IFJ PAN
Theory Division – Particle Theory
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QUANTUM FIELD THEORY

EXERCISES 2

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1 Interaction Picture

1. *Time-evolution operator*

   (a) Consider the time-evolution operator

   $$U(t,t_0) = e^{iH_0(t-t_0)}e^{-iH(t-t_0)} \, , \tag{1.1}$$

   where H_0 is the free field hamiltonian, $H = H_0 + H_{\text{int}}$ the interacting Hamiltonian and t_0 the reference time in the definition of the interaction-picture field:

   $$\phi_I(t,x) = e^{iH_0(t-t_0)}\phi(t_0,\vec{x})e^{-iH_0(t-t_0)} \, . \tag{1.2}$$

   For a general time argument t' we have

   $$U(t,t') = e^{iH_0(t-t_0)}e^{-iH(t-t')}e^{-iH_0(t'-t_0)} \, . \tag{1.3}$$

   Show that this operator obeys the identities

   $$U(t_1,t_2)U(t_2,t_3) = U(t_1,t_3) \quad \text{and} \quad U(t_1,t_3)\left[U(t_2,t_3)\right]^\dagger = U(t_1,t_2) \, . \tag{1.4}$$

2. *Ground-state*

   (a) In the lecture we discussed the construction of the ground state $|\Omega\rangle$ in terms of interaction-picture fields,

   $$|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1} U(t_0,-T)\,|0\rangle \, . \tag{1.5}$$

   Familiarise yourself with the arguments in this construction and show in analogous manner that

   $$\langle\Omega| = \lim_{T\to\infty(1-i\epsilon)} |0\rangle\, U(T,t_0)\left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1} \, . \tag{1.6}$$