Fixed-order predictions for top-quark pair production and decay at the LHC

Cavendish/DAMTP seminar

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16th May 2019

Cavendish Laboratory

Outline

NNLO QCD ingredients and technologies

- Introduction / Generalities (recap)
- STRIPPER - IR subtraction framework (probably unfamiliar)

$\Downarrow$ Phenomenological application

Top-quark pair production and decay @ LHC

- Stable top-quark production (review)
- Production $\otimes$ decay (pheno)
    - Leptonic differential distributions
    - Spin-correlation observables

Fixed-order: Beyond the Born approximation

Kinematical features

Experimental precision

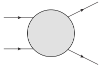
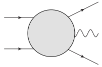
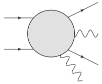
WHY SHOULD WE CARE ABOUT NNLO QCD?

Perturbative convergence

New channels

Fixed-order: Collider observables in QCD

- any process at colliders is specified by final states, and cuts on these final states
- parton-hadron duality is used, but partons being massless can be emitted at will
- it is necessary to sum (incoherently) over processes with a different number of final partons

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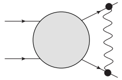
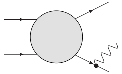
- exchange or emission of partons lead to divergences



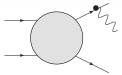
virtual - UV/IR

virtual momentum arbitrarily large/small



real - IR soft

gluon energy arbitrarily small

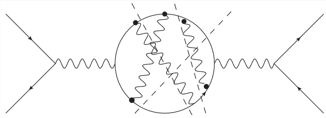


real - IR collinear

angle between partons arbitrarily small

Fixed-order: Kinoshita-Lee-Nauenberg theorem

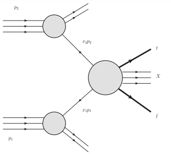
- the theorem states that for "suitably averaged" transition probabilities (cross sections), the result is finite
- particular case is given by electron-positron annihilation



- after cuts: the different contributions are divergent, but the self energy itself is finite, and the total cross-section is just its imaginary part
- the averaging is obtained by integrating the cross section with a "jet function" F_n dependent on the momenta of the partons (or mesons and hadrons)
- F_n is required to be "infrared safe", i.e. the value for a soft or collinear degenerate configuration of $n+1$ is the same as the value for the equivalent n partons

Fixed-order: Collinear factorization

- unfortunately, in hadronic collisions, the initial states are not properly averaged
- instead a factorization theorem is used, e.g. for top quark pair production:



$$\sigma_{h_1 h_2 \to t\bar{t}}(s, m_t^2) = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2$$

$$\phi_{i,h_1}(x_1, \mu_F^2)\phi_{j/h_2}(x_2, \mu_F^2)\hat{\sigma}_{ij}(\hat{s}, m_t^2, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

- the divergences of the initial state collinear radiation are absorbed into the (universal) parton distribution functions
- the general formula is

$$\left[\sigma_{ij}(x)/x\right] = \sum [\hat{\sigma}_{kl}(z)/z] \otimes \Gamma_{ki} \otimes \Gamma_{lj} \qquad [f_1 \otimes f_2](x) = \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 f_1(x_1) f_2(x_2) \delta(x - x_1 x_2)$$

$$\Gamma_{ij} = \delta_{ij}\delta(1-x) - \frac{\alpha_s}{\pi}\frac{P_{ij}^{(0)}(x)}{\epsilon} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{1}{2\epsilon^2}\left(\left(P_{ik}^{(0)} \otimes P_{kj}^{(0)}\right)(x) + \beta_0 P_{ij}^{(0)}(x)\right) - \frac{1}{2\epsilon} P_{ij}^{(1)}(x)\right] + \mathcal{O}\left(\alpha_s^3\right)$$

- Consistency of the construction requires a consistent dimensional regularization

- add to the original cross section $\sigma = \sigma^{LO} + \sigma^{NLO}$

$$\sigma^{LO} = \int_m \mathrm{d}\sigma^B \quad , \qquad \sigma^{NLO} \equiv \int \mathrm{d}\sigma^{NLO} = \int_{m+1} \mathrm{d}\sigma^R + \int_m \mathrm{d}\sigma^V$$

an identity involving approximations to the real radiation cross section

$$\sigma^{NLO} = \int_{m+1} \left[\mathrm{d}\sigma^R - \mathrm{d}\sigma^A\right] + \int_{m+1} \mathrm{d}\sigma^A + \int_m \mathrm{d}\sigma^V$$

and regroup the terms as

$$\sigma^{NLO} = \int_{m+1} \left[\left(\mathrm{d}\sigma^R\right)_{\epsilon=0} - \left(\mathrm{d}\sigma^A\right)_{\epsilon=0}\right] + \int_m \left[\mathrm{d}\sigma^V + \int_1 \mathrm{d}\sigma^A\right]_{\epsilon=0}$$

- for $\mathrm{d}\sigma^A$ it must be possible to
  1. obtain the Laurent expansion by integration over the single particle unresolved space (preferably analytically)
  2. approximate $\mathrm{d}\sigma^R$ (preferably pointwise)

Fixed-order: Subtraction at NLO (and beyond?)

NLO Subtraction Schemes

- Dipole Subt. [Catani,Seymour'98]
- FKS [Frixione,Kunst,Signer'95]
- Antenna Subtraction [Kosower'97]
- Nagy-Soper [Nagy,Soper'07]

Generalization to NNLO?

- Nature of singularities known: soft and collinear limits
- NLO (fairly simple):
  - single soft
  - single collinear
- At NNLO? : Many (overlapping) ways to reach soft and collinear limits
- Possible way: Decomposition of the phase space to disentangle them

Fixed-order: NNLO subtraction schemes

Handling real radiation contribution in NNLO calculations
cancellation of infrared divergences

increasing number of available NNLO calculations with a variety of schemes

- qT-slicing [Catani,Grazzini, '07] ,[Ferrera,Grazzini,Tramontano, '11], [Catani,Cieri,DeFlorian,Ferrera,Grazzini,'12], [Gehrmann,Grazzini,Kallweit,Maierhofer,Manteuffel,Rathlev,Torre,'14-15'], [Bonciani,Catani,Grazzini,Sargsyan,Torre,'14-'15]
- N-jettiness slicing [Gaunt,Stahlhofen,Tackmann,Walsh, '15], [Boughezal,Focke,Giele,Liu,Petriello,'15-'16] , [Bougezal,Campell,Ellis,Focke,Giele,Liu,Petriello,'15], [Campell,Ellis,Williams,'16]
- Antenna subtraction [Gehrmann, GehrmannDeRidder,Glover,Heinrich,'05-'08] , [Weinzierl,'08,'09], [Currie,Gehrmann,GehrmannDeRidder,Glover,Pires,'13-'17], [Bernreuther,Bogner,Dekkers,'11,'14], [Abelof,(Dekkers),GehrmannDeRidder,'11-'15], [Abelof,GehrmannDeRidder,Maierhofer,Pozzorini,'14], [Chen,Gehrmann,Glover,Jaquier,'15]
- Colorful subtraction [DelDuca,Somogyi,Troscanyi,'05-'13], [DelDuca,Duhr,Somogyi,Tramontano,Troscanyi,'15]
- Sector-improved residue subtraction (STRIPPER) [Czakon,'10,'11] , [Czakon,Fiedler,Mitov,'13,'15], [Czakon,Heymes,'14] [Czakon,Fiedler,Heymes,Mitov,'16,'17], [Bughezal,Caola,Melnikov,Petriello,Schulze,'13,'14], [Bughezal,Melnikov,Petriello,'11], [Caola,Czernecki,Liang,Melnikov,Szafron,'14], [Bruchseifer,Caola,Melnikov,'13-'14], [Caola, Melnikov, Röntsch,'17]
- ...

Sector-improved residue subtraction

STRIPPER: Basics

Hadronic cross section:

$$\sigma_{h_1 h_2}(P_1, P_2) = \sum \iint_0^1 \mathrm{d}x_1\, \mathrm{d}x_2\, f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

partonic cross section:

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \hat{\sigma}_{ab}^{(1)} + \hat{\sigma}_{ab}^{(2)} + \mathcal{O}\left(\alpha_S^3\right)$$

Contributions with different final state multiplicities and convolutions:

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\mathrm{RR}} + \hat{\sigma}_{ab}^{\mathrm{RV}} + \hat{\sigma}_{ab}^{\mathrm{VV}} + \hat{\sigma}_{ab}^{\mathrm{C2}} + \hat{\sigma}_{ab}^{\mathrm{C1}}$$

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = \text{(single convolution)}\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = \text{(double convolution)}\, \mathrm{F}_{n}$$

Several layers of decomposition

Selector functions:

$$1 = \sum_{i,j} \left[\sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right]$$

Factorization of double soft limits:

$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

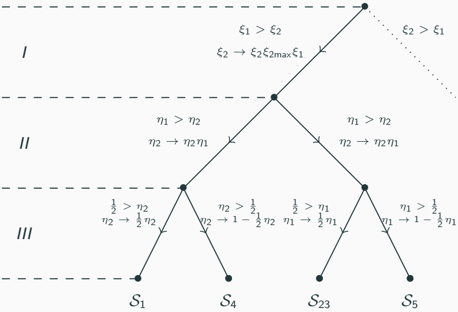
Sector parameterization
Parameterization with respect to the reference parton r:

angles: $\hat{\eta}_i = \frac{1}{2}(1 - \cos\theta_{ir}) \in [0, 1]$

energies: $\hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0, 1]$

4 sub-sectors

Triple collinear factorization



STRIPPER: Compilation

$$\hat{\sigma}^{\text{RR}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}^{(0)}_{n+2} \middle| \mathcal{M}^{(0)}_{n+2} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}^{\text{RV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\text{Re} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(1)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\text{VV}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\text{Re} \left\langle \mathcal{M}^{(0)}_{n} \middle| \mathcal{M}^{(2)}_{n} \right\rangle + \left\langle \mathcal{M}^{(1)}_{n} \middle| \mathcal{M}^{(1)}_{n} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}^{\text{C1}}_{ab} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}^{\text{C2}}_{ab} = (\text{double convolution})\, \mathrm{F}_{n}$$

Sector decomposition and master formula:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\Downarrow$$

$$\left(\sigma^{RR}_{F}, \sigma^{RR}_{SU}, \sigma^{RR}_{DU}\right) \left(\sigma^{RV}_{F}, \sigma^{RV}_{SU}, \sigma^{RV}_{DU}\right) \left(\sigma^{VV}_{F}, \sigma^{VV}_{DU}, \sigma^{VV}_{FR}\right) \left(\sigma^{C1}_{SU}, \sigma^{C1}_{DU}\right) \left(\sigma^{C2}_{DU}, \sigma^{C2}_{FR}\right)$$

$\Downarrow$ 4 dimensional formulation [Czakon,Heymes'14]

$$\left(\sigma^{RR}_{F}\right) \left(\sigma^{RV}_{F}\right) \left(\sigma^{VV}_{F}\right) \left(\sigma^{RR}_{SU}, \sigma^{RV}_{SU}, \sigma^{C1}_{SU}\right) \left(\sigma^{RR}_{DU}, \sigma^{RV}_{DU}, \sigma^{VV}_{DU}, \sigma^{C1}_{DU}, \sigma^{C2}_{DU}\right) \left(\sigma^{RV}_{FR}, \sigma^{VV}_{FR}, \sigma^{C2}_{FR}\right)$$

STRIPPER: Developments

- new phase space parameterization:
  - reduced mis-binning
  - new 4-dimensional formulation
  - ϵ-pole cancelation PS point wise (strong check of construction/implementation)
- C++ framework:
  - automatic generation of necessary contributions/subtraction-terms
  - only one- and two-loop amplitutes as external input (OpenLoops/Recola - interfaces)
  - flexible 'measurement'-system (fastNLO - tables, smearing techniques, efficient scale and PDF variation)

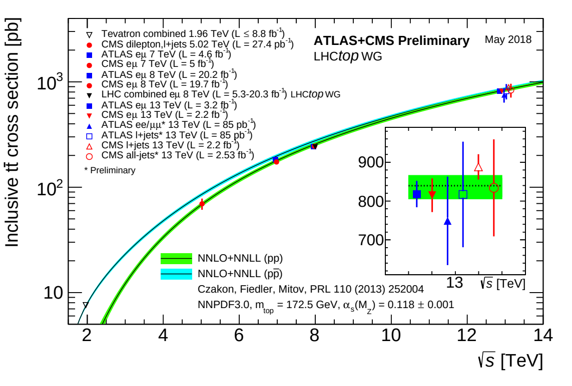
Top quark pair production and decay

Top-quark production at the LHC

- Heaviest known particle $\rightarrow$ special place in the SM
- Abundantly produced at the LHC
  $\rightarrow$ top-quark factory
  $\rightarrow$ high quality and precision data
- Many opportunities to study QCD/SM in high precision
- Many connections to other fields: Higgs, BSM, EW precision
- High perturbative accuracy needed to describe and squeeze out most of the data available $\leftarrow$ this talk

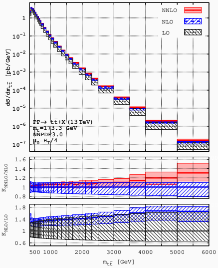
State-of-the-art: Total cross section for $t\bar{t}$ production

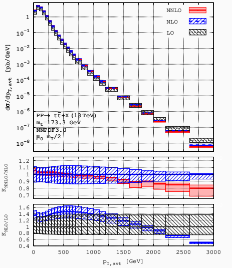
- NNLO QCD + NNLL soft gluon resummation
- Uncertainties of a few percent
- Remarkable agreement with measurements at 7, 8 and 13 TeV



arxiv:1303.6254 [Czakon,Fiedler,Mitov '13]

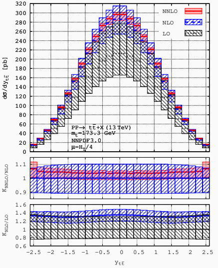
State-of-the-art: Differential $t\bar{t}$ cross sections @ NNLO QCD

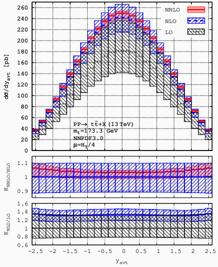




NNLO QCD

- Modification of shape for p_T and $m_{t\bar{t}}$
- Reduction of scale dependence
- choice of dynamical scale is crucial
  $\rightarrow$ extensive study of perturbative convergence

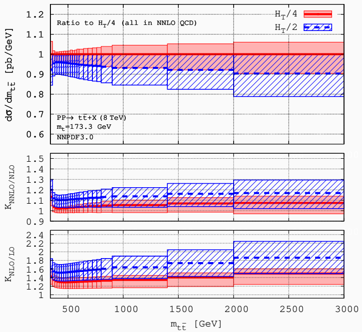


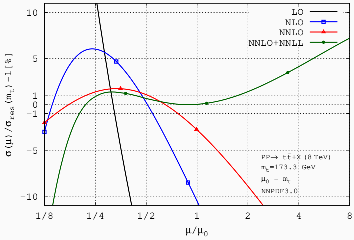


arxiv:1606.03350 [Czakon,Heymes,Mitov '16]

- Renormalization/Factorization scale dependence → major source of theory uncertainty
- What is a sensible scale choice? → possible metric: *principle of fasted convergence*
- Total cross section: $\mu = m_t/2$
- Differential cross sections? Probing a vast energy regime ⇒ dynamical scales
- $H_T/4$ established for most observables (except $m_T/2$ for $p_{T,t}$ distributions)

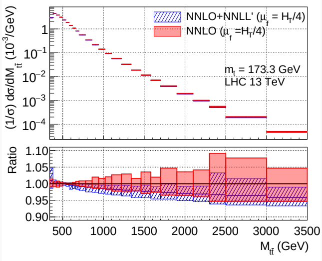
arxiv:1606.03350 [Czakon,Heymes,Mitov '16]

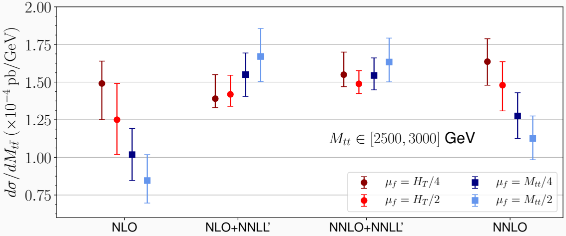




State-of-the-art: Resummation for differential observables

- Advances in resummation for differential observables
- Threshold (low p_T) and small-mass (high p_T - 'boosted tops') logarithms
- Stabilizes results w.r.t. scale choice form
- Results support $H_T/4$ as the 'best' scale since $H_T/4$ seems to capture most of of the resummation features





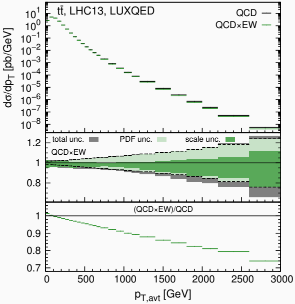
arxiv:1803.07623 [Czakon,Ferroglia,Heymes,Mitov,Pecjak,Scott,Wang,Yang '18]

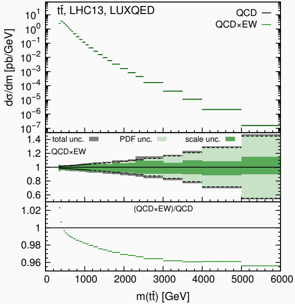
State-of-the-art: NLO-EW corrections

- Studied in additive and multiplicative approach
- Observed strong PDF dependence
- Size of corrections are observable dependent:
  $p_{T,avt}$: up to -25% at high p_T (Sudakov logarithms),
  $>$ NNLO QCD scale dependence for $p_{T,avt} > 500$ GeV
  $y_t, y_{t\bar{t}}$: small effect ($<$ NNLO QCD scale dependence)
- multiplicative approach results in smaller scale dependence

Combination with NNLL' resummation $\rightarrow$ most complete SM description available

arxiv:1705.04105 [Czakon, Heymes, Mitov,Pagani,Tsinikos,Zaro '17]





Production and decay

Production and decay: Towards realistic top-quark states

Elephant in the room:

Top-quarks are not stable and are measured utilising the decay products

- Decay products are measured in fiducial phase space $\rightarrow$ all previous results rely on the extrapolation of the phase space
- The phase space extrapolation relies heavily on MC modeling of the top-quark production and its decay
- The modeling might have more or less subtle impacts on results derived in the extrapolated phase space

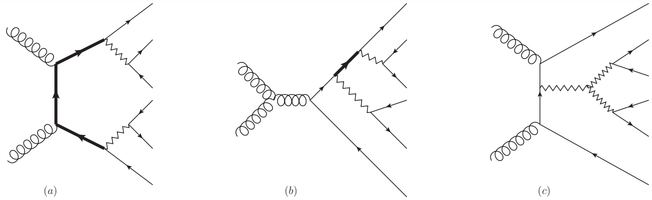
Production and decay: Towards realistic top-quark states

Narrow-Width-Approximation

- Considering limit $\Gamma_t \to 0$
- Factorization of production and decay
- Reduction of complexity by keeping crucial features of decay like spin-correlations
- Expected error of $\mathcal{O}(\Gamma_t/m_t)$

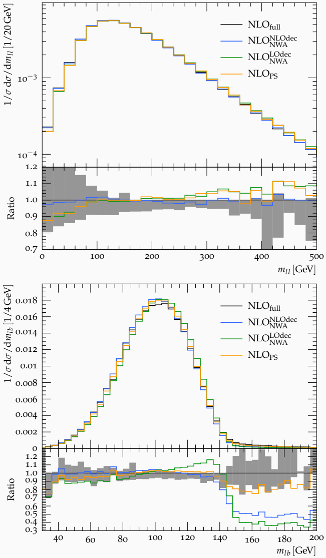
Off-shell calculations

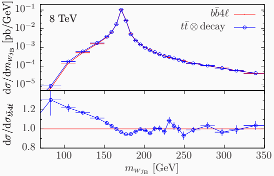
- Considering the complete process: $pp \to \ell^+\ell^-\nu\bar{\nu}b\bar{b} + X$
- Technically challenging due to high multiplicity, difficult phase space
- Off-shell and non-resonant effects important in certain phase space region

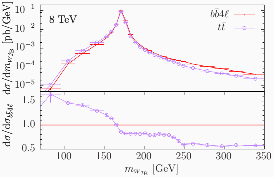


Production and decay: NLO for off-shell $t\bar{t}$

- NLO corrections to full $pp \to \ell^+\ell^-\nu\bar{\nu}b\bar{b} + X$ [5FS: Bevilacqua et al, Denner et al, Heinrich et al, 4FS: Frederix, Cascioli et al]
- Off-shell & non-resonant effects depend strongly on observable
- $\to$ NWA approximation valid for many observables
- Higher order corrections to decay are important!
- Kinematical thresholds and edges are sensitive to off-shell effects $\Rightarrow$ NWA does not give a valid description







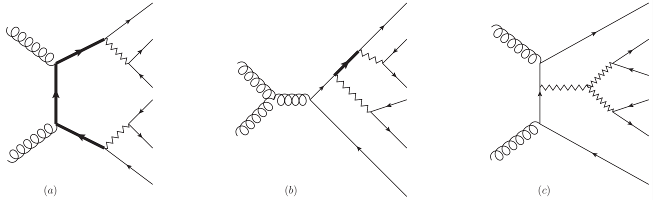
- Matching fixed order calculation to PS
- Technical subtlety: resonance-aware matching. Implementation in POWHEG framework
- Detailed comparison of:
  - "$t\bar{t}$": NWA, NLO production only (industry standard)
  - "$t\bar{t}\otimes$ decay": NWA, NLO production & decay , approximate LO finite width effects
    [Campbell,Ellis,Nason,Re '14]
  - "$b\bar{b}4\ell$": full off-shell
    [Jezo,Nason '15] [Jezo,Lindert,Nason,Oleari,Pozzorini '16]
- Upshot:
  - "$t\bar{t}\otimes$ decay" closer to "$b\bar{b}4\ell$" than "$t\bar{t}$" (in terms of shape and normalization)
  - NLO corrections to decay are crucial for NWA to be reliable to work

Production and decay: Narrow-Width-Approximation

- top-quark have a short life time, decay before hadronization. $\Gamma_t \ll m_t$
- $$\frac{1}{(p^2 - m_t^2)^2 + m_t^2\Gamma_t^2} \xrightarrow{\Gamma_t/m_t \to 0} \frac{\pi\delta(p^2 - m_t^2)}{m_t\Gamma_t}$$
- for on-shell top-quarks:

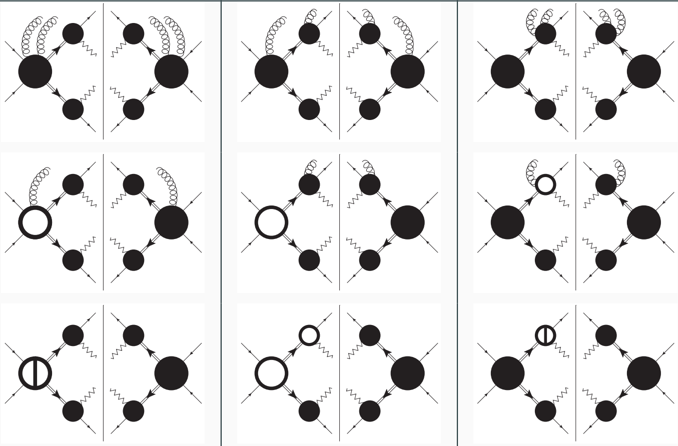
$$\not{p} + m_t = \sum_\lambda u_\lambda(p)\bar{u}_\lambda(p)$$

- factorization of production and decay
- polarized matrix elements required



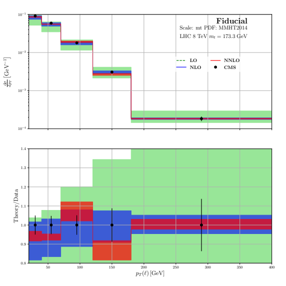
Production and decay: Production and decay in NWA @ NNLO QCD

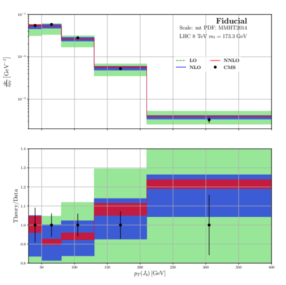
NNLO x LO	NLO x NLO	LO x NNLO



- NNLO QCD correction to NWA with leptonic decays now available
- Extension of the STRIPPER framework used for differential $t\bar{t}$
- Predictions for inclusive and fiducial phase spaces
- Many applications in work: leptonic distributions, top-quark (differential) cross sections in fiducial phase space, top-quark mass extraction
- Study of top-quark spin-correlation

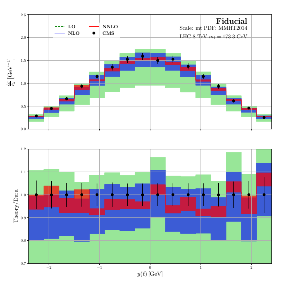
- comparision between CMS data [CMS,1505.04480] and NWA @ NNLO QCD
- good discribtion of many distributions:
    - transverse momentum of ℓ and b-jets

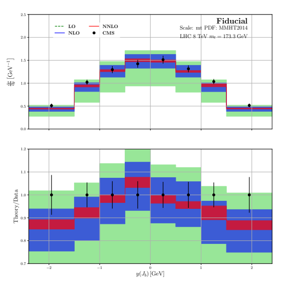




Production and decay: fiducial cross sections at 8 TeV

- comparision between CMS data [CMS,1505.04480] and NWA @ NNLO QCD
- good discribtion of many distributions:
    - transverse momentum of ℓ and b-jets
    - rapidities of ℓ and b-jets





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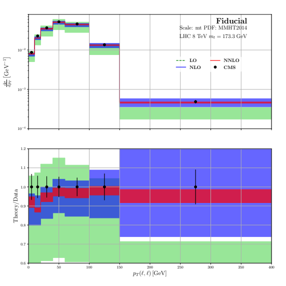
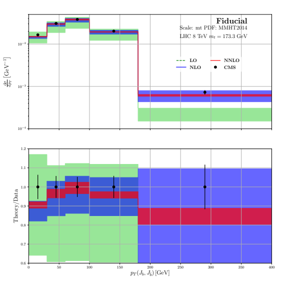
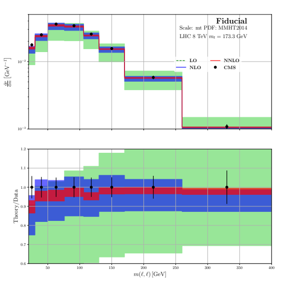
  - transverse momentum of ℓ and b-jets
  - rapidities of ℓ and b-jets
  - transverse momentum of lepton and b-jet pairs

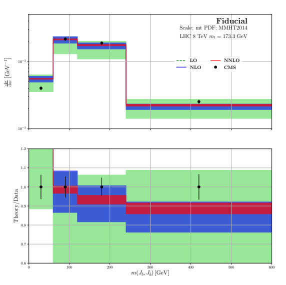




Production and decay: fiducial cross sections at 8 TeV

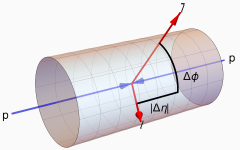
- comparision between CMS data [CMS,1505.04480] and NWA @ NNLO QCD
- good discribtion of many distributions:
  - transverse momentum of ℓ and b-jets
  - rapidities of ℓ and b-jets
  - transverse momentum of lepton and b-jet pairs
  - invariant masses of lepton and b-jet pairs

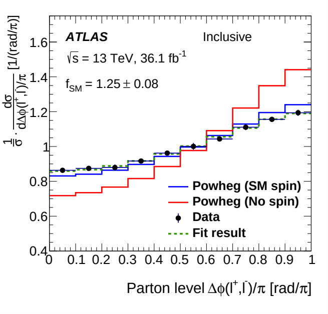




Production and decay: Spin-correlation @ NNLO QCD

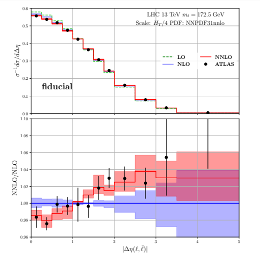
- Direct measurement of top-quark spin density matrix [CMS,PAS TOP-18-006]
  - full spin information
  - systematic difficulties (neutrinos $\to$ top-momenta)
- Leptonic observables are sensitive to $t\bar{t}$ spin-correlations. For example the opening angles of the leptons: $\Delta\Phi_{\ell\ell}$ and $|\Delta\eta_{\ell\ell}|$
- Boosted top favor antiparallel leptons
- Spin correlation counter acts
- Effect of higher corrections?



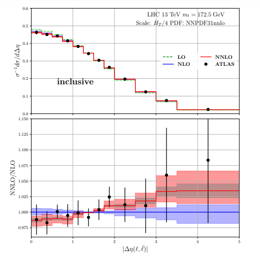


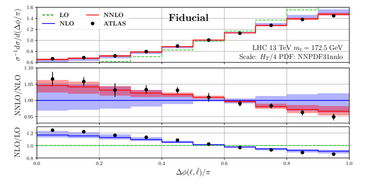
Production and decay: Spin-correlation @ NNLO QCD

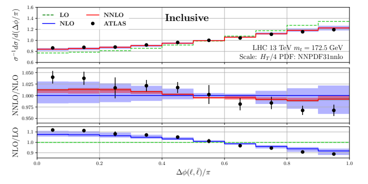
Fiducial phase space



Inclusive phase space





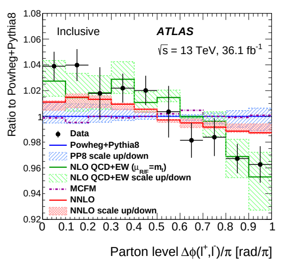


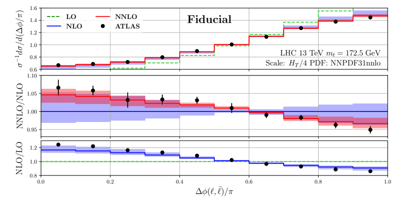
arxiv:1901.05407 [Behring,Czakon,Mitov,Papanastasiou,P '19]

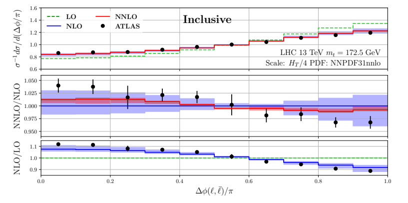
Extrapolation effects?

Production and decay: Spin-correlation @ NNLO QCD

- → extrapolation effect?
- Published results: [arXiv:1903.07570 ATLAS '19] (discrepancy resolved by EW effects?)







arxiv:1901.05407 [Behring,Czakon,Mitov,Papanastasiou,P '19]

Summary & Outlook

Summary

- Top-quark production at the LHC is theoretically very well understood and under control and allows for precision test and parameter extraction of the SM
- Refined calculation (through resummation and/or NLO EW) allow to improve theoretical stability and understanding
- Precision calculations for more realistic final states including the top-quarks decay.
- NNLO QCD predictions including leptonic top-quark decays. Production cross sections and differential distributions in fiducial volumes.

Outlook

- Using precision predictions to get out as much as possible of LHC data
- SM model precision test and parameter estimations: m_t, α_S, PDFs,...
- Incoming NNLO QCD predictions including leptonic decays: differential distributions of decay products $\rightarrow$ overcome penalties of extrapolation