$t\bar{t}$ production and decay at NNLO QCD

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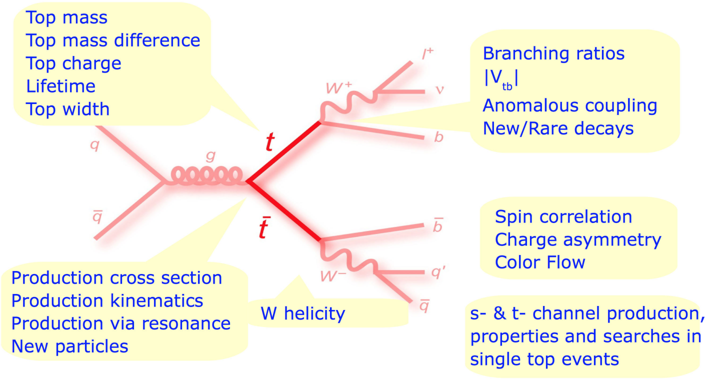
[1]in cooperation with M. Czakon

Introduction

Why is the top-quark (still) interesting?



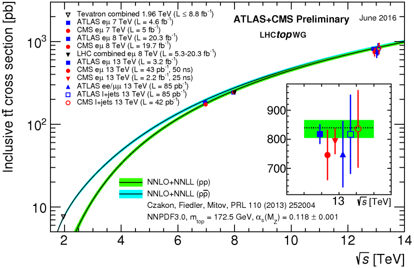
Recent developments

Stable onshell tops and spin summed:

- Total inclusive cross sections @ NNLO+NNLL accuracy [Czakon, Fiedler, Mitov '13]
- Fully differential distributions @ NNLO

Unstable tops + spin correlations:

- Off-shell effects with decays @ NLO + matched parton-shower



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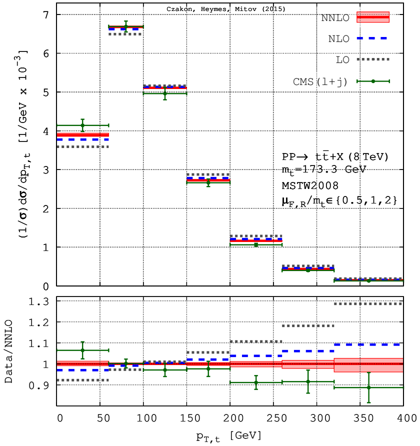

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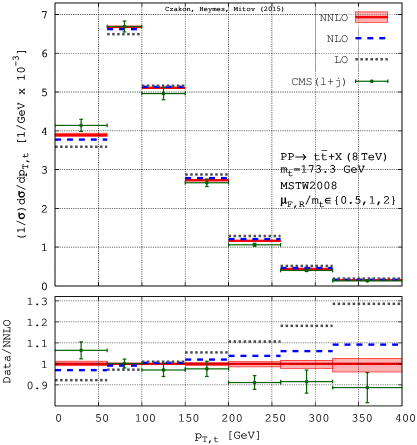
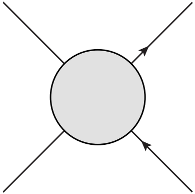


The amplitudes

Feynman diagrams

process in mind: $pp \to t\bar{t}$

production:



Feynman diagrams

process in mind: $t \to b + 2f$

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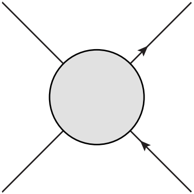
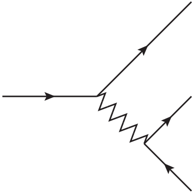


$+$

decay:



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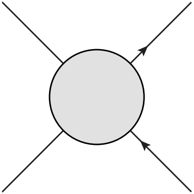

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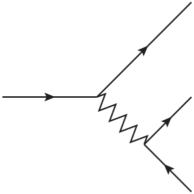
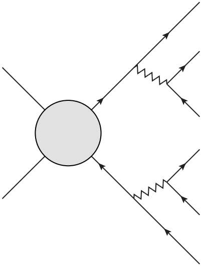
decay:



$=$

combined:



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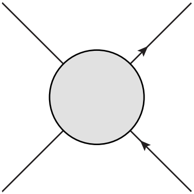

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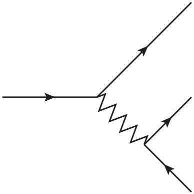
decay:



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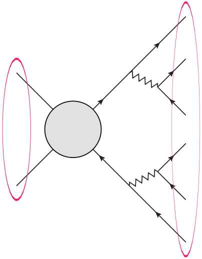

Feynman diagrams

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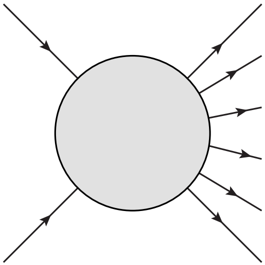

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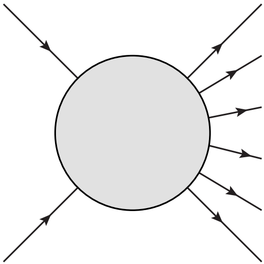


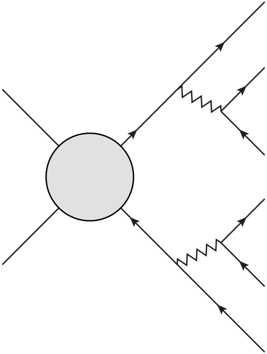
Simplification I

restrict to onshell tops

$\to$ Narrow-Width-Approximation
$\to$ Factorization of production and decay
$\to$ Much simpler production amplitudes
$\to$ Neglecting continuum production

Feynman diagrams

$pp \to t\bar{t} \to b\bar{b} + 4f$



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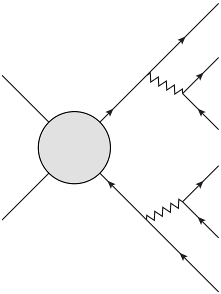
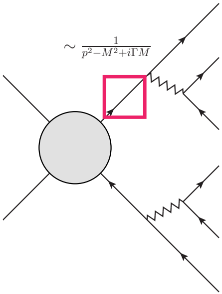
Factorization of decays

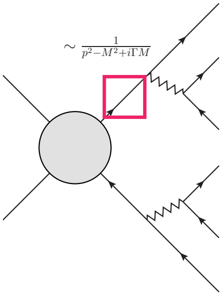
Narrow-Width-Approximation



Narrow-Width-Approximation



Narrow-Width-Approximation



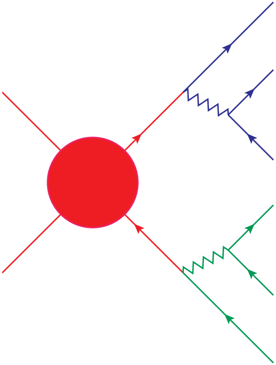
- enters matrix element as:
  $\sim \frac{1}{(p^2 - m^2)^2 + m^2\Gamma^2}$
- For crosssections: Integration over phase-space
- $+$ limit $\Gamma/m \to 0$:

$$\frac{1}{(p^2 - m^2)^2 + m^2\Gamma^2} \to \frac{2\pi}{2m\Gamma}\delta(p^2 - m^2)$$

- On amplitude level:

$$\mathcal{M} = \mathcal{M}_{\text{NWA}} + \mathcal{O}\left(\frac{\Gamma}{m}\right)$$

Amplitude factorization



$$\mathcal{M} = \left(\tilde{A}(t \to b l^+ \nu) \frac{i(\not{p}_t + m)}{p_t^2 - m^2 + im\Gamma_t} \right) \cdot$$
$$\tilde{A}(pp \to \bar{t}t) \cdot$$
$$\left(\frac{i(-\not{p}_{\bar{t}} + m)}{p_{\bar{t}}^2 - m^2 + im\Gamma_t} \tilde{A}(\bar{t} \to \bar{b} l^- \bar{\nu}) \right)$$

Decay spinors

Narrow-Width-Approximation:

$$\frac{i(-\not{p}_{\bar{t}} + m)}{p_{\bar{t}}^2 - m^2 + im\Gamma_t}\tilde{A}(\bar{t} \to \bar{b}l^-\bar{\nu}) \to \frac{i(-\not{p}_{\bar{t}} + m)}{\sqrt{2m\Gamma_t}}\tilde{A}(\bar{t} \to \bar{b}l^-\bar{\nu})$$

Decay spinors

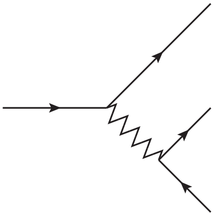
$$\bar{U}(p_t) = \tilde{A}(t \to bl^+\nu)\frac{i(\not{p}_t + m)}{\sqrt{2m\Gamma_t}}$$

$$V(p_{\bar{t}}) = \frac{i(-\not{p}_{\bar{t}} + m)}{\sqrt{2m\Gamma_t}}\tilde{A}(\bar{t} \to \bar{b}l^-\bar{\nu})$$

Amplitude:

$$\mathcal{M} = \bar{U}(p_t)\tilde{A}(pp \to \bar{t}t)V(p_{\bar{t}}) + \mathcal{O}\left(\frac{\Gamma_t}{m}\right)$$

QCD corrections to decay



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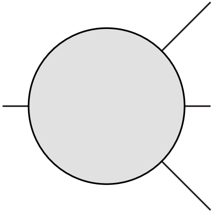

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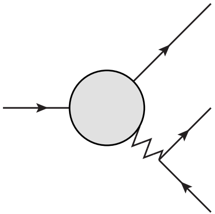

Simplification II

restrict to leptonic top decays

- Vertex corrections (for massless final state):

$$\Gamma^\mu = \frac{g}{\sqrt{2}} \Big\{ \gamma^\mu \left[F_{1L} P_L + F_{1R} P_R \right] + \frac{i\sigma^{\mu\nu} q_\nu}{2m_t} \left[F_{2L} P_R + F_{2R} P_L \right] \Big\}$$

- times W propagator and decay vertex

$$\bar{u}(p_\nu) \frac{i g_W}{\sqrt{2}} \gamma^\nu \frac{(1-\gamma_5)}{2} v(p_{l^+}) \cdot \frac{-i\left(g_{\nu\mu} - \frac{q_\nu q_\mu}{q^2}\right)}{q^2 - m_W^2 + i\Gamma_W m_W} \bar{u}(p_b) i\Gamma^\mu$$

Contributions to amplitude

Production \ Decay	LO	NLO	NNLO
LO			
NLO			
NNLO			

Contributions to amplitude

Production \ Decay	LO	NLO	NNLO
LO			
NLO			
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$t\bar{t}$ production amplitudes

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Goal: Evaluate $\bar{U}(p_t)\tilde{A}(pp \to \bar{t}t)V(p_{\bar{t}})$

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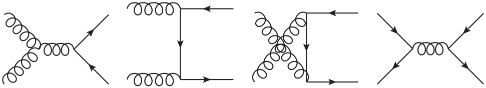
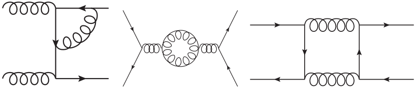
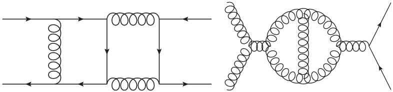
Contributions to $\tilde{A}(pp \to \bar{t}t)$:		gg|qq
LO		3|1
NLO	 ...	33|18
NNLO	 ...	726|190

Structure of born-like amplitudes

Lorentz-structures – Gluon channel

Lorentz-decomposition: $\mathcal{A} = \epsilon_{1\mu}(p_1)\epsilon_{2\nu}(p_2)A^{\mu\nu}$

$A^{\mu\nu}$ is a rank-2 Lorentz tensor
Using:

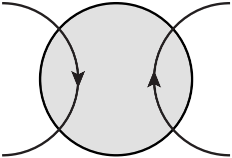
- momentum conservation
- transversality
- equation of motion

leads to 10 independent structures in $d = 4 - 2\epsilon$ dimensions

$$A^{\mu\nu} = \sum_{j=1}^{10} A_j T_j^{\mu\nu}$$

$$\begin{aligned}
T_1^{\mu\nu} &= m\bar{u}_3\gamma^\mu\gamma^\nu v_4 \\
T_2^{\mu\nu} &= \bar{u}_3\gamma^\mu v_4 p_3^\nu \\
T_3^{\mu\nu} &= \bar{u}_3\gamma^\nu v_4 p_3^\mu \\
T_4^{\mu\nu} &= m\bar{u}_3 v_4 g^{\mu\nu} \\
T_5^{\mu\nu} &= m^{-1}\bar{u}_3 v_4 p_3^\mu p_3^\nu \\
T_6^{\mu\nu} &= m^{-2}\bar{u}_3 \not{p}_1 v_4 p_3^\mu p_3^\nu \\
T_7^{\mu\nu} &= \bar{u}_3 \not{p}_1 v_4 g^{\mu\nu} \\
T_8^{\mu\nu} &= m^{-1}\bar{u}_3 \not{p}_1 \gamma^\nu v_4 p_3^\mu \\
T_9^{\mu\nu} &= m^{-1}\bar{u}_3 \not{p}_1 \gamma^\mu v_4 p_3^\nu \\
T_{10}^{\mu\nu} &= \bar{u}_3 \not{p}_1 \gamma^\mu\gamma^\nu v_4
\end{aligned}$$

Lorentz-structures – Quark channel



- two disconnected fermion lines
- connection by gluons+loops
- parity conservation $\rightarrow$ no γ_5

$$\tilde{A} = \sum_{i=1}^{7} A_j T_j$$

with $T_j \sim \bar{v}_2 \Gamma_j u_1 \bar{u}_3 \Gamma'_j v_4$

$$T_1 = m^{-1} \bar{v}_2 \not{p}_3 u_1 \bar{u}_3 v_4$$
$$T_2 = m^{-2} \bar{v}_2 \not{p}_3 u_1 \bar{u}_3 \not{p}_1 v_4$$
$$T_3 = \bar{v}_2 \gamma^\mu u_1 \bar{u}_3 \gamma_\mu v_4$$
$$T_4 = \bar{v}_2 \gamma^\mu u_1 \bar{u}_3 \not{p}_1 \gamma_\mu v_4$$
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$$T_6 = m^{-2} \bar{v}_2 \not{p}_3 \gamma^\mu \gamma^\nu u_1 \bar{u}_3 \not{p}_1 \gamma_\mu \gamma_\nu v_4$$
$$T_7 = \bar{v}_2 \gamma^\mu \gamma^\nu \gamma^\rho u_1 \bar{u}_3 \not{p}_1 \gamma_\mu \gamma_\nu \gamma_\rho v_4$$

Color-structures

color decomposition: $\mathcal{A} = \sum_{ij} c_{ij} C_i A_j$

Gluon channel

- Gluons: a, b adjoint rep.
- Quarks: c, d fundamental rep.

$$C_1 = (T^a T^b)_{cd}$$
$$C_2 = (T^b T^a)_{cd}$$
$$C_3 = \mathrm{Tr}\left\{T^a T^b\right\} \delta_{cd}$$

Quark channel

- Quarks: a, b fundamental rep.
- Quarks: c, d fundamental rep.

$$C_1 = \delta_{ac}\delta_{bd}$$
$$C_2 = \delta_{ab}\delta_{cd}$$

Projection

Idea: Construct projectors: $P_j = \sum_l B_{jl}(T_l)^\dagger$

Extracting the B_{jl}:

$$\sum_{spin/pol/col} P_j \mathcal{A} \stackrel{!}{=} A_j$$

leads to system of equations

$$\sum_{l,k} B_{jl} A_k \sum_{spin/pol/col} (T_l)^\dagger T_k = A_j$$

Inversion $\rightarrow$ coefficients B_{jl}

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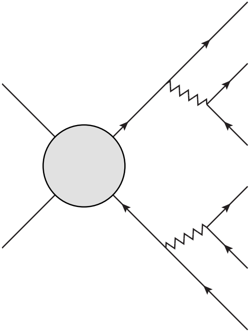
Example: Lorentz-structures gluons

$$\sum_{pol,spin} P_j^{\mu_1\nu_1} \left[\epsilon^*_{1\mu_1}\epsilon^*_{2\nu_1}\epsilon_{1\mu_2}\epsilon_{2\nu_2}\right] A^{\mu_2\nu_2} =$$

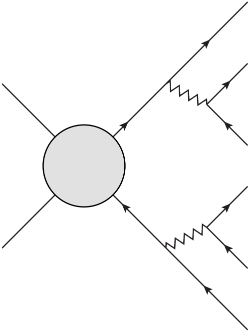
$$\mathrm{Tr}\Bigg[(\not{p}_4 - m)P_j^{\mu_1\nu_1}(\not{p}_3 + m)\cdot \left(-g_{\mu_1\mu_2} + \frac{p_{2\mu_1}p_{1\mu_2} + p_{2\mu_2}p_{1\mu_1}}{p_1\cdot p_2}\right)\cdot \left(-g_{\nu_1\nu_2} + \frac{p_{1\nu_1}p_{2\nu_2} + p_{1\nu_2}p_{2\nu_1}}{p_1\cdot p_2}\right) A^{\mu_2\nu_2}\Bigg]$$

Summary



$$\bar{U}(p_t)\tilde{A}(pp \to \bar{t}t)V(p_{\bar{t}})$$

Summary



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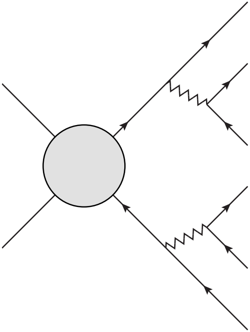
- NWA $\rightarrow$ decay spinors

Summary



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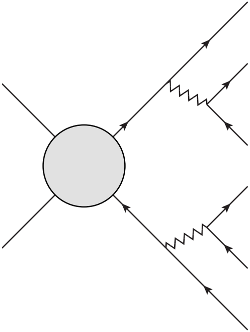
- NWA $\to$ decay spinors
- Lorentz- and color-decomposition

$$\tilde{A} = \sum_{ij} c_{ij} C_i \tilde{A}_j$$

Summary



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- NWA $\to$ decay spinors
- Lorentz- and color-decomposition

$$\tilde{A} = \sum_{ij} c_{ij} C_i \tilde{A}_j$$

- Evaluation of the coefficients
  - functions of $ms = m^2/s, x = t/s, \epsilon$
  - and scalar 2-loop integrals
    $\to$ reuse of numerical integrals from spin summed calculation

Renormalization and finite remainder

UV renormalization and decoupling

$$\left|M_{g,q}(\alpha_S^0, m^0, \epsilon)\right\rangle = 4\pi\alpha_S^0 \left[\left|M_{g,q}^{(0)}(m^0,\epsilon)\right\rangle + \left(\frac{\alpha_S^0}{2\pi}\right)\left|M_{g,q}^{(1)}(m^0,\epsilon)\right\rangle + \left(\frac{\alpha_S^0}{2\pi}\right)^2\left|M_{g,q}^{(2)}(m^0,\epsilon)\right\rangle\right]$$

UV-renormalized amplitude:

$$\left|\mathcal{M}_{g,q}^{R}\left(\alpha_S^{(n_f)}(\mu), m, \mu, \epsilon\right)\right\rangle = \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{-2\epsilon} Z_{g,q} Z_Q \left|M_{g,q}(\alpha_S^0, m^0, \epsilon)\right\rangle$$

- Z_g,Z_q , Z_Q : onshell renormalization constants
- $m^0 = Z_m m$
- $\alpha_S^0 = \left(\frac{e^{\gamma_E}}{4\pi}\right)^{\epsilon} \mu^{2\epsilon} Z_{\alpha_S}^{(n_f)} \alpha_S^{(n_f)}(\mu)$ $\hat{=}$ $\bar{M}S$-scheme with n_f flavours

Decoupling

- $n_f = n_l + n_h$ is not feasible
- decouple the running of α_S from the n_h quarks
- $\alpha_S^{(n_f)} = \zeta_{\alpha_S} \alpha_S^{(n_l)}$

IR divergences and the finite remainder function

$$\left|\mathcal{M}^{fin}_{g,q}\left(\alpha_S^{(n_f)}(\mu), m, \mu, \epsilon\right)\right\rangle = \mathbf{Z}^{-1}_{\mathcal{M}_{g,q}}\left(\{p\}, m, \mu, \epsilon\right)\left|\mathcal{M}^{R}_{g,q}\left(\alpha_S^{(n_f)}(\mu), m, \mu, \epsilon\right)\right\rangle$$

- Amplitude still contains infrared divergences
- Usually: Cancellation with divergences from real-emission contributions
- Complete factorization of IR structure $\rightarrow$ **Z** operator
- $\rightarrow$ finite remainder

- Z can be calculated by its anomalous dimension equation

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$$

- depends on kinematics
- matrix in color space

Summary of progress

Finished

- Projection LO, NLO, NNLO amplitudes
- Finite remainder of all coefficients
- Implementation of Color and Spin Structures in STRIPPER
- Implementation of decay spinors in STRIPPER
- Implementation of LO, NLO coefficients in STRIPPER

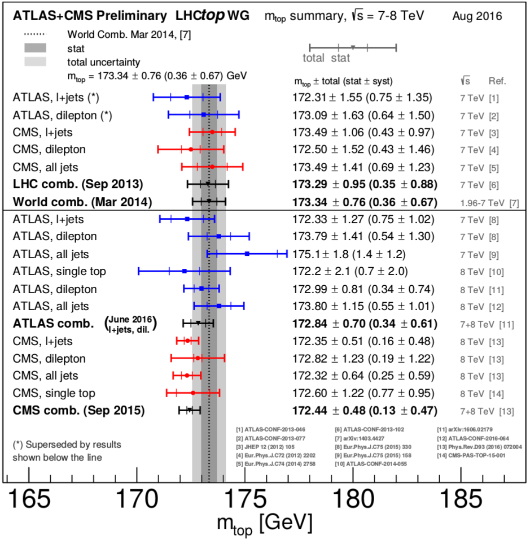
Outlook

- Implementation of NNLO finite remainder
- Real-Virtual and Real-Real contributions
- Implementation of decay phase-space and handling of decay products in STRIPPER
- Merging with the results from Mitov et al.
  $\rightarrow$ QCD corrections to decay

Thank you for your attention.

Backup

Recent results: Measurements



Top quark results from the LHC | Nuno Castro | QCD@LHC 2016

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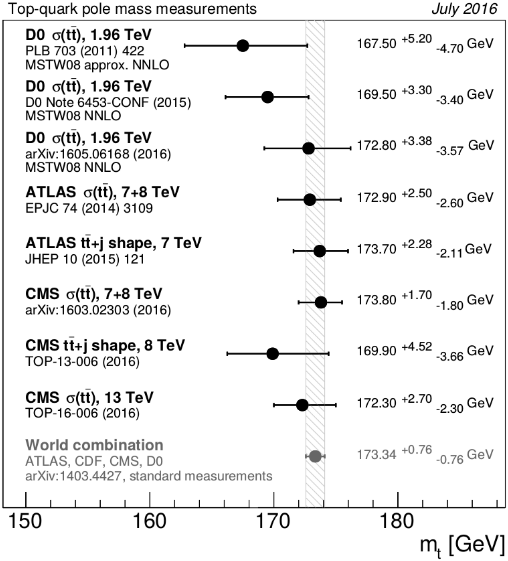

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STRIPPER – SecToR Improved Phase sPacE for real Radiation

The subtraction scheme

- Method of evaluate the double-real emission radiation contribution to NNLO processes
- Decomposition of the phase-space to factorize the singular limits of the amplitude
- Suitable parameterizations to derive (integrated) subtraction terms

The NNLO event generator

- fully differential event generation
- several scales simultaneously
- different pdfs simultaneously
- stable tops
- predecided binned distributions
- fixed top mass=173.3

Why precision predictions for top-quark pair production? $\rightarrow$ Applications

- top quark mass and production cross-sections $\rightarrow$ precision test of SM
- measurements of α_S
- Constrains on PDFs
- SM phase diagram
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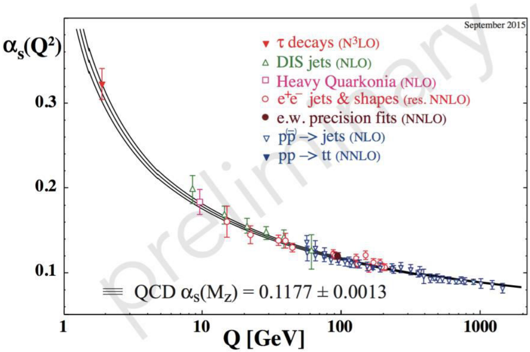

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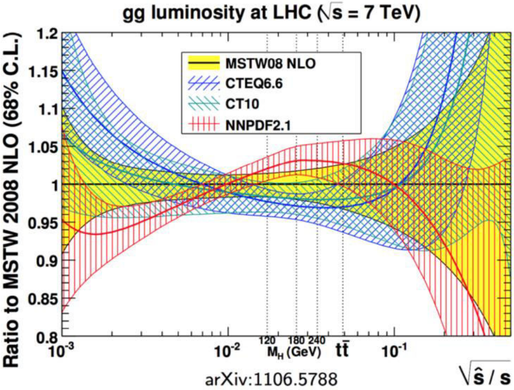

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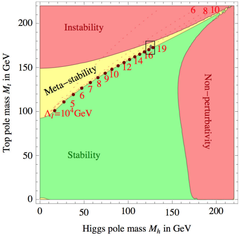

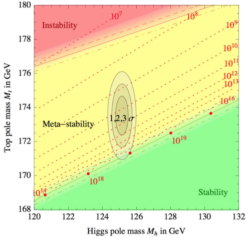

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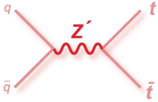
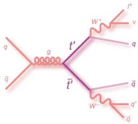
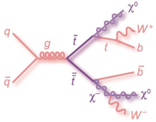
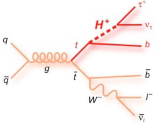
New physics in top quark production



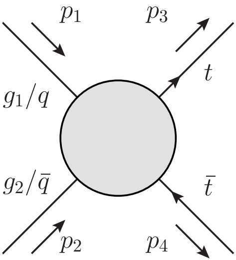




... and decay



Kinematics and polarization



External Momenta

$$p_1^2 = p_2^2 = 0$$
$$p_3^2 = p_4^2 = m^2$$

Mandelstamm variables

$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_2 - p_3)^2$$
$$s + t + u = 2m^2$$

Polarization sum external gluons (axial gauge)

$$\sum_{\text{pol}} \epsilon^*_{i\mu} \epsilon_{i\nu} =$$
$$- g_{\mu\nu} + \frac{n_{i\mu} p_{i\nu} + n_{i\nu} p_{i\mu}}{n_i \cdot p_i}$$

Equation of motion for external (anti)quarks

$$\left(\not{p} - m\right) U = 0$$
$$\left(\not{p} + m\right) V = 0$$

IBP reduction

General two-loop integral:

$$\int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \frac{\mathrm{d}^d l_2}{(2\pi)^d} \prod_i \frac{1}{D_i^{n_i}} \prod_j N_j^{n_j}$$

with $D_i = (\sum p + \sum l)^2 - m^2$ and $N_i = l \cdot p$

Basic Idea of Integration-By-Part (IBP) reduction:

$$\int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \frac{\mathrm{d}^d l_2}{(2\pi)^d} \frac{\partial}{\partial q^\mu} q^\mu I\left(l_1, l_2, \{p_{ext}\}\right) = 0 \text{ with} q = l_1, l_2, \{p_{ext}\}$$

- Relations between different integrals
  $\Rightarrow$ Relate difficult integrals to easy ones
- Reduction to set of master integrals

Differential equations for master integrals

- Differentiation of master integrals with respect to m_s and x: $m_s \frac{\mathrm{d}}{\mathrm{d}m_s} I_i = \ldots \quad x \frac{\mathrm{d}}{\mathrm{d}x} I_i = \ldots$
- IBPs $\rightarrow$ reduce the R.H.S again to masters
  $\rightarrow$ coupled system of first order ODEs

$$m_s \frac{\mathrm{d}}{\mathrm{d}m_s} I_i = \sum c_k I_k$$
$$x \frac{\mathrm{d}}{\mathrm{d}x} I_i = \sum d_k I_k$$

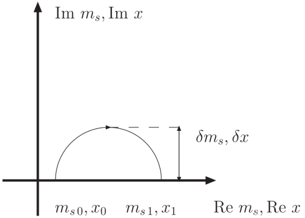
- Boundary conditions $\rightarrow$ Solution

Boundaries

- Analytic expansion around the high energy limes $m_s = \frac{M^2}{s} \rightarrow 0$
- Using Mellin-Barns representations and a lot of handwork to extract a series in ϵ,$m_s s$ and x for each integral
- Expanding the differential equations also in *ep* and solve the algebraic system
- $\rightarrow$ deep expansions in *ms* and x

Numerical evaluation of master integrals

- Using the differential equations to integrate numerically from the pre-calculated boundary conditions
- leaving the real numbers and integrate in a complex plane to grid points



The grid

Choice of points:

- $\beta = \sqrt{1 - 4m_s} = i/80$ for $i = 1, ..., 79$
- 42 points for x: Gauss-Konrod points in available phase-space