Improvements of the sector-improved residue subtraction scheme

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Introduction

Sector-decomposition

New phase space construction

4 dimensional formulation

C++ implementation of STRIPPER

Summary

Predictions from higher order perturbation theory

Ultimate Goal: describe measurements for high energy collisions

- Model $\rightarrow$ QFT
- predictions $\rightarrow$ perturbation theory
- (simplified) idea: higher orders $\rightarrow$ better predictions
- higher order introduce UV and IR divergences
  - need for regularization (dimensional regularization, mass,...) and renormalization (introduction of additional scale μ)
  - methods of handling IR divergences

$\Rightarrow$ increasing complexity of calculations

The Les Houches wishlist

List of processes of phenomenological interest

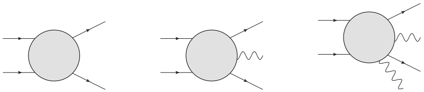
process	NLO	NNLO	N^3LO
$pp \to H$	$\surd$	$(\surd)_{HEFT}$	$(\surd)_{HEFT}$
$pp \to H + j$	$\surd$	$\surd$	
$pp \to H + 2j$	$\surd$	$(\surd)_{VBF}$	
$pp \to H + 3j$	$\surd$		
$pp \to V$	$\surd$	$\surd$	!
$pp \to V + j$	$\surd$	$\surd$	
$pp \to V + 2j$	$\surd$	!	
$pp \to t\bar{t}$	$\surd$	$\surd$	
$pp \to t\bar{t} + j$	$\surd$	!	
$pp \to 2j$	$\surd$	$\surd$	
$pp \to 3j$	$\surd$	!	
...			

current bottlenecks:

- Two-loop $\geq$ 5-point functions
- Handling of real radiation contributions

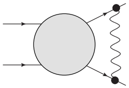
Collider observables in QCD

- any process at colliders is specified by final states, and cuts on these final states
- parton-hadron duality is used, but partons being massless can be emitted at will
- it is necessary to sum (incoherently) over processes with a different number of final partons



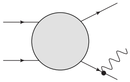
...

- exchange or emission of partons lead to divergences



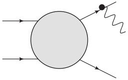
virtual - UV/IR

virtual momentum arbitrarily large/small



real - IR soft

gluon energy arbitrarily small

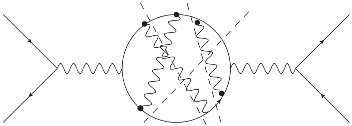


real - IR collinear

angle between partons arbitrarily small

Kinoshita-Lee-Nauenberg theorem

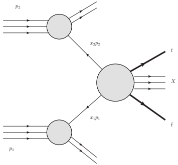
- the theorem states that for "suitably averaged" transition probabilities (cross sections), the result is finite
- particular case is given by electron-positron annihilation



- after cuts: the different contributions are divergent, but the self energy itself is finite, and the total cross-section is just its imaginary part
- the averaging is obtained by integrating the cross section with a "jet function" F_n dependent on the momenta of the partons (or mesons and hadrons)
- F_n is required to be "infrared safe", i.e. the value for a soft or collinear degenerate configuration of $n+1$ is the same as the value for the equivalent n partons

Factorization

- unfortunately, in hadronic collisions, the initial states are not properly averaged
- instead a factorization theorem is used, e.g. for top quark pair production:



$$\sigma_{h_1 h_2 \to t\bar{t}}(s, m_t^2) = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2$$

$$\phi_{i,h_1}(x_1, \mu_F^2)\phi_{j/h_2}(x_2, \mu_F^2)\hat{\sigma}_{ij}(\hat{s}, m_t^2, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

- the divergences of the initial state collinear radiation are absorbed into the (universal) parton distribution functions
- the general formula is

$$\left[\sigma_{ij}(x)/x\right] = \sum [\hat{\sigma}_{kl}(z)/z] \otimes \Gamma_{ki} \otimes \Gamma_{lj} \qquad \left[f_1 \otimes f_2\right](x) = \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 f_1(x_1) f_2(x_2) \delta(x - x_1 x_2)$$

$$\Gamma_{ij} = \delta_{ij}\delta(1-x) - \frac{\alpha_s}{\pi}\frac{P_{ij}^{(0)}(x)}{\epsilon} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{1}{2\epsilon^2}\left(\left(P_{ik}^{(0)} \otimes P_{kj}^{(0)}\right)(x) + \beta_0 P_{ij}^{(0)}(x)\right) - \frac{1}{2\epsilon} P_{ij}^{(1)}(x)\right] + \mathcal{O}\left(\alpha_s^3\right)$$

- Consistency of the construction requires a consistent dimensional regularization

The general idea of subtraction

- add to the original cross section $\sigma = \sigma^{LO} + \sigma^{NLO}$

$$\sigma^{LO} = \int_m \mathrm{d}\sigma^B \quad , \qquad \sigma^{NLO} \equiv \int \mathrm{d}\sigma^{NLO} = \int_{m+1} \mathrm{d}\sigma^R + \int_m \mathrm{d}\sigma^V$$

  an identity involving approximations to the real radiation cross section

$$\sigma^{NLO} = \int_{m+1} \left[\mathrm{d}\sigma^R - \mathrm{d}\sigma^A\right] + \int_{m+1} \mathrm{d}\sigma^A + \int_m \mathrm{d}\sigma^V$$

  and regroup the terms as

$$\sigma^{NLO} = \int_{m+1} \left[\left(\mathrm{d}\sigma^R\right)_{\epsilon=0} - \left(\mathrm{d}\sigma^A\right)_{\epsilon=0}\right] + \int_m \left[\mathrm{d}\sigma^V + \int_1 \mathrm{d}\sigma^A\right]_{\epsilon=0}$$

- for $\mathrm{d}\sigma^A$ it must be possible to
  1. obtain the Laurent expansion by integration over the single particle unresolved space (preferably analytically)
  2. approximate $\mathrm{d}\sigma^R$ (preferably pointwise)

Subtraction at NLO (and beyond?)

NLO Subtraction Schemes

- Dipole Subt. [Catani,Seymour'98]
- FKS [Frixione,Kunst,Signer'95]
- Antenna Subtraction [Kosower'97]
- Nagy-Soper [Nagy,Soper'07]

Generalization to NNLO?

- Nature of singularities known: soft and collinear limits
- NLO (fairly simple):
  - single soft
  - single collinear
- At NNLO? : Many (overlapping) ways to reach soft and collinear limits
- Possible way: Decomposition of the phase space to disentangle them

NNLO subtraction schemes

Handling real radiation contribution in NNLO calculations
cancellation of infrared divergences

increasing number of available NNLO calculations with a variety of schemes

- qT-slicing [Catani,Grazzini, '07] ,[Ferrera,Grazzini,Tramontano, '11], [Catani,Cieri,DeFlorian,Ferrera,Grazzini,'12], [Gehrmann,Grazzini,Kallweit,Maierhofer,Manteuffel,Rathlev,Torre,'14-15'], [Bonciani,Catani,Grazzini,Sargsyan,Torre,'14-'15]
- N-jettiness slicing [Gaunt,Stahlhofen,Tackmann,Walsh, '15], [Boughezal,Focke,Giele,Liu,Petriello,'15-'16] , [Bougezal,Campell,Ellis,Focke,Giele,Liu,Petriello,'15], [Campell,Ellis,Williams,'16]
- Antenna subtraction [Gehrmann, GehrmannDeRidder,Glover,Heinrich,'05-'08] , [Weinzierl,'08,'09], [Currie,Gehrmann,GehrmannDeRidder,Glover,Pires,'13-'17], [Bernreuther,Bogner,Dekkers,'11,'14], [Abelof,(Dekkers),GehrmannDeRidder,'11-'15], [Abelof,GehrmannDeRidder,Maierhofer,Pozzorini,'14], [Chen,Gehrmann,Glover,Jaquier,'15]
- Colorful subtraction [DelDuca,Somogyi,Troscanyi,'05-'13], [DelDuca,Duhr,Somogyi,Tramontano,Troscanyi,'15]
- Sector-improved residue subtraction (STRIPPER) [Czakon,'10,'11] , [Czakon,Fiedler,Mitov,'13,'15], [Czakon,Heymes,'14] [Czakon,Fiedler,Heymes,Mitov,'16,'17], [Bughezal,Caola,Melnikov,Petriello,Schulze,'13,'14], [Bughezal,Melnikov,Petriello,'11], [Caola,Czernecki,Liang,Melnikov,Szafron,'14], [Bruchseifer,Caola,Melnikov,'13-'14], [Caola, Melnikov, Röntsch,'17]

Formulation

Hadronic cross section:

$$\sigma_{h_1 h_2}(P_1, P_2) = \sum \iint_0^1 \mathrm{d}x_1\, \mathrm{d}x_2\, f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

partonic cross section:

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \hat{\sigma}_{ab}^{(1)} + \hat{\sigma}_{ab}^{(2)} + \mathcal{O}\left(\alpha_S^3\right)$$

Contributions with different final state multiplicities and convolutions:

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\mathrm{RR}} + \hat{\sigma}_{ab}^{\mathrm{RV}} + \hat{\sigma}_{ab}^{\mathrm{VV}} + \hat{\sigma}_{ab}^{\mathrm{C2}} + \hat{\sigma}_{ab}^{\mathrm{C1}}$$

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_{n}$$

Sector decomposition

Several layers of decomposition

Selector functions:

$$1 = \sum_{i,j} \left[\sum_{k} \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right]$$

Factorization of double soft limits:

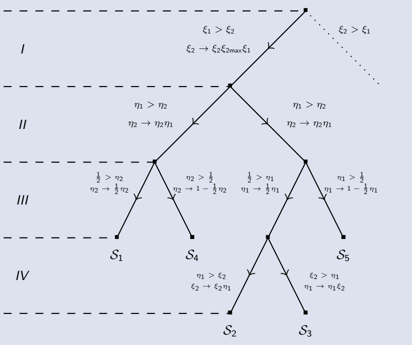
$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

Sector parameterization

Parameterization with respect to the reference parton r:
angles: $\hat{\eta}_i = \frac{1}{2}(1 - \cos\theta_{ir}) \in [0,1]$
energies: $\hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$

originally: 5 sub-sectors

Triple collinear factorization



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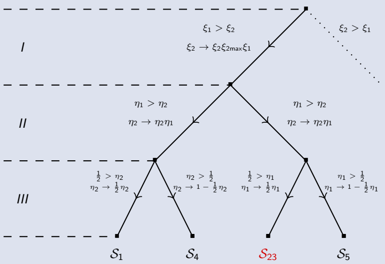
now: 4 sub-sectors

Triple collinear factorization



Caola, Melnikov, Röntsch [hep-ph:1702.01352v1]

STRIPPER

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_{n}$$

Sector decomposition and master formula:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_{+}}_{\text{reg. + sub.}}$$

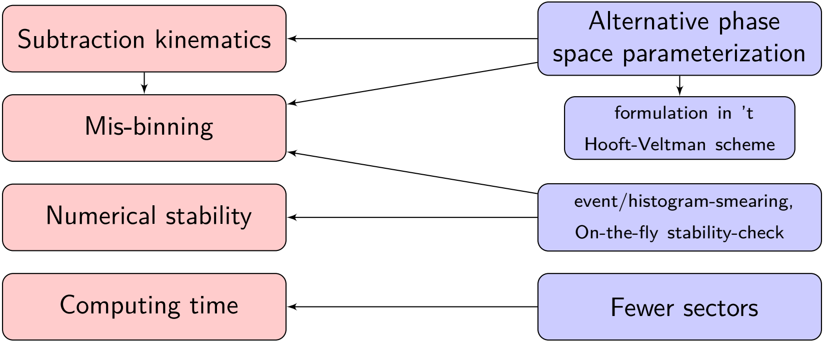
$$\Downarrow$$

$$\left(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}\right) \quad \left(\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}\right) \quad \left(\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}\right) \quad \left(\sigma_{SU}^{C1}, \sigma_{DU}^{C1}\right) \quad \left(\sigma_{DU}^{C2}, \sigma_{FR}^{C2}\right)$$

$\Downarrow$ 4 dim formulation [Czakon,Heymes'14]

$$\left(\sigma_F^{RR}\right) \quad \left(\sigma_F^{RV}\right) \quad \left(\sigma_F^{VV}\right) \quad \left(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}\right) \quad \left(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2}\right) \quad \left(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}\right)$$

How to improve the STRIPPER subtraction scheme?



Motto

Optimizing by minimizing

New phase space construction: Idea

Goal

Phase space construction with a minimal # of subtraction kinematics

Old construction

- Start with unresolved partons
- Fill remaining phase space with Born configuration

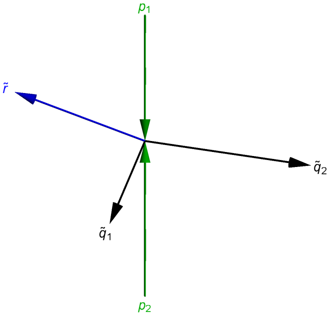
$\rightarrow$ Non-minimal # kinematic configurations
(e.g. single soft and collinear limits yield different configurations)

New construction

- Start with Born configuration
- Add unresolved partons (u_i)
- Cleverly adjust Born configuration to accommodate the u_i

New phase space construction

Mapping from $n+2$ to Born configuration: $\{P, r_j, u_k\} \to \{\tilde{P}, \tilde{r}_j\}$
modification of [Frixone,Webber'02] or [Frixione,Nason,Oleari'07]



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i:
  $\{\tilde{P}, \tilde{r}_j, u_k\} \to \{P, r_j, u_k\}$
- Preserve $q^2 = \tilde{q}^2\,,\;\; \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born phase space configuration

New phase space construction

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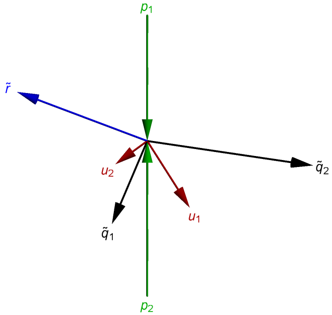

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- Generate unresolved partons u_i

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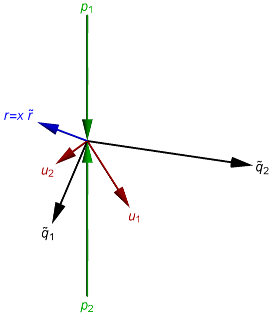

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Main steps:

- Generate Born phase space configuration
- Generate unresolved partons u_i
- Rescale reference momentum

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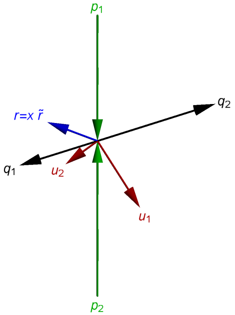

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- Preserve $q^2 = \tilde{q}^2\,,\;\; \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

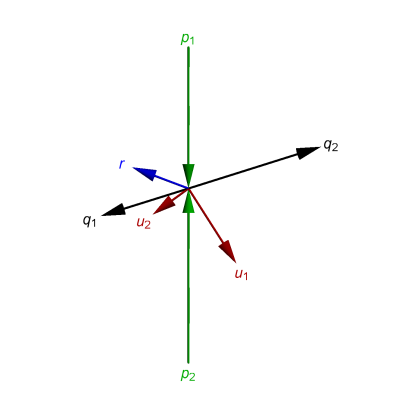
Main steps:

- Generate Born phase space configuration
- Generate unresolved partons u_i
- Rescale reference momentum
- Boost non-reference momenta of the Born configuration

Behaviour in singular limits

Collinear limit of u_2

(sector 1, $\eta_2 \to 0$)



Soft limit of u_2

(sector 1, $\xi_2 \to 0$)

$\to$ Both singular limits approach the same kinematic configuration

Behaviour in singular limits

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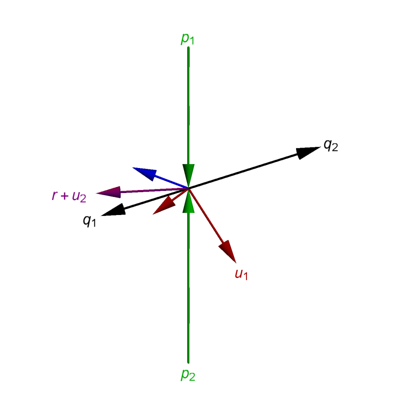

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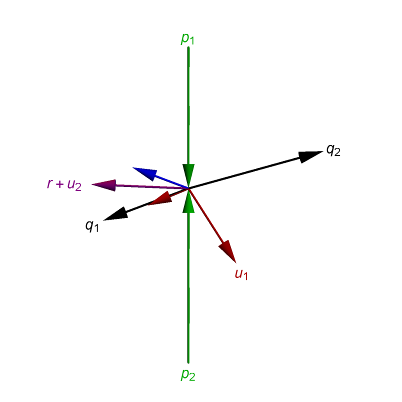

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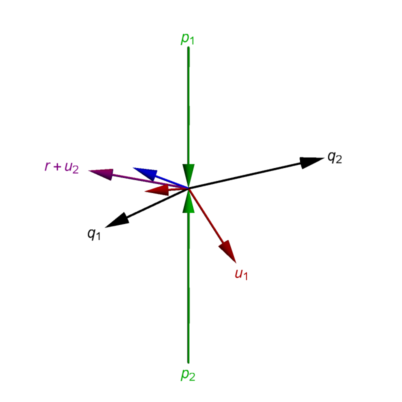

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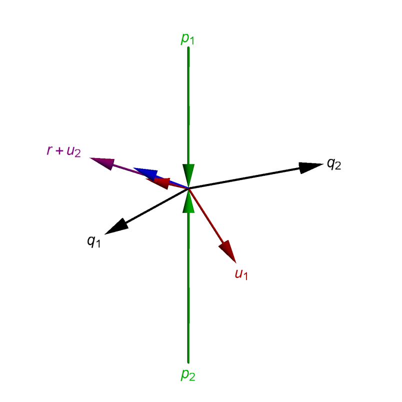

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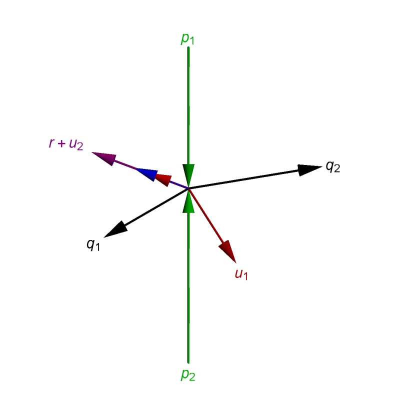

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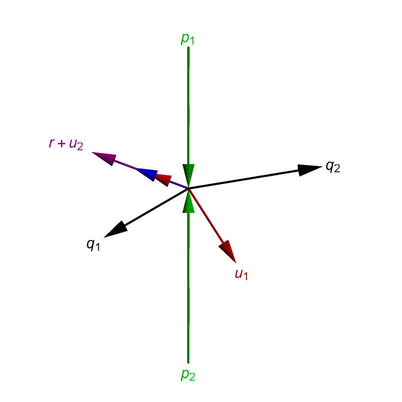

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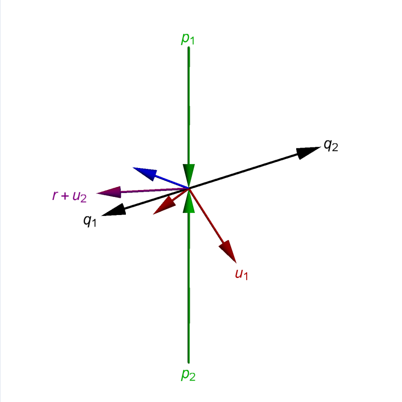

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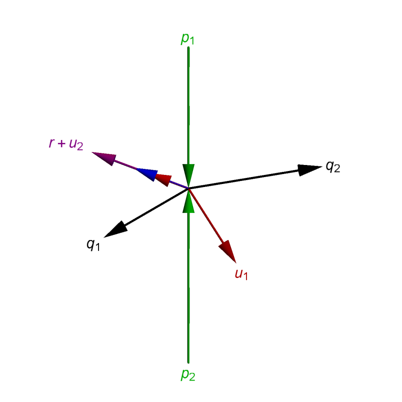

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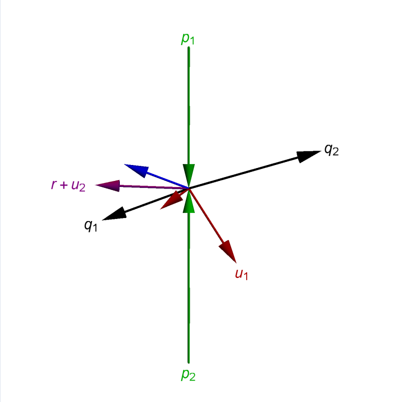

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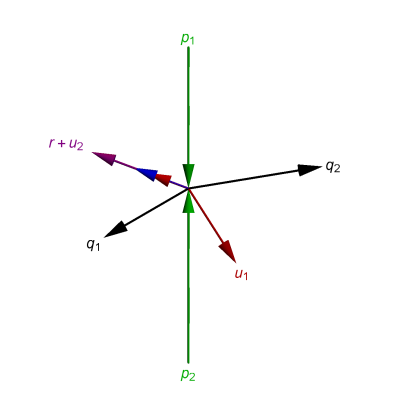
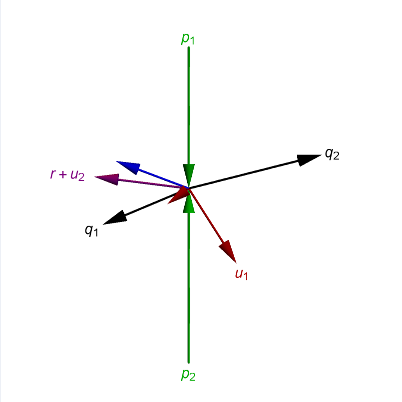
Collinear limit of u_2

(sector 1, $\eta_2 \to 0$)



Soft limit of u_2

(sector 1, $\xi_2 \to 0$)

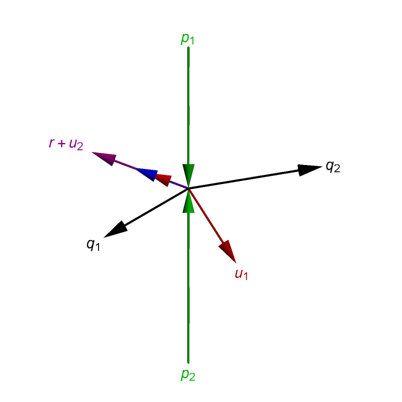


$\to$ Both singular limits approach the same kinematic configuration

Behaviour in singular limits

Collinear limit of u_2

(sector 1, $\eta_2 \to 0$)



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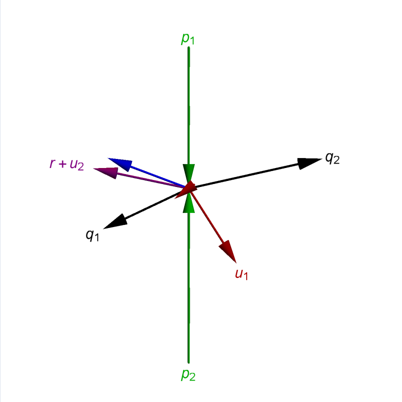

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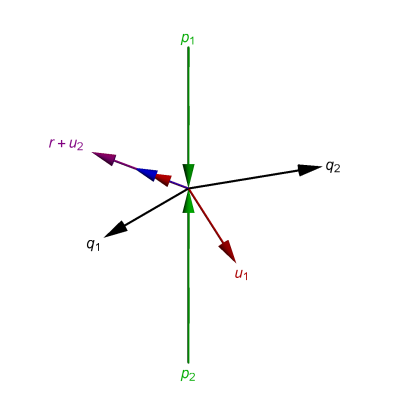

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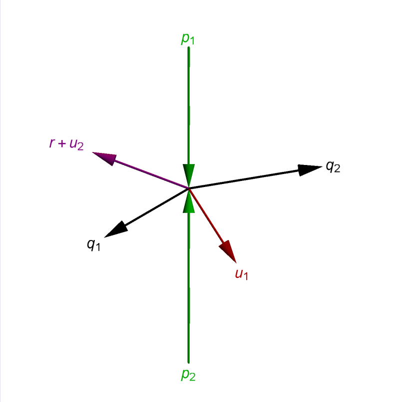
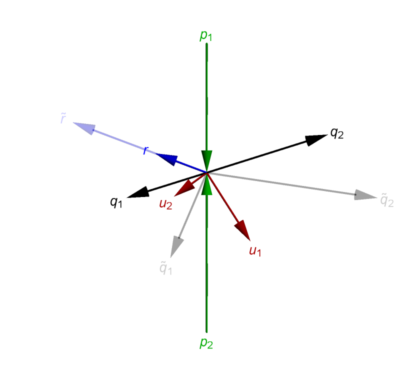


→ Both singular limits approach the same kinematic configuration

Behaviour in singular limits

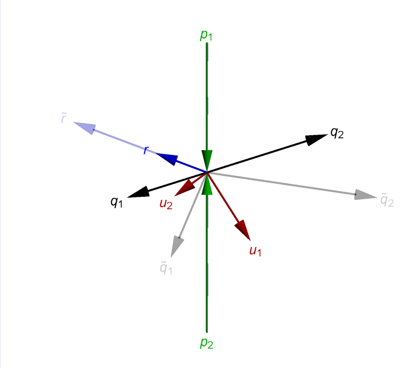
Triple collinear limit of u_1 & u_2

(sector 1, $\eta_1 \to 0$)



Double soft limit of u_1 & u_2

(sector 1, $\xi_1 \to 0$)

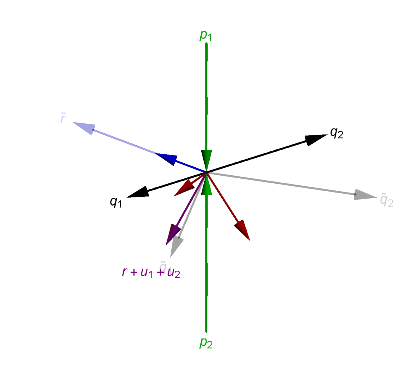


$\to$ Both double unresolved limits approach the Born configuration

Behaviour in singular limits

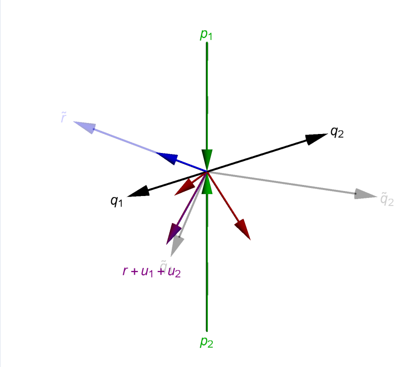
Triple collinear limit of u_1 & u_2

(sector 1, $\eta_1 \to 0$)



Double soft limit of u_1 & u_2

(sector 1, $\xi_1 \to 0$)



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Behaviour in singular limits

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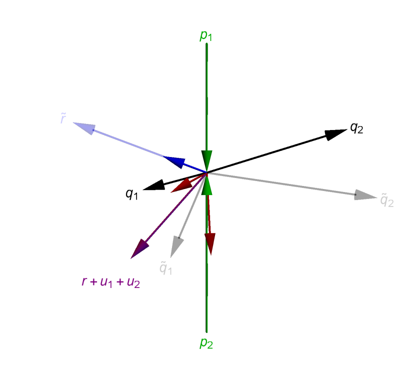

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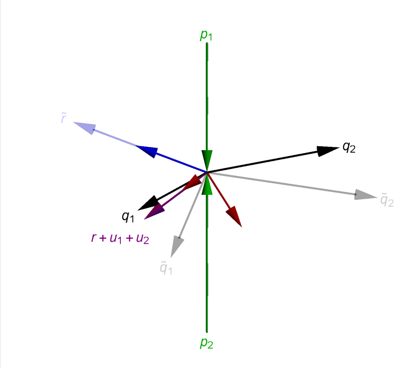

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Behaviour in singular limits

Triple collinear limit of u_1 & u_2

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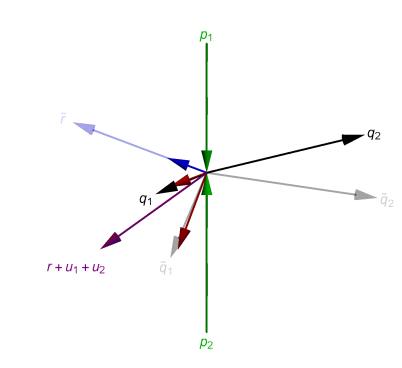

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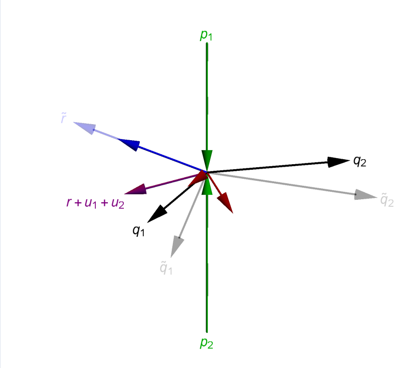

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Behaviour in singular limits

Triple collinear limit of u_1 & u_2

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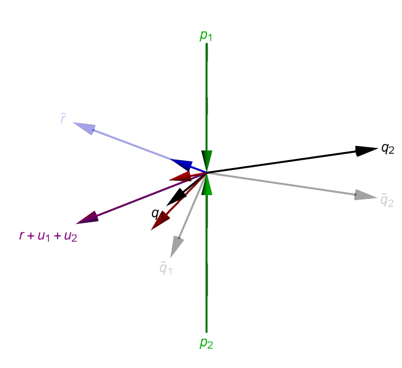

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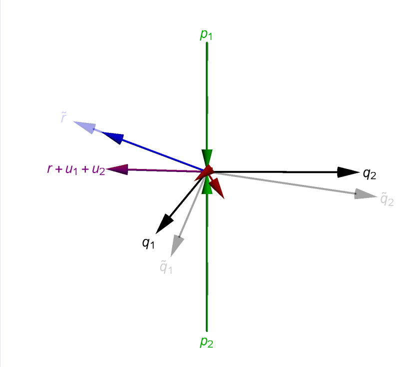

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Behaviour in singular limits

Triple collinear limit of u_1 & u_2

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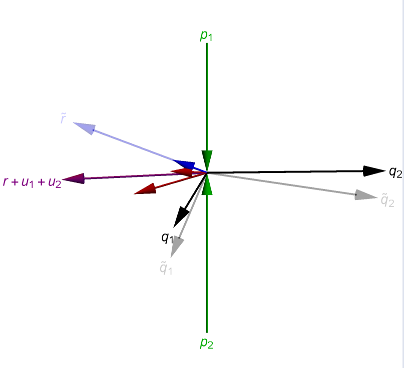
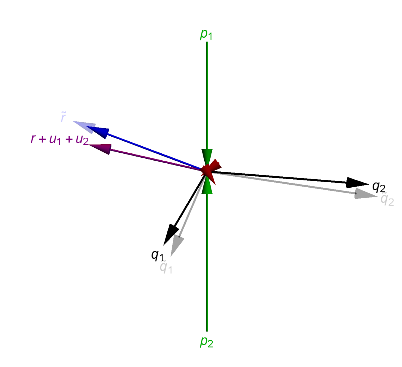


Double soft limit of u_1 & u_2

(sector 1, $\xi_1 \to 0$)

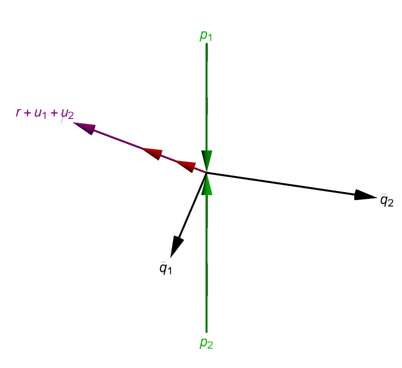


$\rightarrow$ Both double unresolved limits approach the Born configuration

Behaviour in singular limits

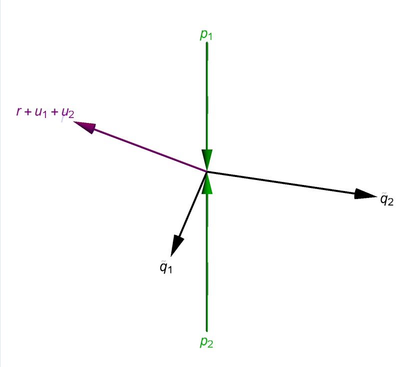
Triple collinear limit of u_1 & u_2

(sector 1, $\eta_1 \to 0$)



Double soft limit of u_1 & u_2

(sector 1, $\xi_1 \to 0$)



$\to$ Both double unresolved limits approach the Born configuration

Consequences

Features

- Minimal number of subtraction kinematics
- Only one DU configuration
  $\rightarrow$ pole cancellation for each Born phase space point
- Expected improved convergence of invariant mass distributions, since $\tilde{q}^2 = q^2$

Unintentional features

- Construction in lab frame
- Original construction of 't Hooft Veltman corrections [Czakon,Heymes'14] is spoiled

4 dimensional formulation

Separately finite contributions

Finite parts:

σ_F^{RR}

σ_F^{RV}

σ_F^{VV}

Finite remainder parts:

$\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}$

Single (SU) and double (DU) unresolved parts:

$\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}$ $\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{C1}, \sigma_{DU}^{VV}, \sigma_{DU}^{C2}$

$\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}$

$\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{C1}, \sigma_{DU}^{VV}, \sigma_{DU}^{C2}$

The measurement function

Observables: Implemented by infrared safe measurement function (MF) F_m

Infrared property in STRIPPER context:

- $\{x_i\} \to 0 \leftrightarrow$ single unresolved limit
  $\Rightarrow \mathrm{F}_{n+2} \to \mathrm{F}_{n+1}$
- $\{x_i\} \to 0 \leftrightarrow$ double unresolved limit
  $\Rightarrow \mathrm{F}_{n+2} \to \mathrm{F}_n$
  $\Rightarrow \mathrm{F}_{n+1} \to \mathrm{F}_n$

Tool for new formulation in the 't Hooft Veltman scheme:

Parameterized MF $\mathrm{F}^{\alpha}_{n+1}$

- $\mathrm{F}^{\alpha}_n \equiv 0$ for $\alpha \neq 0$
  (NLO MF)
- 'arbitrary' F^0_n
  (NNLO MF)
- $\alpha \neq 0 \Rightarrow \mathrm{DU} = 0$ and SU separately finite

Example: $F^{\alpha}_{n+1} = F_{n+1}\Theta_{\alpha}(\{\alpha_i\})$ with $\Theta_{\alpha} = 0$ if some $\alpha_i < \alpha$

The single unresolved (SU) contribution

$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1}$ where $\sigma_{SU}^{c} = \int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{c}} F_{n+1} + I_{n}^{\mathrm{c}} F_{n} \right)$

NLO measurement function ($\alpha \neq 0$):

$$\int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{RR}} + I_{n+1}^{\mathrm{RV}} + I_{n+1}^{\mathrm{C1}} \right) F_{n+1}^{\alpha} = \text{finite in 4 dim.}$$

All divergences cancel in d-dimensions:

$$\sum_{c} \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1}^{\alpha} \equiv \sum_{c} \mathcal{I}^{c} = 0$$

SU finiteness for $\alpha = 0$

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \underbrace{-\mathcal{I}^{RR} - \mathcal{I}^{RV} - \mathcal{I}^{C1}}_{=0}$$

$$\sigma_{SU}^{c} - \mathcal{I}^{c} = \int \mathrm{d}\Phi_{n+1} \left\{ \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} + I_{n+1}^{c,(0)} \right] F_{n+1} + \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} + I_{n}^{c,(0)} \right] F_{n} \right\}$$
$$- \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1} \Theta_{\alpha}(\{\alpha_i\})$$

SU finiteness for $\alpha = 0$

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \underbrace{- \mathcal{I}^{RR} - \mathcal{I}^{RV} - \mathcal{I}^{C1}}_{=0}$$

$$\sigma_{SU}^{c} - \mathcal{I}^{c} = \int \mathrm{d}\Phi_{n+1} \left\{ \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} + I_{n+1}^{c,(0)} \right] F_{n+1} + \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} + I_{n}^{c,(0)} \right] F_{n} \right\}$$
$$- \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1} \Theta_{\alpha}(\{\alpha_i\})$$
$$= \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)} F_{n+1} + I_{n}^{c,(-2)} F_{n}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)} F_{n+1} + I_{n}^{c,(-1)} F_{n}}{\epsilon} \right] (1 - \Theta_{\alpha}(\{\alpha_i\}))$$
$$+ \int \mathrm{d}\Phi_{n+1} \left[I_{n+1}^{c,(0)} F_{n+1} + I_{n}^{c,(0)} F_{n} \right] + \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} \right] F_{n} \Theta_{\alpha}(\{\alpha_i\})$$

$$=: \underbrace{Z^{c}(\alpha)}_{\text{integrable, zero volume for } \alpha \to 0} + \underbrace{C^{c}}_{\text{no divergencies}} + \underbrace{N^{c}(\alpha)}_{\text{only } F_n \to \text{DU}}$$

The function $N^c(\alpha)$

Looks like slicing, but it is slicing *only* for divergences
$\rightarrow$ no actual slicing parameter in result

Powerlog-expansion:

$$N^c(\alpha) = \sum_{k=0}^{\ell_{\max}} \ln^k(\alpha) N_k^c(\alpha)$$

- all $N_k^c(\alpha)$ regular in α
- start expression independent of $\alpha \Rightarrow$ all logs cancel
- only $N_0^c(0)$ relevant

Putting parts together:

$$\sigma_{SU} - \sum_c N_0^c(0) \text{ and } \sigma_{DU} + \sum_c N_0^c(0)$$

are finite in 4 dimension

$\Downarrow$

SU contribution: $\sigma_{SU} - \sum_c N_0^c = \sum_c C^c$
original expression σ_{SU} in 4-dim without poles, no further ϵ pole cancellation

C++ implementation of STRIPPER

C++ implementation of STRIPPER

Features of the implementation

- General subtraction framework
  - Provides a general set of subtraction terms
  - Tree-level amplitudes are calculated automatically using a Fortran library [van Hameren '09][Bury, van Hameren '15]
  - User has to provide the 1- and 2-loop amplitudes
- Separate evaluation of coefficients of scales and PDFs
  $\rightarrow$ Cheaper calculations with several scales and PDFs
- FastNLO interface
  - Allows to produce tables for fast fits
  - FastNLO tables for $t\bar{t}$ differential distributions released this spring [Czakon, Heymes, Mitov '17]

Summary

Summary

- Minimizing the STRIPPER scheme
- alternative phase space parameterization
- new formulation of 't Hooft Veltman scheme
- tests for a class of processes:
  $pp \to t\bar{t}$, $e^+e^- \to 2, 3j$, t decay, DIS, Drell-Yan, H decays, dijets

Supplements

Factorization and subtraction terms

Single unresolved phase space:

$$\iint_0^1 \mathrm{d}\eta\, \mathrm{d}\xi\; \eta^{a_1 - b_1\epsilon} \xi^{a_2 - b_2\epsilon}$$

Double unresolved phase space:

$$\iiiint_0^1 \mathrm{d}\eta_1\, \mathrm{d}\xi_1\, \mathrm{d}\eta_2\, \mathrm{d}\xi_2\; \eta_1^{a_1 - b_1\epsilon} \xi_1^{a_2 - b_2\epsilon} \eta_2^{a_3 - b_3\epsilon} \xi_2^{a_4 - b_4\epsilon}$$

Factorized singular limits:

$$\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}} \tilde{\mu}(\{x_i\}) \underbrace{\prod x_i^{a_i+1} \langle \mathcal{M}_{n+2} | \mathcal{M}_{n+2} \rangle}_{\text{regular}} \mathrm{F}_{n+2}$$

Regularisation:

Master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 \mathrm{d}x \left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x) - f(0)}{x^{1+b\epsilon}}$$

Calculation of $N_0^c(0)$

For each sector/contribution:

1. extraction of $\mathrm{d}\Phi_{n+1}$ from $\mathrm{d}\Phi_{n+2}\,|_{\text{SU pole}}$ (only for RR contribution)

$$\mathrm{d}\Phi_{n+2}\,|_{\text{SU pole}} = \Big(\underbrace{\mathrm{d}\Phi_n\,\mathrm{d}^d\mu(u_1)}_{\mathrm{d}\Phi_{n+1}}\mathrm{d}^d\mu(u_2)\Big)\Big|_{u_2\text{col/soft}}$$

2. expansion in ϵ up to ϵ^{-1} (except $\mathrm{d}\Phi_{n+1}$): $\mathrm{d}^d\Phi_{n+1}\left(\frac{\cdots}{\epsilon^2}+\frac{\cdots}{\epsilon}\right)$
3. Identifying $\ln^k(\alpha)$'s from x_i integrations over Θ function

$$\Theta_\alpha(\hat{\eta}, u^0) = \Theta(\hat{\eta}-\alpha)\Theta(\hat{\xi}u_{max}/E_{norm}-\alpha)$$

   $\rightarrow$ discard them

4. perform integration over Θ-functions of non-canceling and non-vanishing (in $\alpha \rightarrow 0$ limit) terms

PS

common starting point for all phase spaces :

$$\mathrm{d}\Phi_n = \mathrm{d}Q^2 \left[\prod_{j=1}^{n_{fr}} \mu_0(r_j) \prod_{k=1}^{n_u} \mu_0(u_k) \delta_+ \left(\left(P - \sum_{j=1}^{n_{fr}} r_j - \sum_{k=1}^{n_u} u_k \right)^2 - Q^2 \right) \right] \prod_{i=1}^{n_q} \mu_{m_i}(q_i) (2\pi)^d \delta^{(d)} \left(\sum_{i=1}^{n_q} q_i - q \right)$$

$$\text{with } \mu_m(k) \equiv \frac{\mathrm{d}^d k}{(2\pi)^d} \, 2\pi \delta\left(k^2 - m^2\right) \theta\left(k^0\right),$$

n : # final state particles, n_{fr} : # final state references, n_u : # additional partons