Towards top-quark pair production and decay at NNLO QCD

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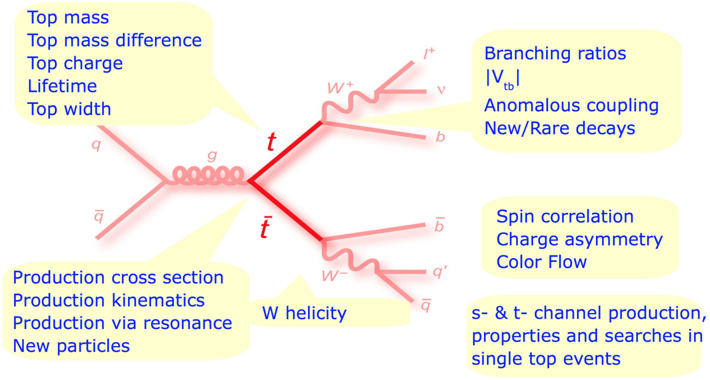
Introduction

Why top-quark physics?



Theoretical developments

Stable onshell tops and spin summed:

- Total inclusive cross sections @ NNLO+NNLL accuracy

  [Czakon, Fiedler, Mitov '13]

- Fully differential distributions @ NNLO

  [Czakon, Fiedler, Heymes, Mitov '16]

- + EW corrections

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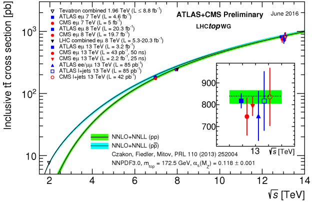
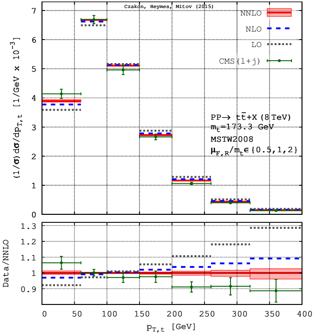
Unstable tops + spin correlations:

- Approximate NNLO + NNLO decay

  [Gao,Papanastasiou '17]





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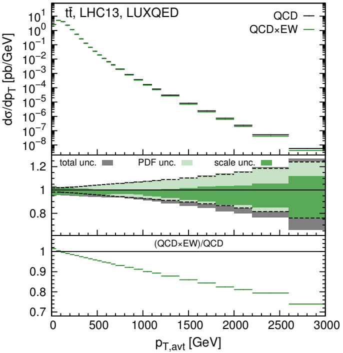

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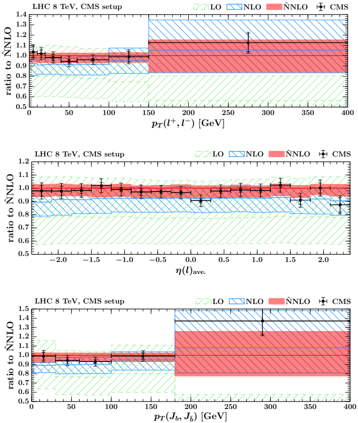
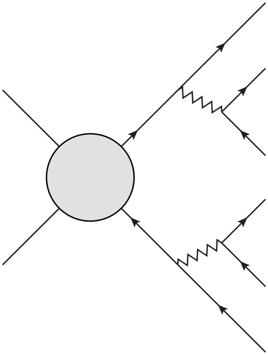


Goal: $t\bar{t}$ production and decay at NNLO QCD

Narrow-Width-Approximation

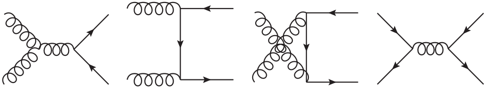
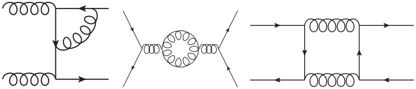
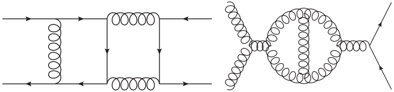
- On-shell top-quarks
- Factorization of top-decay
- Separations of QCD corrections
- Keep spin correlations



$\rightarrow$ polarised $t\bar{t}$-production amplitudes

Polarised $t\bar{t}$ production amplitudes

$t\bar{t}$ production amplitudes

<table>
<tr><th colspan="2">Contributions to $\mathcal{M}(gg(q\bar{q}) \to \bar{t}t)$:</th><th>$gg|q\bar{q}$</th></tr>
<tr><td>LO</td><td></td><td>3|1</td></tr>
<tr><td>NLO</td><td> . . .</td><td>33|18</td></tr>
<tr><td>NNLO</td><td> . . .</td><td>726|190</td></tr>
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Decomposition into color- and Lorentz-structures $\rightarrow$ full color- and spin information

Lorentz structures

Gluon channel

$$\mathcal{M} = \epsilon_{1\mu}(p_1)\epsilon_{2\nu}(p_2)M^{\mu\nu}$$

$M^{\mu\nu}$ is a rank-2 Lorentz tensor

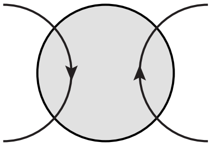
- Momentum conservation
- Transversality
- Equation of motion
- Parity conservation $\rightarrow$ no γ_5

8 independent structures

($d = 4$ dimensions)

$$M^{\mu\nu} = \sum_{j=1}^{8} M_j T_j^{\mu\nu}$$

Quark channel



- Two disconnected fermion lines
- Connection by gluons+loops

4 independent structures

$$\mathcal{M} = \sum_{i=1}^{4} M_j T_j$$

with $T_j \sim \bar{v}_2 \Gamma_j u_1 \bar{u}_3 \Gamma'_j v_4$

Color structures

Color decomposition: $\mathcal{M} = \sum_{ij} c_{ij} C_i M_j$

Gluon channel
color representations

- Gluons: a, b adjoint
- Quarks: c, d fundamental

$$C_1 = (T^a T^b)_{cd}$$
$$C_2 = (T^b T^a)_{cd}$$
$$C_3 = \mathrm{Tr}\left\{T^a T^b\right\} \delta_{cd}$$

Quark channel
color representations

- Quarks: a, b fundamental
- Quarks: c, d fundamental

$$C_1 = \delta_{ac}\delta_{bd}$$
$$C_2 = \delta_{ab}\delta_{cd}$$

Projection

Construct projectors: $P_j = \sum_l B_{jl} (T_l)^\dagger$

Extracting the B_{jl}:

$$\sum_{spin/pol,col} P_j \mathcal{A} \stackrel{!}{=} A_j$$

leads to system of equations

$$\sum_{l,k} B_{jl} A_k \sum_{spin/pol,col} (T_l)^\dagger T_k = A_j$$

Inversion $\rightarrow$ coefficients B_{jl}

Short summary

$$\mathcal{M} = \sum_{ij} c_{ij} C_i M_j$$

- Gluon: 3(color) $\cdot$ 8(spin)
  Quark: 2(color) $\cdot$ 4(spin)
  $\rightarrow$ combined 32 structures
- Scalar coefficients c_{ij}:
  - Rational function of $m_s = m_t^2/s$, $x = t/s$ and ϵ
  - Scalar Feynman integrals

Evaluation of coefficients

Integration by parts identities (IBP)

$$\int \mathrm{d}^d k_1 \mathrm{d}^d k_2 \frac{\partial}{\partial k_j^\mu} \left(p_l \prod \frac{(p \cdot k)_i^{b_i}}{(q_i^2 + m_i^2)^{a_i}} \right)$$

$\mathcal{O}(10^4)$ scalar Feynman integrals
$\rightarrow$ 422 master integrals

Master integrals

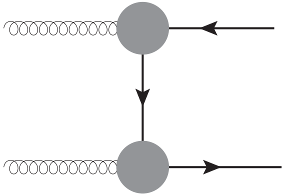
- Partially canonicalized new

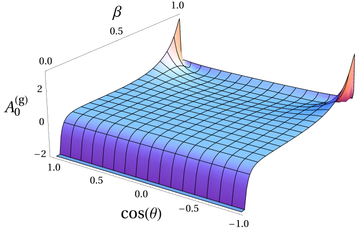
analogous to [Czakon '08],[Czakon,Fiedler,Mitov '13]

- Differential equations generated by IBPs
- High energy expansion as boundary condition
- Numerical integration for 'bulk' region
  $\rightarrow$ Interpolation grid
- Threshold expansion for $\beta = \sqrt{1 - 4m_s} \rightarrow 0$

Master integrals

t channel diagrams





t-channel diagrams

- Divergences for $\cos\Theta \to \pm 1$ for high energies
  $\to$ complicates numerical integration
- Improve numerical stability by canonicalization of involved masters
- $\epsilon - \mathrm{d}\log$ form $\to$ expect more stable numerical evaluation

Partial canonical basis for master integrals

Idea:

- Perform rational basis transformation $\vec{f} = \hat{T}(\epsilon, \vec{x})\vec{f}_{\text{old}}$ such that DEQs have the form

$$\mathrm{d}\vec{f} = \epsilon\, \mathrm{d}\hat{A}(\vec{x})\, \vec{f}$$

- Simple formal solution: $\vec{f} = \exp\left(\epsilon \int \mathrm{d}\hat{A}\right) \vec{f}_0$

Top-pair case: 422 masters

- Canonical basis for subset of master integrals:
  - No elliptic integrals involved
  - No transformation of kinematic variables needed

$\rightarrow$ 65 masters directly canonicalizable

- Using CANONICA [Meyer '16,'17]

Differential equations for master integrals

- Differential equations with respect to m_s and x:

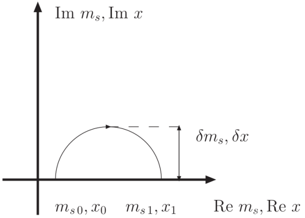
$$m_s \frac{\mathrm{d}}{\mathrm{d}m_s} I_i = \sum c_k I_k$$
$$x \frac{\mathrm{d}}{\mathrm{d}x} I_i = \sum d_k I_k$$

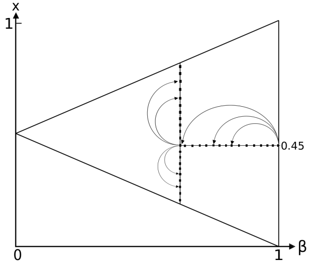
Boundaries

- Expansion around the high energy limit $m_s = \frac{m_t^2}{s} \to 0$
- Using Mellin-Barns representations and a lot of handwork to extract a series in ϵ and m_s for each integral
- Expanding the differential equations in ϵ
- deep power-log expansions in m_s for fixed x
  $\to$ algebraic system

Numerical evaluation of master integrals

- Using the differential equations to integrate numerically from the pre-calculated boundary conditions
- Leaving the real numbers and integrate in a complex plane to grid points





The grid

Choice of points:

- $\beta = \sqrt{1 - 4m_s} = i/80$ for $i = 1, ..., 79$
- 42 points for x: Gauss-Kronrod points in available phase-space

Threshold expansion

- Express DEQs in $\beta = \sqrt{1 - 4m_s}$
- Solve with pow-log ansatz for fixed $x = x_0$ (points given by interpolation grid)

$$I_i(\beta, x_0) = \sum \sum c_{imn} \beta^n \ln^m \beta$$

$n \in [-15, 51], m \in [0, 8]$

- Constrains by DEQ $\rightarrow$ eliminates most of the coefficients
- Match free coefficients to result from numerical integration

Finite remainder function

IR divergences and the finite remainder function

$$|\mathcal{M}_n\rangle = \mathbf{Z}(\epsilon, \{p_i\}, \{m_i\}, \mu_R)\,|\mathcal{F}\rangle$$

- Complete factorization of IR structure → **Z** operator

$$\left|\mathcal{M}_n^{(0)}\right\rangle = \left|\mathcal{F}_n^{(0)}\right\rangle$$
$$\left|\mathcal{M}_n^{(1)}\right\rangle = \mathbf{Z}^{(1)}\left|\mathcal{M}_n^{(0)}\right\rangle + \left|\mathcal{F}_n^{(1)}\right\rangle$$
$$\left|\mathcal{M}_n^{(2)}\right\rangle = \mathbf{Z}^{(2)}\left|\mathcal{M}_n^{(0)}\right\rangle + \mathbf{Z}^{(1)}\left|\mathcal{F}_n^{(1)}\right\rangle + \left|\mathcal{F}_n^{(2)}\right\rangle$$

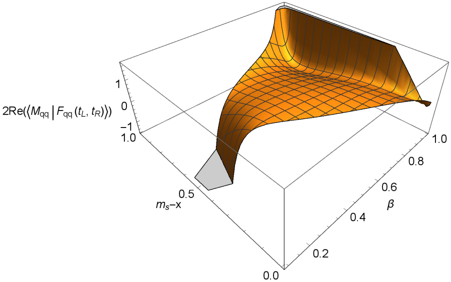
- **Z** can be calculated by its anomalous dimension equation

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$$

- Depends on kinematics and operator on color space
→ Projection on color and spin structures

Finite remainder for polarised tops

2-Loop finite remainder for:
$q\bar{q} \to t_L \bar{t}_R$



2-Loop finite remainder for:
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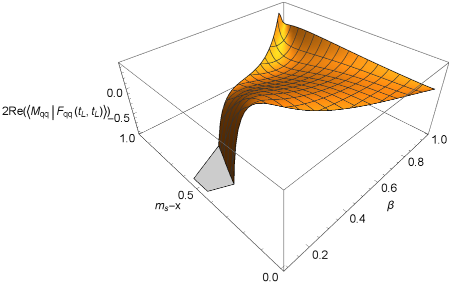


(work in progress)

Summary of progress

Finished

- Projection LO, NLO, NNLO amplitudes
- Finite remainder of all coefficients
- Improved set of master integrals
- Kinematical expansions of coefficients
- Implementation of coefficients, color and spin structures in STRIPPER

Outlook

- Usage of amplitudes within STRIPPER
  $\leftarrow$ Talk by Arnd Behring
- Implementation of decay phase-space and handling of decay products in STRIPPER
- QCD-corrections to decay

Backup

STRIPPER – SecToR Improved Phase sPacE for real Radiation

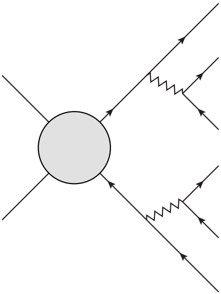
The subtraction scheme

- Method of evaluate the double-real emission radiation contribution to NNLO processes
- Decomposition of the phase-space to factorize the singular limits of the amplitude
- Suitable parameterizations to derive (integrated) subtraction terms

The NNLO event generator

- automated up to small driver program
- fully differential event generation
- several scales simultaneously
- different pdfs simultaneously
- stable tops
- pre-decided binned distributions

Narrow-Width-Approximation



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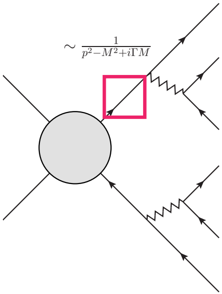

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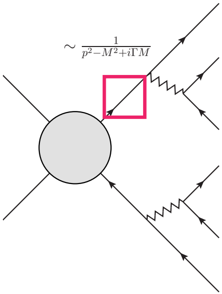


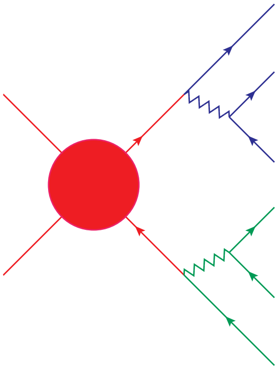
- enters matrix element as:
  $\sim \frac{1}{(p^2-m^2)^2+m^2\Gamma^2}$
- For cross sections: Integration over phase-space
- $+$ limit $\Gamma/m \to 0$:

$$\frac{1}{(p^2 - m^2)^2 + m^2\Gamma^2} \to \frac{2\pi}{2m\Gamma}\delta(p^2 - m^2)$$

- On amplitude level:

$$\mathcal{M} = \mathcal{M}_{\text{NWA}} + \mathcal{O}\left(\frac{\Gamma}{m}\right)$$

Amplitude factorization



$$\mathcal{M} = \left(\tilde{A}(t \to bl^+\nu)\frac{i(\not{p}_t + m)}{p_t^2 - m^2 + im\Gamma_t}\right)\cdot$$
$$\tilde{A}(pp \to \bar{t}t)\cdot$$
$$\left(\frac{i(-\not{p}_{\bar{t}} + m)}{p_{\bar{t}}^2 - m^2 + im\Gamma_t}\tilde{A}(\bar{t} \to \bar{b}l^-\bar{\nu})\right)$$

Decay spinors

Narrow-Width-Approximation:

$$\frac{i(-\not{p}_{\bar{t}} + m)}{p_{\bar{t}}^2 - m^2 + im\Gamma_t}\tilde{A}(\bar{t} \to \bar{b}l^-\bar{\nu}) \to \frac{i(-\not{p}_{\bar{t}} + m)}{\sqrt{2m\Gamma_t}}\tilde{A}(\bar{t} \to \bar{b}l^-\bar{\nu})$$

Decay spinors

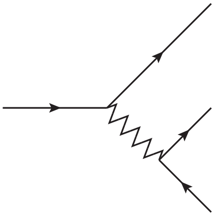
$$\bar{U}(p_t) = \tilde{A}(t \to bl^+\nu)\frac{i(\not{p}_t + m)}{\sqrt{2m\Gamma_t}}$$

$$V(p_{\bar{t}}) = \frac{i(-\not{p}_{\bar{t}} + m)}{\sqrt{2m\Gamma_t}}\tilde{A}(\bar{t} \to \bar{b}l^-\bar{\nu})$$

Amplitude:

$$\mathcal{M} = \bar{U}(p_t)\tilde{A}(pp \to \bar{t}t)V(p_{\bar{t}}) + \mathcal{O}\left(\frac{\Gamma_t}{m}\right)$$

QCD corrections to decay



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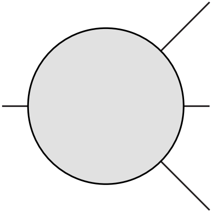

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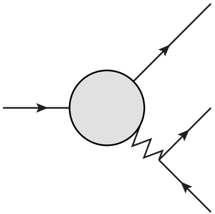


Simplification II

restrict to leptonic top decays

- Vertex corrections (for massless final state):

$$\Gamma^\mu = \frac{g}{\sqrt{2}} \{ \gamma^\mu \left[F_{1L} P_L + F_{1R} P_R \right] + \frac{i\sigma^{\mu\nu} q_\nu}{2m_t} \left[F_{2L} P_R + F_{2R} P_L \right] \}$$

- times W propagator and decay vertex

$$\bar{u}(p_\nu) \frac{i g_W}{\sqrt{2}} \gamma^\nu \frac{(1-\gamma_5)}{2} v(p_{l^+}) \cdot \frac{-i(g_{\nu\mu} - \frac{q_\nu q_\mu}{q^2})}{q^2 - m_W^2 + i\Gamma_W m_W} \bar{u}(p_b) i\Gamma^\mu$$

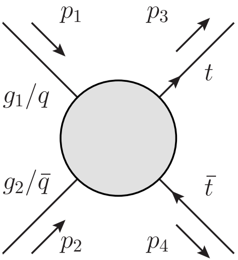
Contributions to amplitude

Production \ Decay	LO	NLO	NNLO
LO			
NLO			
NNLO			

Contributions to amplitude

Production \ Decay	LO	NLO	NNLO
LO			
NLO			
NNLO			

Kinematics and polarization



External Momenta

$$p_1^2 = p_2^2 = 0$$
$$p_3^2 = p_4^2 = m^2$$

Mandelstamm variables

$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_2 - p_3)^2$$
$$s + t + u = 2m^2$$

Polarization sum external gluons (axial gauge)

$$\sum_{\text{pol}} \epsilon^*_{i\mu}\epsilon_{i\nu} =$$
$$-g_{\mu\nu} + \frac{n_{i\mu}p_{i\nu} + n_{i\nu}p_{i\mu}}{n_i \cdot p_i}$$

Equation of motion for external (anti)quarks

$$\left(\not{p} - m\right) U = 0$$
$$\left(\not{p} + m\right) V = 0$$

IBP reduction

General two-loop integral:

$$\int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \frac{\mathrm{d}^d l_2}{(2\pi)^d} \prod_i \frac{1}{D_i^{n_i}} \prod_j N_j^{n_j}$$

with $D_i = (\sum p + \sum l)^2 - m^2$ and $N_i = l \cdot p$

Basic Idea of Integration-By-Part (IBP) reduction:

$$\int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \frac{\mathrm{d}^d l_2}{(2\pi)^d} \frac{\partial}{\partial q^\mu} q^\mu I\left(l_1, l_2, \{p_{ext}\}\right) = 0 \text{ with} q = l_1, l_2, \{p_{ext}\}$$

- Relations between different integrals
  $\Rightarrow$ Relate difficult integrals to easy ones
- Reduction to set of master integrals

UV renormalization and decoupling

$$\left|M_{g,q}(\alpha_S^0, m^0, \epsilon)\right\rangle = 4\pi\alpha_S^0 \left[\left|M_{g,q}^{(0)}(m^0,\epsilon)\right\rangle + \left(\frac{\alpha_S^0}{2\pi}\right)\left|M_{g,q}^{(1)}(m^0,\epsilon)\right\rangle + \left(\frac{\alpha_S^0}{2\pi}\right)^2\left|M_{g,q}^{(2)}(m^0,\epsilon)\right\rangle\right]$$

UV-renormalized amplitude:

$$\left|\mathcal{M}_{g,q}^R\left(\alpha_S^{(n_f)}(\mu), m, \mu, \epsilon\right)\right\rangle = \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{-2\epsilon} Z_{g,q} Z_Q \left|M_{g,q}(\alpha_S^0, m^0, \epsilon)\right\rangle$$

- Z_g, Z_q , Z_Q : onshell renormalization constants
- $m^0 = Z_m m$
- $\alpha_S^0 = \left(\frac{e^{\gamma_E}}{4\pi}\right)^{\epsilon}\mu^{2\epsilon} Z_{\alpha_S}^{(n_f)}\alpha_S^{(n_f)}(\mu)$ $\hat{=}$ $\overline{M}S$-scheme with n_f flavours

Decoupling

- $n_f = n_l + n_h$ is not feasible
- decouple the running of α_S from the n_h quarks
- $\alpha_S^{(n_f)} = \zeta_{\alpha_S}\alpha_S^{(n_l)}$