NNLO QCD predictions for 2 to 3 processes

Rene Poncelet
In collaboration with Michal Czakon, Alexander Mitov and Herschel Chawdhry

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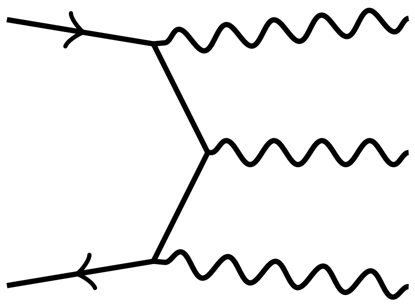
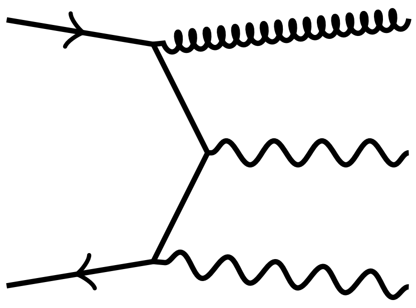
Outline

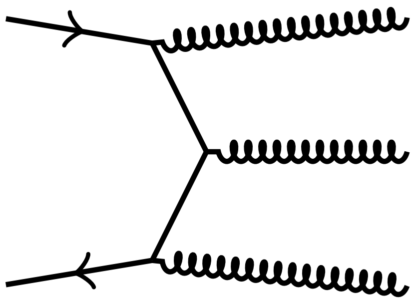
→ NNLO QCD pheno for 2 to 3 processes



$pp \to \gamma\gamma\gamma + X$



$pp \to g\gamma\gamma + X$



$pp \to ggg + X$

→ Sector-improved residue subtraction

→ 5-point amplitudes

Precision vs. Multiplicity @ the LHC

Why are we interested in NNLO QCD for $2 \to 3$ processes?

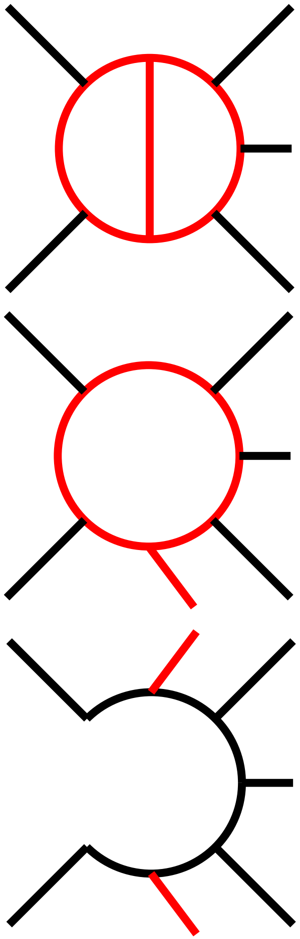
Phenomenological aspects:

- For $2 \to 2$ NNLO QCD (+NLO EW) huge success for many measurements! In some cases N3LO on the wish list.
- Next phase of LHC → enough statistics to actually resolve $2 \to 3$ NNLO?!
    - Massless processes a clear case!
    - But also heavy processes H/V+2j, ttH, ttV, VVV, … call for NNLO predictions!

Theory aspects:

- Development of NNLO QCD technology (amplitudes & subtraction) crucial work on the road towards NNLO event simulation.
- Necessary ingredient for differential $2 \to 2$ N3LO QCD

NNLO QCD prediction beyond 2 → 2



2 → 3 Two-loop amplitudes:

- (Non-) planar 5 point massless ‘pheno ready’
  [Chawdry’19’20’21,Abreu’20’21,Agarwal’21,Badger’21]
  fast progress in the last half year
  → triggered by efficient MI representation [Chicherin’20]
- 5 point with one external mass [Abreu’20,Syrrakos’20,Canko’20,Badger’21]

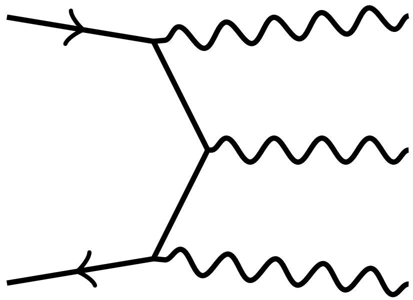
Many leg, IR stable one-loop amplitudes → OpenLoops [Buccioni’19]

Cross sections → Combination with real radiation

- Various NNLO subtraction schemes are available:
  qT-slicing [Catain’07], N-jettiness slicing [Gaunt‘15/Boughezal‘15], Antenna [Gehrmann’05-’08], Colorful [DelDuca‘05-’15], Projetction [Cacciari‘15], Geometric [Herzog‘18], Unsubtraction [Aguilera-Verdugo’19], Nested collinear [Caola’17], Sector-improved residue subtraction [Czakon’10-’14]

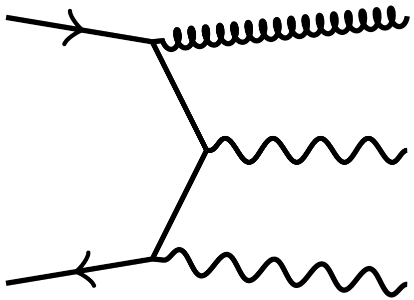
Phenomenological applications

Three photons



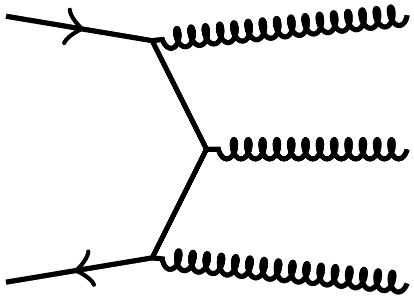
$pp \to \gamma\gamma\gamma + X$

Two photons plus jet



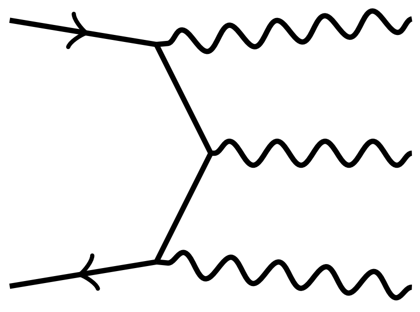
$pp \to g\gamma\gamma + X$

Three jets



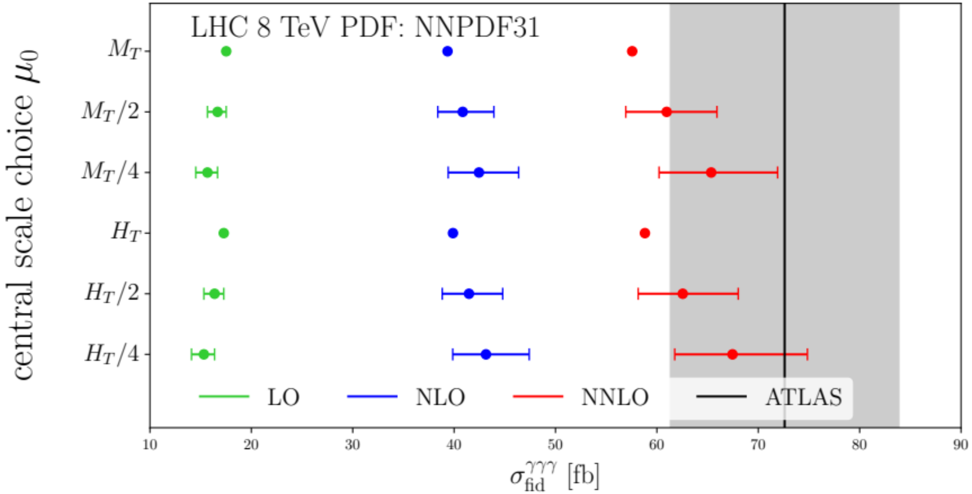
$pp \to ggg + X$

Three photon production



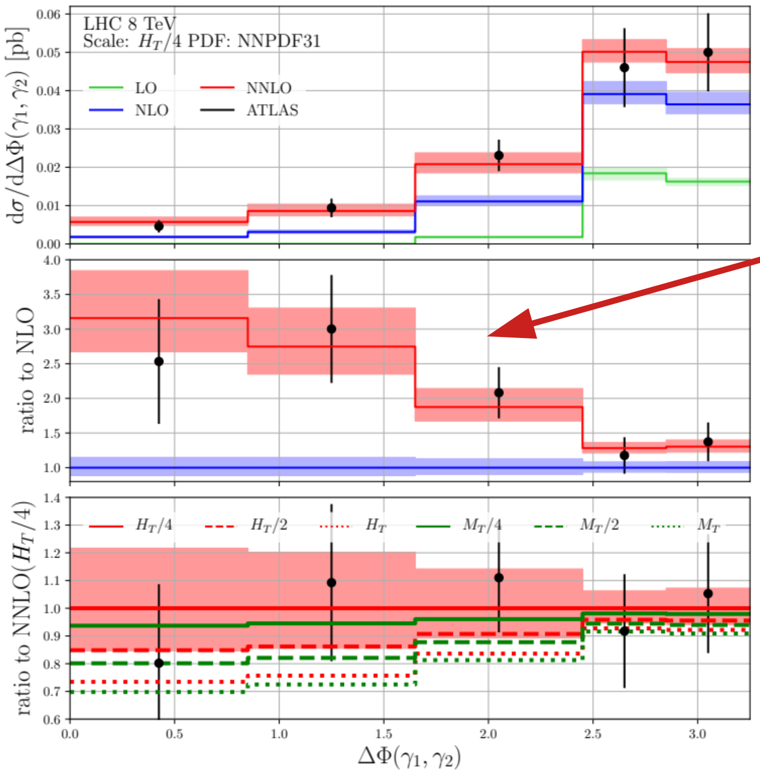
$pp \to \gamma\gamma\gamma + X$

- First NNLO QCD 2 → 3 cross sections: [Chawdhry'19],[Kallweit'20]
- Simplest among the 2 → 3 massless cases: colour singlet
- Planar Two-loop virtuals:
  $2\,\mathrm{Re}(\mathcal{M}^{(0)^\dagger}\mathcal{F}^{(2)})$ with 'original' pentagon functions [Henn'18]
  → Fast helicity amplitudes: [Abreu'20],[Chawdhry'20]

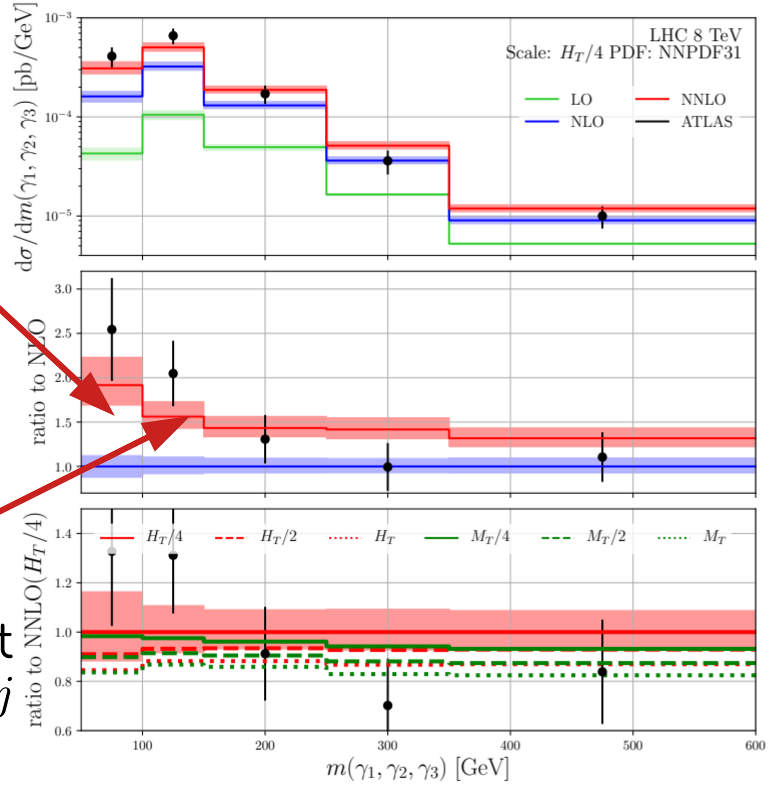


- Large NNLO/NLO K-factors
- Similar behaviour as $pp \to \gamma\gamma$
- NNLO QCD corrections essential for theory/data comparison
- Contribution of 2-loop amps small ≈ 1%

Three photon production



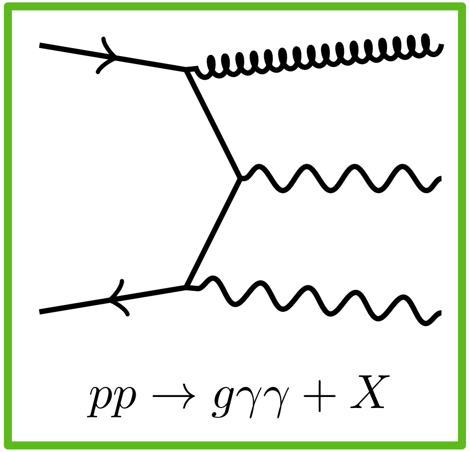
Corrections to shape and normalization

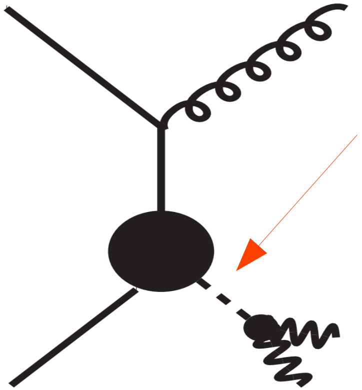


Typical for colour singlets: Scale uncertainty stays large. Very different for $pp \to j\gamma\gamma, pp \to jjj$

Diphoton plus jet production

- Photon pair production @ LHC is of particular interest:
  - Main background to cleanest Higgs decay channel
- Inclusive diphoton show large NNLO QCD corrections
  - Perturbative convergence @ N3LO?
    First steps: [Chen's talk at RADCOR+Loopfest2021]
  - Diphoton plus jet @ NNLO QCD ($p_T(\gamma\gamma) \to$ 0 limit)
- $p_T(\gamma\gamma)$ spectrum itself interesting for Higgs → γγ :
  - Higgs – p_T measurements resolve local Higgs couplings → BSM searches
  - Angular diphoton observables → spin measurements





Diphoton plus jet - setup

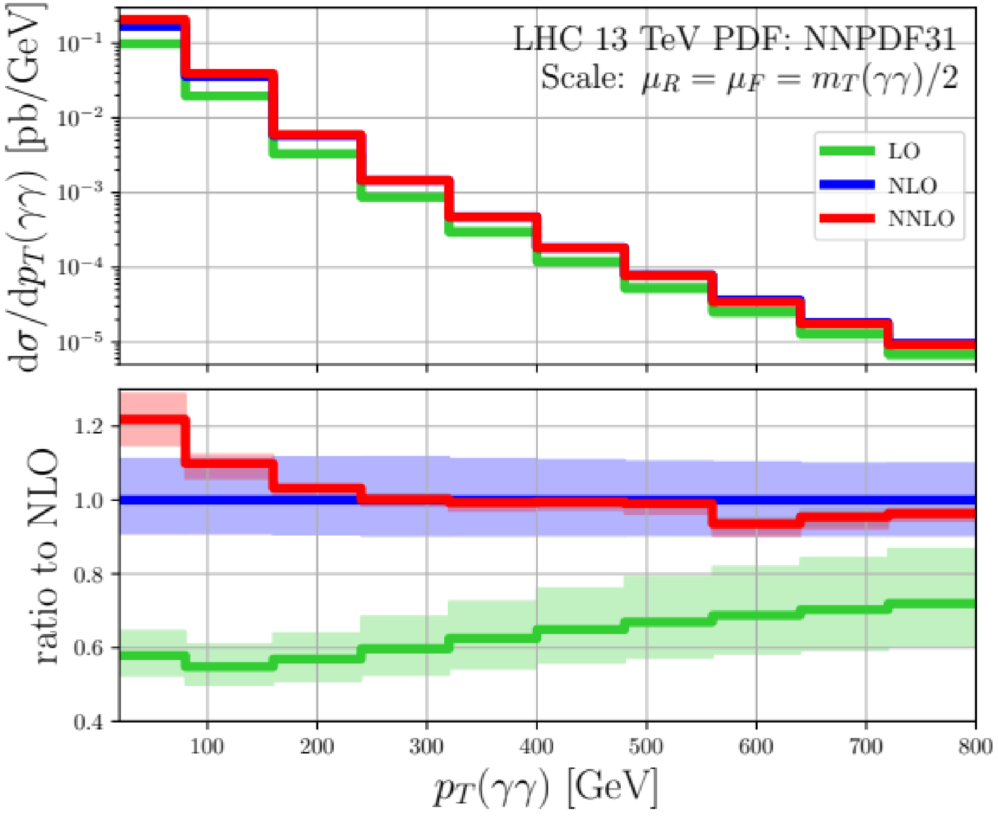
[Chawdry'21]: Inspired by Higgs → γγ measurement phase spaces

- Smooth photon isolation criteria: $E_T = 10\text{ GeV}, R_\gamma = 0.4, \Delta R(\gamma,\gamma) > 0.4$
- $p_T(\gamma_1) > 30\text{ GeV}, p_T(\gamma_2) > 18\text{ GeV and } |y(\gamma)| < 2.4$
- $m(\gamma\gamma) > 90\text{ GeV and } p_T(\gamma\gamma) > 20\text{ GeV}$, below resummation important
- No further restrictions on jets (IR safety from $p_T(\gamma\gamma)$ cut)

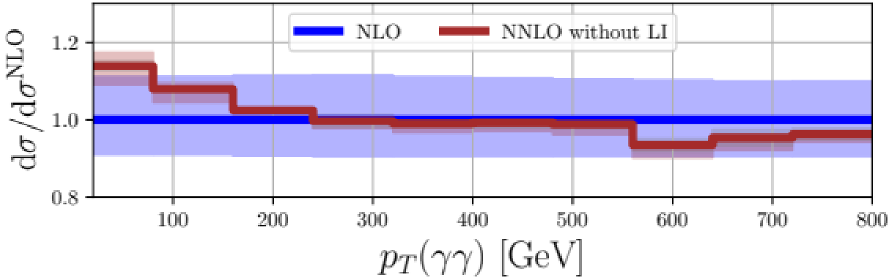
Technicalities:

- LHC 13 TeV, PDF: NNPDF31, Scale: $\mu_R^2 = \mu_F^2 = \frac{1}{4} m_T^2(\gamma\gamma) = \frac{1}{4}(m(\gamma\gamma)^2 + p_T(\gamma\gamma)^2)$
- 5 massless flavours and top-quarks (in all one-loop amps)
- Approximation of two-loop amps:
  $2\,\mathrm{Re}(\mathcal{M}^{(0)^\dagger}\mathcal{F}^{(2)}) + \mathcal{F}^{(1)^\dagger}\mathcal{F}^{(1)}$ without top-quark loops
  and $2\,\mathrm{Re}(\mathcal{M}^{(0)^\dagger}\mathcal{F}^{(2)})$ in leading colour limit [Chawdhry'21]
  → Update to full colour planned [Agarwal'21]

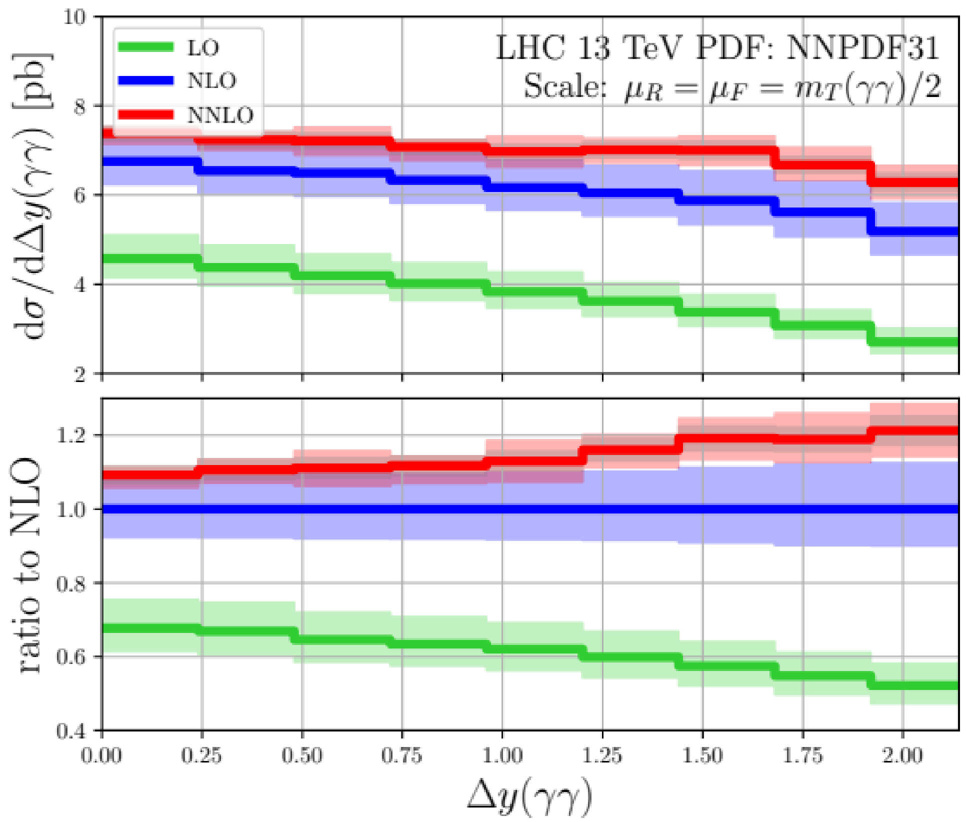
Diphoton plus jet – pT spectrum

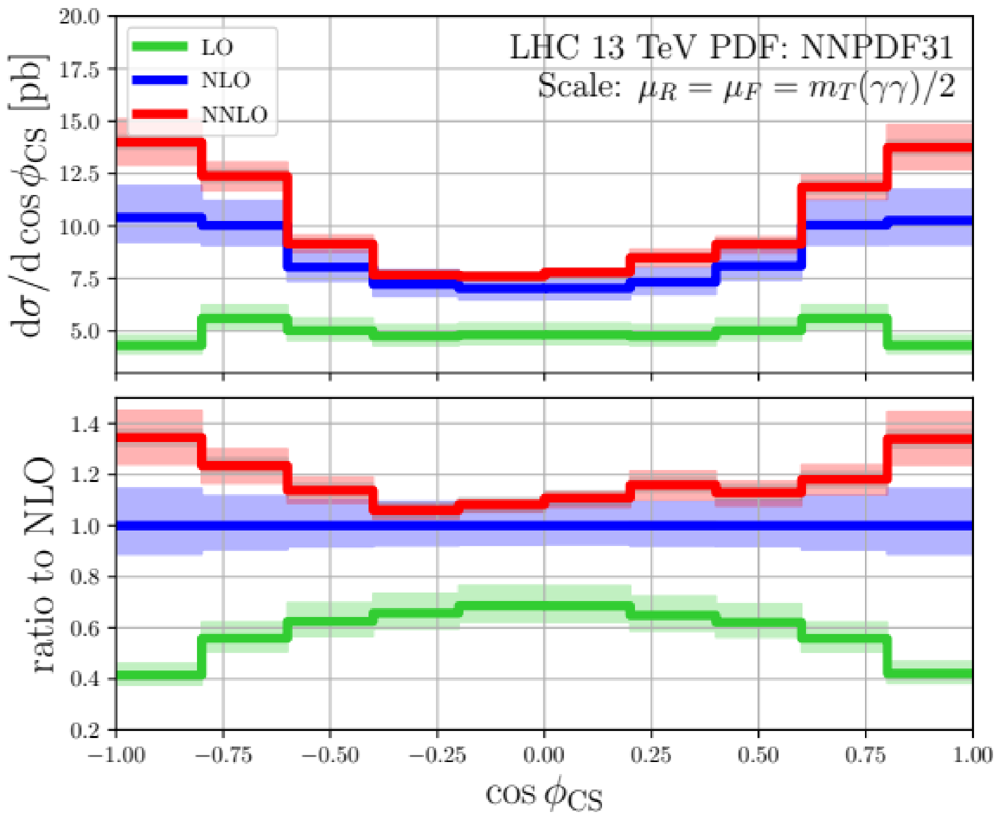


- Beautiful perturbative convergence
- Scale dependence:
  NLO: ~10%
  NNLO: ~1-2%
- Low p_T region:
  - ? Resummation for $p_T(\gamma\gamma)/m(\gamma\gamma) \ll 1$
  - Strong effect from the loop induced!



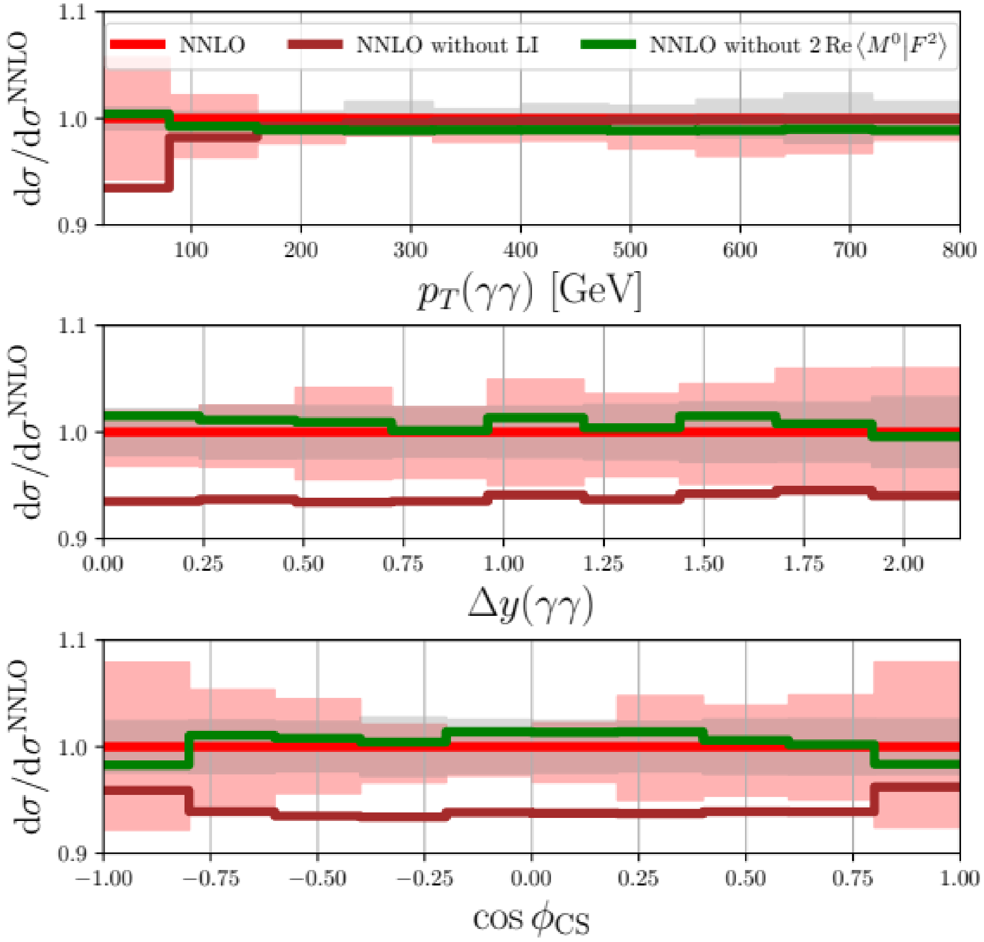
Diphoton plus jet – Angular observables





Note: Normalization affected by low p_T behaviour

Diphoton plus jet – two-loop contribution

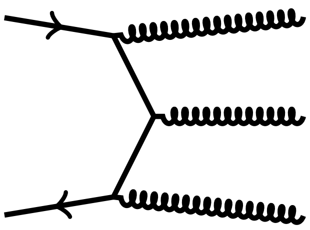


- Two-loop contribution (green line) <~1%,
- Loop induced contribution:
  - sizeable effects for low p_T, vanishes for high p_T
  - flat effect in 'bulk' observables
  - Dominant source of scale dependence
  - NLO QCD correction (formally N3LO) relevant,
    ~~missing piece~~: $gg \to g\gamma\gamma$ two-loop [Badger'21]

Three jet production

- Multi-jet rates provide a unique possibility to test (perturbative) QCD
- Parameter extraction:
  - Measurements of α_s from event shapes and jet rate ratios
    → energy scale dependence → test of α_s running
  - PDF extraction → high-x gluon
- Multi-jet signatures are background for many SM signatures.
- Allow to probe broad ranges of energy scales for heavy new physics
- Large cross sections → large statistics
  In practice only limited by systematics!

  → Theory uncertainties: missing higher orders, resummation, NP-physics, ...



$pp \to ggg + X$

Three jet production

Advances in perturbative QCD allow precision predictions for multi-jet rates

NNLO QCD predictions for two and three jet rates

- NNLO QCD di-jet production known:
  - Gluons only [Gehrmann-De Ridder'13], partially leading colour [Currie'16]
  - Complete [Czakon'18] → sub-leading colour effects < 1-2%
- NNLO QCD tri-jet production:
  - Bottleneck double virtual amplitudes: recently published in leading colour approximation [Abreu'21]
  - Handling of real radiation:
    - Sector-improved residue subtraction conceptually capable
    - Computationally very challenging!

Three jet production - Setup

Setup:

- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets with:
  - $p_T(j) > 60 \text{ GeV and } |y(j)| < 4.4$
  - $H_{T,2} = p_T(j_1) + p_T(j_2) > 250 \text{ GeV}$
- Scales: $\mu_R = \mu_F = \hat{H}_T = \sum_{\text{partons}} p_T$

R32 ratios:

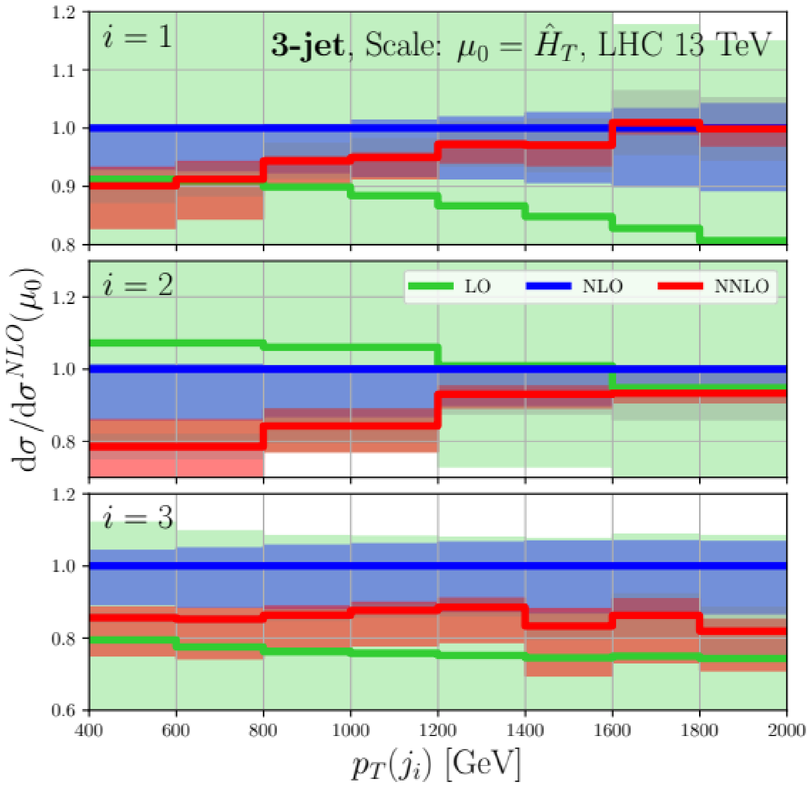
- $$R_{3/2}(X, \mu_R, \mu_F) = \frac{\mathrm{d}\sigma_3(\mu_R, \mu_F)/\mathrm{d}X}{\mathrm{d}\sigma_2(\mu_R, \mu_F)/\mathrm{d}X}$$
- Scale dependence is determined by correlated variation

Only Approximation made: $\mathcal{R}^{(2)}(\mu_R^2) = 2\,\mathrm{Re}\left[\mathcal{M}^{\dagger(0)}\mathcal{F}^{(2)}\right](\mu_R^2) + \left|\mathcal{F}^{(1)}\right|^2(\mu_R^2) \equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^{4} c_i \ln^i\left(\frac{\mu_R^2}{s_{12}}\right)$

→ taken from [Abreu'21]

$$\mathcal{R}^{(2)}(s_{12}) \approx \mathcal{R}^{(2)l.c.}(s_{12})$$

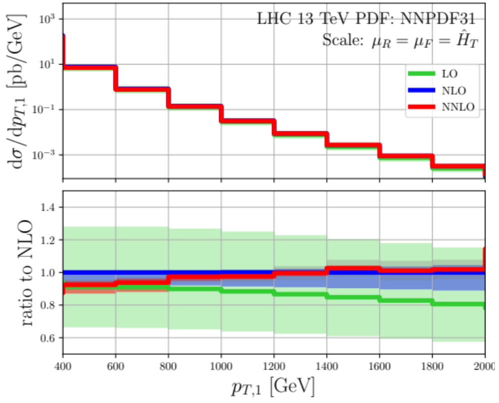
Three jet production – transverse jet momenta

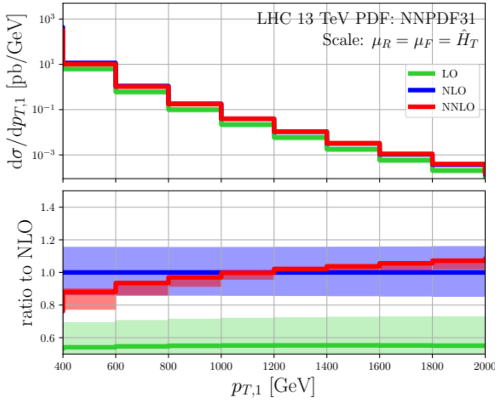


- $p_T(j_2)$:
  → suffers from slow MC convergence, larger binning
  → shows reasonable perturbative convergence
- $p_T(j_3)$:
  → fast MC convergence
  → flat k-factor

Caveat:
→ Scale choice based on full event
→ reasonable for $p_T(j_1)$ and $p_T(j_2)$
→ $p_T(j_3) \ll p_T(j_1) + p_T(j_2)$
  → potentially large hierarchy?
→ investigation with 'jet-based' scale useful

Three jet production - R32(pT1)





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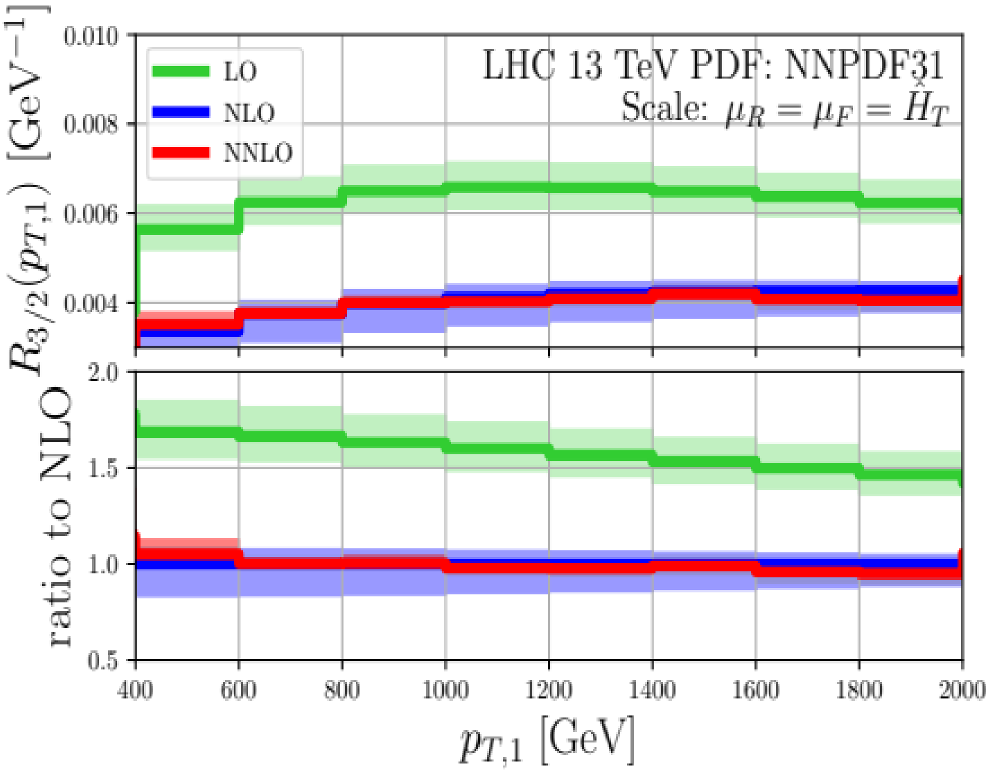
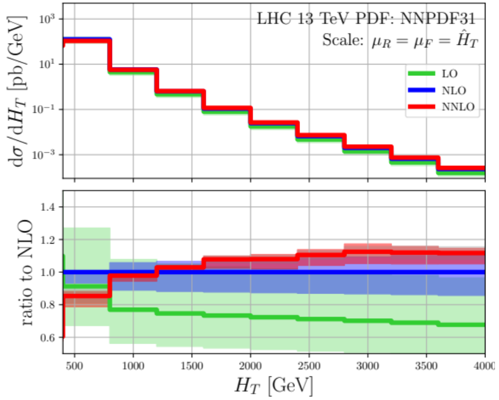
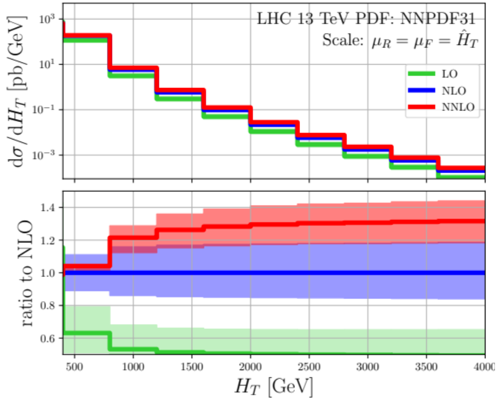


Three jet production - R32(HT)





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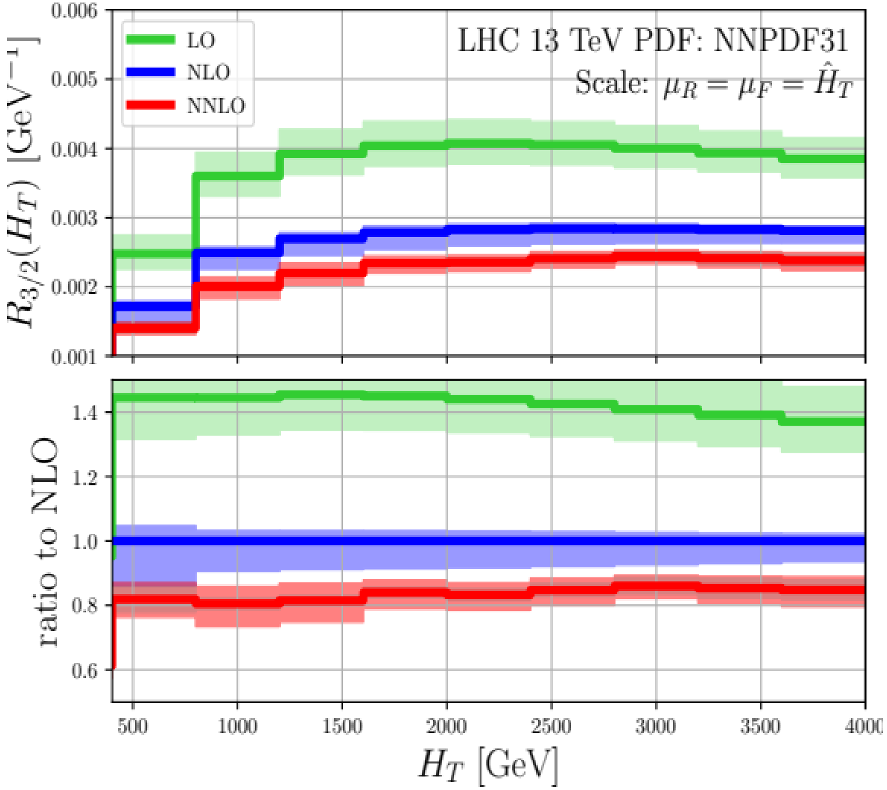


$$H_T = \sum_{\text{jets}} p_T$$

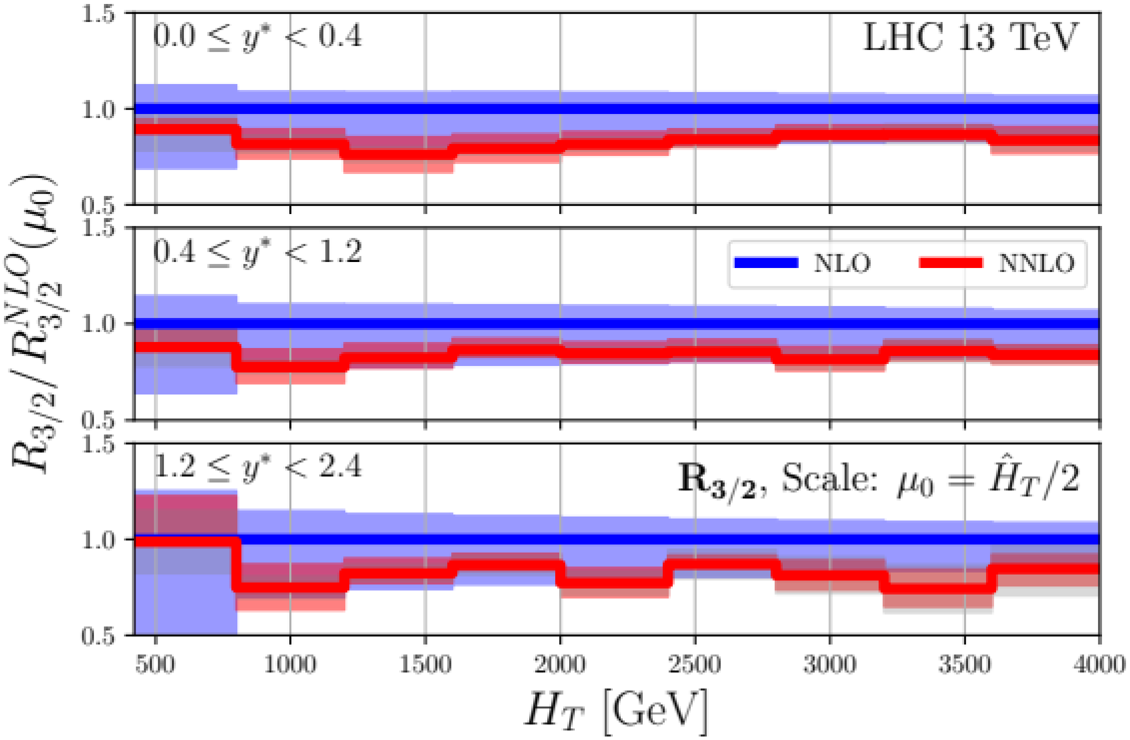
Scale dependence correlated in ratio

→ reduction of scale dependence

→ flat k-factor

→ scale bands in ratio barely overlap

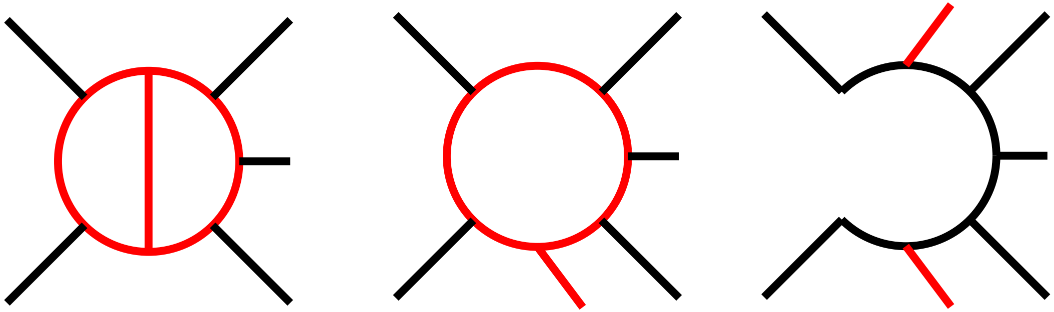
Three jet production – R32(HT,y*)



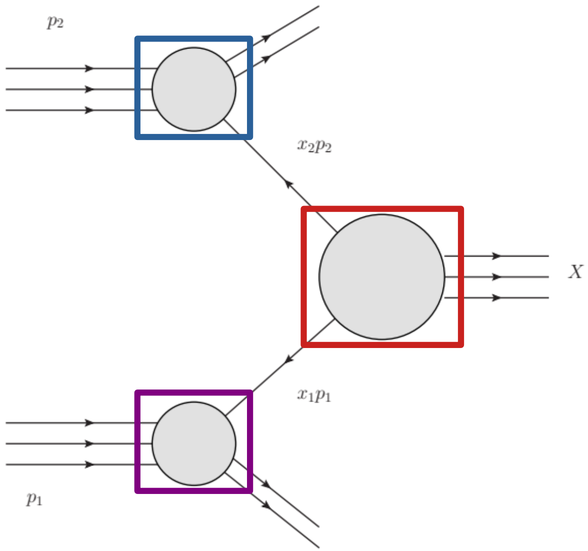
Double differential w.r.t. $y^* = |y(j_1) - y(j_2)|/2$

Different central scale choice: $\hat{H}_T/2$

Sector-improved residue subtraction



Hadronic cross section



Hadronic cross section:

$$\sigma_{h_1h_2\to X} = \sum_{ij}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\phi_{i,h_1}(x_1,\mu_F^2)\phi_{j/h_2}(x_2,\mu_F^2)\hat{\sigma}_{ij\to X}(\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)$$

Parton distribution functions

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

Partonic cross section beyond LO

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

Contributions with different multiplicities and # convolutions:

$$\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab}$$

Each term separately IR divergent. But sum is:

→ finite

→ regularization scheme independent

Considering CDR ($d = 4 - 2\epsilon$):

→ Laurent expansion: $\hat{\sigma}^{C}_{ab} = \sum_{i=-4}^{0} c_i \epsilon^i + \mathcal{O}(\epsilon)$

$$\hat{\sigma}^{\mathrm{RR}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}^{(0)}_{n+2} \middle| \mathcal{M}^{(0)}_{n+2} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}^{\mathrm{RV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(1)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\mathrm{VV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n} \middle| \mathcal{M}^{(2)}_{n} \right\rangle + \left\langle \mathcal{M}^{(1)}_{n} \middle| \mathcal{M}^{(1)}_{n} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}^{\mathrm{C1}}_{ab} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\mathrm{C2}}_{ab} = (\text{double convolution})\, \mathrm{F}_{n}$$

Sector decomposition I

Considering working in CDR:

→ Virtuals are usually done in this regularization

→ Real radiation:

→ Very difficult integrals, analytical impractical (except very simple cases)!

→ Numerics not possible, integrals are divergent: ε-poles!

How to extract these poles? → Sector decomposition!

Divide and conquer the phase space:

$$1 = \sum_{i,j} \left[\sum_{k} \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \quad \Rightarrow \quad \hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \sum_{i,j} \left[\sum_{k} \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

Sector decomposition II

Divide and conquer the phase space:

→ Each $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$ has simpler divergences.

Soft and collinear (w.r.t parton k,l) of partons i and j

→ Parametrization w.r.t. reference parton:

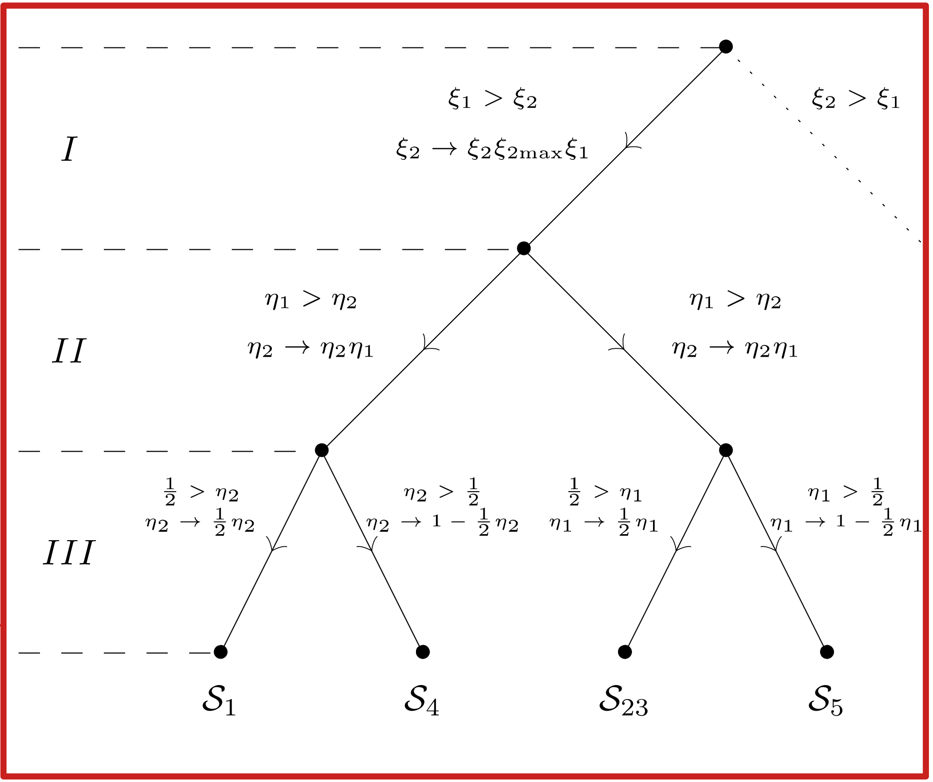
$$\hat{\eta}_i = \frac{1}{2}(1-\cos\theta_{ir}) \in [0,1] \qquad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$$

→ Subdivide to factorize divergences

→ double soft factorization:

$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

→ triple collinear factorization



[Czakon'10,Caola'17]

Sector decomposition III

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\,\mathcal{S}_{kl,m}\left\langle \mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2} = \sum_{\text{sub-sec.}}\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i\,\underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}}\,\mathrm{d}\tilde{\mu}(\{x_i\})\underbrace{\prod x_i^{a_i+1}\langle \mathcal{M}_{n+2}|\mathcal{M}_{n+2}\rangle}_{\text{regular}}\mathrm{F}_{n+2}$$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 \mathrm{d}x\,\left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x)-f(0)}{x^{1+b\epsilon}}$$

Finite NNLO cross section

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_{n}$$

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

⬇ sector decomposition and master formula

$$\left(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}\right) \quad \left(\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}\right) \quad \left(\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}\right) \quad \left(\sigma_{SU}^{C1}, \sigma_{DU}^{C1}\right) \quad \left(\sigma_{DU}^{C2}, \sigma_{FR}^{C2}\right)$$

⬇ re-arrangement of terms → 4-dim. formulation [Czakon'14,Czakon'19]

$$\underline{\left(\sigma_F^{RR}\right)} \quad \underline{\left(\sigma_F^{RV}\right)} \quad \underline{\left(\sigma_F^{VV}\right)} \quad \underline{\left(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}\right)} \quad \underline{\left(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2}\right)} \quad \underline{\left(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}\right)}$$

separately finite: ε poles cancel

Improved phase space generation

Phase space cut and differential observable introduce

mis-binning : mismatch between kinematics in subtraction terms

$\rightarrow$ leads to increased variance of the integrand

$\rightarrow$ slow Monte Carlo convergence

New phase space parametrization [Czakon'19]:

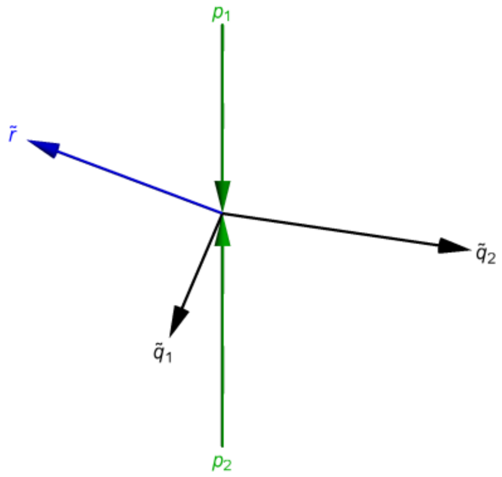
Minimization of # of different subtraction kinematics in each sector

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i : $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,, \quad \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

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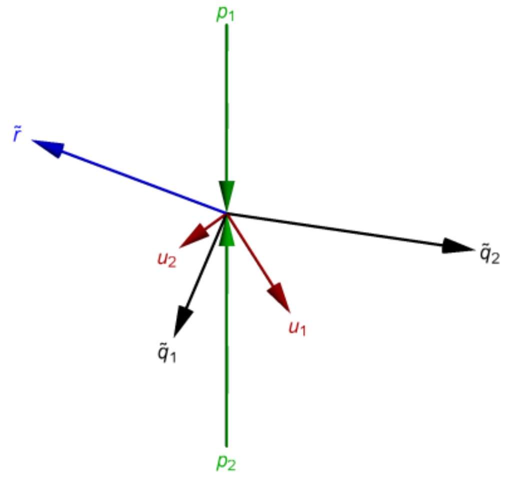

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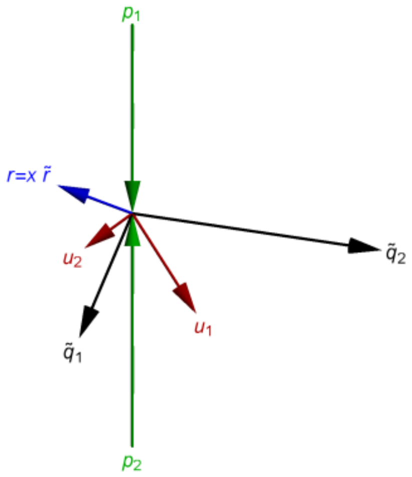

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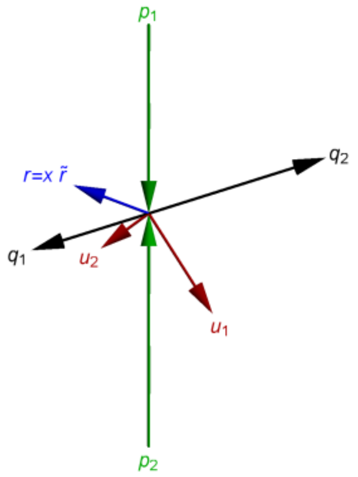

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Main steps:

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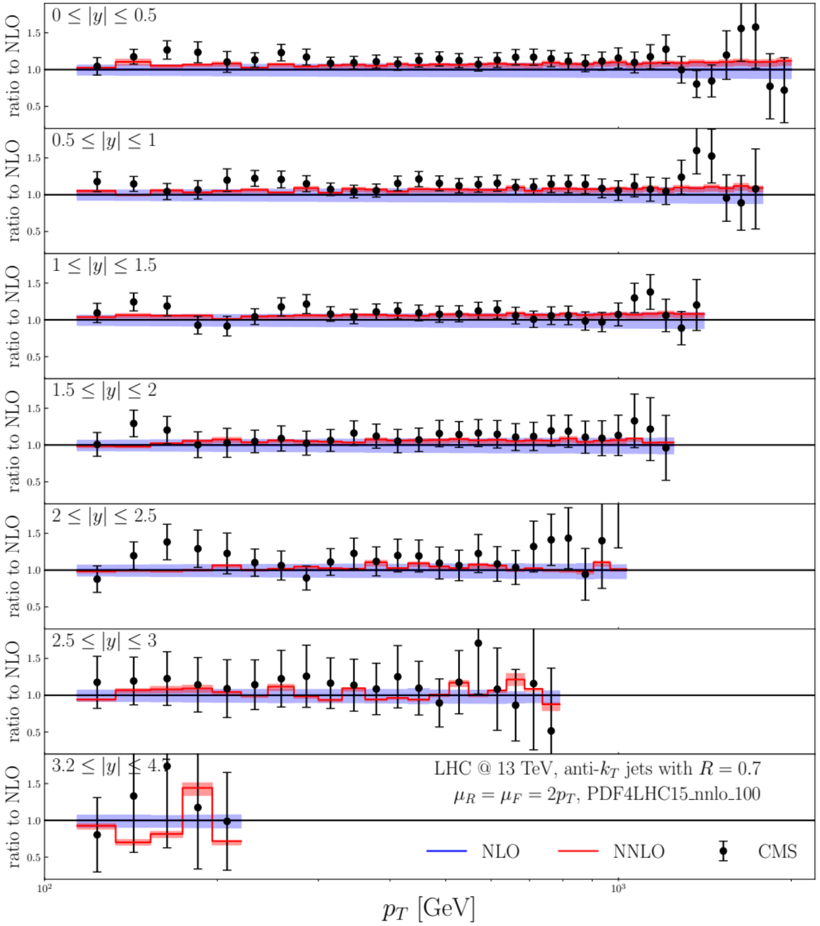
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

Ultimate check: single inclusive jets

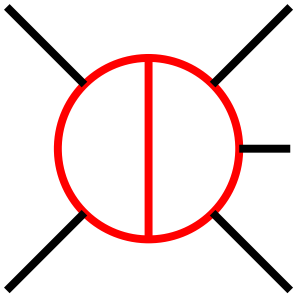
- Well studied observable:
  - NNLOJet [Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires '16-19]
- Full colour [Czakon'19]:
  - Tests all possible IR subtraction terms
  - Comparison to NNLOJet results:
    - Found full agreement within MC error
    - Puts bounds on sub-leading colour terms ~1-2 %



Further technical developments

- Narrow-width-approximation and Double-Pole-Approximation for resonant particles:
  - Top-quark pairs + decays [Czakon'19'20]
  - W+W- polarization [Poncelet'21]
- Automated interfaces to OpenLoops, Recola and Njet
- Implementation of state-of-the-art twoloop matrix-elements:
  - 2→ 2(1) : pp → VV, pp→ Vj, pp → H (j), e+e- → jets, DIS
  - 2 → 3: Pp → 3y, pp → 2y + j, pp → 3j
- Fragmentation of massless partons into hadrons
  - First application to pp → tt +X → l+l- v v~ B + X (NWA) [Czakon'21]
- Countless small improvements in terms of organization and efficiency

Five-point amplitudes



Five-point amplitudes - Overview

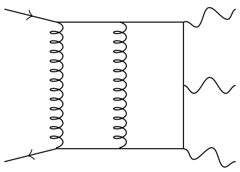
The all massless case:

- $pp \to jjj$
  - Euclidean results: insights in rational structure of amplitudes [Abreu'19]
  - Physical phase space [Abreu'21]:
    - based on 'pentagon-functions' by Chicherin and Sotnikov
    - efficient evaluation times (~1sec)→ 'pheno-ready'
- $pp \to \gamma\gamma\gamma$
  - First, squared matrix elements with 'pentagon-functions' by [Gehrmann'18]. Very slow, however usable for pheno application [Chawdhry'19].
  - Helicity amplitudes with new 'pentagon-functions' [Abreu'20,Chawdhry'20]
- $pp \to \gamma\gamma j$
  - Squared matrix element in planar limit [Agarwal'21]
  - Helicity amplitudes in planar limit [Chawdhry'21]
  - Both in full glory [Agarwal'21] + gg induced [Badger'21]
- $pp \to \gamma jj$ ← untouched territory so far...

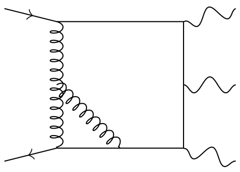
Planar five-point amplitudes

$$q\bar{q} \to \gamma\gamma\gamma$$

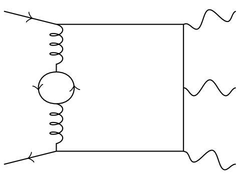
- 3 independent helicities
- QED x QCD → leading color ≠ planar



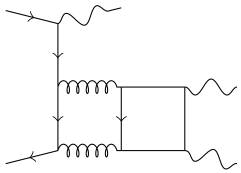
$\sim C_F^2$



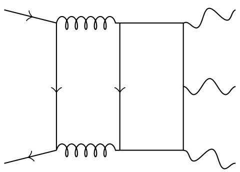
$\sim C_F C_A$



$\sim C_F n_f$



$\sim C_F \sum_{q'} Q_q'^2$

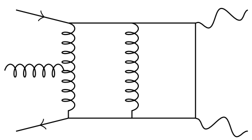


$\sim C_F \sum_{q'} Q_q'^3$

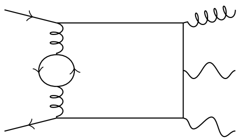
$$\mathcal{F}^{(2)}(q\bar{q} \to \gamma\gamma\gamma)\Big|_{\text{planar}} = Q_q^3 N_c^2 \left(\mathcal{F}^{C_F^2} + 2\mathcal{F}^{C_F C_A} \right) + Q_q^3 C_F n_f \mathcal{F}^{C_F n_f}$$

$$q\bar{q} \to g\gamma\gamma \quad qg \to q\gamma\gamma$$

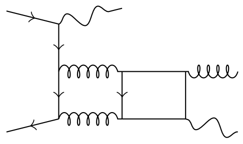
- Kinematics: $\{s_{ij}\} = \{s_{12}, s_{23}, s_{34}, s_{45}, s_{51}\}$
  $\mathrm{tr}_5 = 4i\epsilon(p_1, p_2, p_3, p_4)$



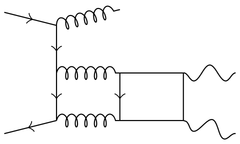
$\sim \mathcal{F}^{(2),Q_q^2,N_c^2}$



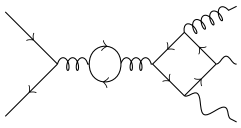
$\sim \mathcal{F}^{(2),Q_q^2,n_f}$



$\sim \mathcal{F}^{(2),Q_q Q_{q'} N_c}$



$\sim \mathcal{F}^{(2),Q_{q'}^2 N_c}$



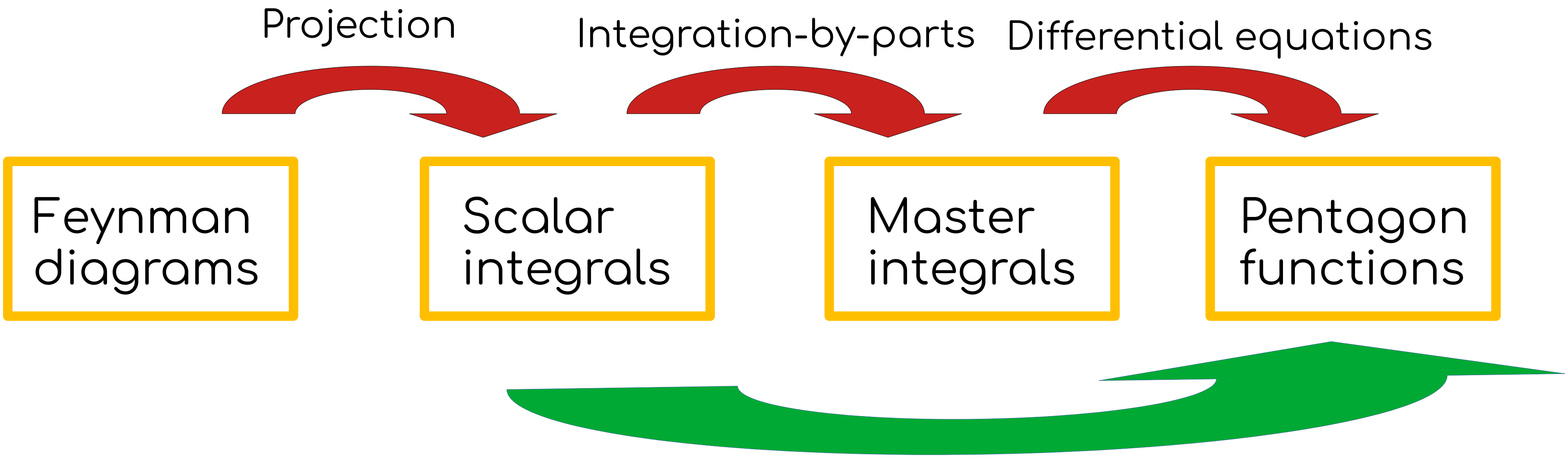
$\sim \mathcal{F}^{(2),Q_{q'}^2 n_f}$

$$\mathcal{F}^{(2)}(q\bar{q} \to \gamma\gamma g)\Big|_{\text{planar}} = Q_q^2 N_c^2 \left(\mathcal{F}^{(2),Q_q^2,N_c^2} + \frac{n_f}{N_c} \mathcal{F}^{(2),Q_q^2,n_f} \right) + Q_{l,2} n_f \mathcal{F}^{(2),Q_{q'}^2,n_f}$$

$\boxed{}$ = non-planar diagrams at LC

Our framework

Old school approach:



Automated framework using finite fields to avoid expression swell based on Firefly [Klappert'19'20]

Projection

Projection to helicity amplitudes based on [Chen '19]

Spin structure of $q\bar{q} \to \gamma\gamma\gamma$ and $q\bar{q} \to g\gamma\gamma$:
$$\mathcal{M}^{\bar{h}} = \epsilon^{*\mu}_{3,h_3}\epsilon^{*\nu}_{4,h_4}\epsilon^{*\rho}_{5,h_5}\bar{v}(h_2)\Gamma_{\mu\nu\rho}u(h_1)$$

Explicit representation of polarization vectors in terms of momenta (d=4):

$$\epsilon^{\mu}_{i,h} = \frac{1}{\sqrt{2}}(\epsilon^{\mu}_{i,X} + hi\epsilon^{\mu}_{i,Y})$$

Ansatz:

$$\epsilon^{\mu}_{i,X} = c^{X}_{i,1}p^{\mu}_{1} + c^{X}_{i,2}p^{\mu}_{2} + c^{X}_{i,3}p^{\mu}_{i}$$
$$\Rightarrow \epsilon^{\mu}_{i,Y} = \mathcal{N}_{i,Y}\epsilon^{\mu}{}_{\nu\rho\sigma}q^{\nu}p^{\rho}_{i}\epsilon^{\sigma}_{i,X}$$

Constraints:

$$(\epsilon_{i,X})^2 = -1, \quad \epsilon_{i,X}\cdot q = 0, \quad \epsilon_{i,X}\cdot p_i = 0$$

Spinors expressed through trace:

$$\mathcal{M} = \bar{v}(p_2,h_2)\Gamma u(p_1,h_1) = \mathrm{Tr}\Big\{(u\otimes\bar{v})\,\Gamma\Big\}$$
$$(u\otimes\bar{v})_{\alpha\beta} = \frac{\bar{u}Nv}{\bar{u}Nv}(u\otimes\bar{v})_{\alpha\beta} = \frac{1}{\mathcal{N}}[(u\otimes\bar{u})N(v\otimes\bar{v})]_{\alpha\beta}$$

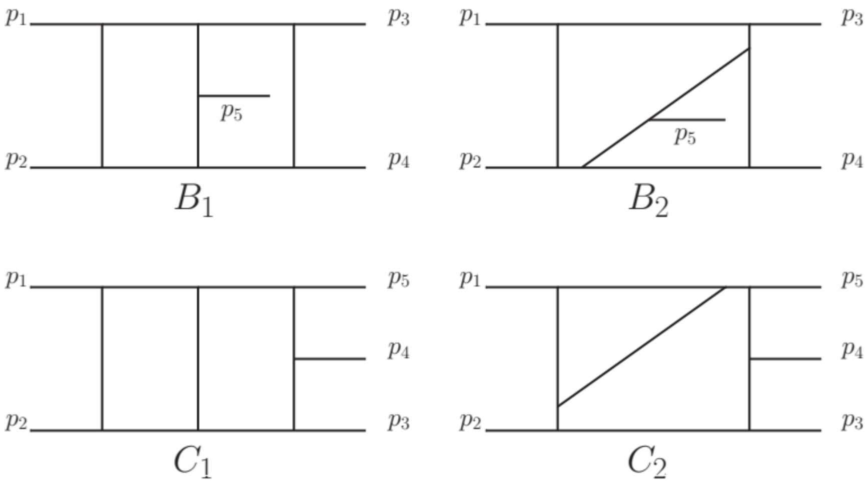
Application to Feynman diagrams → scalar expression: $\mathcal{M} = \sum c(\{s_{ij}\},\mathrm{tr}_5,d)I(\{s_{ij}\},d)$

Note: bare amplitudes are scheme-dependent, finite remainders are not

Integration-by-parts identities and masters

Analytically derived IBP tables [Chawdhry'18]:
→ Strategy: solve 1 master integral at a time
→ Crossed kinematics by finite field numerics
→ Translated to UT basis in [Chicherin'20]

$$I(\{s_{ij}\},d) = \sum \tilde{c}(\{s_{ij}\},d)\mathrm{UT}(\{s_{ij}\},d)$$



Representation of master integrals in terms of 'pentagon-functions' of weight i : $\vec{t_i}$

$$\mathrm{UT}(\{s_{ij}\},d) = \sum_{i=0}^{4} \left(\vec{c_i}\cdot\vec{t_i}\right)\epsilon^i$$

Amplitudes! Assemble!

All bits known analytically, but adding them up is cumbersome...

Using the increasingly adapted finite field approach (using Firefly):

→ evaluating all components in finite field points

→ doing the sums

→ reconstruct the finite remainder amplitude:

$$\mathcal{F} = \mathcal{M} - \mathrm{IR/UV} = Q_q^2 \mathcal{F}^{(0),Q_q^2}\left(1 + \left(\frac{\alpha_s}{4\pi}\right)\left(C_F \mathcal{R}^{(1),Q_q^2,C_F} + \frac{T_F}{C_A}\mathcal{R}^{(1),Q_q^2,T_F/C_A} + T_F \frac{Q_{l,2}}{Q_q^2}\mathcal{R}^{(1),Q_{q'}^2,T_F}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left(N_c^2 \mathcal{R}^{(2),Q_q^2,N_c^2} + N_c n_f \mathcal{R}^{(2),Q_q^2,n_f} + n_f \frac{Q_{l,2}}{Q_q^2}\mathcal{R}^{(2),Q_{q'}^2,n_f}\right)\right)$$

$$\mathcal{R}^{(\ell),i,c} = \sum_e \boxed{r_e^{(\ell),i,c}}\,\boxed{t_e}$$

$\boxed{t_e}$: Combinations of transcendental functions

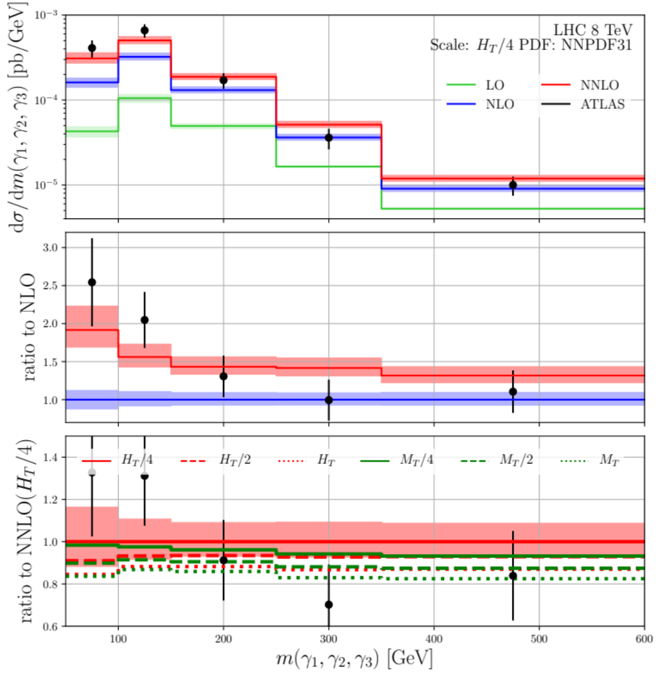
$\boxed{r_e^{(\ell),i,c}}$: rational in s_{ij} and linear in tr_5

→ Exploiting Q-linear relations among rationals:

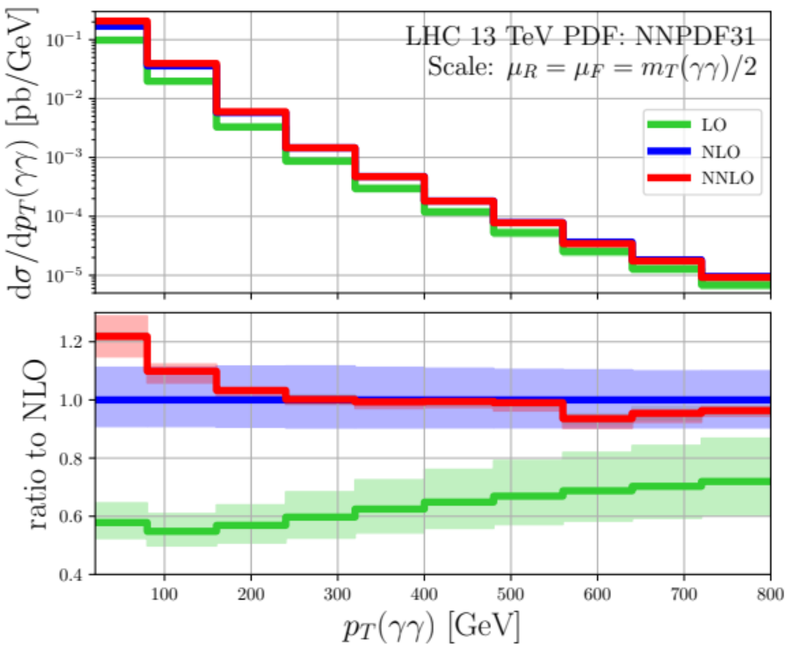
$q\bar{q} \to g\gamma\gamma$	# tot./ # ind.
$\mathcal{R}^{+-----,(2),Q_q^2,N_c^2}$	96 / 33
$\mathcal{R}^{+-----,(2),Q_q^2,n_f}$	48 / 22
$\mathcal{R}^{+-----,(2),Q_{q'}^2,n_f}$	6 / 2
$\mathcal{R}^{+-+--,(2),Q_q^2,N_c^2}$	7266 / 66
$\mathcal{R}^{+-+--,(2),Q_q^2,n_f}$	504 / 27
$\mathcal{R}^{+-+--,(2),Q_{q'}^2,n_f}$	58 / 8
$\mathcal{R}^{+--+-,(2),Q_q^2,N_c^2}$	7252 / 101
$\mathcal{R}^{+--+-,(2),Q_q^2,n_f}$	736 / 59
$\mathcal{R}^{+--+-,(2),Q_{q'}^2,n_f}$	58 / 8

Closing the loop

$pp \to \gamma\gamma\gamma$

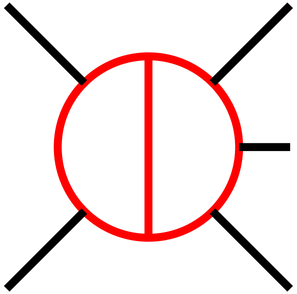


$pp \to \gamma\gamma j$

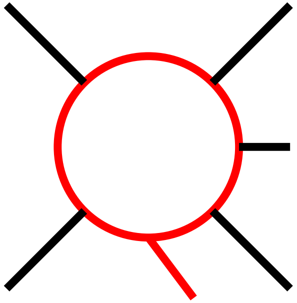


Summary & Outlook

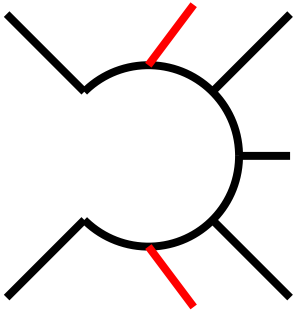
Summary and Outlook



First NNLO QCD phenomenology results for $2 \to 3$ processes
- Three photons, di-photon plus jet, three jets
- A plethora of phenomenological applications pending…
- Computational efficiency?



Sector improved residue subtraction
- Pragmatic divide and conquer technique
- Many technical improvements: phase space, NWA & DPA, oneloop-interfaces, fragmentation,…



Five-point amplitudes in projection approach
- $pp \to \gamma\gamma\gamma \quad pp \to \gamma\gamma j$
- Beyond LC ?! [Agarwal'21]
- Application to $pp \to jjj \quad \text{and} \quad pp \to \gamma jj$?

Summary and Outlook

Thank you for your attention!

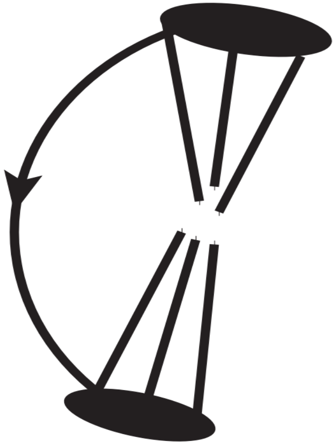
Backup

Three jet production – azimuthal decorrelation

Kinematic constraints on the azimuthal separation between the two leading jets ($\varphi12$)

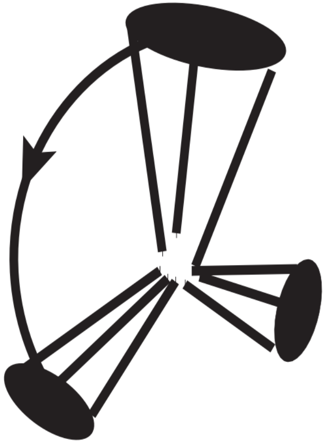
Dijet:

$\varphi12 = \pi$



Trijet:

$\varphi12 > 2/3\pi$



$\varphi12$ sensitive to the jet multiplicity:

2j: $\varphi12 = \pi$

3j: $\varphi12 > 2/3\pi$

4j: unconstrained

Study of the ratio

R32(HT,y*,φMax) =
(dσ3(φ<φMax)/dHT/dy*)/(dσ2/dHT/dy*)

With y* = |y1-y2|/2

Three jet production – R32(HT,y*,φMax)

NNLO/NLO K-factor smaller than NLO/LO
Scale dependence is reduced

φMax

NLO 4-jet

y*