Polarised amplitudes for top quark pair production at NNLO

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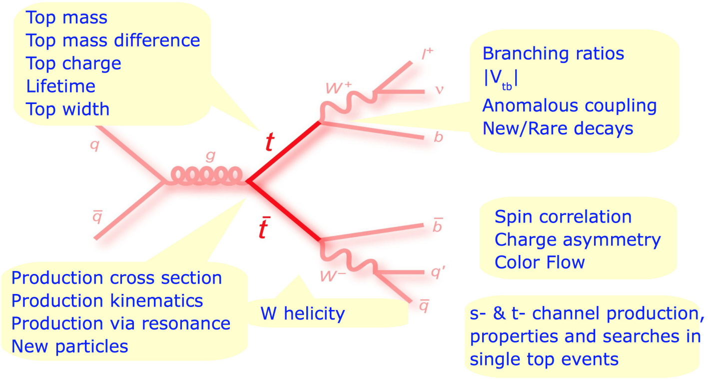
[1] in cooperation with M. Czakon

Introduction

Why is the top-quark (still) interesting?



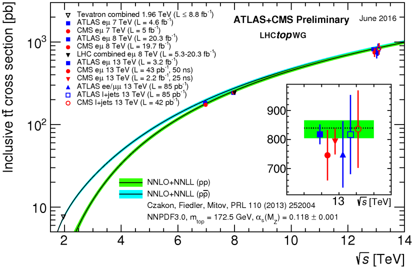
Theoretical developments

Stable onshell tops and spin summed:

- Total inclusive cross sections @ NNLO+NNLL accuracy [Czakon, Fiedler, Mitov '13]
- Fully differential distributions @ NNLO [Czakon, Fiedler, Heymes, Mitov '16]

Unstable tops + spin correlations:

- Off-shell effects with decays @ NLO + matched parton-shower [Bevilacqua, Hartanto, Kraus, Worek '16]



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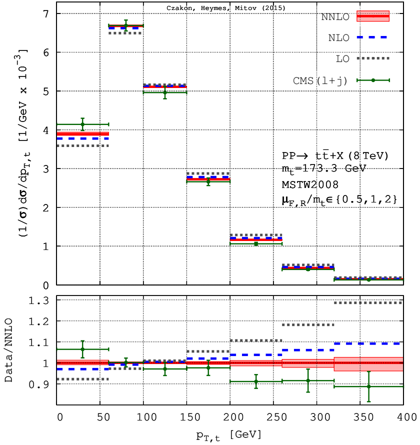

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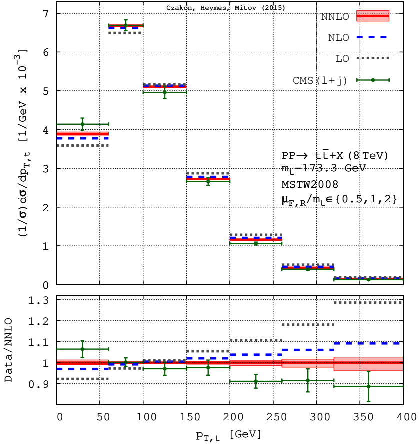
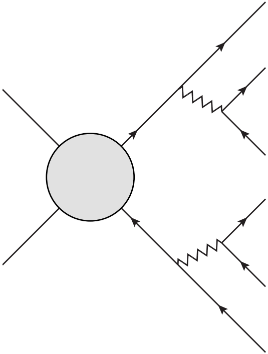


Goal: $t\bar{t}$ production and decay at NNLO QCD

Narrow-Width-Approximation

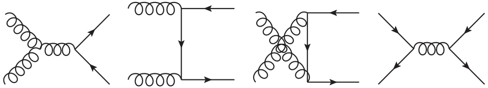
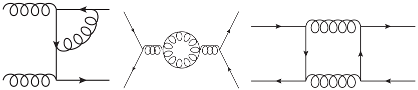
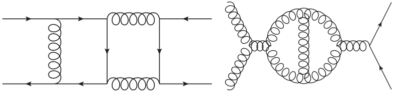
- On-shell top-quarks
- Factorization of top-decay
- Separations of QCD corrections
- Keep spin correlations



$\rightarrow$ polarised $t\bar{t}$-production amplitudes

Polarised $t\bar{t}$ production amplitudes

$t\bar{t}$ production amplitudes

Contributions to $\mathcal{M}(gg(q\bar{q}) \to \bar{t}t)$:		$gg \mid q\bar{q}$
LO		$3 \mid 1$
NLO	 . . .	$33 \mid 18$
NNLO	 . . .	$726 \mid 190$

Decomposition into color- and Lorentz-structures → full color- and spin information

Lorentz structures

Gluon channel

$$\mathcal{M} = \epsilon_{1\mu}(p_1)\epsilon_{2\nu}(p_2)M^{\mu\nu}$$

$M^{\mu\nu}$ is a rank-2 Lorentz tensor

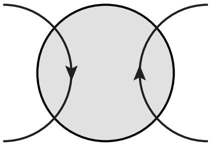
- Momentum conservation
- Transversality
- Equation of motion
- Parity conservation $\rightarrow$ no γ_5

10 independent structures

($d = 4 - 2\epsilon$ dimensions)

$$M^{\mu\nu} = \sum_{j=1}^{10} M_j T_j^{\mu\nu}$$

Quark channel



- Two disconnected fermion lines
- Connection by gluons+loops

7 independent structures

$$\tilde{M} = \sum_{i=1}^{7} M_j T_j$$

with $T_j \sim \bar{v}_2 \Gamma_j u_1 \bar{u}_3 \Gamma_j' v_4$

Color structures

Color decomposition: $\mathcal{M} = \sum_{ij} c_{ij} C_i M_j$

Gluon channel
color representations

- Gluons: a, b adjoint
- Quarks: c, d fundamental

$$C_1 = (T^a T^b)_{cd}$$
$$C_2 = (T^b T^a)_{cd}$$
$$C_3 = \mathrm{Tr}\left\{T^a T^b\right\} \delta_{cd}$$

Quark channel
color representations

- Quarks: a, b fundamental
- Quarks: c, d fundamental

$$C_1 = \delta_{ac}\delta_{bd}$$
$$C_2 = \delta_{ab}\delta_{cd}$$

Projection

Construct projectors: $P_j = \sum_l B_{jl} (T_l)^\dagger$

Extracting the B_{jl}:

$$\sum_{spin/pol/col} P_j \mathcal{A} \stackrel{!}{=} A_j$$

leads to system of equations

$$\sum_{l,k} B_{jl} A_k \sum_{spin/pol/col} (T_l)^\dagger T_k = A_j$$

Inversion $\rightarrow$ coefficients B_{jl}

Short summary

$$\mathcal{M} = \sum_{ij} c_{ij} C_i M_j$$

- Gluon: 3(color) $\cdot$ 10(spin)
  Quark: 2(color) $\cdot$ 7(spin)
  $\rightarrow$ 44 combined structures
- Scalar coefficients:
  - Rational function of $m_s = m_t^2/s$, $x = -t/s$ and ϵ
  - Scalar Feynman integrals

Evaluation of coefficients

Integration by parts identities (IBP)

$$\int \mathrm{d}^d k_1 \mathrm{d}^d k_2 \frac{\partial}{\partial k_j^\mu} \left(p_l \prod \frac{(p \cdot k)_i^{b_i}}{(q_i^2 + m_i^2)^{a_i}} \right)$$

$\mathcal{O}(10^4)$ scalar Feynman integrals
$\rightarrow$ 422 master integrals

Master integrals

- Differential equations generated by IBPs
- Pre-calculated boundary conditions (high energy limit)
- Numerical integration
  $\rightarrow$ Interpolation grid

Use of already existing master integrals

IR divergences and the finite remainder function

$$|\mathcal{M}_n\rangle = \mathbf{Z}(\epsilon, \{p_i\}, \{m_i\}, \mu_R)\, |\mathcal{F}\rangle$$

- Complete factorization of IR structure → $\mathbf{Z}$ operator

$$\begin{aligned}
\left|\mathcal{M}_n^{(0)}\right\rangle =& \left|\mathcal{F}_n^{(0)}\right\rangle \\
\left|\mathcal{M}_n^{(1)}\right\rangle =& \mathbf{Z}^{(1)} \left|\mathcal{M}_n^{(0)}\right\rangle + \left|\mathcal{F}_n^{(1)}\right\rangle \\
\left|\mathcal{M}_n^{(2)}\right\rangle =& \mathbf{Z}^{(2)} \left|\mathcal{M}_n^{(0)}\right\rangle \\
&+ \mathbf{Z}^{(1)} \left|\mathcal{F}_n^{(1)}\right\rangle + \left|\mathcal{F}_n^{(2)}\right\rangle
\end{aligned}$$

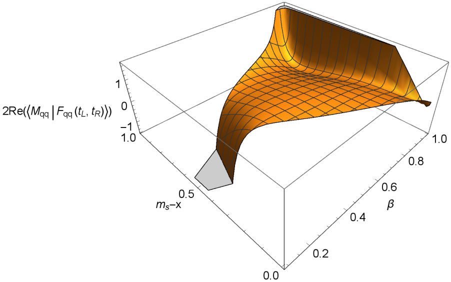
- $\mathbf{Z}$ can be calculated by its anomalous dimension equation

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{Z} = -\mathbf{\Gamma}\mathbf{Z}$$

- Depends on kinematics and operator on color space
→ Projection on color and spin structures

Finite remainder for polarised tops

2-Loop finite remainder for:
$q\bar{q} \to t_L \bar{t}_R$



2-Loop finite remainder for:
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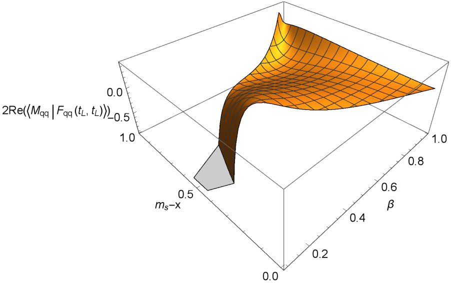


(work in progress)

Summary of progress

Finished

- Projection LO, NLO, NNLO amplitudes
- Finite remainder of all coefficients
- Implementation of coefficients, color and spin structures in STRIPPER

Outlook

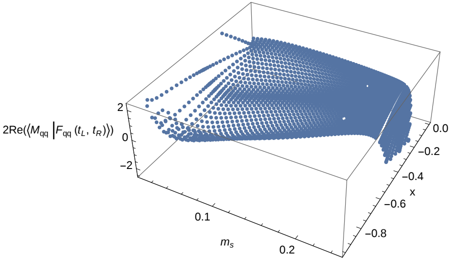
- Implementation of decay phase-space and handling of decay products in STRIPPER
- Merging with the results from Mitov et al.
  $\rightarrow$ QCD corrections to decay

Thank you for your attention.

Backup

Finite remainder for polarised tops – sampling points

2-Loop finite remainder for:
$q\bar{q} \to t_L \bar{t}_R$



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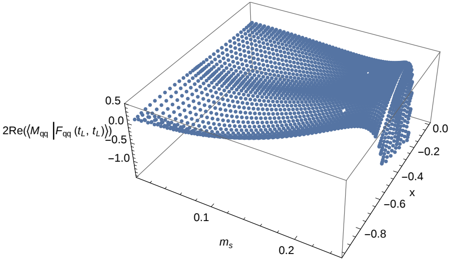


(work in progress)

STRIPPER – SecToR Improved Phase sPacE for real Radiation

The subtraction scheme

- Method of evaluate the double-real emission radiation contribution to NNLO processes
- Decomposition of the phase-space to factorize the singular limits of the amplitude
- Suitable parameterizations to derive (integrated) subtraction terms

The NNLO event generator

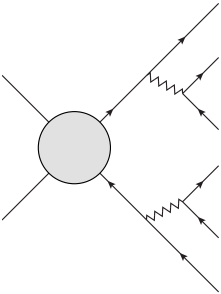
- fully differential event generation
- several scales simultaneously
- different pdfs simultaneously
- stable tops
- predecided binned distributions
- fixed top mass=173.3

Lorentzstructures

$$T_1^{\mu\nu} = m\bar{u}_3\gamma^\mu\gamma^\nu v_4$$
$$T_2^{\mu\nu} = \bar{u}_3\gamma^\mu v_4 p_3^\nu$$
$$T_3^{\mu\nu} = \bar{u}_3\gamma^\nu v_4 p_3^\mu$$
$$T_4^{\mu\nu} = m\bar{u}_3 v_4 g^{\mu\nu}$$
$$T_5^{\mu\nu} = m^{-1}\bar{u}_3 v_4 p_3^\mu p_3^\nu$$
$$T_6^{\mu\nu} = m^{-2}\bar{u}_3 \not{p}_1 v_4 p_3^\mu p_3^\nu$$
$$T_7^{\mu\nu} = \bar{u}_3 \not{p}_1 v_4 g^{\mu\nu}$$
$$T_8^{\mu\nu} = m^{-1}\bar{u}_3 \not{p}_1 \gamma^\nu v_4 p_3^\mu$$
$$T_9^{\mu\nu} = m^{-1}\bar{u}_3 \not{p}_1 \gamma^\mu v_4 p_3^\nu$$
$$T_{10}^{\mu\nu} = \bar{u}_3 \not{p}_1 \gamma^\mu\gamma^\nu v_4$$

$$T_1 = m^{-1}\bar{v}_2 \not{p}_3 u_1 \bar{u}_3 v_4$$
$$T_2 = m^{-2}\bar{v}_2 \not{p}_3 u_1 \bar{u}_3 \not{p}_1 v_4$$
$$T_3 = \bar{v}_2\gamma^\mu u_1 \bar{u}_3 \gamma_\mu v_4$$
$$T_4 = \bar{v}_2\gamma^\mu u_1 \bar{u}_3 \not{p}_1 \gamma_\mu v_4$$
$$T_5 = m^{-1}\bar{v}_2 \not{p}_3 \gamma^\mu\gamma^\nu u_1 \bar{u}_3 \gamma_\mu\gamma_\nu v_4$$
$$T_6 = m^{-2}\bar{v}_2 \not{p}_3 \gamma^\mu\gamma^\nu u_1 \bar{u}_3 \not{p}_1 \gamma_\mu\gamma_\nu v_4$$
$$T_7 = \bar{v}_2\gamma^\mu\gamma^\nu\gamma^\rho u_1 \bar{u}_3 \not{p}_1 \gamma_\mu\gamma_\nu\gamma_\rho v_4$$

Narrow-Width-Approximation



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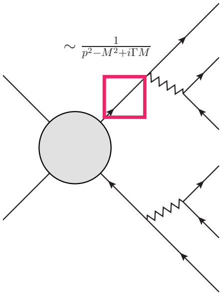

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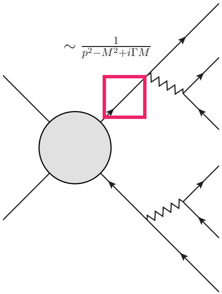


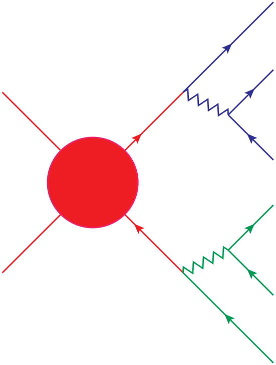
- enters matrix element as:
  $\sim \frac{1}{(p^2-m^2)^2+m^2\Gamma^2}$
- For crosssections: Integration over phase-space
- $+$ limit $\Gamma/m \to 0$:

$$\frac{1}{(p^2-m^2)^2+m^2\Gamma^2} \to \frac{2\pi}{2m\Gamma}\delta(p^2-m^2)$$

- On amplitude level:

$$\mathcal{M} = \mathcal{M}_{\mathrm{NWA}} + \mathcal{O}\left(\frac{\Gamma}{m}\right)$$

Amplitude factorization



$$\mathcal{M} = \left(\tilde{A}(t \to b l^+ \nu) \frac{i(\not{p}_t + m)}{p_t^2 - m^2 + im\Gamma_t} \right) \cdot$$
$$\tilde{A}(pp \to \bar{t}t) \cdot$$
$$\left(\frac{i(-\not{p}_{\bar{t}} + m)}{p_{\bar{t}}^2 - m^2 + im\Gamma_t} \tilde{A}(\bar{t} \to \bar{b} l^- \bar{\nu}) \right)$$

Decay spinors

Narrow-Width-Approximation:

$$\frac{i(-\not{p}_{\bar{t}} + m)}{p_{\bar{t}}^2 - m^2 + im\Gamma_t} \tilde{A}(\bar{t} \to \bar{b} l^- \bar{\nu}) \to \frac{i(-\not{p}_{\bar{t}} + m)}{\sqrt{2m\Gamma_t}} \tilde{A}(\bar{t} \to \bar{b} l^- \bar{\nu})$$

Decay spinors

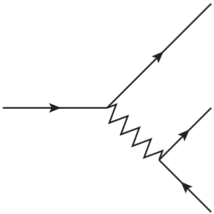
$$\bar{U}(p_t) = \tilde{A}(t \to b l^+ \nu) \frac{i(\not{p}_t + m)}{\sqrt{2m\Gamma_t}}$$

$$V(p_{\bar{t}}) = \frac{i(-\not{p}_{\bar{t}} + m)}{\sqrt{2m\Gamma_t}} \tilde{A}(\bar{t} \to \bar{b} l^- \bar{\nu})$$

Amplitude:

$$\mathcal{M} = \bar{U}(p_t) \tilde{A}(pp \to \bar{t} t) V(p_{\bar{t}}) + \mathcal{O}\left(\frac{\Gamma_t}{m}\right)$$

QCD corrections to decay



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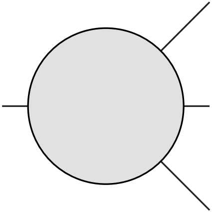

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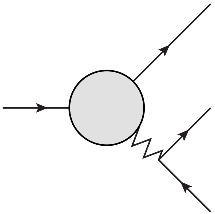


Simplification II

restrict to leptonic top decays

- Vertex corrections (for massless final state):

$$\Gamma^\mu = \frac{g}{\sqrt{2}} \left\{ \gamma^\mu \left[F_{1L} P_L + F_{1R} P_R \right] + \frac{i\sigma^{\mu\nu} q_\nu}{2 m_t} \left[F_{2L} P_R + F_{2R} P_L \right] \right\}$$

- times W propagator and decay vertex

$$\bar{u}(p_\nu) \frac{i g_W}{\sqrt{2}} \gamma^\nu \frac{(1-\gamma_5)}{2} v(p_{l^+}) \cdot \frac{-i\left(g_{\nu\mu} - \frac{q_\nu q_\mu}{q^2}\right)}{q^2 - m_W^2 + i\Gamma_W m_W} \bar{u}(p_b) i \Gamma^\mu$$

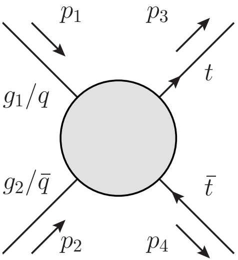
Contributions to amplitude

Production \ Decay	LO	NLO	NNLO
LO			
NLO			
NNLO			

Contributions to amplitude

Production \ Decay	LO	NLO	NNLO
LO			
NLO			
NNLO			

Kinematics and polarization



External Momenta

$$p_1^2 = p_2^2 = 0$$
$$p_3^2 = p_4^2 = m^2$$

Mandelstamm variables

$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_2 - p_3)^2$$
$$s+t+u = 2m^2$$

Polarization sum external gluons (axial gauge)

$$\sum_{\text{pol}} \epsilon^*_{i\mu}\epsilon_{i\nu} =$$
$$-g_{\mu\nu} + \frac{n_{i\mu}p_{i\nu} + n_{i\nu}p_{i\mu}}{n_i \cdot p_i}$$

Equation of motion for external (anti)quarks

$$\left(\not{p} - m\right) U = 0$$
$$\left(\not{p} + m\right) V = 0$$

IBP reduction

General two-loop integral:

$$\int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \frac{\mathrm{d}^d l_2}{(2\pi)^d} \prod_i \frac{1}{D_i^{n_i}} \prod_j N_j^{n_j}$$

with $D_i = (\sum p + \sum l)^2 - m^2$ and $N_i = l \cdot p$

Basic Idea of Integration-By-Part (IBP) reduction:

$$\int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \frac{\mathrm{d}^d l_2}{(2\pi)^d} \frac{\partial}{\partial q^\mu} q^\mu I\left(l_1, l_2, \{p_{ext}\}\right) = 0 \text{ with} q = l_1, l_2, \{p_{ext}\}$$

- Relations between different integrals
  $\Rightarrow$ Relate difficult integrals to easy ones
- Reduction to set of master integrals

Differential equations for master integrals

- Differentiation of master integrals with respect to m_s and x: $m_s \frac{\mathrm{d}}{\mathrm{d}m_s} I_i = \dots \quad x \frac{\mathrm{d}}{\mathrm{d}x} I_i = \dots$
- IBPs $\rightarrow$ reduce the R.H.S again to masters
  $\rightarrow$ coupled system of first order ODEs

$$m_s \frac{\mathrm{d}}{\mathrm{d}m_s} I_i = \sum c_k I_k$$
$$x \frac{\mathrm{d}}{\mathrm{d}x} I_i = \sum d_k I_k$$

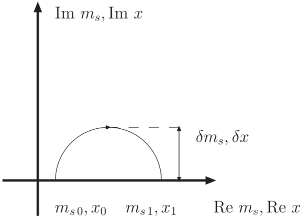
- Boundary conditions $\rightarrow$ Solution

Boundaries

- Analytic expansion around the high energy limes $m_s = \frac{M^2}{s} \rightarrow 0$
- Using Mellin-Barns representations and a lot of handwork to extract a series in ϵ,$m_s s$ and x for each integral
- Expanding the differential equations also in *ep* and solve the algebraic system
- $\rightarrow$ deep expansions in *ms* and x

Numerical evaluation of master integrals

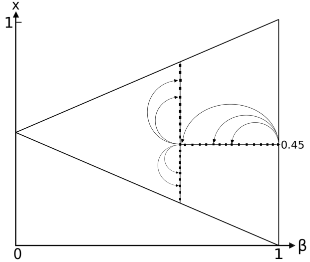
- Using the differential equations to integrate numerically from the pre-calculated boundary conditions
- leaving the real numbers and integrate in a complex plane to grid points



The grid

Choice of points:

- $\beta = \sqrt{1 - 4m_s} = i/80$ for $i = 1, ..., 79$
- 42 points for x: Gauss-Konrod points in available phase-space



UV renormalization and decoupling

$$\left|M_{g,q}(\alpha_S^0, m^0, \epsilon)\right\rangle = 4\pi\alpha_S^0 \left[\left|M_{g,q}^{(0)}(m^0,\epsilon)\right\rangle + \left(\frac{\alpha_S^0}{2\pi}\right)\left|M_{g,q}^{(1)}(m^0,\epsilon)\right\rangle + \left(\frac{\alpha_S^0}{2\pi}\right)^2\left|M_{g,q}^{(2)}(m^0,\epsilon)\right\rangle\right]$$

UV-renormalized amplitude:

$$\left|\mathcal{M}_{g,q}^{R}\left(\alpha_S^{(n_f)}(\mu), m, \mu, \epsilon\right)\right\rangle = \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{-2\epsilon} Z_{g,q} Z_Q \left|M_{g,q}(\alpha_S^0, m^0, \epsilon)\right\rangle$$

- Z_g,Z_q , Z_Q : onshell renormalization constants
- $m^0 = Z_m m$
- $\alpha_S^0 = \left(\frac{e^{\gamma_E}}{4\pi}\right)^{\epsilon}\mu^{2\epsilon} Z_{\alpha_S}^{(n_f)}\alpha_S^{(n_f)}(\mu)$ $\hat{=}$ $\overline{M}S$-scheme with n_f flavours

Decoupling

- $n_f = n_l + n_h$ is not feasible
- decouple the running of α_S from the n_h quarks
- $\alpha_S^{(n_f)} = \zeta_{\alpha_S}\alpha_S^{(n_l)}$

Why precision predictions for top-quark pair production? $\rightarrow$ Applications

- top quark mass and production cross-sections $\rightarrow$ precision test of SM
- measurements of α_S
- Constrains on PDFs
- SM phase diagram
- BSM searches in production and decay

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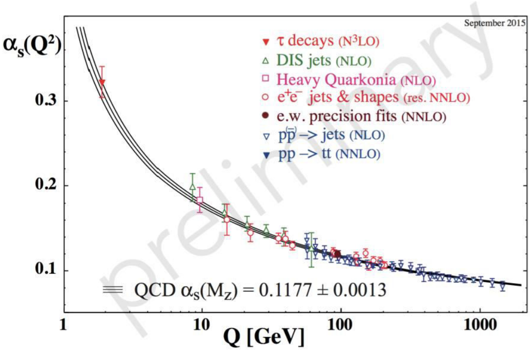

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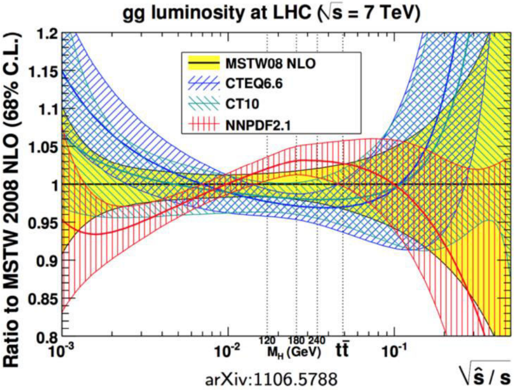

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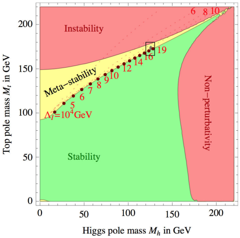
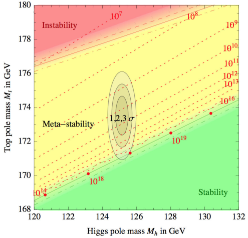
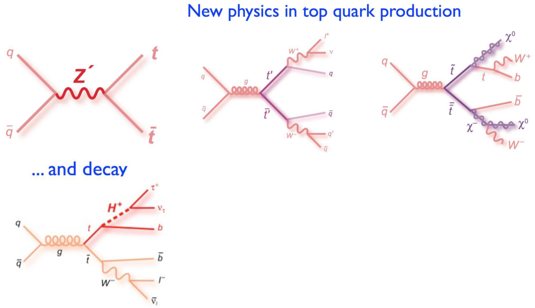


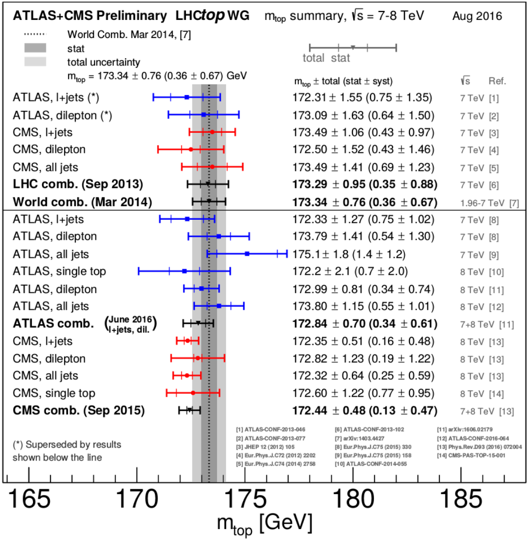


Why precision predictions for top-quark pair production? $\rightarrow$ Applications

- top quark mass and production cross-sections $\rightarrow$ precision test of SM
- measurements of α_S
- Constrains on PDFs
- SM phase diagram
- BSM searches in production and decay



Recent results: Measurements



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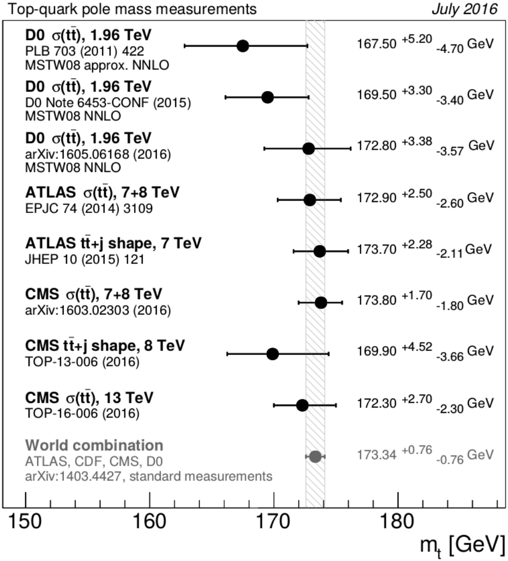
Recent results: Measurements



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