Three photon production at the LHC: Amplitudes and Phenomenology

Oxford Dalitz Seminar

Rene Poncelet

in collaboration with H. Chawdhry, M. Czakon and A. Mitov.

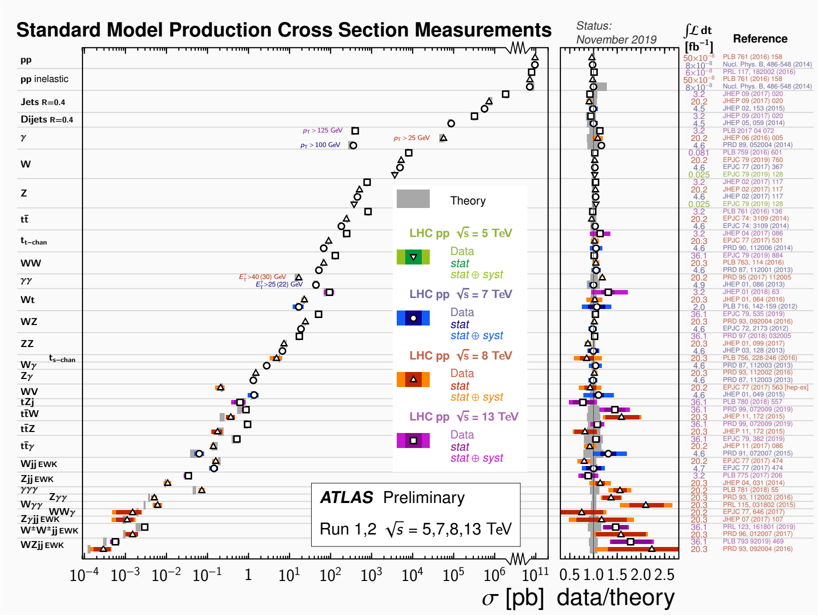
13th Feburary 2020

Cavendish Laboratory

Outline

- Introduction
- Phenomenology
- Massless 5-point amplitudes at two-loop

Precision at the LHC



NNLO QCD

Tremendous progress in NNLO QCD calculation in the past decade

State-of-the-art:

- All (Standard Model) $2 \to 2$ processes calculated
- Phenomenology: SM precision measurements and parameter estimation, PDF determination, ...

$\rightarrow$ Valuable input for the LHC physics program!

- $2 \to 3$ is the natural step beyond. Many efforts on-going

Not quite comparable to the 'NLO revolution' yet, lack of automated:

1. Real radiation contributions $\rightarrow$ subtraction schemes
2. Two-loop matrix elements

Subtractions schemes

Handling real radiation contribution in NNLO calculations
cancellation of infra-red divergences

increasing number of available NNLO calculations with a variety of schemes

- qT-slicing [Catani,Grazzini, '07] ,[Ferrera,Grazzini,Tramontano, '11], [Catani,Cieri,DeFlorian,Ferrera,Grazzini,'12], [Gehrmann,Grazzini,Kallweit,Maierhofer,Manteuffel,Rathlev,Torre,'14-15'], [Bonciani,Catani,Grazzini,Sargsyan,Torre,'14-'15], [Grazzini "MATRIX" '17-'19]
- N-jettiness slicing [Gaunt,Stahlhofen,Tackmann,Walsh, '15], [Boughezal,Focke,Giele,Liu,Petriello,'15-'16] , [Bougezal,Campell,Ellis,Focke,Giele,Liu,Petriello,'15], [Campell,Ellis,Williams,'16]
- Antenna subtraction [Gehrmann, GehrmannDeRidder,Glover,Heinrich,'05-'08] , [Weinzierl,'08,'09], [Currie,Gehrmann,GehrmannDeRidder,Glover,Pires,'13-'17], [Bernreuther,Bogner,Dekkers,'11,'14], [Abelof,(Dekkers),GehrmannDeRidder,'11-'15], [Abelof,GehrmannDeRidder,Maierhofer,Pozzorini,'14], [Chen,Gehrmann,Glover,Jaquier,'15]
- Colorful subtraction [DelDuca,Somogyi,Troscanyi,'05-'13], [DelDuca,Duhr,Somogyi,Tramontano,Troscanyi,'15]
- Sector-improved residue subtraction (STRIPPER) [Czakon,'10,'11] , [Czakon,Fiedler,Mitov,'13,'15], [Czakon,Heymes,'14] [Czakon,Fiedler,Heymes,Mitov,'16,'17], [Bughezal,Caola,Melnikov,Petriello,Schulze,'13,'14], [Bughezal,Melnikov,Petriello,'11], [Caola,Czernecki,Liang,Melnikov,Szafron,'14], [Bruchseifer,Caola,Melnikov,'13-'14], [Caola, Melnikov, Röntsch,'17-'19]
- Projection-to-Born [Cacciari et al '15], [Dreyer,Karlberg '18], Geometric [Herzog '18], Unsubtraction [Aguilera-Verdugo et al '19], . . .

Subtraction beyond $2 \to 2$ with the STRIPPER c++ framework

- Jet-production at NNLO QCD [Czakon,van Hameren,Mitov,Poncelet'19]: full set of subtraction terms in action
- Fully automated generation of subtraction terms
- Straight-forward user interface:
  - Generation of required contributions
  - Combination of equivalent contributions $\to$ minimize computational setup
  - Automated interfaces to OpenLoops2 [Buccioni et al. '19] and Recola2 [Denner et al. '16-17], including available correlators

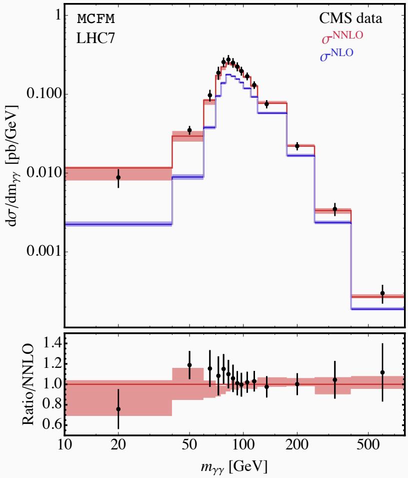
The framework is ready for the future

Introduction: Why $pp \to \gamma\gamma\gamma$

- Potentially simplest $2 \to 3$ process:
  - Simple IR-structure
  - Allows for tool development
  - First experience with the two-loop matrix elements
  - Two-loop amplitudes: fewer diagrams and channels than jets

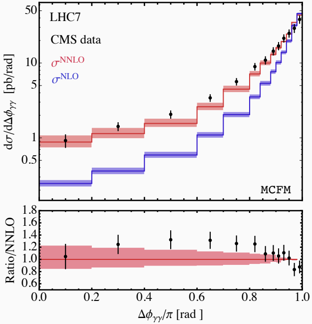
$\to$ Proof-of-principle calculation

- Phenomenologically interesting:
  - LHC measurements show significant deviations from standard NLO predictions
  - Large NNLO/NLO K-factors have been observed in photon pair production [Catani et al 11],[Campbell et al 16]

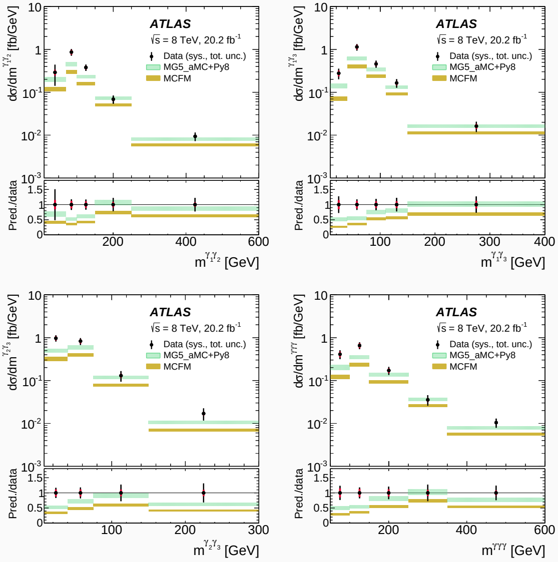


[Campbell, Ellis , Li, Williams 16]

- Giant K-factor $\sigma^{NNLO}/\sigma^{NLO} \approx 2$
- Significant impact on shape of differential distribution
- Good agreement between NNLO QCD and data (CMS 7 TeV)



Detailed differential measurements by ATLAS [1712.07291 ATLAS]



NNLO QCD corrections to three-photon production at the LHC

[Chawdhry, Czakon, Mitov, Poncelet '19]

- Detailed differential measurements by ATLAS [1712.07291 ATLAS]

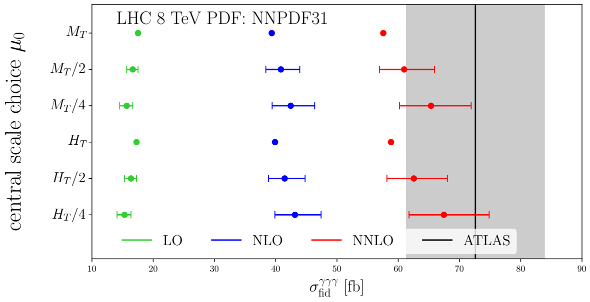
Setup:

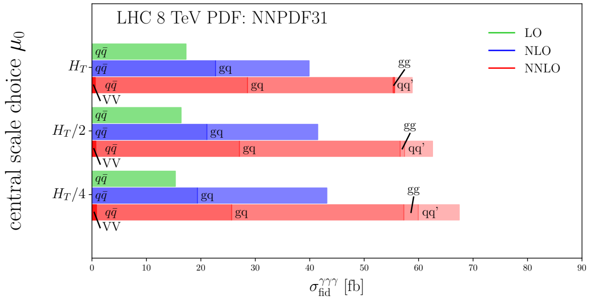
- E_T ($= p_T$) cut for the three photons: $E_{T,\gamma_1} > 27$ GeV, $E_{T,\gamma_2} > 22$ GeV, $E_{T,\gamma_3} > 15$ GeV
- Rapidity: All photons have $|\eta_\gamma| < 2.37$ (+exclusion of $1.37 < |\eta_\gamma| < 1.56$)
- Separation of photons: The angular distance between each two photons ΔR is required to be > 0.45
- Invariant mass: $m_{\gamma\gamma\gamma} > 50$ GeV
- Photon isolation: Using the Frixione [Frixione '98] isolation as indicated for the MadGraph@NLO setup. This means $R_0 = 0.4$, $E_T^{iso} > 10$ GeV and $\chi(R) = (1 - \cos(\Delta R))/(1 - \cos(\Delta R_0))$.
- PDF set: *NNPDF31_nnlo_as_0118*
- Scales:

$$\mu_0 = m_{\perp,\gamma\gamma\gamma} = \sqrt{p_\gamma^2 + (p_{\gamma,T})^2} \quad \text{with} \quad p_\gamma = \sum_{i=1}^{3} p_{\gamma_i} \ ,$$

$$\mu_0 = H_T/4 = \frac{1}{4} \sum p_{\gamma_i,T}$$

3 Photons @ LHC: Fiducial cross section

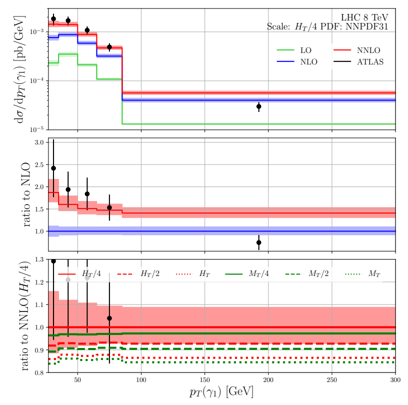


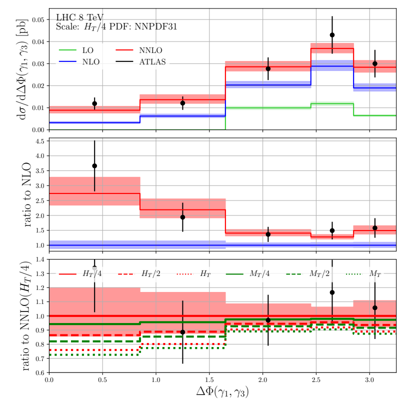


- Huge K-factors: $\sigma^{\text{NLO}}/\sigma^{\text{LO}} \approx 2-3$ $\sigma^{\text{NNLO}}/\sigma^{\text{NLO}} \approx 1.5$
- Significant improvement in description of data
- Scale dependence dominated by gg/qq' channels

3 Photons @ LHC: Differential distributions

- Not only normalization $\to$ significant effects on the shape
- Remarkable agreement of measurement with NNLO QCD

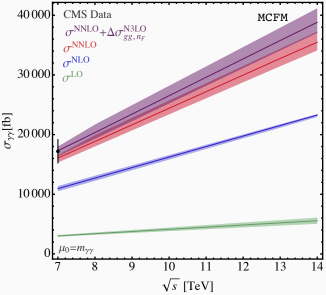




3 Photons @ LHC: Perturbative convergence

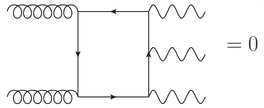
$pp \to \gamma\gamma$

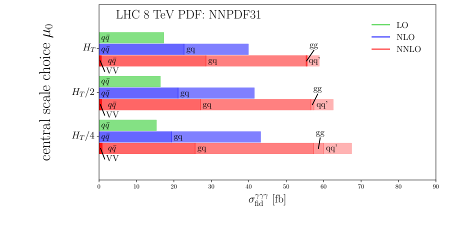
- Similar large K-factors in $\gamma\gamma$
  [Catani, Cieri, de Florian, Ferrera, Grazzini 11]
  [Campbell, Ellis , Li, Williams 16]
- $gg \to \gamma\gamma$ 1-loop box
  $\sim +10\%$ cross section
  +sizeable NLO corrections ($\Delta\sigma^{N3LO}_{gg,n_f}$)



$pp \to \gamma\gamma\gamma$

- gg channel contributes through
  $gg \to \gamma\gamma\gamma q\bar{q}$ @ NNLO QCD
- $gg \to \gamma\gamma\gamma$ 1-loop box does vanish
  (Furry's Theorem)

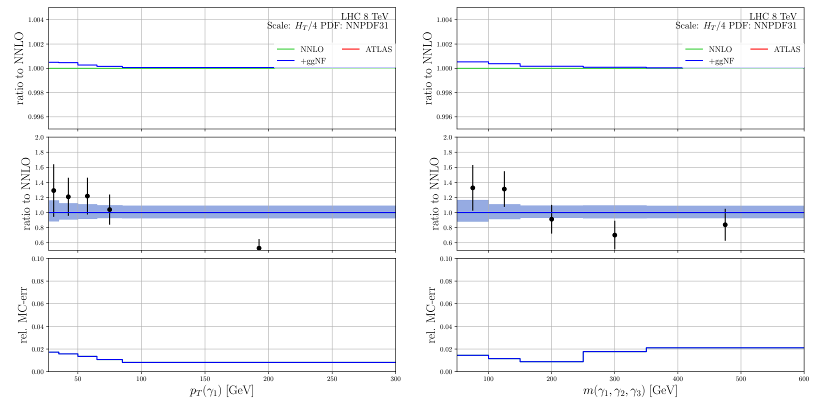




3 Photons @ LHC: Perturbative convergence

- $gg \to \gamma\gamma\gamma$ 1-loop box vanishes (Furry's Theorem)
- But $gg \to \gamma\gamma\gamma g$ does not, and is separately finite N^{3}LO contribution

$\to$ negligible!



$$pp \to \gamma\gamma\gamma$$

- NNLO QCD corrections are vital for comparisons of data with SM
  - Improved normalization
  - Very good agreement of differential cross section between NNLO QCD and ATLAS data
- Huge K-factors $\sigma^{\text{NNLO}}/\sigma^{\text{NLO}} \approx 1.5$

  → Very similar to Photon pair production

  → but without *gg*-box contribution
- Scale dependence driven by 'LO' contributions

  → $H_T/4$ dynamical scale choice (similar to the scale used in top-pair production)

3 Photons @ LHC: Phenomenology-Summary

$$pp \to \gamma\gamma\gamma$$

- NNLO QCD corrections are vital for comparisons of data with SM
  - Improved normalization
  - Very good agreement of differential cross section between NNLO QCD and ATLAS data
- Huge K-factors $\sigma^{\text{NNLO}}/\sigma^{\text{NLO}} \approx 1.5$

  $\to$ Very similar to Photon pair production

  $\to$ but without gg-box contribution
- Scale dependence driven by 'LO' contributions

  $\to$ $H_T/4$ dynamical scale choice (similar to the scale used in top-pair production)

What is about the other $2 \to 3$ processes?

Main bottle-neck are the two-loop amplitudes!

Five-point amplitudes in the IBP approach

$2 \to 3$ Phenomenology @ NNLO QCD

State-of-art:

$\to$ no general algorithm for numerical evaluation (like cut-based one-loop amps)

$\to$ case-by-case study

- Massless case:
  - Masters known
    - planar: numerical implementation [Gehrmann et al '18]
    - non-planar: differential equations [Chicherin et al'19]
  - Reductions:
    - planar: known analytically [Chawdhry et al '18]
    - non-planar: work in progress ([Guan, Liu, Ma '19]?)
- Massless 5-point amplitudes:
  - $pp \to \gamma\gamma\gamma$ (N_c^3 contribution done)
  - $pp \to \gamma\gamma j$
  - $pp \to \gamma jj$
  - $pp \to jjj$

    (planar, euclidean region only [Abreu,Dormans,Febres Cordero,Ita,Page,Sotnikov '19])
- Massive particles?

  more scales $\Leftrightarrow$ more difficult (a lot of unknowns: Reductions, Masters)

The traditional approach to amplitudes

Feynman diagrams

$\downarrow$

Tensor reduction $\rightarrow$ scalar integrals

$\downarrow$

IBP reduction of scalar integrals to masters

$\downarrow$

Expression of masters in terms of function basis (Polylogarithms, etc.)

$\downarrow$

Evaluable (Semi-) Analytic expression of the matrix element

The traditional approach to amplitudes

Feynman diagrams

$\downarrow$

Tensor reduction $\rightarrow$ scalar integrals

$\downarrow$

IBP reduction of scalar integrals to masters

$\downarrow$

Expression of masters in terms of function basis (Polylogarithms, etc.)

$\downarrow$

Evaluable (Semi-) Analytic expression of the matrix element

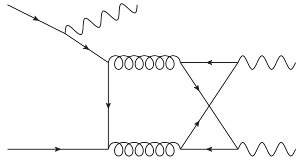
5-point 2-loop: $q\bar{q} \to \gamma\gamma\gamma$ matrix element

- Diagram generation with DiaGen [Czakon, private code] $\to$ 1200 diagrams
- Color and fermion-loop structure of the matrix element:

$$\bar{\sum} 2\mathrm{Re}\left\langle \mathcal{M}^{(0)} \middle| \mathcal{M}^{(2)} \right\rangle = \mathcal{M}^{(\mathrm{lc},1)} C_F^2 C_A + \mathcal{M}^{(\mathrm{lc},2)} C_F C_A^2 + \mathcal{M}^{(\mathrm{f})} C_A C_F + \mathcal{M}^{(\mathrm{np})} (N_c - 1/N_c)$$

- Color decomposition in the leading colour approximation

$$\bar{\sum} 2\mathrm{Re}\left\langle \mathcal{M}^{(0)} \middle| \mathcal{M}^{(2)} \right\rangle_{\mathrm{l.-c.}} \approx N_c^3 \left(\mathcal{M}^{(\mathrm{lc},1)} + \mathcal{M}^{(\mathrm{lc},2)} \right)$$



- neglecting $\mathcal{M}^{(\mathrm{f})}$ contribution ($\sim n_f N_c^2$), contains non-planar contribution

5-point 2-loop: Scalar integrals

- All $k_1 \cdot k_2$ and $k_i \cdot p_j$ expressed through inverse propagators
- 11-propagator integral:

$$
\begin{aligned}
B[\vec{a}] &= \Big\{k_1^2, k_2^2, (k_1+p_1)^2, (k_1+p_1+p_2)^2, \\
&\quad (k_2-p_3)^2, (k_2-k_1-p_3)^2, \\
&\quad (k_2-k_1-p_1-p_2+p_4)^2, (k_2+p_4)^2, \\
&\quad (k_2+p_1+p_2)^2, (k_2+p_1)^2, (k_1+p_3)^2\Big\} \\
C[\vec{a}] &= \Big\{k_1^2, k_2^2, (k_1+p_1+p_2)^2, (k_1-k_2)^2, \\
&\quad (k_2+p_1)^2, (k_2+p_1+p_2)^2, (k_2-p_3)^2, \\
&\quad (k_1+p_1+p_2-p_3)^2, (k_1+p_1+p_2-p_3-p_4)^2, \\
&\quad (k_2-p_3-p_4)^2, (k_1+p_1)^2\Big\}
\end{aligned}
$$

$$\Rightarrow \mathcal{M}^{(C)} = \sum c_i(\{s_{ij}\}, \epsilon) I(\vec{a}_i) \quad I \text{ either B or C}$$

The traditional approach to amplitudes

Feynman diagrams

$\downarrow$

Tensor reduction $\rightarrow$ scalar integrals

$\downarrow$

IBP reduction of scalar integrals to masters

$\downarrow$

Expression of masters in terms of function basis (Polylogarithms, etc.)

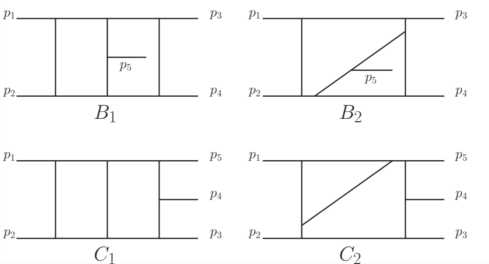
$\downarrow$

Evaluable (Semi-) Analytic expression of the matrix element

Massless 5-point 2-loop: IBP identities and reduction

Topologies for massless 5-point amplitudes

$$B_1 = B\,[1,1,1,1,1,1,1,1,0,0,0]$$
$$B_2 = B\,[1,1,1,1,0,1,1,1,0,0,1]$$
$$C_1 = C\,[1,1,1,1,1,1,0,1,1,0,0]$$
$$C_2 = C\,[1,1,1,1,1,0,0,1,1,0,1]$$
$$C_3 = C\,[1,0,1,1,1,1,0,1,0,0,1]$$



- Reduction of planar topologies up to numerator power -5 available: [Chawdhry,Lim,Mitov '18]
  - Memory and CPU intensive venture
  - 'divide and conquer': solve IBPs for one master at a time $\rightarrow$ easy to parallelize and reduced memory consumption
- Non-planar topologies: work ongoing, but is constraint by available CPU hours, recent developments [Guan, Liu, Ma '19]

The traditional approach to amplitudes

Feynman diagrams

$\downarrow$

Tensor reduction $\rightarrow$ scalar integrals

$\downarrow$

IBP reduction of scalar integrals to masters

$\downarrow$

Expression of masters in terms of function basis (Polylogarithms, etc.)

$\downarrow$

Evaluable (Semi-) Analytic expression of the matrix element

5-point 2-loop: Master Integrals

Master integrals expressed through planar 'pentagon-function'-basis

[Gehrmann,Henn,Presti '18]

- DEQ for masters $\vec{I} = \sum_k \epsilon^k \vec{I}^{(k)}$ in ϵ-form: $\mathrm{d}\vec{I}^{(k+1)}(s_{ij}) = \mathrm{d}\tilde{A}(s_{ij})\vec{I}^{(k)}(s_{ij})$
- $\vec{I}^{(k)}$ can be expressed through iterated integrals
- Independent functions:

Weight	Functions
1	1
2	1
3	4
4	11

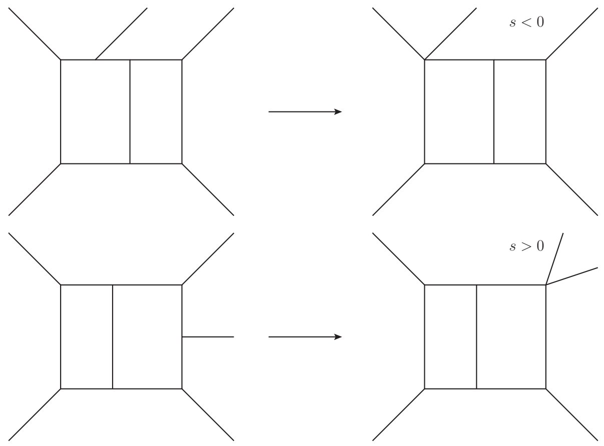
Note: Independent in the sense that $\ln(s_{12}) \leftrightarrow \ln(s_{23})$

- Different boundary constants for different physical regions

5-point 2-loop: Crossings

Large set of functions ($\sim$ 1800 @ Weight 4, including lower weight products) due to momenta permutations

The difference to an euclidean amplitude (particularly in sub topologies):



5-point 2-loop: The amplitude in terms functions

$$\mathcal{M}^{(C)} = \sum_{k=-4}^{0} \sum_{l} c_l^k(\{s_{ij}\}) f_l^k \epsilon^k$$

- Building amplitude together $\rightarrow$ computationally intensive
- Large cancellation between in the rational coefficient $\mathcal{O}(1GB) \rightarrow \mathcal{O}(1MB)$ after simplifying
- Usage of rational reconstruction software FiniteFlow [Peraro '19] to sum up coefficients $c_l^k(\{s_{ij}\})$ (but under usage of the analytical reduction result)
- Cancellation of UV and IR poles checked analytically

5-point 2-loop: Evaluation

- Rational c++ implementation of coefficients with the help of CLN library to avoid loss numerical precision
- Usage of 'pentagon-function' implementation by [Gehrmann,Henn,Presti '18]
- 10 to 50 minutes per phase space point, 30k points evaluated (unweighted Born PS points)
- Coefficients relatively fast (~ 1 min), the functions take most of the time (numerical integration)
- Stability checks by changing integration precision
- Additional checks with interpolation software **GPTree** [Kasabov '19] to detect numerical instabilities

5-point 2-loop: Beyond $pp \to \gamma\gamma\gamma | N_c^3$

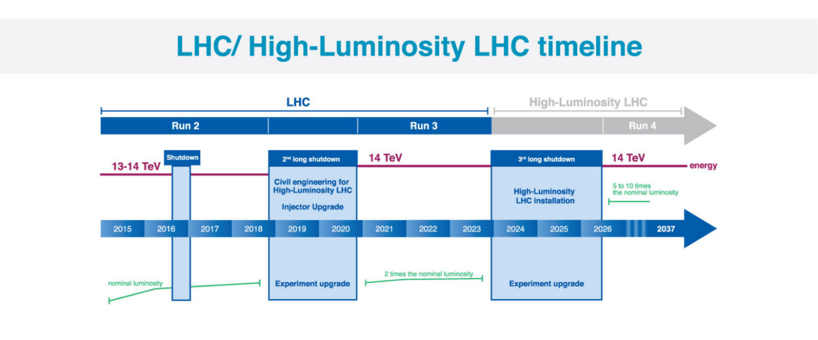
- Same technology applicable to the non-planar contributions (as soon as reductions are available, Master representation?)
- The other 5-point amplitudes:
    - $pp \to \gamma\gamma j$
    - $pp \to \gamma jj$
    - $pp \to jjj$

  are feasible as well (at least in the leading colour limit)

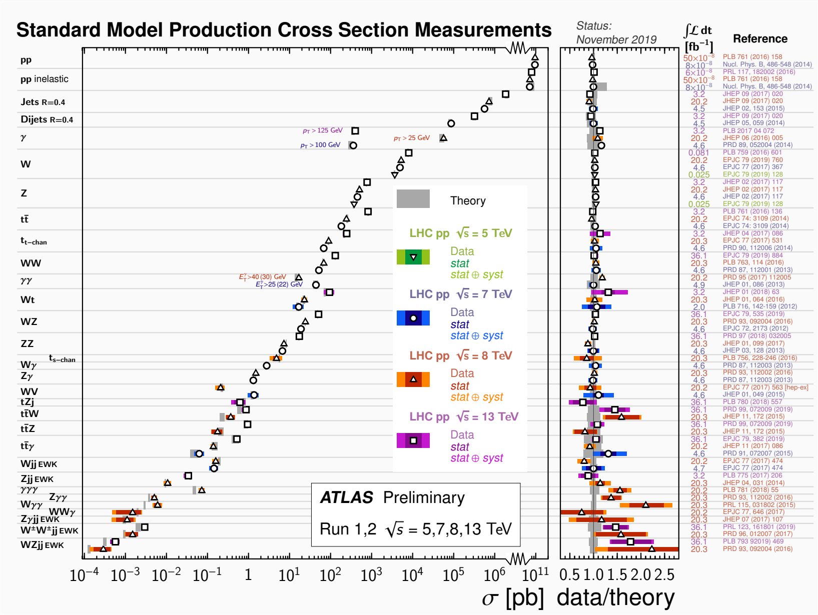
The Future

- LHC-HL ⇒ more and more statistics which allow for:
  - Unprecedented precision in SM measurements
  - More exclusive selections
  - Rare processes measured qualitatively and quantitatively
- Precision predictions for multi-leg processes become more and more important!

Perturbative QCD is the backbone of theory predictions at the LHC



Precision at the LHC



Conclusions and Outlook

$2 \to 3$ Phenomenology

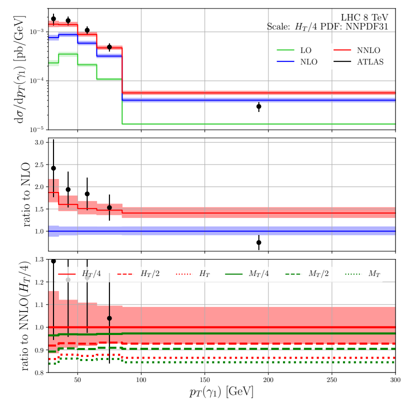
- First process: $pp \to \gamma\gamma\gamma$
- Shows importance of NNLO QCD beyond $2 \to 2$
- Will certainly become more and more important with more experimental statistics

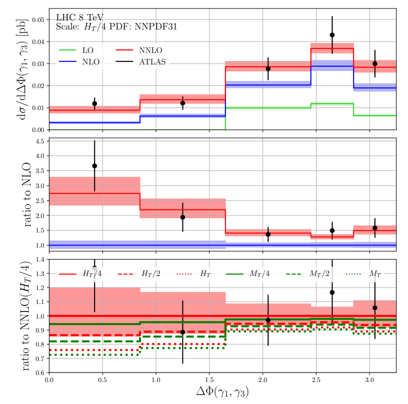
Advances for 5-point amplitudes:

- Application of IBP reductions for $pp \to \gamma\gamma\gamma$
- Finite remainder constructed and ready for use
- Certainly not the end of the story, many more amplitudes feasible with same techniques (5 partons, 4 partons + photon, 3 partons + 2 photons)

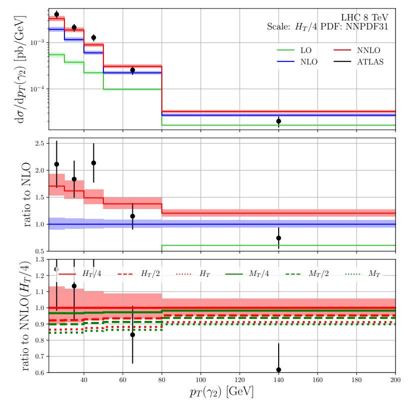
Backup

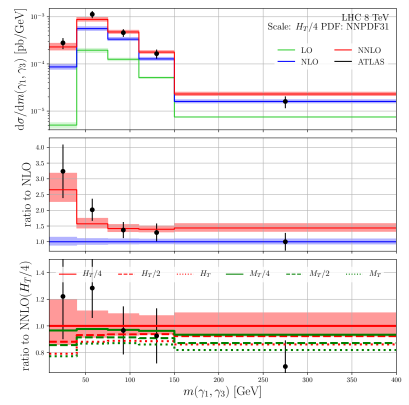
- Not only normalization → significant effects on the shape
- Remarkable agreement of measurement with NNLO QCD





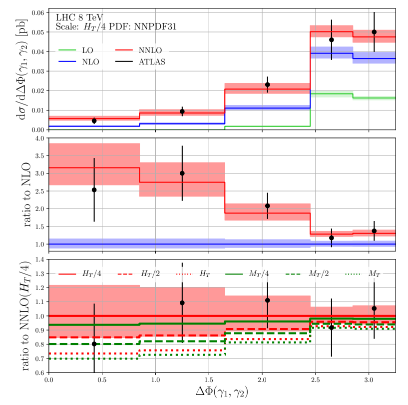
- Not only normalization $\rightarrow$ significant effects on the shape
- Remarkable agreement of measurement with NNLO QCD

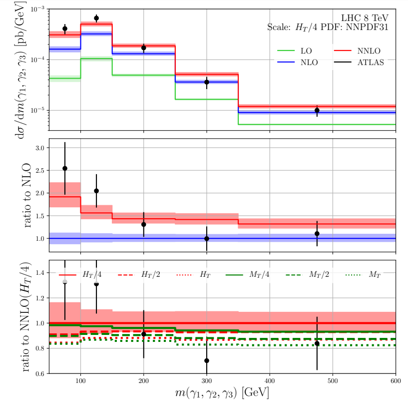




3 Photons @ LHC: Differential distributions

- Not only normalization → significant effects on the shape
- Remarkable agreement of measurement with NNLO QCD





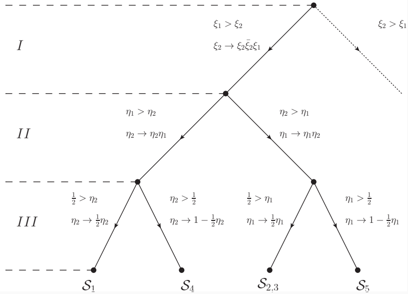
STRIPPER: Minimal sector-improved residue subtraction

Refined formulation of the sector-improved residue subtraction

[Czakon '10 '11][Czakon,Heymes '14][Czakon,van Hameren,Mitov,Poncelet '19]

- New phase space parametrization:
  - Starts from Born kinematics → additional radiation accommodated by rescaling and boosts
  - Generates minimal set of subtraction kinematics in each sector
  - Only one (!) double unresolved kinematic (= Born kinematic)
- Minimal set of sectors
- New 4-dimensional formulation:
  - New method to determine necessary counter terms
  - Numerical pole cancellation for each Born phase space point

Sector decomposition:



STRIPPER: Single-inclusive jet cross sections

First complete NNLO QCD calculation for inclusive jet production

[Czakon,van Hameren,Mitov,Poncelet]

Many publications and studies by the NNLOJET collaboration:

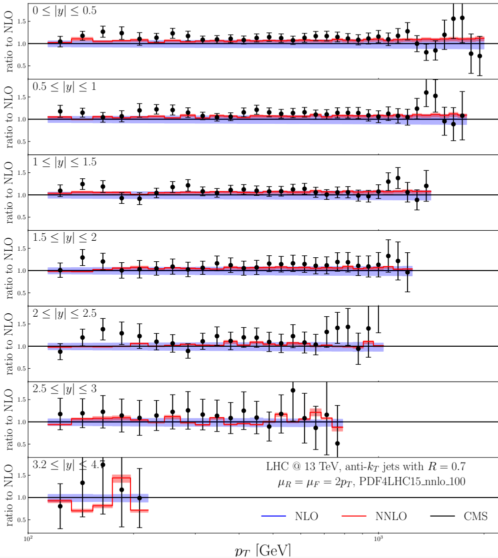
[Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires '16-19]

- Antenna subtraction formalism
- Leading colour approximation for channels with quarks (expected to be a good approximation)
- Extensive analysis of renormalization scale setting and dependence:
  - Cancellation between different n-jet samples!
  - Distinguish 'jet'- and 'event'-type scales:
  - Inclusive jet observables: $\mu = p_T$ for each jet
- Very good description of LHC data for various observables: inclusive jets, various di-jet observables.

Technically very challenging process.
Contains the full set of NNLO IR singularities!

STRIPPER: Single-inclusive jet cross sections

- First full NNLO QCD calculation at 13 TeV
- Quite slow convergence: 350k CPU hours → optimization potential!
- Comparison to NNLOJET: sub-leading colour effects within MC errors, thus indeed small
- K-factors public



STRIPPER: Single-inclusive jet cross sections

- First full NNLO QCD calculation at 13 TeV
- Quite slow convergence: 350k CPU hours → optimization potential!
- Comparison to NNLOJET: sub-leading colour effects within MC errors, thus indeed small
- K-factors public