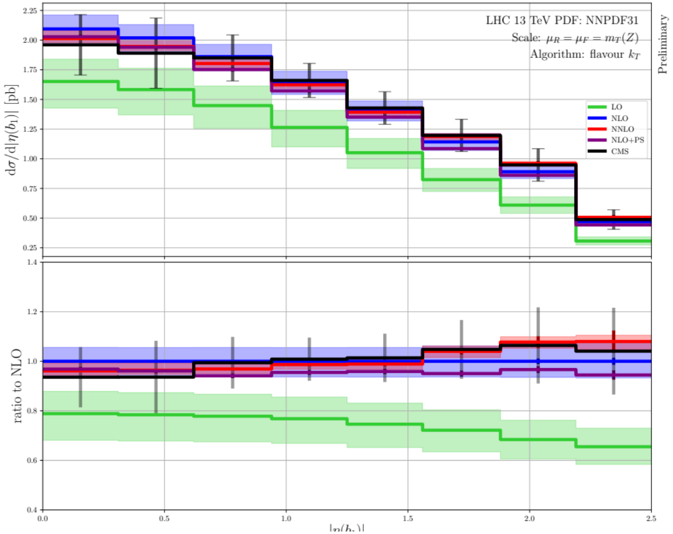
Phenomenology: Tunable parameter

Benchmark process: $pp \to Z(ll) + b\text{-jet}$

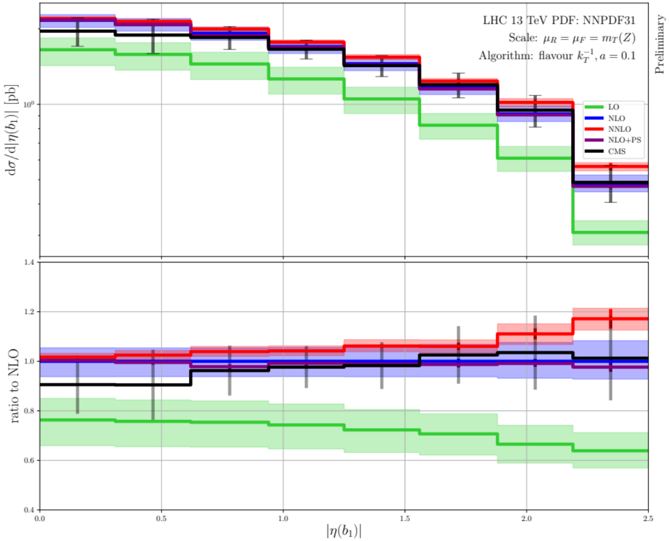
Tunable parameter a:

- Limit a → 0 <=> original anti-kT (IR unsafe)
- Large a <=> large modification of cluster sequence

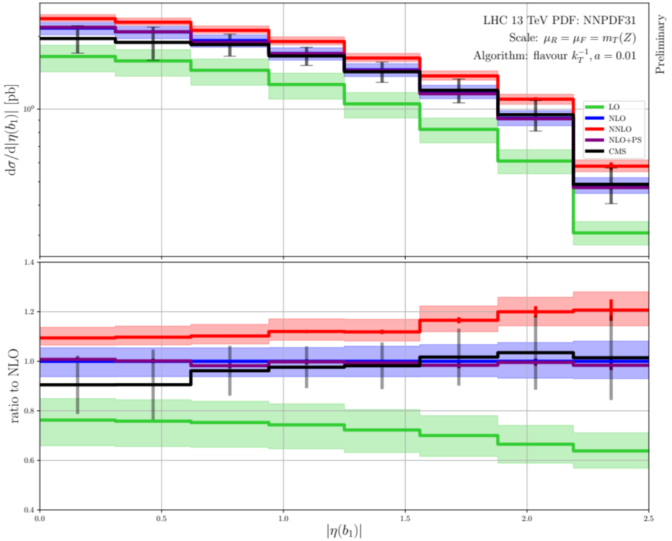
Flavour kT:



Flavour anti-kT: $a = 0.1$

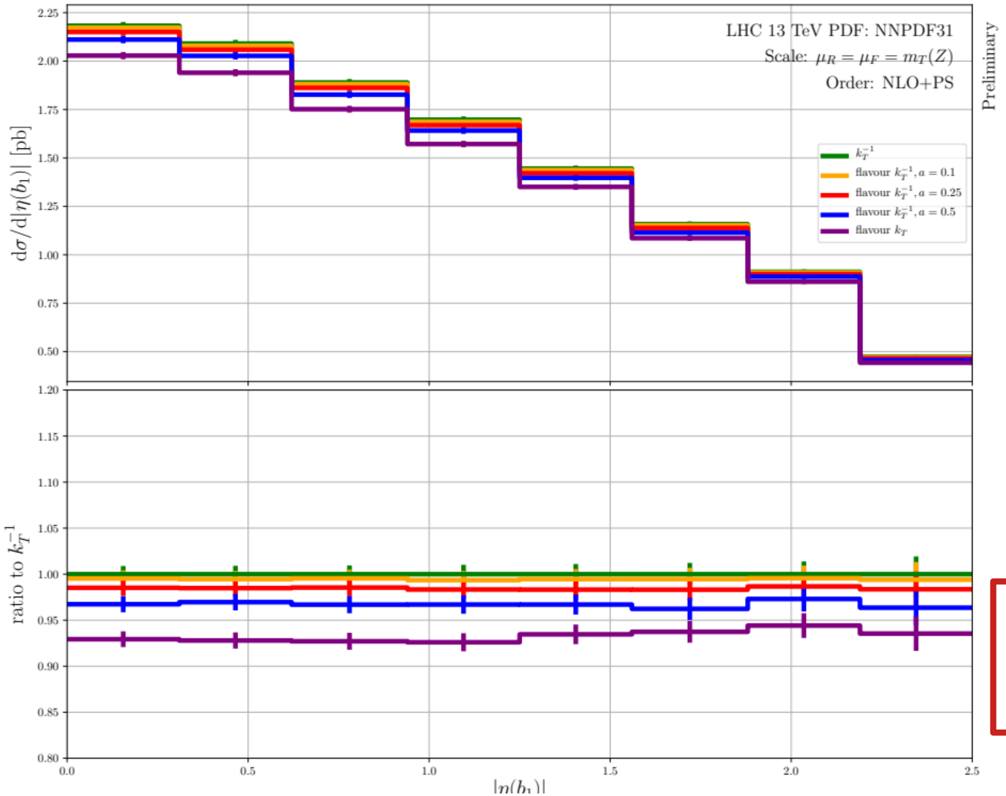


Flavour anti-kT: $a = 0.01$



Phenomenology: Tunable parameter II

What happens in the presence of many flavoured partons? → NLO PS



Tunable parameter a:

- Flavour anti-kT results are similar to standard anti-kT
  → small unfolding factors
- Flavour-kT has larger difference

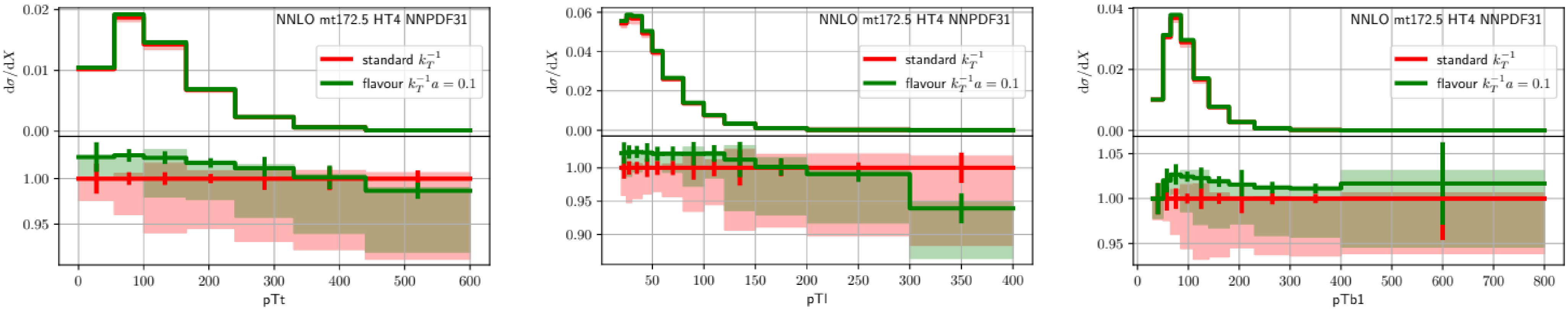
Combine with perturbative convergence:
→ a~0.1 is a good candidate

b-jets in top-pair production&decay

NNLO QCD corrections [Czakon'20] to: $pp \to t(\to b\bar{\ell}\nu)\bar{t}(\to \bar{b}\ell\bar{\nu}) + X$

Flavour sensitive channels like: $pp \to t\bar{t}b\bar{b} \to \bar{\ell}\nu\ell\bar{\nu}\, b\bar{b}b\bar{b}$

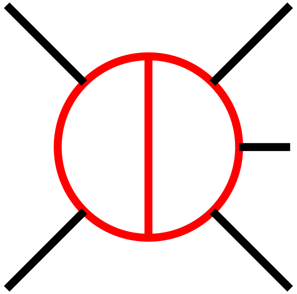
Small numerical impact from extra bbar emissions
in pp → bbar [Catani'20] and single-top production [Berger '17'18, Campbell '20]
→ naive treatment via cut-off procedure



Naive 'cut-off' treatment vs. proposed IR safe flavour anti-kT

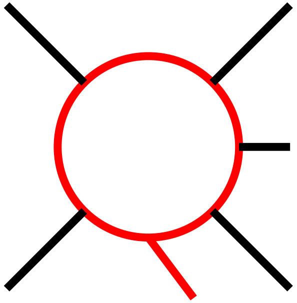
Summary & Outlook

Summary and Outlook



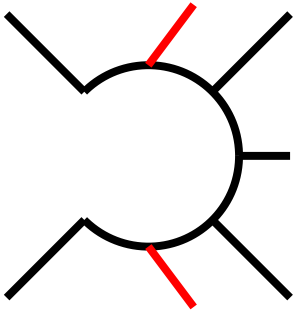
Precision jet observables allow for many pheno applications!

- First NNLO QCD phenomenology results for three jet production R32 ratios, azimuthal decorrelation, event-shapes
- Future application to alphaS extraction



Sector improved residue subtraction

- Pragmatic divide and conquer technique
- Many technical improvements: phase space, NWA & DPA, oneloop-interfaces, fragmentation,…



Flavoured jet observables

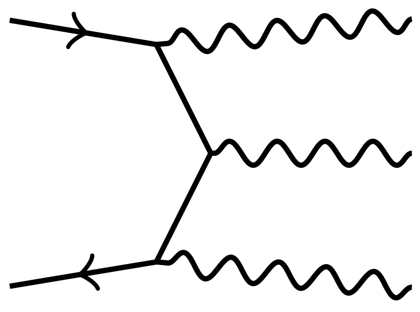
- New proposed flavour safe version of anti-kT
- Phenomenological applications to Z+b-jet, top-quark pairs
- Many more applications ahead: W+c-jet, open-b's,…

Summary and Outlook

Thank you for your attention!

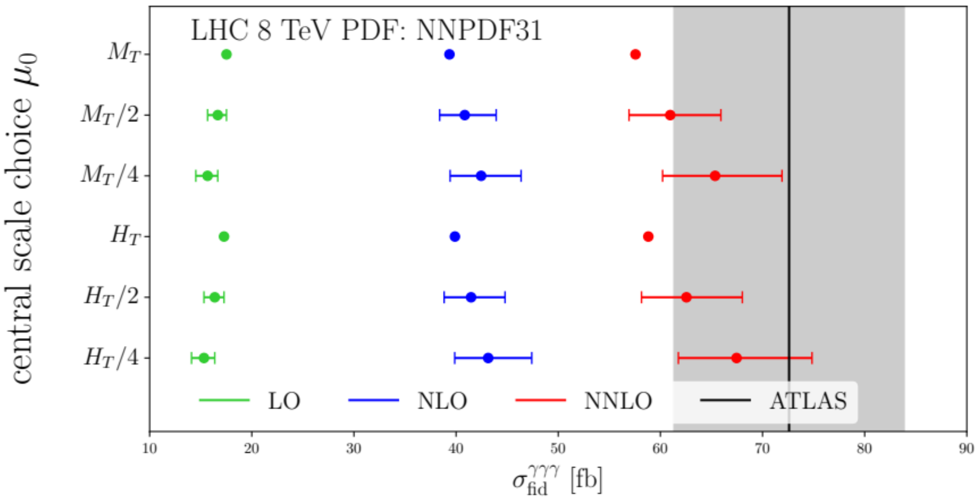
Backup

Three photon production



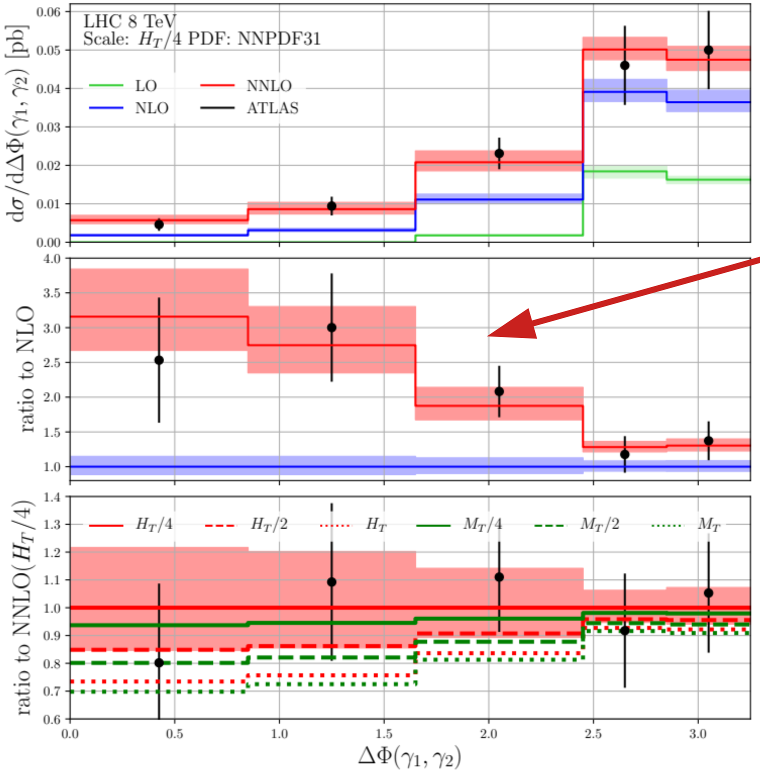
$pp \to \gamma\gamma\gamma + X$

- First NNLO QCD 2 → 3 cross sections: [Chawdhry'19],[Kallweit'20]
- Simplest among the 2 → 3 massless cases: colour singlet
- Planar Two-loop virtuals:
  $2\,\mathrm{Re}(\mathcal{M}^{(0)\dagger}\mathcal{F}^{(2)})$ with 'original' pentagon functions [Henn'18]
  → Fast helicity amplitudes: [Abreu'20],[Chawdhry'20]

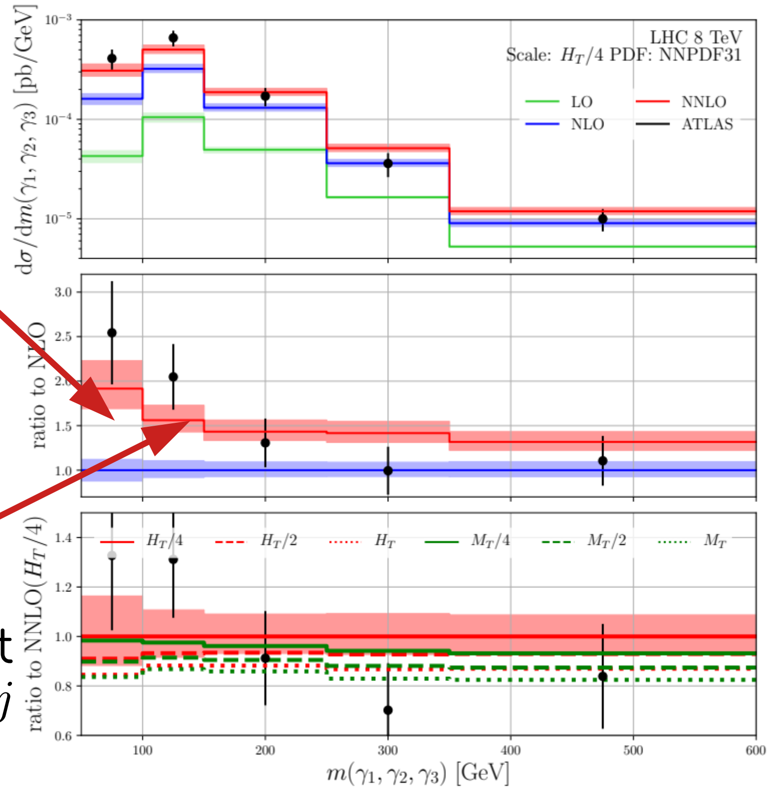


- Large NNLO/NLO K-factors
- Similar behaviour as $pp \to \gamma\gamma$
- NNLO QCD corrections essential for theory/data comparison
- Contribution of 2-loop amps small ≈ 1%

Three photon production



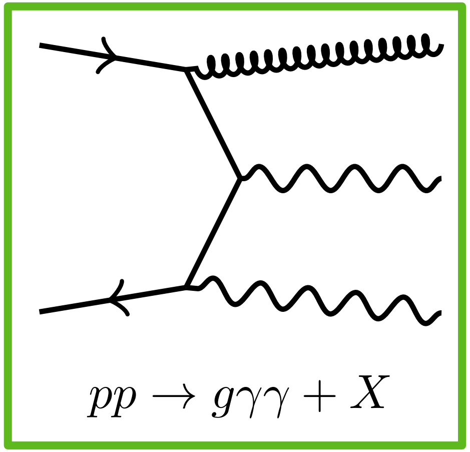
Corrections to shape and normalization

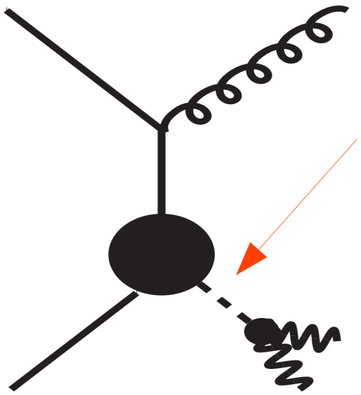


Typical for colour singlets: Scale uncertainty stays large. Very different for $pp \to j\gamma\gamma, pp \to jjj$

Diphoton plus jet production

- Photon pair production @ LHC is of particular interest:
  - Main background to cleanest Higgs decay channel
- Inclusive diphoton show large NNLO QCD corrections
  - Perturbative convergence @ N3LO?
    First steps: [Chen's talk at RADCOR+Loopfest2021]
  - Diphoton plus jet @ NNLO QCD ($p_T(\gamma\gamma) \to$ 0 limit)
- $p_T(\gamma\gamma)$ spectrum itself interesting for Higgs → γγ :
  - Higgs – p_T measurements resolve local Higgs couplings → BSM searches
  - Angular diphoton observables → spin measurements





Diphoton plus jet - setup

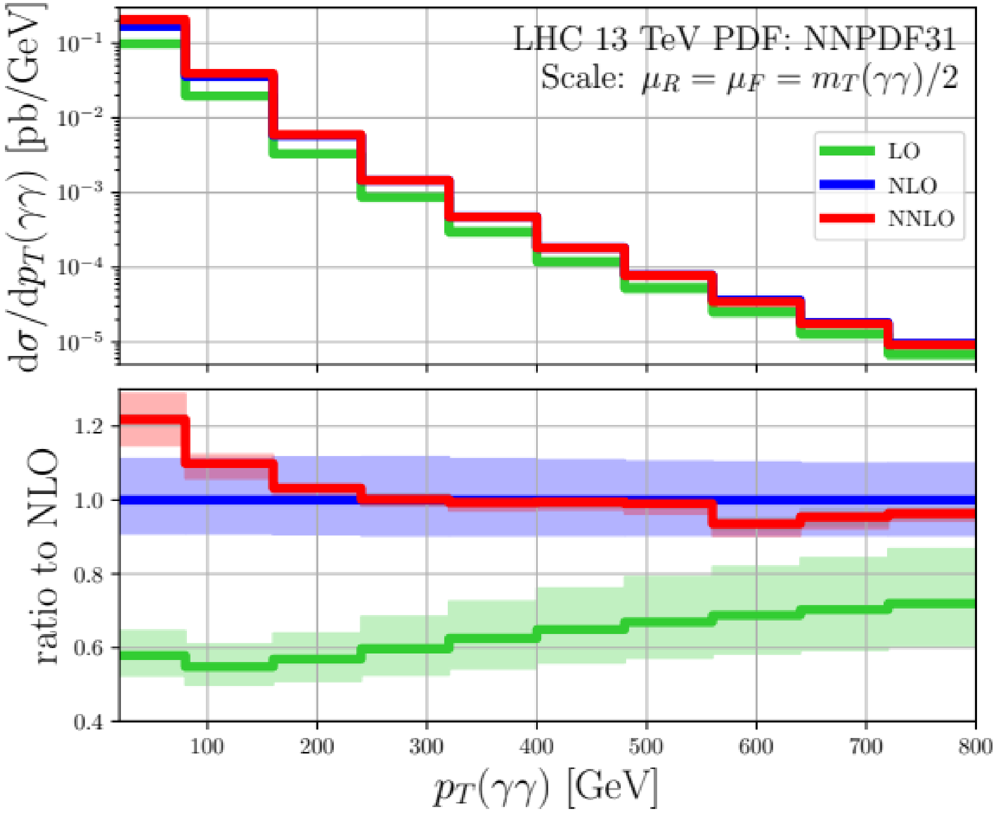
[Chawdry'21]: Inspired by Higgs → γγ measurement phase spaces

- Smooth photon isolation criteria: $E_T = 10~\mathrm{GeV}, R_\gamma = 0.4, \Delta R(\gamma,\gamma) > 0.4$
- $p_T(\gamma_1) > 30~\mathrm{GeV}, p_T(\gamma_2) > 18~\mathrm{GeV}$ and $|y(\gamma)| < 2.4$
- $m(\gamma\gamma) > 90~\mathrm{GeV}$ and $p_T(\gamma\gamma) > 20~\mathrm{GeV}$, below resummation important
- No further restrictions on jets (IR safety from $p_T(\gamma\gamma)$ cut)

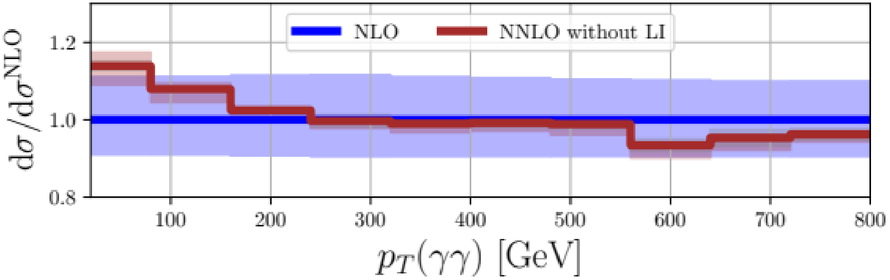
Technicalities:

- LHC 13 TeV, PDF: NNPDF31, Scale: $\mu_R^2 = \mu_F^2 = \frac{1}{4} m_T^2(\gamma\gamma) = \frac{1}{4}(m(\gamma\gamma)^2 + p_T(\gamma\gamma)^2)$
- 5 massless flavours and top-quarks (in all one-loop amps)
- Approximation of two-loop amps:
  $2\,\mathrm{Re}(\mathcal{M}^{(0)^\dagger}\mathcal{F}^{(2)}) + \mathcal{F}^{(1)^\dagger}\mathcal{F}^{(1)}$ without top-quark loops
  and $2\,\mathrm{Re}(\mathcal{M}^{(0)^\dagger}\mathcal{F}^{(2)})$ in leading colour limit [Chawdhry'21]
  → Update to full colour planned [Agarwal'21]

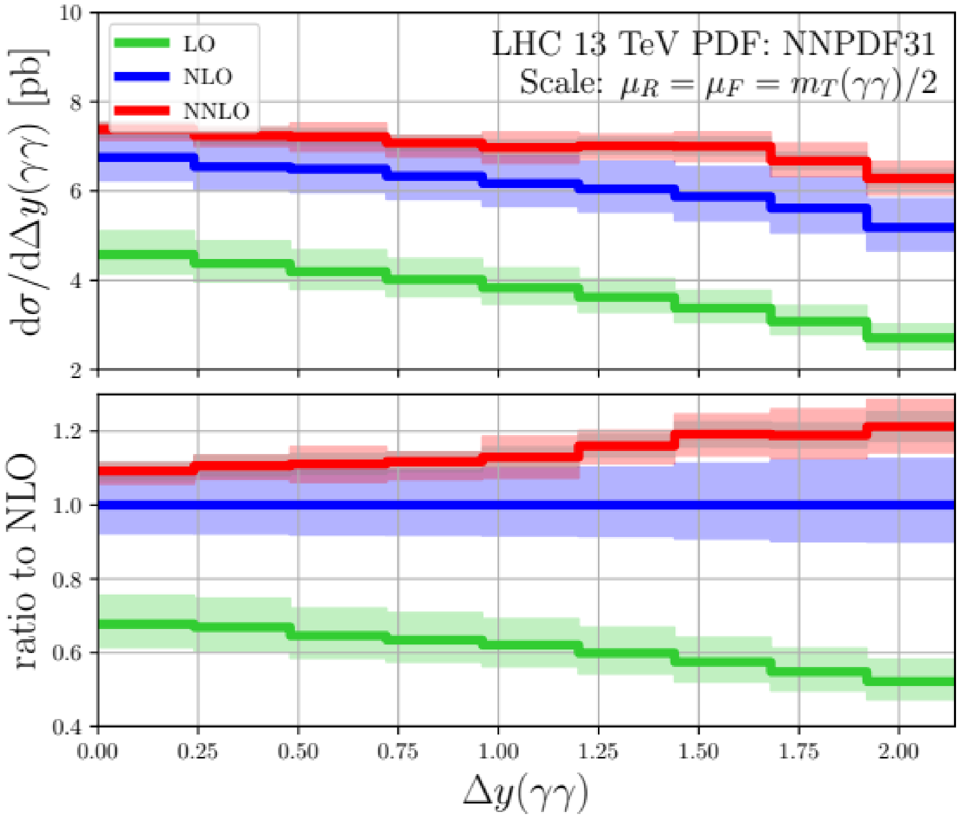
Diphoton plus jet – pT spectrum

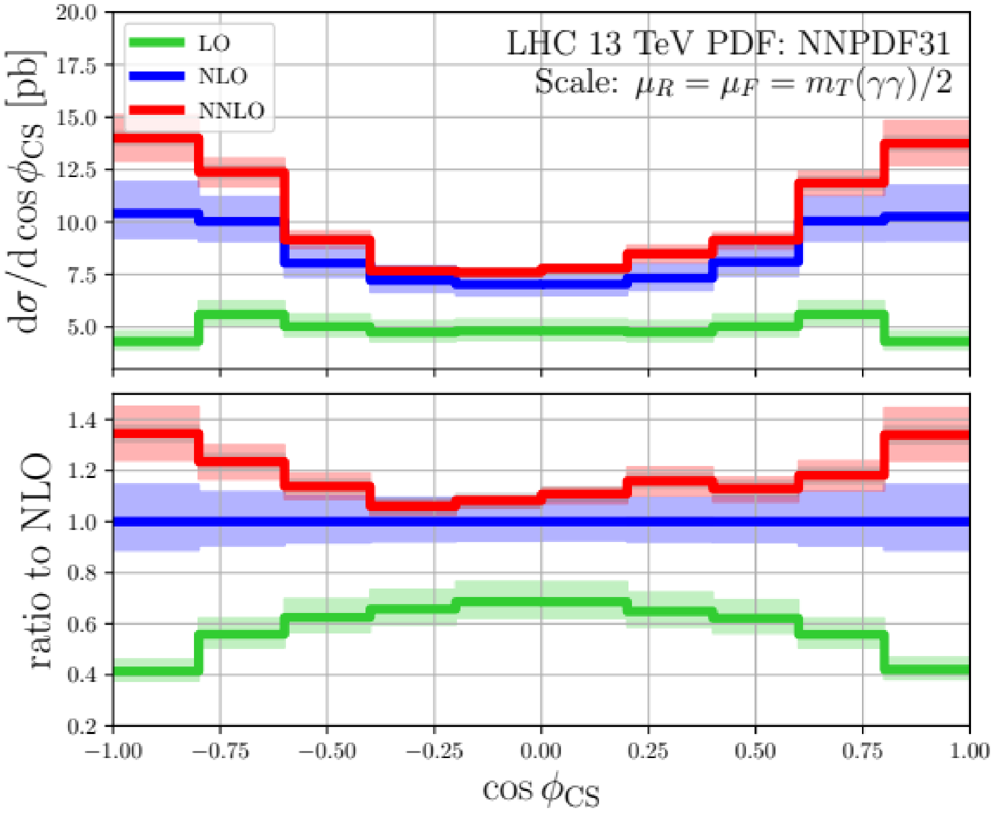


- Beautiful perturbative convergence
- Scale dependence:
  NLO: ~10%
  NNLO: ~1-2%
- Low p_T region:
  - ? Resummation for $p_T(\gamma\gamma)/m(\gamma\gamma) \ll 1$
  - Strong effect from the loop induced!



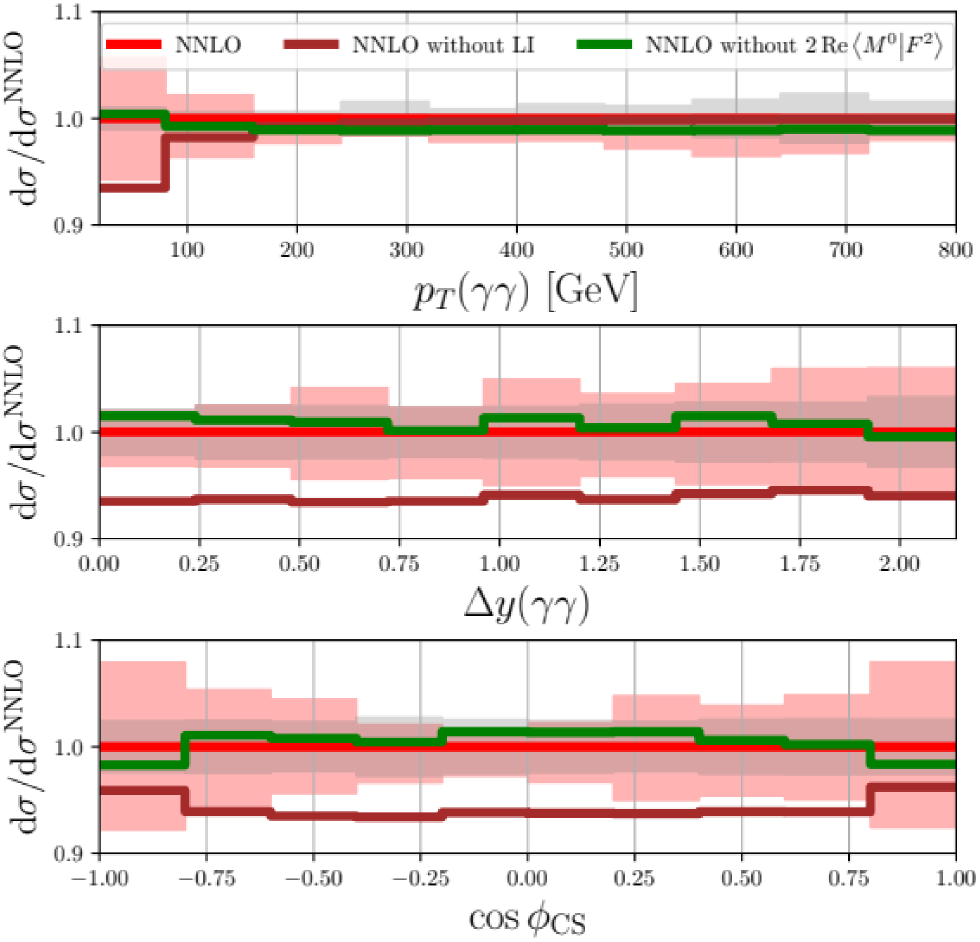
Diphoton plus jet – Angular observables





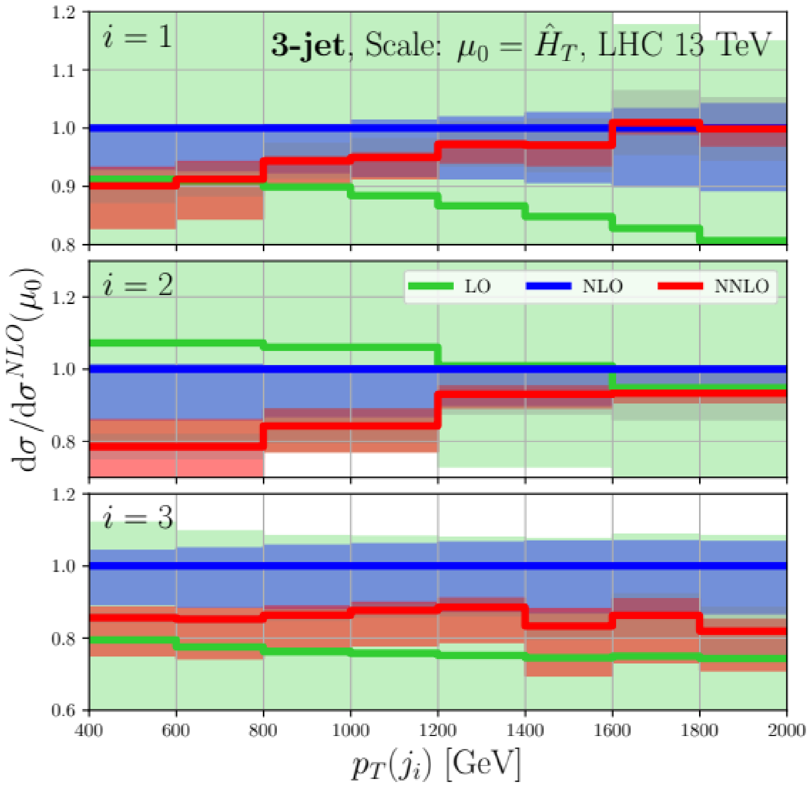
Note: Normalization affected by low p_T behaviour

Diphoton plus jet – two-loop contribution



- Two-loop contribution (green line) <~1%,
- Loop induced contribution:
  - sizeable effects for low p_T, vanishes for high p_T
  - flat effect in 'bulk' observables
  - Dominant source of scale dependence
  - NLO QCD correction (formally N3LO) relevant, ~~missing piece~~: $gg \to g\gamma\gamma$ two-loop [Badger'21]

Three jet production – transverse jet momenta



- $p_T(j_2)$:
  → suffers from slow MC convergence, larger binning
  → shows reasonable perturbative convergence
- $p_T(j_3)$:
  → fast MC convergence
  → flat k-factor

Caveat:

→ Scale choice based on full event
→ reasonable for $p_T(j_1)$ and $p_T(j_2)$
→ $p_T(j_3) \ll p_T(j_1) + p_T(j_2)$
→ potentially large hierarchy?
→ investigation with 'jet-based' scale useful

Sector decomposition II

Divide and conquer the phase space:

→ Each $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$ has simpler divergences.
Soft and collinear (w.r.t parton k,l) of partons i and j

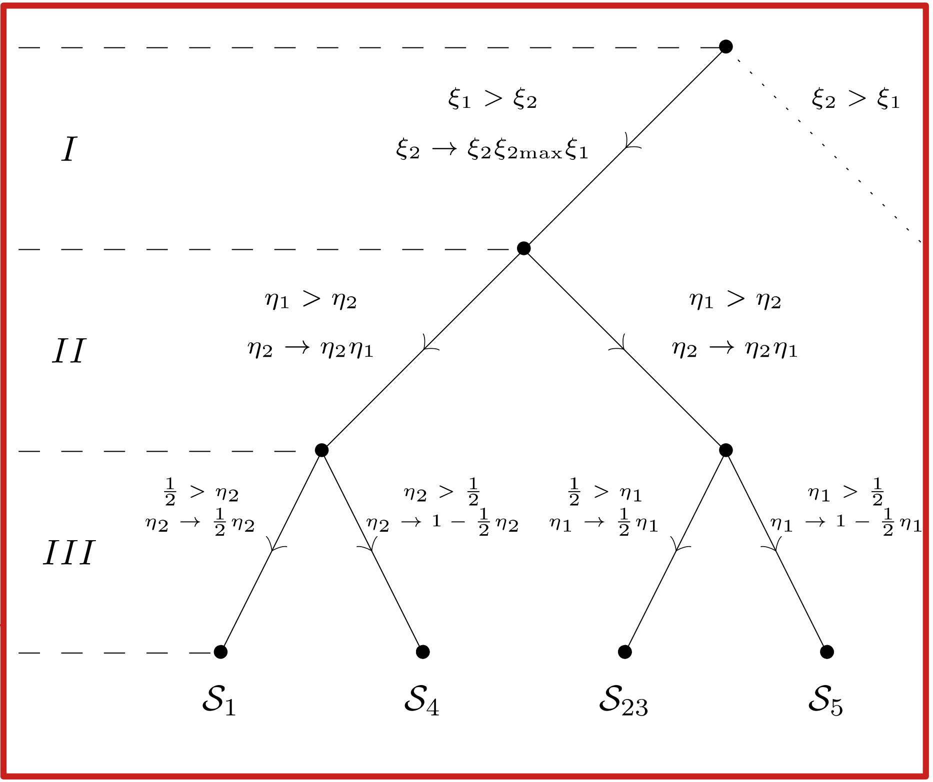
→ Parametrization w.r.t. reference parton:

$$\hat{\eta}_i = \frac{1}{2}(1-\cos\theta_{ir}) \in [0,1] \qquad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$$

→ Subdivide to factorize divergences

→ double soft factorization:
$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$

→ triple collinear factorization



[Czakon'10,Caola'17]

Improved phase space generation

Phase space cut and differential observable introduce

mis-binning : mismatch between kinematics in subtraction terms

$\rightarrow$ leads to increased variance of the integrand

$\rightarrow$ slow Monte Carlo convergence

New phase space parametrization [Czakon'19]:

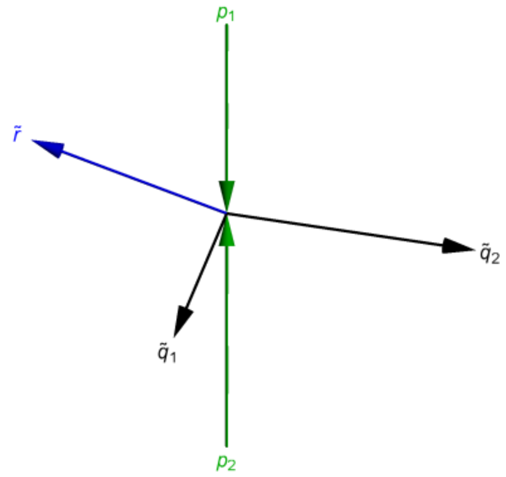
Minimization of # of different subtraction kinematics in each sector

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i : $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,,\ \ \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

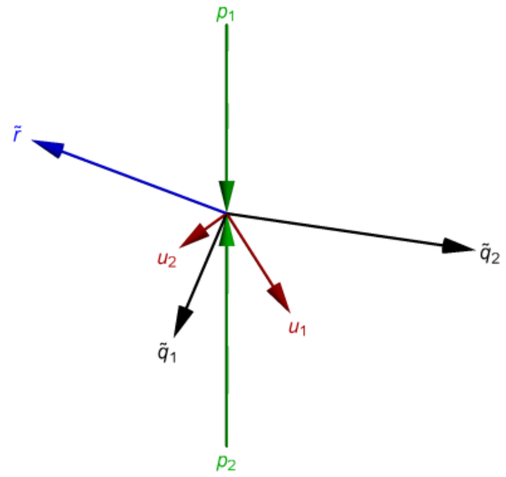
- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



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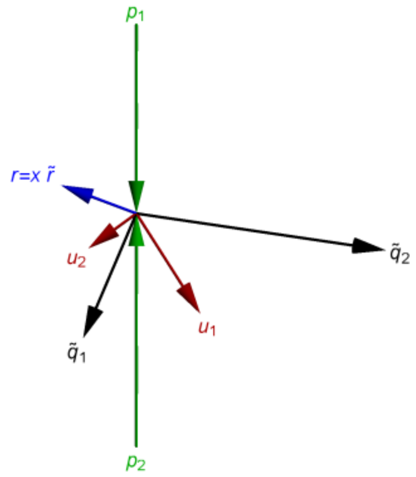

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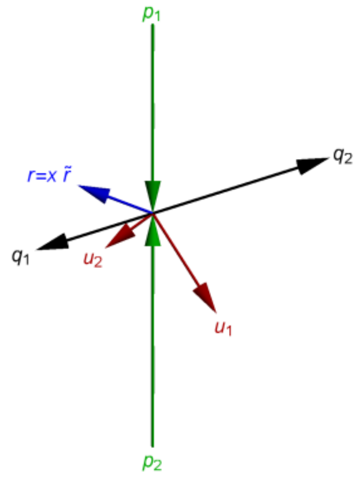

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