NNLO QCD calculations with the sector-improved residue subtraction scheme

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Predictions from higher order perturbation theory

Ultimate Goal: describe measurements for high energy collisions

- Model $\rightarrow$ QFT
- predictions $\rightarrow$ perturbation theory
- (simplified) idea: higher orders $\rightarrow$ better predictions
- higher order introduce UV and IR divergences
  - need for regularization (dimensional regularization, mass,...) and renormalization (introduction of additional scale μ)
  - methods of handling IR divergences

$\Rightarrow$ increasing complexity of calculations

The Les Houches wishlist

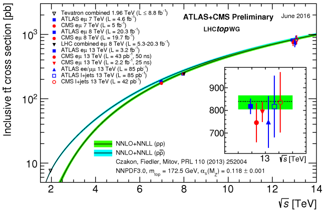
List of process of phenomenological interest

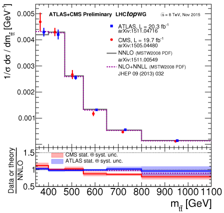
process	NLO	NNLO	N^3LO
$pp \to H$	$\surd$	$(\surd)_{HEFT}$	$(\surd)_{HEFT}$
$pp \to H + j$	$\surd$	$\surd$	
$pp \to H + 2j$	$\surd$	$(\surd)_{VBF}$	
$pp \to H + 3j$	$\surd$		
$pp \to V$	$\surd$	$\surd$	!
$pp \to V + j$	$\surd$	$\surd$	
$pp \to V + 2j$	$\surd$	!	
$pp \to t\bar{t}$	$\surd$	$\surd$	
$pp \to t\bar{t} + j$	$\surd$	!	
$pp \to 2j$	$\surd$	$\surd$	
$pp \to 3j$	$\surd$	!	
...			

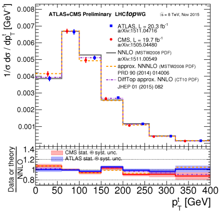
Is it is worth the effort....?

Total and Differential Cross Sections

- We are well in the hadron collider precision measurement territory !!!
- …for a few years now







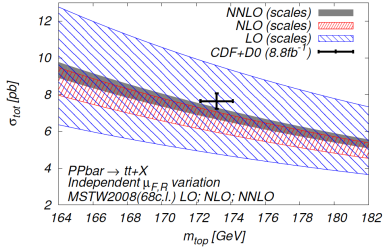
LHC	σ(tt) [pb]	L [fb^{-1}]	N_{event}
7 TeV	172.676	5	8.6 x 10^5
8 TeV	246.652	19.7	4.8 x 10^6
13 TeV	807.296	2.3	1.8 x 10^6

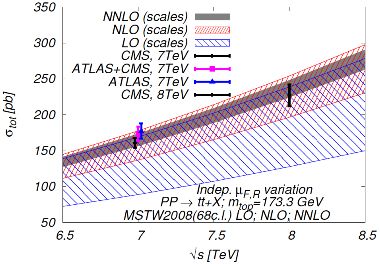
LHC Run 1

LHC Run 2

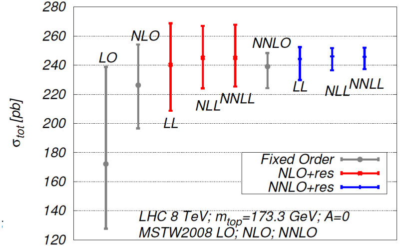
3

Perturbation Theory Convergence





Scale variation



Concurrent uncertainties:

Scales	**~ 3%**
pdf (at 68%cl)	**~ 2-3%**
α_S (parametric)	**~ 1.5%**
m_{top} (parametric)	**~ 3%**

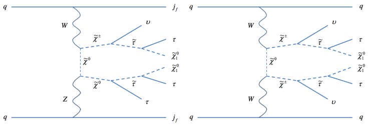
Soft gluon resummation makes a difference: **5% → 3%**

MC, Fiedler, Mitov PRL`13

Search for Supersymmetry…

- An example of the importance of top-quark cross sections as background
- Search for supersymmetry in the vector-boson fusion topology in proton-proton collisions

LHC @ 8 TeV



CERN-PH-EP/2015-213
2015/09/01

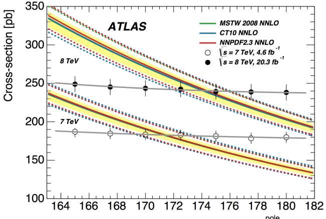
Process	$\mu^\pm\mu^\mp jj$	$e^\pm\mu^\mp jj$	$\mu^\pm\tau_h^\mp jj$	$\tau_h^\pm\tau_h^\mp jj$
Z+jets	4.3 ± 1.7	$3.7^{+2.1}_{-1.9}$	19.9 ± 2.9	12.3 ± 4.4
W+jets	<0.1	$4.2^{+3.3}_{-2.5}$	17.3 ± 3.0	2.0 ± 1.7
VV	2.8 ± 0.5	3.1 ± 0.7	2.9 ± 0.5	0.5 ± 0.2
$t\bar{t}$	24.0 ± 1.7	$19.0^{+2.3}_{-2.4}$	11.7 ± 2.8	–
QCD	—	—	—	6.3 ± 1.8
Higgs boson	1.0 ± 0.1	1.1 ± 0.5	—	1.1 ± 0.1
VBF Z	—	—	—	0.7 ± 0.2
Total	32.2 ± 2.4	$31.1^{+4.6}_{-4.1}$	51.8 ± 5.1	22.9 ± 5.1
Observed	31	22	41	31

4

Top Quark Mass From σ_{tt}

Experiment	pole mass	data
D0 [25]	$169.1^{+5.9}_{-5.2}$ GeV	$p\bar{p}$, 1.96 TeV, 1 fb^{-1}
Langenfeld et al. [24]	$168.9^{+3.5}_{-3.4}$ GeV	idem
D0 [26]	$167.5\ ^{+5.4}_{-4.9}$ GeV	$p\bar{p}$, 1.96 TeV, 5.3 fb^{-1}
CMS [32]	176.7 ± 2.9 GeV	pp, 7 TeV, 5 fb^{-1}
ATLAS [28]	$172.9\ ^{+2.5}_{-2.6}$ GeV	pp, 7 TeV, 5 fb^{-1} + 8 TeV, 20 fb^{-1}
CMS [29]	$173.6\ ^{+1.7}_{-1.8}$ GeV	idem
CMS [29]	$173.9\ ^{+1.8}_{-1.9}$ GeV	idem
CMS [29]	$174.1\ ^{+2.1}_{-2.2}$ GeV	idem
ATLAS [37]	$173.7\ ^{+2.3}_{-2.1}$ GeV	pp, 7 TeV, 5 fb^{-1}

arXiv:1406.5375



Collected by M. Vos `14

…and the Strong Coupling Constant

NNLL + NNLO with NNPDF23

Exp.	E_{CM} [GeV]	$\alpha_s(M_Z)$	Exp.	scale	PDF	m_{top}	E_{beam}	total
ATLAS	7000	0.1207	±0.0017	±0.0014	±0.0014	±0.0018	±0.0009	±0.0033
ATLAS	8000	0.1168	±0.0018	±0.0015	±0.0013	±0.0018	±0.0008	±0.0033
CMS	7000	0.1184	±0.0016	±0.0014	±0.0014	±0.0018	±0.0008	±0.0032
CMS	8000	0.1174	±0.0017	±0.0015	±0.0013	±0.0018	±0.0008	±0.0033
CDF&D0	1960	0.1201	±0.0032	±0.0013	±0.0010	±0.0013	±0.0000	±0.0038
unweigted	**average**	**0.1187**						

Workshop on high-precision α_s measurements: from LHC to FCC-ee
CERN, 13 October 2015

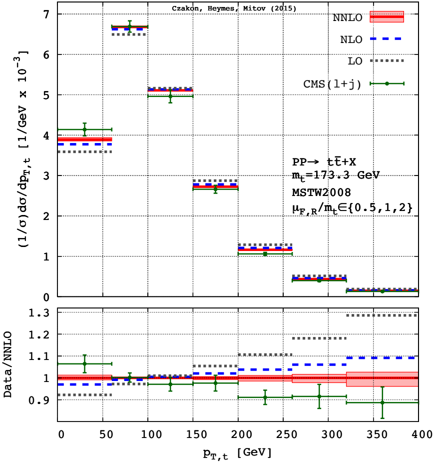
α_s from σ(ttbar): preliminary new results

Gavin Salam (CERN), work in progress
with Siggi Bethke, Günther Dissertori and Thomas Klijnsma

Differential Distributions @ LHC

- Transverse momentum important for PDF fits
- Even with fixed scale the agreement with data quite good
- Apparently convergence poor in normalized distributions

MC, Heymes, Mitov PRL`15



Top quark pair production and decay

Theoretical developments

Stable onshell tops and spin summed:

- Total inclusive cross sections @ NNLO+NNLL accuracy

  [Czakon, Fiedler, Mitov '13]

- Fully differential distributions @ NNLO

  [Czakon, Fiedler, Heymes, Mitov '16]

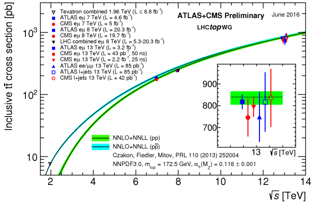
- + EW corrections

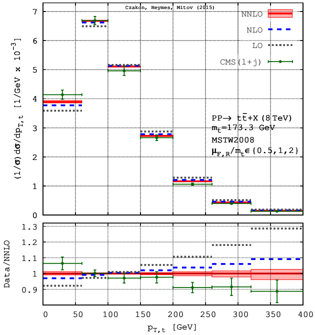
  [Czakon, Heymes, Mitov, Pagani, Tsinikos, Zaro '17]

Unstable tops + spin correlations:

- Approximate NNLO + NNLO decay

  [Gao,Papanastasiou '17]





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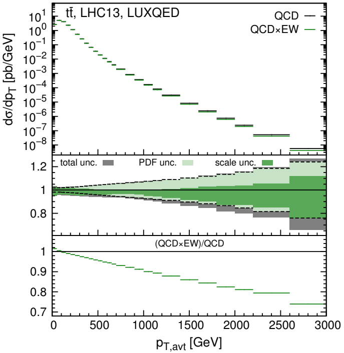

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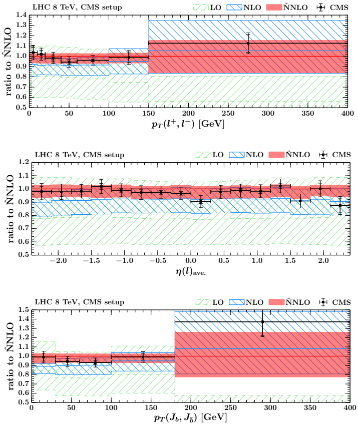
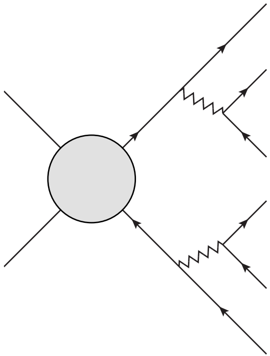


Goal: $t\bar{t}$ production and decay at NNLO QCD

Narrow-Width-Approximation

- On-shell top-quarks
- Factorization of top-decay
- Separations of QCD corrections
- Keep spin correlations



$\rightarrow$ polarised $t\bar{t}$-production amplitudes

Polarised $t\bar{t}$ production amplitudes

Gluon channel

$$\mathcal{M} = \epsilon_{1\mu}(p_1)\epsilon_{2\nu}(p_2)M^{\mu\nu}$$

$M^{\mu\nu}$ is a rank-2 Lorentz tensor

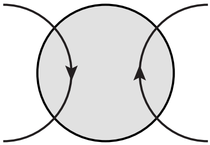
- Momentum conservation
- Transversality
- Equation of motion
- Parity conservation $\rightarrow$ no γ_5

8 independent structures

($d = 4$ dimensions)

$$M^{\mu\nu} = \sum_{j=1}^{8} M_j T_j^{\mu\nu}$$

Quark channel



- Two disconnected fermion lines
- Connection by gluons+loops

4 independent structures

$$\mathcal{M} = \sum_{i=1}^{4} M_j T_j$$

with $T_j \sim \bar{v}_2 \Gamma_j u_1 \bar{u}_3 \Gamma'_j v_4$

Color structures

Color decomposition: $\mathcal{M} = \sum_{ij} c_{ij} C_i M_j$

Gluon channel
color representations

- Gluons: a, b adjoint
- Quarks: c, d fundamental

$$C_1 = (T^a T^b)_{cd}$$
$$C_2 = (T^b T^a)_{cd}$$
$$C_3 = \mathrm{Tr}\left\{T^a T^b\right\} \delta_{cd}$$

Quark channel
color representations

- Quarks: a, b fundamental
- Quarks: c, d fundamental

$$C_1 = \delta_{ac}\delta_{bd}$$
$$C_2 = \delta_{ab}\delta_{cd}$$

Projection

Construct projectors: $P_j = \sum_l B_{jl} (T_l)^\dagger$

Extracting the B_{jl}:

$$\sum_{spin/pol,col} P_j \mathcal{A} \stackrel{!}{=} A_j$$

leads to system of equations

$$\sum_{l,k} B_{jl} A_k \sum_{spin/pol,col} (T_l)^\dagger T_k = A_j$$

Inversion $\rightarrow$ coefficients B_{jl}

Short summary

$$\mathcal{M} = \sum_{ij} c_{ij} C_i M_j$$

- Gluon: 3(color) $\cdot$ 8(spin)
  Quark: 2(color) $\cdot$ 4(spin)
  $\rightarrow$ combined 32 structures
- Scalar coefficients c_{ij}:
  - Rational function of $m_s = m_t^2/s$, $x = t/s$ and ϵ
  - Scalar Feynman integrals

Evaluation of coefficients

Integration by parts identities (IBP)

$$\int \mathrm{d}^d k_1 \mathrm{d}^d k_2 \frac{\partial}{\partial k_j^\mu} \left(p_l \prod \frac{(p \cdot k)_i^{b_i}}{(q_i^2 + m_i^2)^{a_i}} \right)$$

$\mathcal{O}(10^4)$ scalar Feynman integrals
$\rightarrow$ 422 master integrals

Master integrals

- Partially canonicalized new in collaboration with Long Chen

analogous to [Czakon '08],[Czakon,Fiedler,Mitov '13]

- Differential equations generated by IBPs
- High energy expansion as boundary condition
- Numerical integration for 'bulk' region
  $\rightarrow$ Interpolation grid
- Threshold expansion for $\beta = \sqrt{1 - 4m_s} \rightarrow 0$

Finite remainder function

IR divergences and the finite remainder function

$$|\mathcal{M}_n\rangle = \mathbf{Z}(\epsilon, \{p_i\}, \{m_i\}, \mu_R)\, |\mathcal{F}\rangle$$

- Complete factorization of IR structure → **Z** operator

$$\begin{aligned}
\left|\mathcal{M}_n^{(0)}\right\rangle &= \left|\mathcal{F}_n^{(0)}\right\rangle \\
\left|\mathcal{M}_n^{(1)}\right\rangle &= \mathbf{Z}^{(1)} \left|\mathcal{M}_n^{(0)}\right\rangle + \left|\mathcal{F}_n^{(1)}\right\rangle \\
\left|\mathcal{M}_n^{(2)}\right\rangle &= \mathbf{Z}^{(2)} \left|\mathcal{M}_n^{(0)}\right\rangle \\
&\quad + \mathbf{Z}^{(1)} \left|\mathcal{F}_n^{(1)}\right\rangle + \left|\mathcal{F}_n^{(2)}\right\rangle
\end{aligned}$$

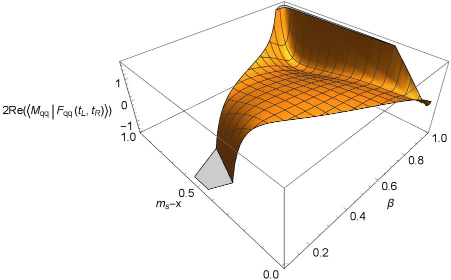
- **Z** can be calculated by its anomalous dimension equation

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{Z} = -\mathbf{\Gamma}\mathbf{Z}$$

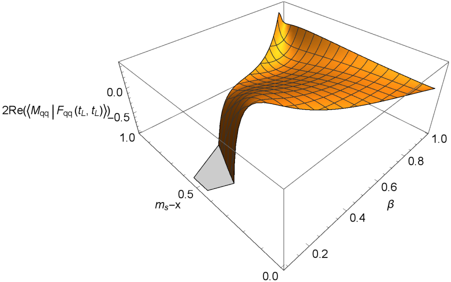
- Depends on kinematics and operator on color space
→ Projection on color and spin structures

Finite remainder for polarised tops

2-Loop finite remainder for:
$q\bar{q} \to t_L \bar{t}_R$



2-Loop finite remainder for:
$q\bar{q} \to t_R \bar{t}_R$



(work in progress)

Summary - top quark pair production and decay

Finished

- Projection LO, NLO, NNLO amplitudes
- Finite remainder of all coefficients
- Improved set of master integrals
- Kinematical expansions of coefficients
- Implementation of coefficients, color and spin structures in STRIPPER

Outlook

- Usage of amplitudes within STRIPPER
- Implementation of decay phase-space and handling of decay products in STRIPPER
- QCD-corrections to decay

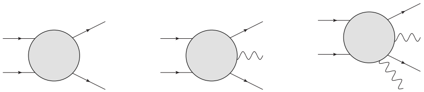
$\rightarrow$ publication in preparation

Subtraction Schemes

Sector-improved residue subtraction scheme

Collider observables in QCD

- any process at colliders is specified by final states, and cuts on these final states
- parton-hadron duality is used, but partons being massless can be emitted at will
- it is necessary to sum (incoherently) over processes with a different number of final partons



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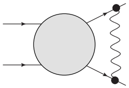
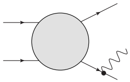
- exchange or emission of partons lead to divergences



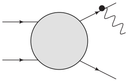
virtual - UV/IR

virtual momentum arbitrarily large/small



real - IR soft

gluon energy arbitrarily small

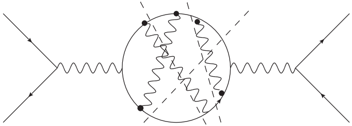


real - IR collinear

angle between partons arbitrarily small

Kinoshita-Lee-Nauenberg theorem

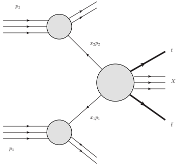
- the theorem states that for "suitably averaged" transition probabilities (cross sections), the result is finite
- particular case is given by electron-positron annihilation



- after cuts: the different contributions are divergent, but the self energy itself is finite, and the total cross-section is just its imaginary part
- the averaging is obtained by integrating the cross section with a "jet function" F_n dependent on the momenta of the partons (or mesons and hadrons)
- F_n is required to be "infrared safe", i.e. the value for a soft or collinear degenerate configuration of $n+1$ is the same as the value for the equivalent n partons

Factorization

- unfortunately, in hadronic collisions, the initial states are not properly averaged
- instead a factorization theorem is used, e.g. for top quark pair production:



$$\sigma_{h_1 h_2 \to t\bar{t}}(s, m_t^2) = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2$$
$$\phi_{i,h_1}(x_1, \mu_F^2)\phi_{j/h_2}(x_2, \mu_F^2)\hat{\sigma}_{ij}(\hat{s}, m_t^2, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

- the divergences of the initial state collinear radiation are absorbed into the (universal) parton distribution functions
- the general formula is

$$\left[\sigma_{ij}(x)/x\right] = \sum [\hat{\sigma}_{kl}(z)/z] \otimes \Gamma_{ki} \otimes \Gamma_{lj} \qquad \left[f_1 \otimes f_2\right](x) = \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 f_1(x_1) f_2(x_2) \delta(x - x_1 x_2)$$

$$\Gamma_{ij} = \delta_{ij}\delta(1-x) - \frac{\alpha_s}{\pi}\frac{P_{ij}^{(0)}(x)}{\epsilon} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{1}{2\epsilon^2}\left(\left(P_{ik}^{(0)} \otimes P_{kj}^{(0)}\right)(x) + \beta_0 P_{ij}^{(0)}(x)\right) - \frac{1}{2\epsilon} P_{ij}^{(1)}(x)\right] + \mathcal{O}\left(\alpha_s^3\right)$$

- Consistency of the construction requires a consistent dimensional regularization

The general idea of subtraction

- add to the original cross section $\sigma = \sigma^{LO} + \sigma^{NLO}$

$$\sigma^{LO} = \int_m d\sigma^B \quad , \qquad \sigma^{NLO} \equiv \int d\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

  an identity involving approximations to the real radiation cross section

$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^R - d\sigma^A \right] + \int_{m+1} d\sigma^A + \int_m d\sigma^V$$

  and regroup the terms as

$$\sigma^{NLO} = \int_{m+1} \left[\left(d\sigma^R \right)_{\epsilon=0} - \left(d\sigma^A \right)_{\epsilon=0} \right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0}$$

- for $d\sigma^A$ it must be possible to
  1. obtain the Laurent expansion by integration over the single particle unresolved space (preferably analytically)
  2. approximate $d\sigma^R$ (preferably pointwise)

Subtraction at NLO (and beyond?)

NLO Subtraction Schemes

- Dipole Subt. [Catani,Seymour'98]
- FKS [Frixione,Kunst,Signer'95]
- Antenna Subtraction [Kosower'97]
- Nagy-Soper [Nagy,Soper'07]

Generalization to NNLO?

- Nature of singularities known: soft and collinear limits
- NLO (fairly simple):
  - single soft
  - single collinear
- At NNLO? : Many (overlapping) ways to reach soft and collinear limits
- Possible way: Decomposition of the phase space to disentangle them

NNLO subtraction schemes

Handling real radiation contribution in NNLO calculations
cancellation of infrared divergences

increasing number of available NNLO calculations with a variety of schemes

- qT-slicing [Catani,Grazzini, '07] ,[Ferrera,Grazzini,Tramontano, '11], [Catani,Cieri,DeFlorian,Ferrera,Grazzini,'12], [Gehrmann,Grazzini,Kallweit,Maierhofer,Manteuffel,Rathlev,Torre,'14-15'], [Bonciani,Catani,Grazzini,Sargsyan,Torre,'14-'15]
- N-jettiness slicing [Gaunt,Stahlhofen,Tackmann,Walsh, '15], [Boughezal,Focke,Giele,Liu,Petriello,'15-'16] , [Bougezal,Campell,Ellis,Focke,Giele,Liu,Petriello,'15], [Campell,Ellis,Williams,'16]
- Antenna subtraction [Gehrmann, GehrmannDeRidder,Glover,Heinrich,'05-'08] , [Weinzierl,'08,'09], [Currie,Gehrmann,GehrmannDeRidder,Glover,Pires,'13-'17], [Bernreuther,Bogner,Dekkers,'11,'14], [Abelof,(Dekkers),GehrmannDeRidder,'11-'15], [Abelof,GehrmannDeRidder,Maierhofer,Pozzorini,'14], [Chen,Gehrmann,Glover,Jaquier,'15]
- Colorful subtraction [DelDuca,Somogyi,Troscanyi,'05-'13], [DelDuca,Duhr,Somogyi,Tramontano,Troscanyi,'15]
- Sector-improved residue subtraction (STRIPPER) [Czakon,'10,'11] , [Czakon,Fiedler,Mitov,'13,'15], [Czakon,Heymes,'14] [Czakon,Fiedler,Heymes,Mitov,'16,'17], [Bughezal,Caola,Melnikov,Petriello,Schulze,'13,'14], [Bughezal,Melnikov,Petriello,'11], [Caola,Czernecki,Liang,Melnikov,Szafron,'14], [Bruchseifer,Caola,Melnikov,'13-'14], [Caola, Melnikov, Röntsch,'17]

Formulation

Hadronic cross section:

$$\sigma_{h_1 h_2}(P_1, P_2) = \sum \iint_0^1 \mathrm{d}x_1\, \mathrm{d}x_2\, f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

partonic cross section:

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \hat{\sigma}_{ab}^{(1)} + \hat{\sigma}_{ab}^{(2)} + \mathcal{O}\left(\alpha_S^3\right)$$

Contributions with different final state multiplicities and convolutions:

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\mathrm{RR}} + \hat{\sigma}_{ab}^{\mathrm{RV}} + \hat{\sigma}_{ab}^{\mathrm{VV}} + \hat{\sigma}_{ab}^{\mathrm{C2}} + \hat{\sigma}_{ab}^{\mathrm{C1}}$$

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_{n}$$

Sector decomposition

Several layers of decomposition for $\mathrm{d}\Phi_{n+2}$

Selector functions:

$$1 = \sum_{i,j} \left[\sum_{k} \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right]$$

Factorization of double soft limits:

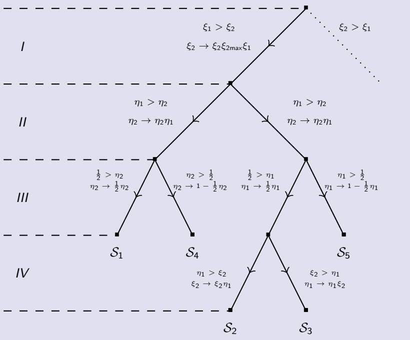
$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

Sector parameterization

Parameterization with respect to the reference parton r:
angles: $\hat{\eta}_i = \frac{1}{2}(1 - \cos\theta_{ir}) \in [0,1]$
energies: $\hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$

originally: 5 sub-sectors

Triple collinear factorization



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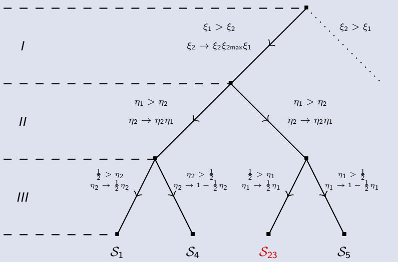
now: 4 sub-sectors

Triple collinear factorization



Caola, Melnikov, Röntsch [hep-ph:1702.01352v1]

STRIPPER

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$$\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$$

$$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_{n}$$

Sector decomposition and master formula:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_{+}}_{\text{reg. + sub.}}$$

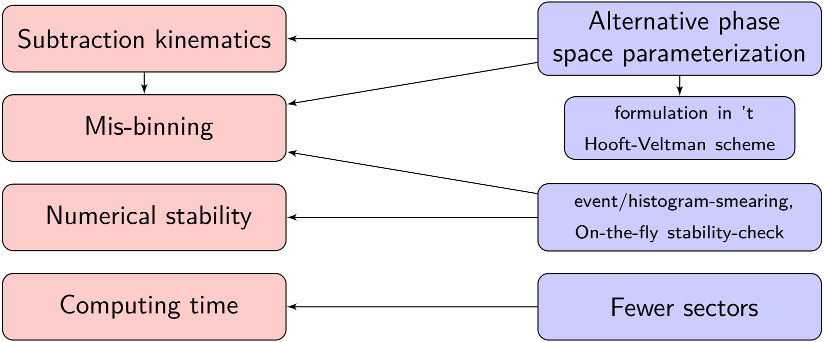
$$\Downarrow$$

$$\left(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}\right) \quad \left(\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}\right) \quad \left(\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}\right) \quad \left(\sigma_{SU}^{C1}, \sigma_{DU}^{C1}\right) \quad \left(\sigma_{DU}^{C2}, \sigma_{FR}^{C2}\right)$$

$\Downarrow$ 4 dim formulation [Czakon,Heymes'14]

$$\left(\sigma_F^{RR}\right) \quad \left(\sigma_F^{RV}\right) \quad \left(\sigma_F^{VV}\right) \quad \left(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}\right) \quad \left(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2}\right) \quad \left(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}\right)$$

How to improve the STRIPPER subtraction scheme?



Motto

Optimizing by minimizing

New phase space construction: Idea

Goal

Phase space construction with a minimal # of subtraction kinematics

Old construction

- Start with unresolved partons
- Fill remaining phase space with Born configuration

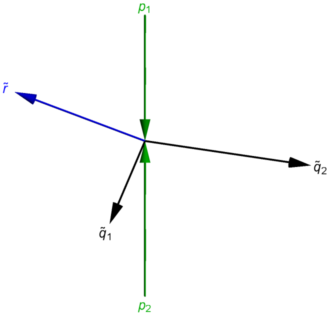
$\rightarrow$ Non-minimal # kinematic configurations
(e.g. single soft and collinear limits yield different configurations)

New construction

- Start with Born configuration
- Add unresolved partons (u_i)
- Cleverly adjust Born configuration to accommodate the u_i

New phase space construction

Mapping from $n+2$ to Born configuration: $\{P, r_j, u_k\} \to \{\tilde{P}, \tilde{r}_j\}$
modification of [Frixone,Webber'02] or [Frixione,Nason,Oleari'07]



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i:
  $\{\tilde{P}, \tilde{r}_j, u_k\} \to \{P, r_j, u_k\}$
- Preserve $q^2 = \tilde{q}^2\,,\;\; \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born phase space configuration

New phase space construction

Mapping from $n+2$ to Born configuration: $\{P, r_j, u_k\} \to \{\tilde{P}, \tilde{r}_j\}$
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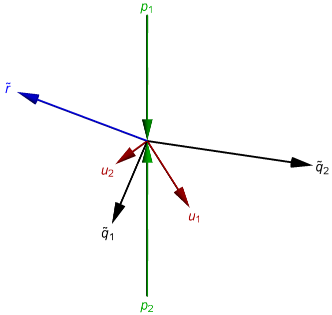

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Main steps:

- Generate Born phase space configuration
- Generate unresolved partons u_i

New phase space construction

Mapping from $n+2$ to Born configuration: $\{P, r_j, u_k\} \to \{\tilde{P}, \tilde{r}_j\}$
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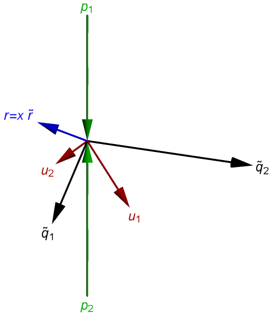

Requirements:

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- Invertible for fixed u_i:
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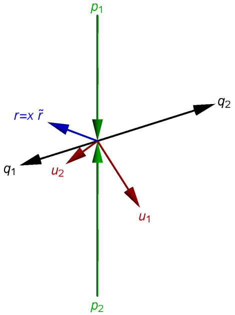
- Preserve $q^2 = \tilde{q}^2\,,\ \ \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born phase space configuration
- Generate unresolved partons u_i
- Rescale reference momentum

New phase space construction

Mapping from $n+2$ to Born configuration: $\{P, r_j, u_k\} \to \{\tilde{P}, \tilde{r}_j\}$
modification of [Frixone,Webber'02] or [Frixione,Nason,Oleari'07]



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i:
  $\{\tilde{P}, \tilde{r}_j, u_k\} \to \{P, r_j, u_k\}$
- Preserve $q^2 = \tilde{q}^2\,,\;\; \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

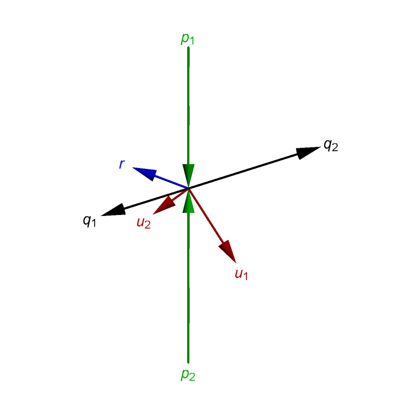
Main steps:

- Generate Born phase space configuration
- Generate unresolved partons u_i
- Rescale reference momentum
- Boost non-reference momenta of the Born configuration

Behaviour in singular limits

Collinear limit of u_2

(sector 1, $\eta_2 \to 0$)



Soft limit of u_2

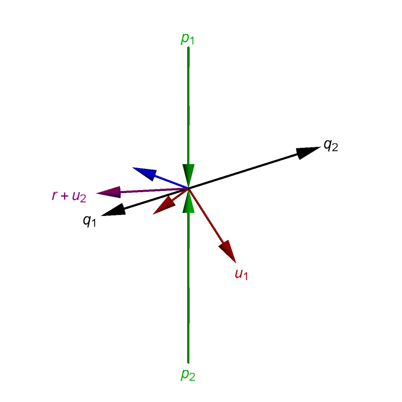
(sector 1, $\xi_2 \to 0$)

$\to$ Both singular limits approach the same kinematic configuration

Behaviour in singular limits

Collinear limit of u_2

(sector 1, $\eta_2 \to 0$)



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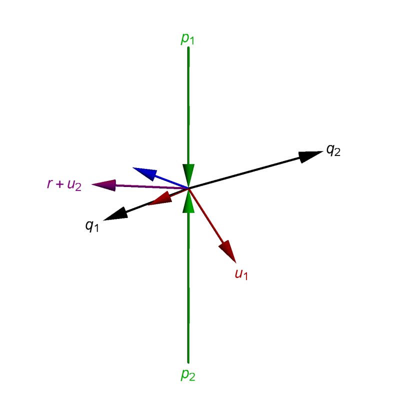

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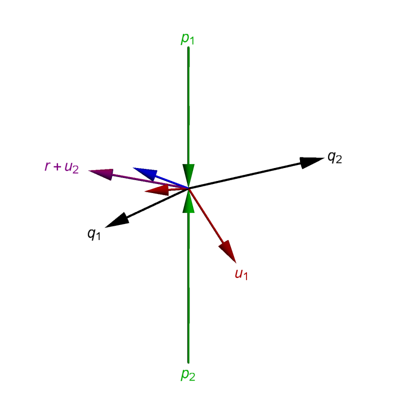

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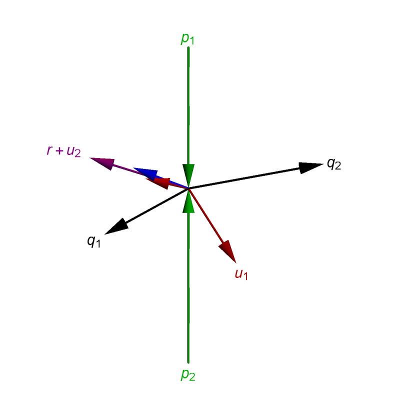

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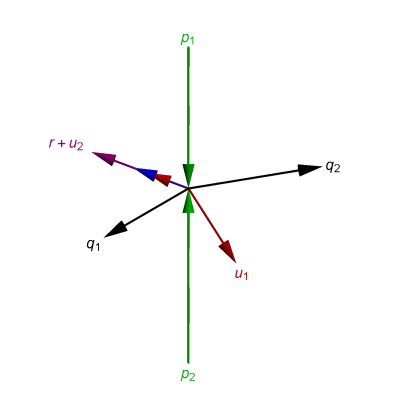

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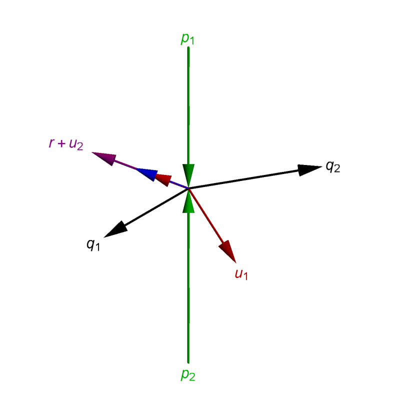

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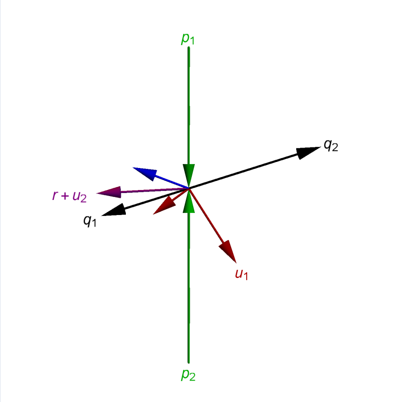

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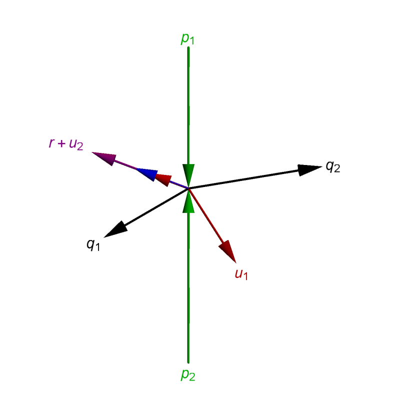

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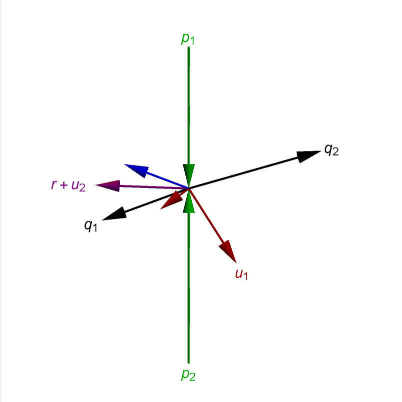

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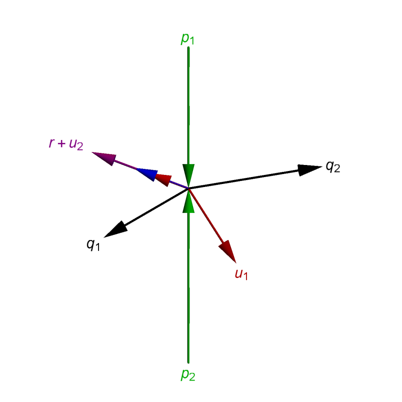

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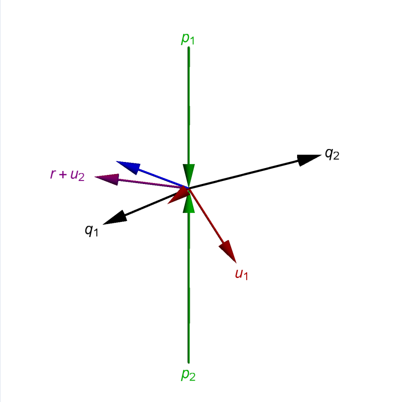

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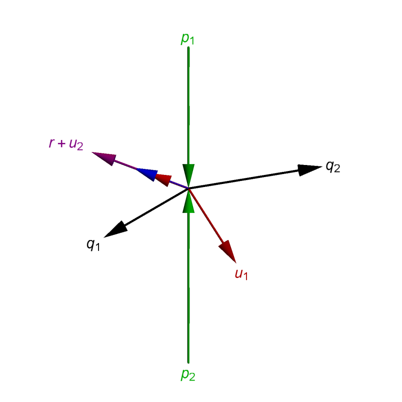

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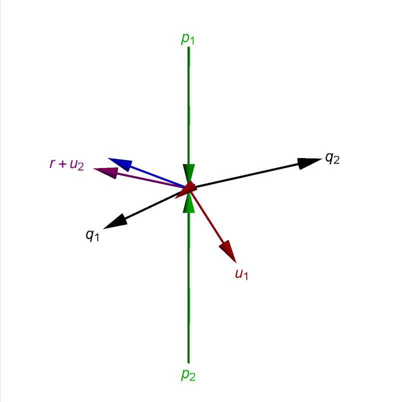

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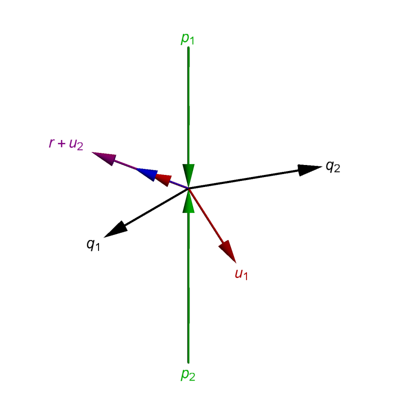

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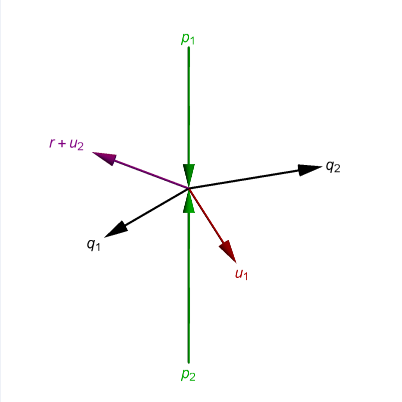
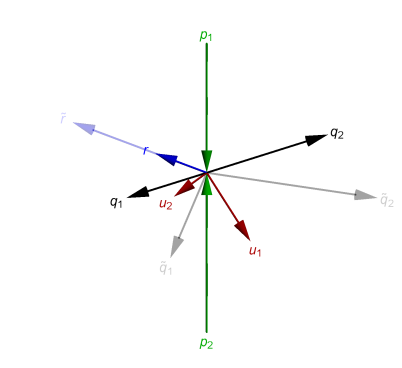


$\rightarrow$ Both singular limits approach the same kinematic configuration

Behaviour in singular limits

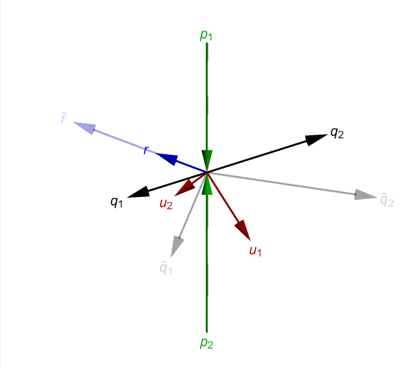
Triple collinear limit of u_1 & u_2

(sector 1, $\eta_1 \to 0$)



Double soft limit of u_1 & u_2

(sector 1, $\xi_1 \to 0$)

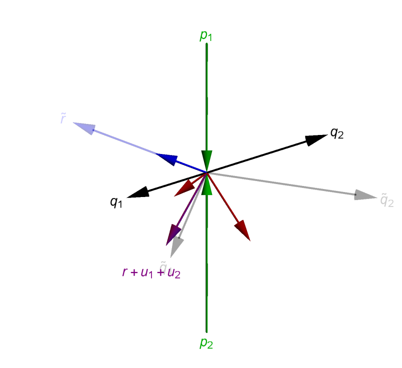


$\to$ Both double unresolved limits approach the Born configuration

Behaviour in singular limits

Triple collinear limit of u_1 & u_2

(sector 1, $\eta_1 \to 0$)



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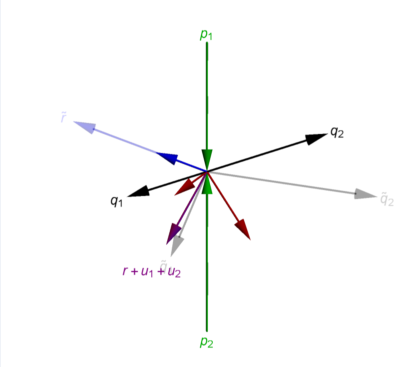

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Behaviour in singular limits

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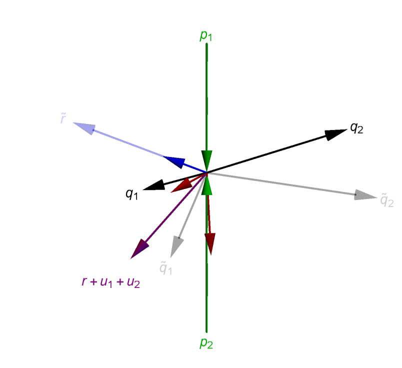

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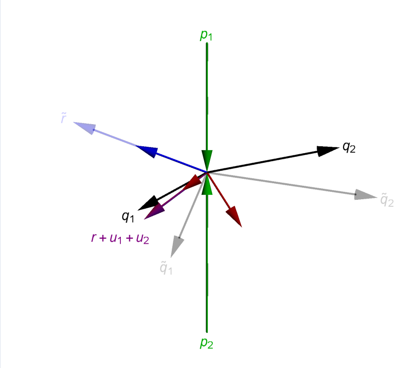

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Behaviour in singular limits

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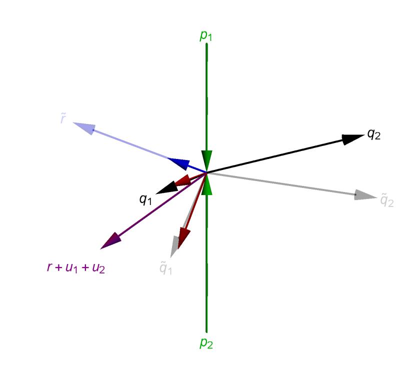

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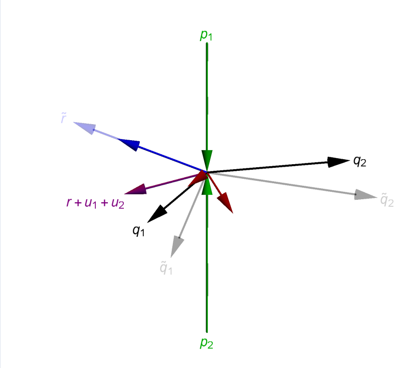

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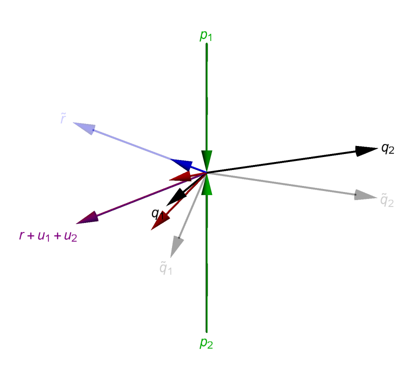

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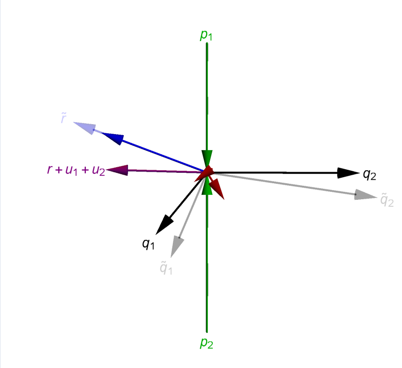

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Behaviour in singular limits

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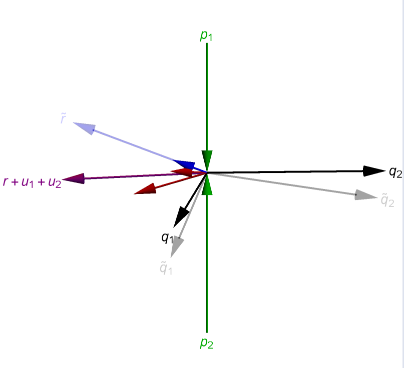

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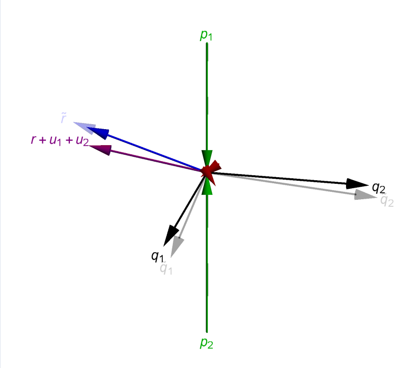

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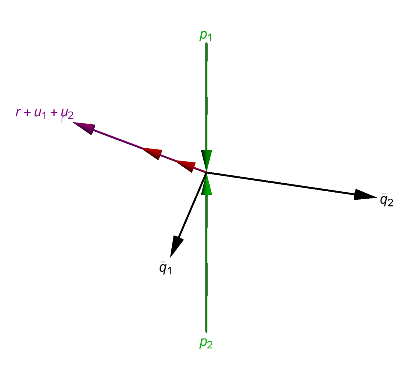

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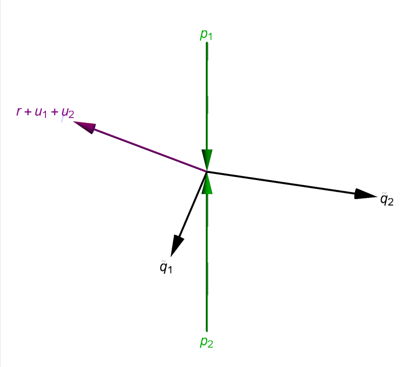

$\rightarrow$ Both double unresolved limits approach the Born configuration

Consequences

Features

- Minimal number of subtraction kinematics
- Only one DU configuration
  $\rightarrow$ pole cancellation for each Born phase space point
- Expected improved convergence of invariant mass distributions, since $\tilde{q}^2 = q^2$

Unintentional features

- Construction in lab frame
- Original construction of 't Hooft Veltman corrections [Czakon,Heymes'14] is spoiled

4 dimensional formulation

't Hooft Veltman scheme

Treat resolved particles in 4 dimensions (momenta and polarisations)

- Avoid unnecessary ϵ-orders of the matrix elements
- Avoid growth of dimensionality of phase space integrals

Make resolved phase space 4-dim. using measurement function, e.g.

$$F_n \to F_n \mathcal{N}^{-(n-1)\epsilon} \prod_{i=1}^{n-1} \delta^{(-2\epsilon)}(q_i)$$

Finite parts: σ_F^{RR} σ_F^{RV} σ_F^{VV}

Finite remainder parts: $\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}$

Single (SU) and double (DU) unresolved parts:

$\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}$ $\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{C1}, \sigma_{DU}^{VV}, \sigma_{DU}^{C2}$

$\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}$ $\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{C1}, \sigma_{DU}^{VV}, \sigma_{DU}^{C2}$

't Hooft Veltman scheme

Goal: Make SU and DU separately finite

Idea: Move "divergent parts" of SU to DU before applying 'tHV scheme

- SU contribution: $\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1}$ with

$$\sigma_{SU}^{c} = \int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{c}} F_{n+1} + I_{n}^{\mathrm{c}} F_{n} \right)$$

- We know: NLO cross section is finite
  $\rightarrow$ F_{n+1} part of SU is finite: Poles cancel between RR, RV and C1 (with NLO measurement function)
- With NNLO measurement function: Additional poles arise
  $\rightarrow$ SU no longer finite by itself
- Non-cancelling ϵ poles are generated by terms with F_n
  $\rightarrow$ can be moved to DU

$\rightarrow$ Task: Identify non-cancelling parts of SU

't Hooft Veltman scheme

Task: Identify non-cancelling parts of SU

- Use parametrised measurement functions

$$F_{n+1}^{\alpha} = F_{n+1}\,\theta\left(\min_{i,j}\eta_{ij} - \alpha\right)\,\theta\left(\min_{i}\frac{u_i^0}{E_{\text{norm}}} - \alpha\right)$$

- Construct:

$$\sigma_{\text{SU}}^{c} - \mathcal{I}_c^{\alpha} = \int \mathrm{d}\Phi_{n+1}\left(I_{n+1}F_{n+1} + I_nF_n - [I_{n+1}]_{1/\epsilon^2,1/\epsilon}F_{n+1}^{\alpha}\right)$$

- Rearrangements allow to extract the non-cancelling part:

$$N^c(\alpha) = \int \mathrm{d}\Phi_{n+1}\,[I_n]_{1/\epsilon^2,1/\epsilon}F_n\,\theta_\alpha$$

- Analytically extract divergences ($\ln^k\alpha$) and cancel them exactly
- Take limit $\alpha \to 0$ to remove dependence on α
- Subtract from σ_{SU} and add to σ_{DU}
  $\to$ separately finite SU and DU contributions
  $\to$ ready for application of 'tHV scheme

Looks like slicing, but it is slicing *only* for divergences
$\to$ no actual slicing parameter in the result

C++ implementation of STRIPPER

C++ implementation of STRIPPER

Features of the implementation

- General subtraction framework
  - Provides a general set of subtraction terms
  - Tree-level amplitudes are calculated automatically using a Fortran library [van Hameren '09][Bury, van Hameren '15]
  - User has to provide the 1- and 2-loop amplitudes
- Separate evaluation of coefficients of scales and PDFs
  $\rightarrow$ Cheaper calculations with several scales and PDFs
- FastNLO interface
  - Allows to produce tables for fast fits
  - FastNLO tables for $t\bar{t}$ differential distributions released this spring [Czakon, Heymes, Mitov '17]

Example: Driver program

```
// define initial state and (multiple) PDFs
InitialState initial("p","p",Ecms,Emin);
initial.include(LHAPDFsetName);
// define (multiple) scales
ScalesList scales(FixedScales(mt,mt,"muR = mt, muF = mt"));
scales.include(DynamicalScalesHT4(1.,1.));
// set up observables to be calculated
Measurement measurement;
measurement.include(TransverseMomentum({"t"}),
                    {{Histogram::bins(40,0.,2000.)}});
// initialise MC generator and specify contribution to calculate
Generator generator(incoming,scales,measurement);
generator.include({{"g","g"},{"t","t~","g","g"}},2,2,0,0,false);
// run integration with 10^6 points
generator.run(1000000);
// write results
ofstream xml("ttbar.xml");
generator.measurement().print(xml);
xml.close();
```

Summary - STRIPPER

Summary

- Minimizing the STRIPPER scheme
- alternative phase space parameterization
- new formulation of 't Hooft Veltman scheme
- tests for a class of processes:
  $pp \to t\bar{t}$, $e^+e^- \to 2, 3j$, t decay, DIS, Drell-Yan, H decays, dijets

Supplements

Factorization and subtraction terms

Single unresolved phase space:

$$\iint_0^1 \mathrm{d}\eta\, \mathrm{d}\xi\; \eta^{a_1 - b_1 \epsilon} \xi^{a_2 - b_2 \epsilon}$$

Double unresolved phase space:

$$\iiiint_0^1 \mathrm{d}\eta_1\, \mathrm{d}\xi_1\, \mathrm{d}\eta_2\, \mathrm{d}\xi_2\; \eta_1^{a_1 - b_1 \epsilon} \xi_1^{a_2 - b_2 \epsilon} \eta_2^{a_3 - b_3 \epsilon} \xi_2^{a_4 - b_4 \epsilon}$$

Factorized singular limits:

$$\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}} \tilde{\mu}(\{x_i\})$$

$$\underbrace{\prod x_i^{a_i+1} \langle \mathcal{M}_{n+2} | \mathcal{M}_{n+2} \rangle}_{\text{regular}} \mathrm{F}_{n+2}$$

Regularisation:

Master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 \mathrm{d}x \left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x) - f(0)}{x^{1+b\epsilon}}$$

Calculation of $N_0^c(0)$

For each sector/contribution:

1. extraction of $\mathrm{d}\Phi_{n+1}$ from $\mathrm{d}\Phi_{n+2}\,|_{\text{SU pole}}$ (only for RR contribution)

$$\mathrm{d}\Phi_{n+2}\,|_{\text{SU pole}} = \Big(\underbrace{\mathrm{d}\Phi_n\,\mathrm{d}^d\mu(u_1)}_{\mathrm{d}\Phi_{n+1}}\mathrm{d}^d\mu(u_2)\Big)\Big|_{u_2\text{col/soft}}$$

2. expansion in ϵ up to ϵ^{-1} (except $\mathrm{d}\Phi_{n+1}$): $\mathrm{d}^d\Phi_{n+1}\left(\frac{\cdots}{\epsilon^2}+\frac{\cdots}{\epsilon}\right)$
3. Identifying $\ln^k(\alpha)$'s from x_i integrations over Θ function

$$\Theta_\alpha(\hat{\eta},u^0) = \Theta(\hat{\eta}-\alpha)\Theta(\hat{\xi}u_{max}/E_{norm}-\alpha)$$

   $\rightarrow$ discard them
4. perform integration over Θ-functions of non-canceling and non-vanishing (in $\alpha \rightarrow 0$ limit) terms

PS

common starting point for all phase spaces :

$$\mathrm{d}\Phi_n = \mathrm{d}Q^2 \left[\prod_{j=1}^{n_{fr}} \mu_0(r_j) \prod_{k=1}^{n_u} \mu_0(u_k) \delta_+ \left(\left(P - \sum_{j=1}^{n_{fr}} r_j - \sum_{k=1}^{n_u} u_k\right)^2 - Q^2\right)\right] \prod_{i=1}^{n_q} \mu_{m_i}(q_i)(2\pi)^d \delta^{(d)}\left(\sum_{i=1}^{n_q} q_i - q\right)$$

$$\text{with } \mu_m(k) \equiv \frac{\mathrm{d}^d k}{(2\pi)^d} 2\pi\delta\left(k^2 - m^2\right)\theta\left(k^0\right),$$

n : # final state particles, n_{fr} : # final state references, n_u : # additional partons