NNLO QCD calculations with the Sector-improved residue subtraction scheme

XXVI Cracow EPIPHANY Conference

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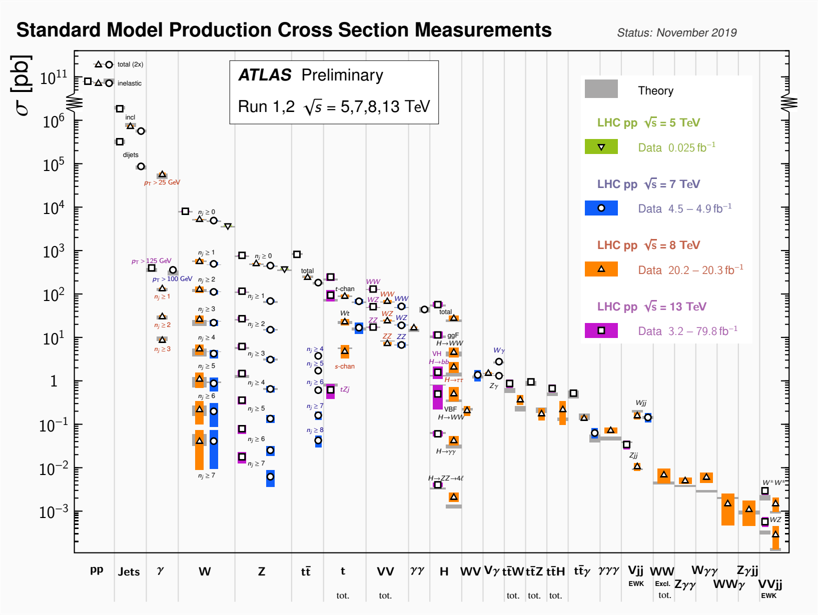
Cavendish Laboratory

Precision at the LHC



NNLO QCD

Tremendous progress in NNLO QCD calculation in the past decade

State-of-the-art:

- All (Standard Model) $2 \to 2$ processes calculated
- Phenomenology: SM precision measurements and parameter estimation, PDF determination, ...
- $\to$Valuable input for the LHC physics program!

Not quite comparable to the 'NLO revolution' yet, lack of automated

1. Real radiation contributions $\to$ subtraction schemes
2. Two-loop matrix elements

Subtractions schemes

Handling real radiation contribution in NNLO calculations
cancellation of infrared divergences

increasing number of available NNLO calculations with a variety of schemes

- qT-slicing [Catani,Grazzini, '07] ,[Ferrera,Grazzini,Tramontano, '11], [Catani,Cieri,DeFlorian,Ferrera,Grazzini,'12], [Gehrmann,Grazzini,Kallweit,Maierhofer,Manteuffel,Rathlev,Torre,'14-15'], [Bonciani,Catani,Grazzini,Sargsyan,Torre,'14-'15], [Grazzini "MATRIX" '17-'19]
- N-jettiness slicing [Gaunt,Stahlhofen,Tackmann,Walsh, '15], [Boughezal,Focke,Giele,Liu,Petriello,'15-'16] , [Bougezal,Campell,Ellis,Focke,Giele,Liu,Petriello,'15], [Campell,Ellis,Williams,'16]
- Antenna subtraction [Gehrmann, GehrmannDeRidder,Glover,Heinrich,'05-'08] , [Weinzierl,'08,'09], [Currie,Gehrmann,GehrmannDeRidder,Glover,Pires,'13-'17], [Bernreuther,Bogner,Dekkers,'11,'14], [Abelof,(Dekkers),GehrmannDeRidder,'11-'15], [Abelof,GehrmannDeRidder,Maierhofer,Pozzorini,'14], [Chen,Gehrmann,Glover,Jaquier,'15]
- Colorful subtraction [DelDuca,Somogyi,Troscanyi,'05-'13], [DelDuca,Duhr,Somogyi,Tramontano,Troscanyi,'15]
- Sector-improved residue subtraction (STRIPPER) [Czakon,'10,'11] , [Czakon,Fiedler,Mitov,'13,'15], [Czakon,Heymes,'14] [Czakon,Fiedler,Heymes,Mitov,'16,'17], [Bughezal,Caola,Melnikov,Petriello,Schulze,'13,'14], [Bughezal,Melnikov,Petriello,'11], [Caola,Czernecki,Liang,Melnikov,Szafron,'14], [Bruchseifer,Caola,Melnikov,'13-'14], [Caola, Melnikov, Röntsch,'17-'19]
- Projection-to-Born [Cacciari et al '15], [Dreyer,Karlberg '18], Geometric [Herzog '18], Unsubtraction [Aguilera-Verdugo et al '19], . . .

Sector-improved residue subtraction

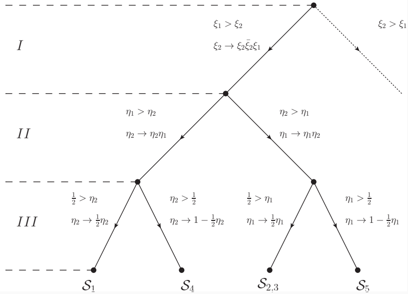
STRIPPER framework: Advances and Application

Refined formulation of the sector-improved residue subtraction

[Czakon '10 '11][Czakon,Heymes '14][Czakon,van Hameren,Mitov,Poncelet '19]

- New phase space parameterization:
  - Starts from Born kinematics → additional radiation accommodated by rescaling and boosts
  - Generates minimal set of subtraction kinematics in each sector
  - Only one (!) double unresolved kinematic (= Born kinematic)
- Minimal set of sectors
- New 4-dimensional formulation:
  - New method to determine necessary counter terms
  - Numerical pole cancellation for each Born phase space point

Sector decomposition:



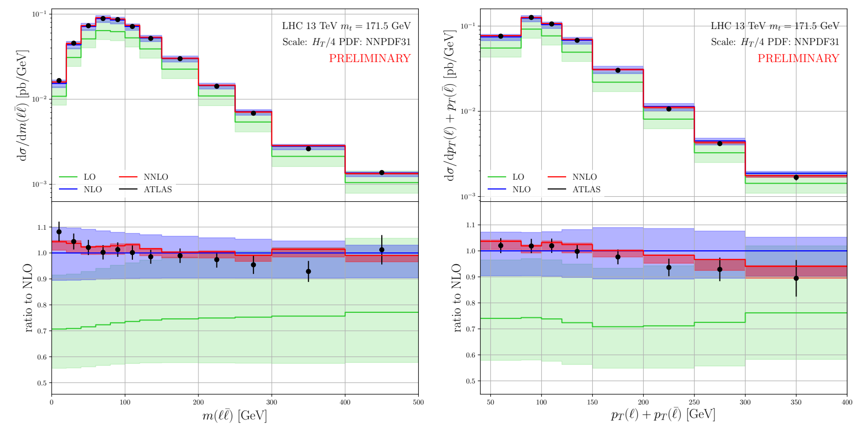
STRIPPER: Top-quark pair production and NWA decay

First NNLO QCD calculation of top-quark production including decays

- Narrow-Width-Approximation: Combination of NNLO QCD in $t\bar{t}$ production and decay
- Phenomenological application: Top-quark pair spin correlations [Behring,Czakon,Mitov,Papanastasiou,Poncelet '19]
  Background: ATLAS observed large deviations from NLO predictions [arXiv:1903.07570 ATLAS '19]
  $\rightarrow$ Subtle discussion of fiducial phase space and b-jet modelling
- Goal in the future: Indirect top-quark mass measurements
  $\rightarrow$ New leptonic differential measurements in fiducial phase space [ATLAS, arXiv:1910.08819], extrapolation of b-jet phase space

STRIPPER: Top-quark pair production and NWA decay

- Nice description of data
- top-quark mass dependence? Caveat: model dependence of b-jet phase space extrapolation ← needs to be understood



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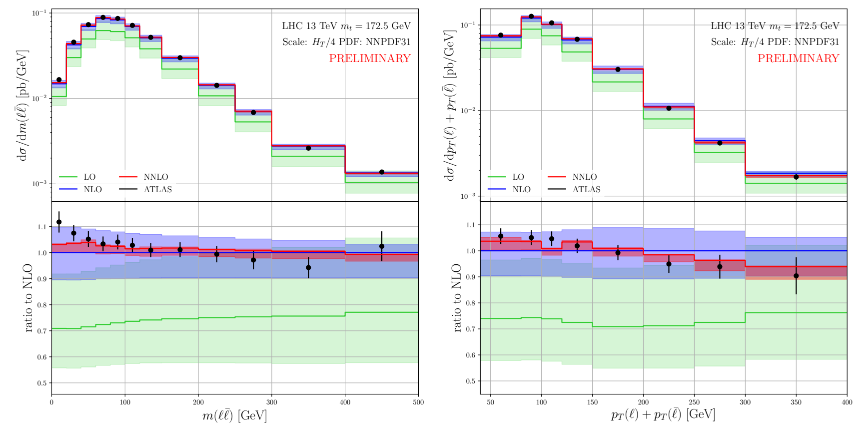


STRIPPER: Single-inclusive jet cross sections

First complete NNLO QCD calculation for inclusive jet production

[Czakon,van Hameren,Mitov,Poncelet]

Many publications and studies by the NNLOJET collaboration:

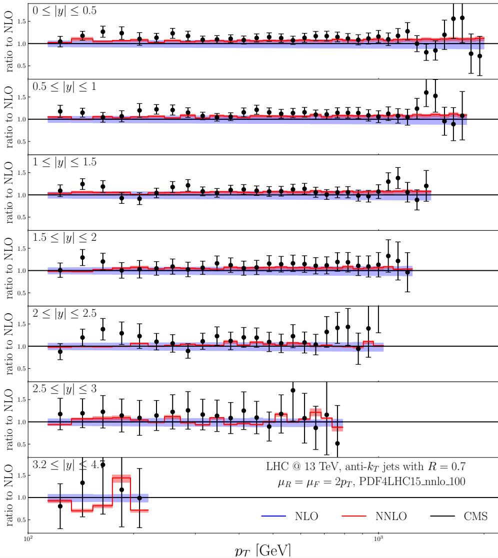
[Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires '16-19]

- Antenna subtraction formalism
- Leading color approximation for channels with quarks (expected to be a good approximation)
- Extensive analysis of renormalization scale setting and dependence:
  - Cancellation between different n-jet samples!
  - Distinguish 'jet'- and 'event'-type scales:
  - Inclusive jet observables: $\mu = p_T$ for each jet
- Very good description of LHC data for various observables: inclusive jets, various di-jet observables.

Technically very challenging process.
Contains the full set of NNLO IR singularities!

STRIPPER: Single-inclusive jet cross sections

- First full NNLO QCD calculation at 13 TeV
- Quite slow convergence: 350k CPU hours → optimization potential!
- Comparison to NNLOJET: sub-leading color effects within MC errors, thus indeed small
- K-factors public



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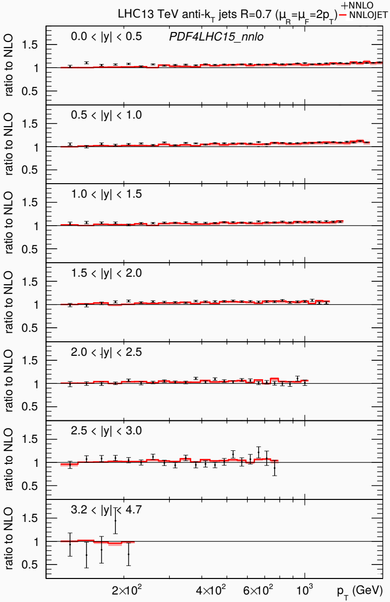


STRIPPER: Subtraction beyond $2 \to 2$

- Jet-production: full set of subtraction terms in action
- Fully automated generation of subtraction terms
- Straight-forward user interface:
  - Generation of required contributions
  - Combination of equivalent contributions $\to$ minimize computational setup
  - Easy extensible interfaces to OpenLoops(2) [Buccioni et al. '19] and Recola [Denner et al. '16-17]
- First $2 \to 3$ calculation: $pp \to \gamma\gamma\gamma$ [Chawdhry, Czakon, Mitov, Poncelet '19]

The framework gets ready for the future

Multileg twoloop matrix elements

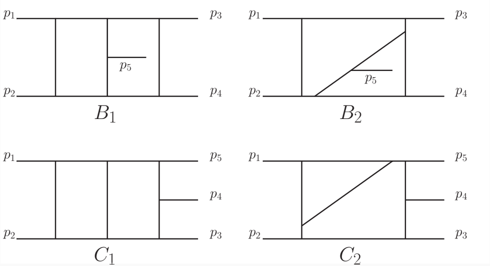
Five-point amplitudes in the IBP approach

First application: $pp \to \gamma\gamma\gamma$ at NNLO QCD [Chawdhry, Czakon, Mitov, Poncelet '19]

5-point 2-loop: IBP identities and reduction

Topologies for massless 5-point amplitudes

- 2 non-/3 planar topologies
- 113 Masters in B1
  75 Masters in B2
  61 Masters in C1
  28 Masters in C2



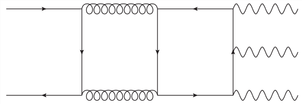
- Reduction of planar topologies up to numerator power -5 available: [Chawdhry,Lim,Mitov '18]
  - Memory and CPU intensive venture
  - 'divide and conquer': solve IBPs for one master at a time → easy to parallelize and reduced memory consumption
- Non-planar topologies: work ongoing, but is constraint by available CPU hours, recent developments [Guan, Liu, Ma '19]

5-point 2-loop: First application: $q\bar{q} \to \gamma\gamma\gamma$

- Diagram generation with DiaGen [Czakon, private code] $\to \sim 1000$ diagrams
- Decomposition of matrix element:

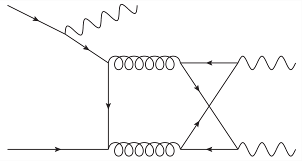
$$\bar{\sum} 2\mathrm{Re}\left\langle \mathcal{M}^{(0)} \middle| \mathcal{M}^{(2)} \right\rangle = \mathcal{M}^{(\mathrm{lc},1)} C_F^2 C_A + \mathcal{M}^{(\mathrm{lc},2)} C_F C_A^2 + \mathcal{M}^{(\mathrm{f})} C_A C_F + \mathcal{M}^{(\mathrm{np})} (N_c - 1/N_c)$$

- Interesting: vanishing contribution from diagrams of type:



- Color decomposition in the leading color approximation

$$\bar{\sum} 2\mathrm{Re}\left\langle \mathcal{M}^{(0)} \middle| \mathcal{M}^{(2)} \right\rangle_{\text{l.-c.}} \approx N_c^3 \left(\mathcal{M}^{(\mathrm{lc},1)} + \mathcal{M}^{(\mathrm{lc},2)} \right)$$



- neglecting $\mathcal{M}^{(\mathrm{f})}$ contribution, mixing with non-planar contribution

5-point 2-loop: First application: $q\bar{q} \to \gamma\gamma\gamma$

- Master integrals expressed through planar 'pentagon-function'-basis [Gehrmann,Henn,Presti '18]
- Quite large set of functions due to numerous momenta permutations
- Computationally most intensive part: insertion of IBPs and Masters and simplification of the rational coefficients!
- Usage of rational reconstruction software FiniteFlow [Peraro '19] to sum up coefficients
- Cancellation of UV and IR poles checked analytically
- Rational c++ implementation of coefficients
- Usage of 'pentagon-function' implementation by [Gehrmann,Henn,Presti '18]
- 10 to 50 minutes per phase space point, 30k points evaluated (unweighted Born PS points)
- Additional checks with interpolation software **GPTree** to detect numerical instabilities

$2 \to 3$ NNLO QCD phenomenology

[Chawdhry, Czakon, Mitov, Poncelet '19]

- First $2 \to 3$ application $pp \to \gamma\gamma\gamma$
- Detailed differential measurements by ATLAS available on HepData [1712.07291 ATLAS]
- Clear discrepancies between NLO QCD and data

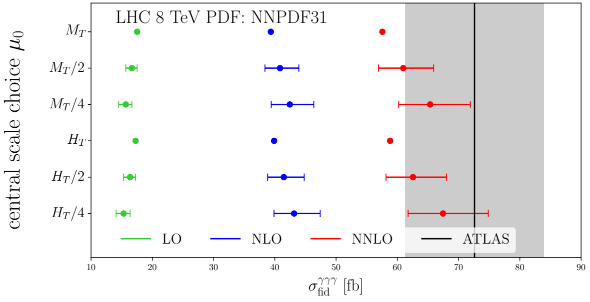
Setup:

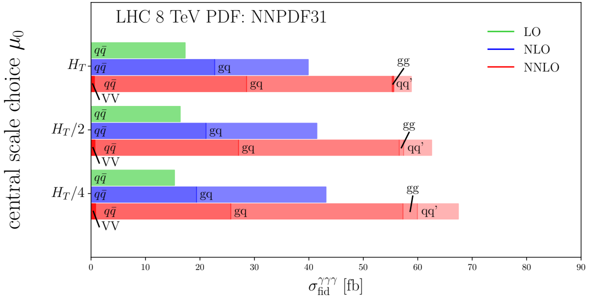
- E_T $(= p_T)$ cut for the three photons: $E_{T,\gamma_1} > 27$ GeV, $E_{T,\gamma_2} > 22$ GeV, $E_{T,\gamma_3} > 15$ GeV
- Rapidity: All photons have $|\eta_\gamma| < 2.37$ (+exclusion of $1.37 < |\eta_\gamma| < 1.56$)
- Separation of photons: The angular distance between each two photons ΔR is required to be > 0.45
- Invariant mass: $m_{\gamma\gamma\gamma} > 50$ GeV
- Photon isolation: Using the Frixione [Frixione '98] isolation as indicated for the MadGraph@NLO setup. This means $R_0 = 0.4$, $E_T^{iso} > 10$ GeV and $\chi(R) = (1 - \cos(\Delta R))/(1 - \cos(\Delta R_0))$.
- PDF set: *NNPDF31_nnlo_as_0118*
- Scales:

$$\mu_0 = m_{\perp,\gamma\gamma\gamma} = \sqrt{p_\gamma^2 + (p_{\gamma,T})^2} \quad \text{with} \quad p_\gamma = \sum_{i=1}^{3} p_{\gamma_i} \ ,$$

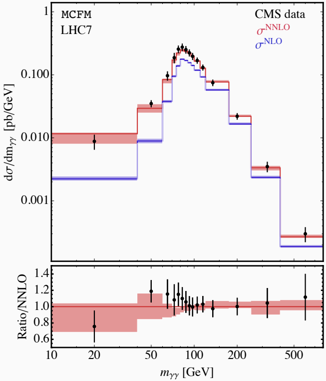
$$\mu_0 = H_T/4 = \frac{1}{4} \sum p_{\gamma_i,T}$$

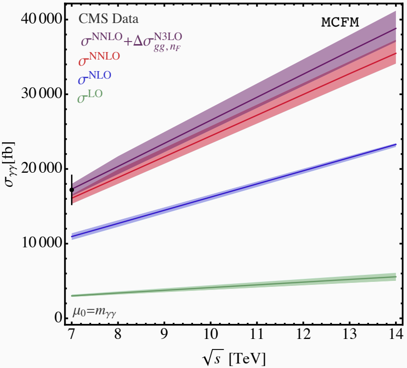
$2 \to 3$ Phenomenology: Fiducial cross section



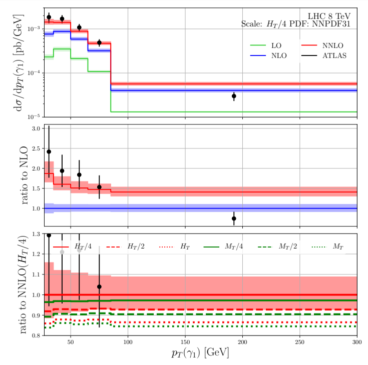


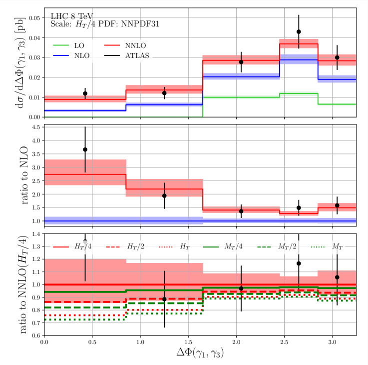
- Similar large K-factors in di-photon production
  [Catani, Cieri, de Florian, Ferrera, Grazzini 11] [Campbell, Ellis , Li, Williams 16]
- Difference: $gg \to \gamma\gamma\gamma$ contribution does vanish





- Not only normalization → significant effects on the shape
- Remarkable agreement of measurement with NNLO QCD





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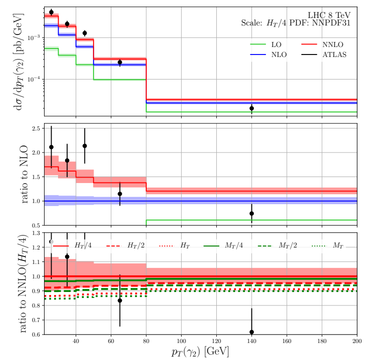

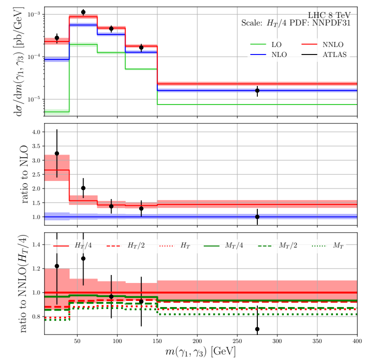

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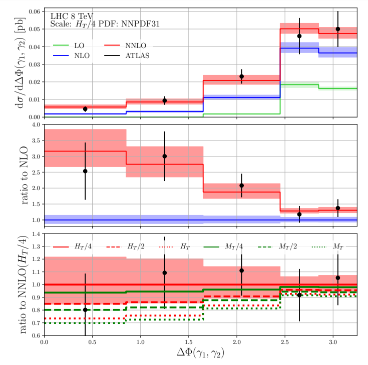

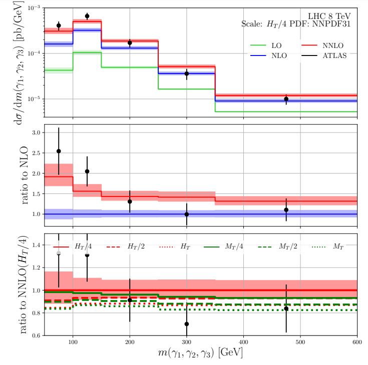

Conclusions and Outlook

STRIPPER: More applications!

- Top-quark plus decay at NNLO QCD $\rightarrow$ spin-correlations
  future: top-quark mass measurements from leptonic distributions
- First complete computation of inclusive jet production
- First $2 \rightarrow 3$ process: $pp \rightarrow \gamma\gamma\gamma$

Advances for 5-point amplitudes:

- Application of IBP reductions for $pp \rightarrow \gamma\gamma\gamma$
- Finite remainder constructed and ready for use
- Certainly not the end of the story, many more amplitudes feasible with same techniques (5 partons, 4 partons + photon, 3 partons + 2 photons)

Backup

STRIPPER: Top-quark pair spin correlation

First NNLO QCD calculation of top-quark production including decays

[Behring,Czakon,Mitov,Papanastasiou,Poncelet '19]

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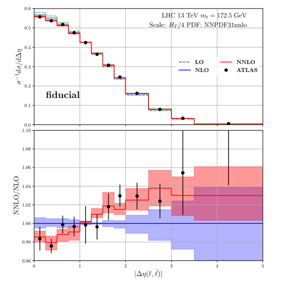
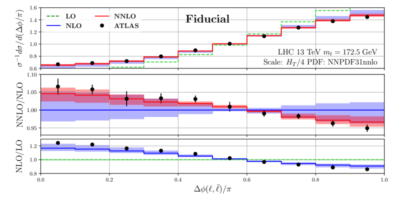
Predictions for fiducial phase space region:





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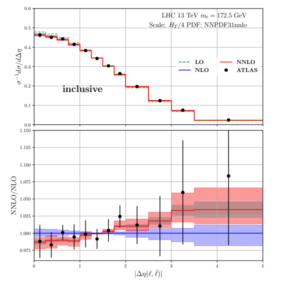
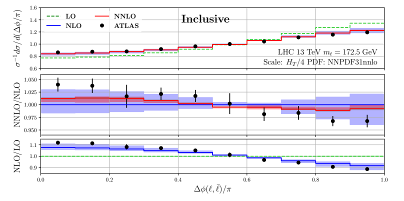
Predictions for inclusive phase space region:





STRIPPER: Top-quark pair production and NWA decay

A lesson in perturbative calculations: Normalized distribution $\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}X}$

- Perturbative expansion:

$$\sigma = \sigma^0 + \alpha_S \sigma^1 + \alpha_S^2 \sigma^2 + \dots$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \frac{\mathrm{d}\sigma^0}{\mathrm{d}X} + \alpha_S \frac{\mathrm{d}\sigma^1}{\mathrm{d}X} + \alpha_S^2 \frac{\mathrm{d}\sigma^2}{\mathrm{d}X} + \dots$$

- Normalized distribution at NNLO:

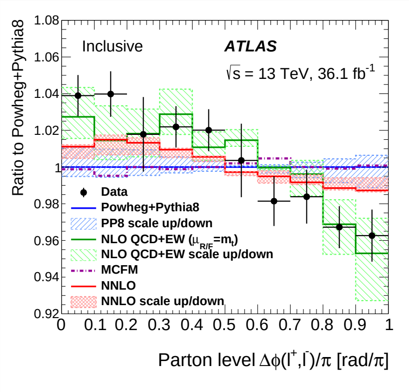
$$R = \frac{1}{\sigma^0 + \alpha_S \sigma^1 + \alpha_S^2 \sigma^2} \left(\frac{\mathrm{d}\sigma^0}{\mathrm{d}X} + \alpha_S \frac{\mathrm{d}\sigma^1}{\mathrm{d}X} + \alpha_S^2 \frac{\mathrm{d}\sigma^2}{\mathrm{d}X} \right) + \mathcal{O}\left(\alpha_S^3\right)$$

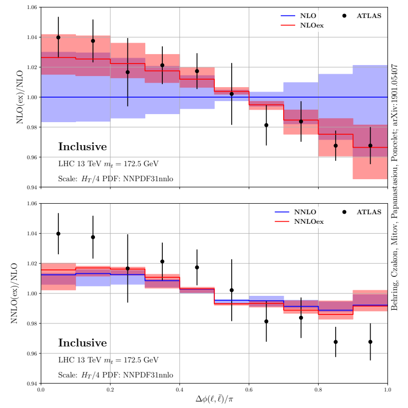
- Expanded ratio:

$$R^{\mathrm{NNLO,exp}} = R^0 + \alpha_S R^1 + \alpha_S^2 R^2 \; ,$$
$$R^0 = \frac{1}{\sigma^0} \frac{\mathrm{d}\sigma^0}{\mathrm{d}X} \; ,$$
$$R^1 = \frac{1}{\sigma^0} \frac{\mathrm{d}\sigma^1}{\mathrm{d}X} - \frac{\sigma^1}{\sigma^0} \frac{1}{\sigma^0} \frac{\mathrm{d}\sigma^0}{\mathrm{d}X} \; ,$$
$$R^2 = \frac{1}{\sigma^0} \frac{\mathrm{d}\sigma^2}{\mathrm{d}X} - \frac{\sigma^1}{\sigma^0} \frac{1}{\sigma^0} \frac{\mathrm{d}\sigma^1}{\mathrm{d}X} + \left(\left(\frac{\sigma^1}{\sigma^0} \right)^2 - \frac{\sigma^2}{\sigma^0} \right) \frac{1}{\sigma^0} \frac{\mathrm{d}\sigma^0}{\mathrm{d}X}$$

STRIPPER: Top-quark pair production and NWA decay

A lesson in perturbative calculations: Normalized distribution $\frac{1}{\sigma}\frac{d\sigma}{dX}$





- Not an EW-effect (which is actually small)
- Everything consistent within scale dependence (7-point variation)
- NNLO QCD resolves this expansion 'ambiguity'