STATISTICAL HADRONIZATION MODEL FOR HEAVY-ION COLLISIONS AT SIS18 ENERGIES

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based on: PRC 102, 054903 (2020)
PRC 00, 004900 (2023) (to appear)

NATIONAL SCIENCE CENTRE
POLAND
SONATA BIS 8 Grant No. 2018/30/E/ST2/00432

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WAYS TO ACCESS THE PHASE DIAGRAM OF QCD

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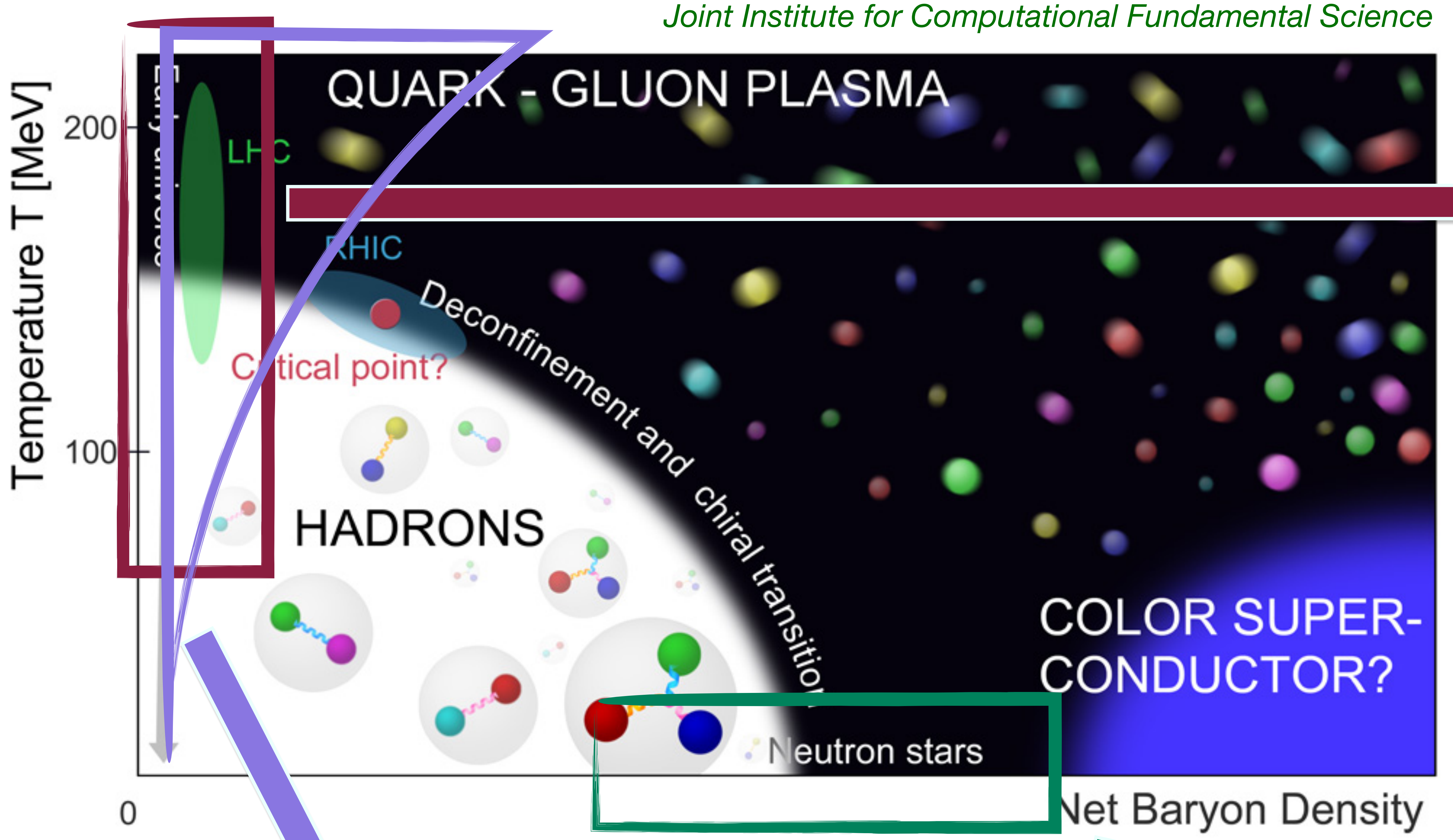
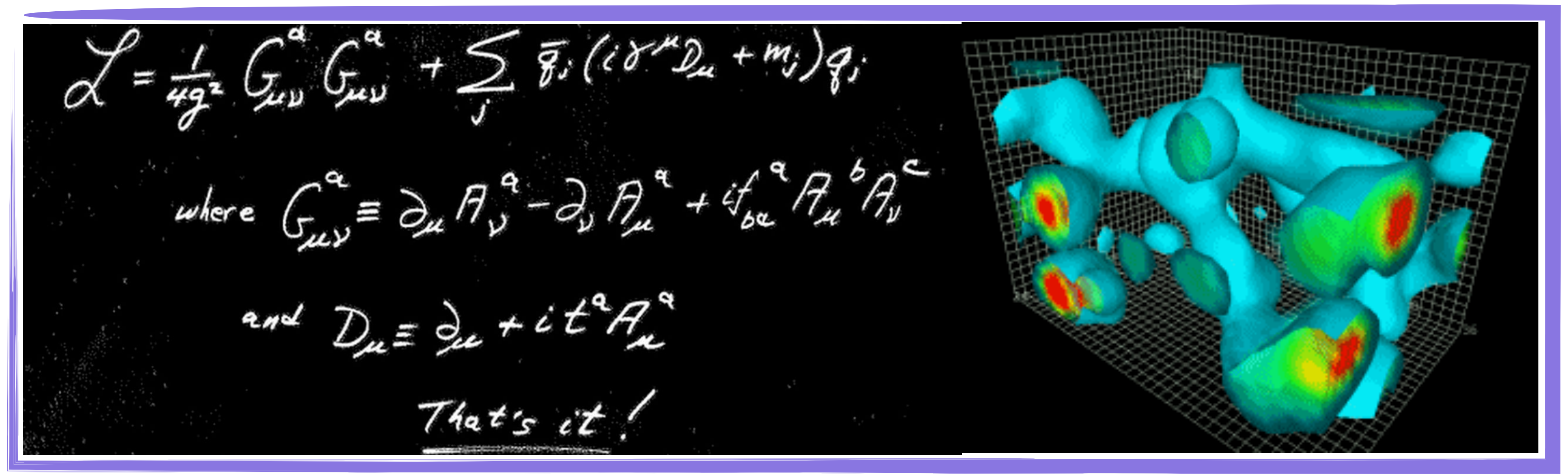


lattice-QCD simulations



$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

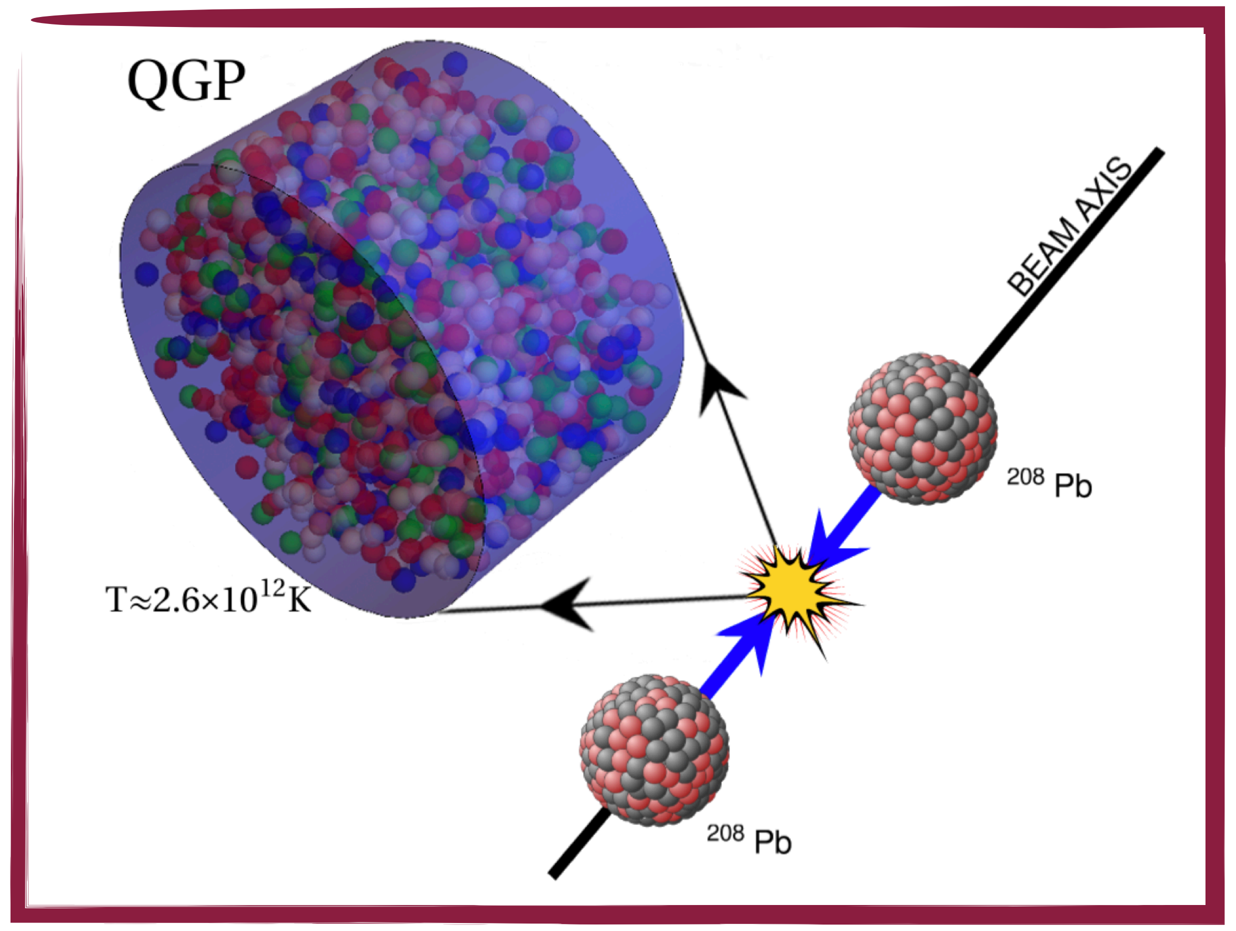
where $G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$

and $D_\mu \equiv \partial_\mu + i t^a A^a_\mu$

That's it!

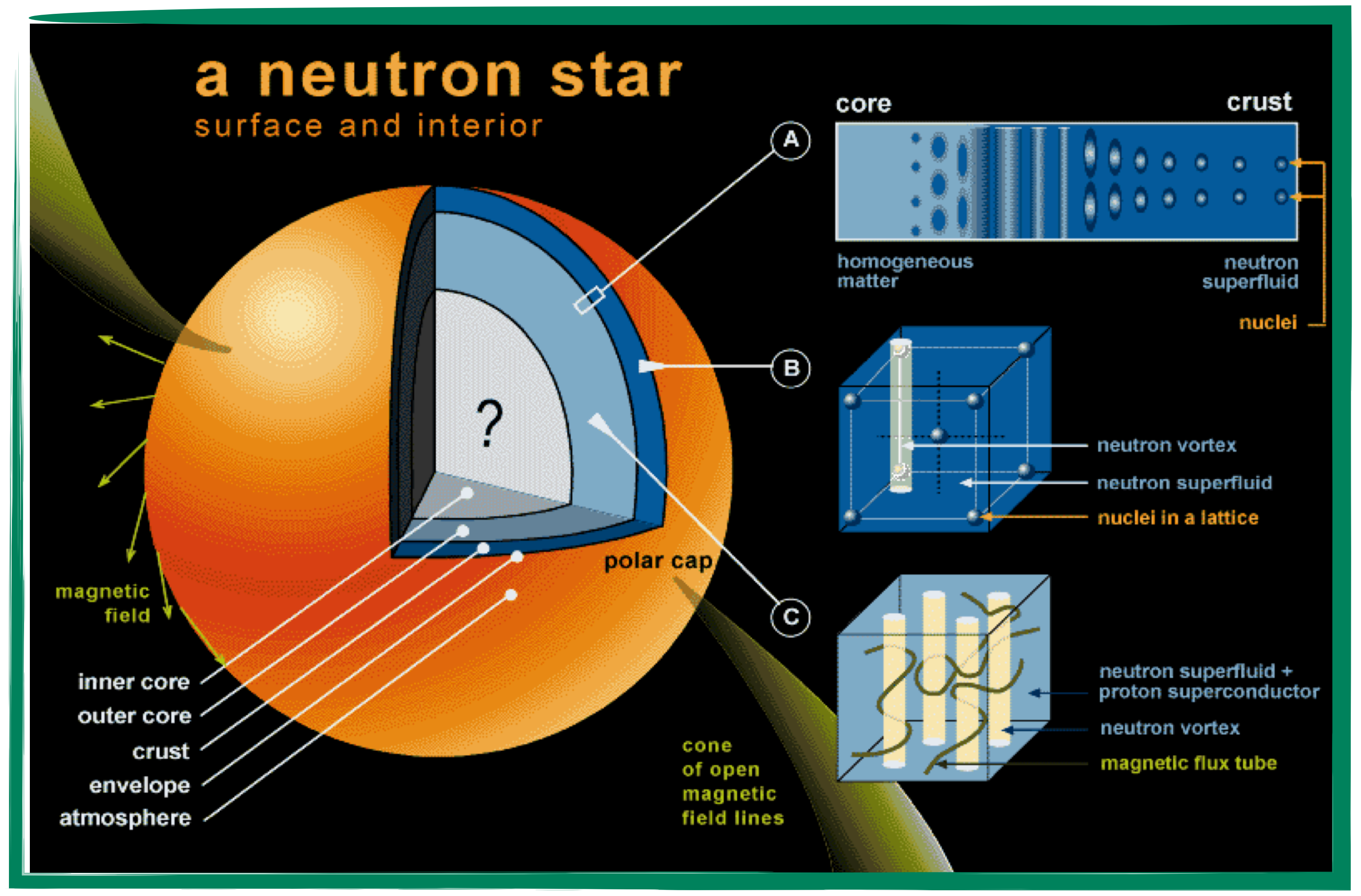
D. Leinweber (www.physics.adelaide.edu.au)

Heavy-ion collision physics



Nature Physics 16, 615–619(2020)

Neutron star physics



D.E. Á. Castillo, talk @RagTime 22

WHAT IS THE STRUCTURE OF QCD PHASE DIAGRAM?

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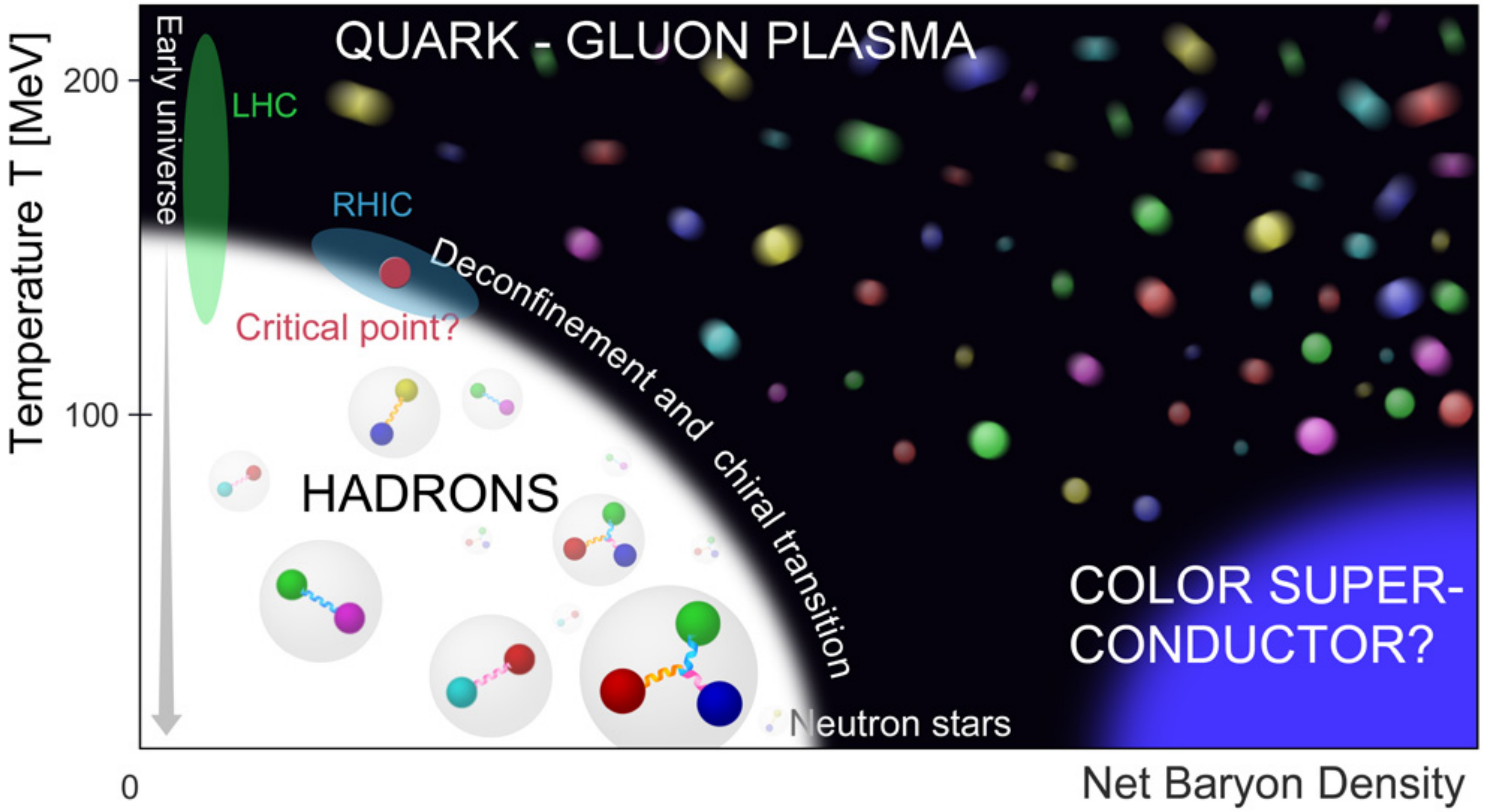


Vanishing μ_B, high T (covered by lattice QCD)

- crossover transition
- no QCD critical point (CP) indicated by lattice QCD at $\mu_B < 400$ MeV, $T > 140$ MeV

Large μ_B, moderate T (covered by QCD inspired models)

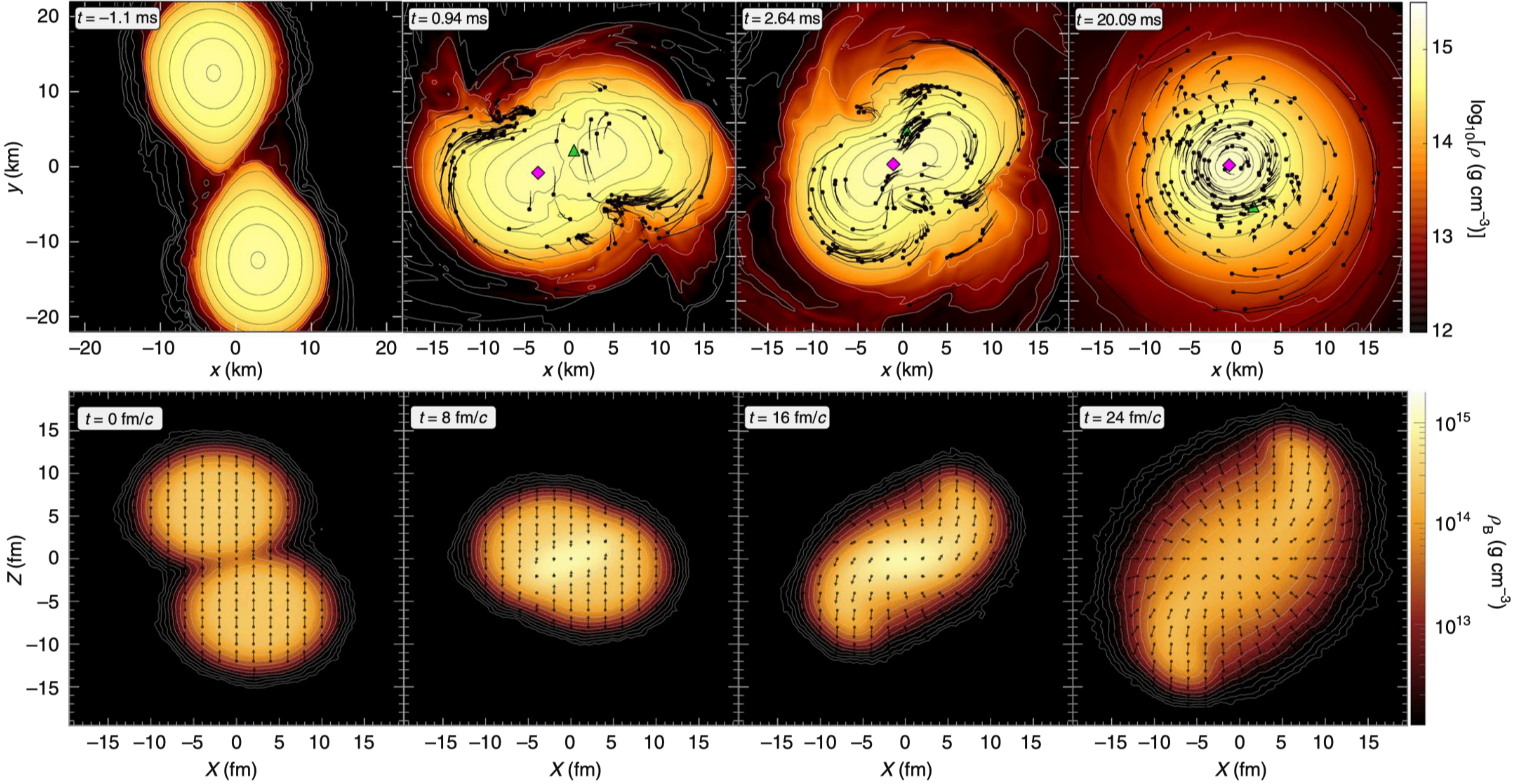
- 1st order transition (?)
- QCD CP (?)

NUCLEAR COLLISIONS ALLOW TO PROBE “WARM” DENSE MATTER

HADES, Nature Phys. 15 (2019) 10, 1040-1045

binary neutron star merger

noncentral Au+Au collision at 2.4 GeV per nucleon pair.



MAPPING PHASE DIAGRAM WITH STATISTICAL HADRONIZATION MODEL

Thermal models of hadron production (based on the idea of **statistical hadronization**) have been very successful in **describing hadron yields in various collision processes.**

J. Cleymans, H. Satz, F. Becattini, M. Gazdzicki, J. Sollfrank, W. Florkowski, W. Broniowski, J. Letessier, J. Rafelski, R. Stock, M. I. Gorenstein, A. Andronic, P. Braun-Munzinger, K. Redlich, and J. Stachel …

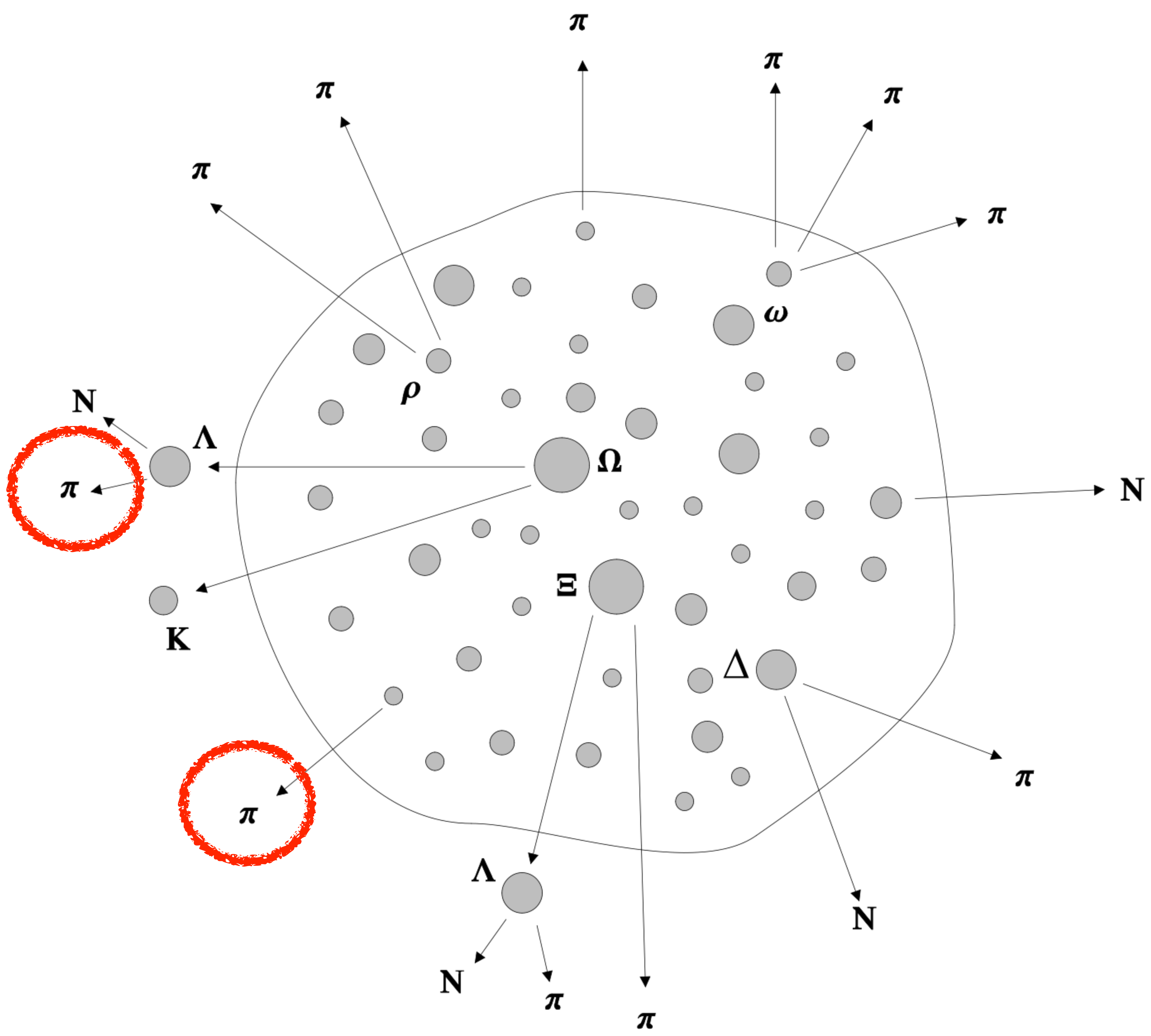
Matter formed at the chemical freeze-out is treated as multicomponent **non-interacting hadron resonance gas.**

In chemical equilibrium multiplicities of particle specie i can be written as:

$$N_i = V n_i = V g_i \int d^3p \; f_i(p)$$

$$f_i(p) = \frac{1}{(2\pi)^3}\left[\exp\left(\frac{E_i(p)-\mu_i}{T}\right)+\epsilon\right]^{-1}$$

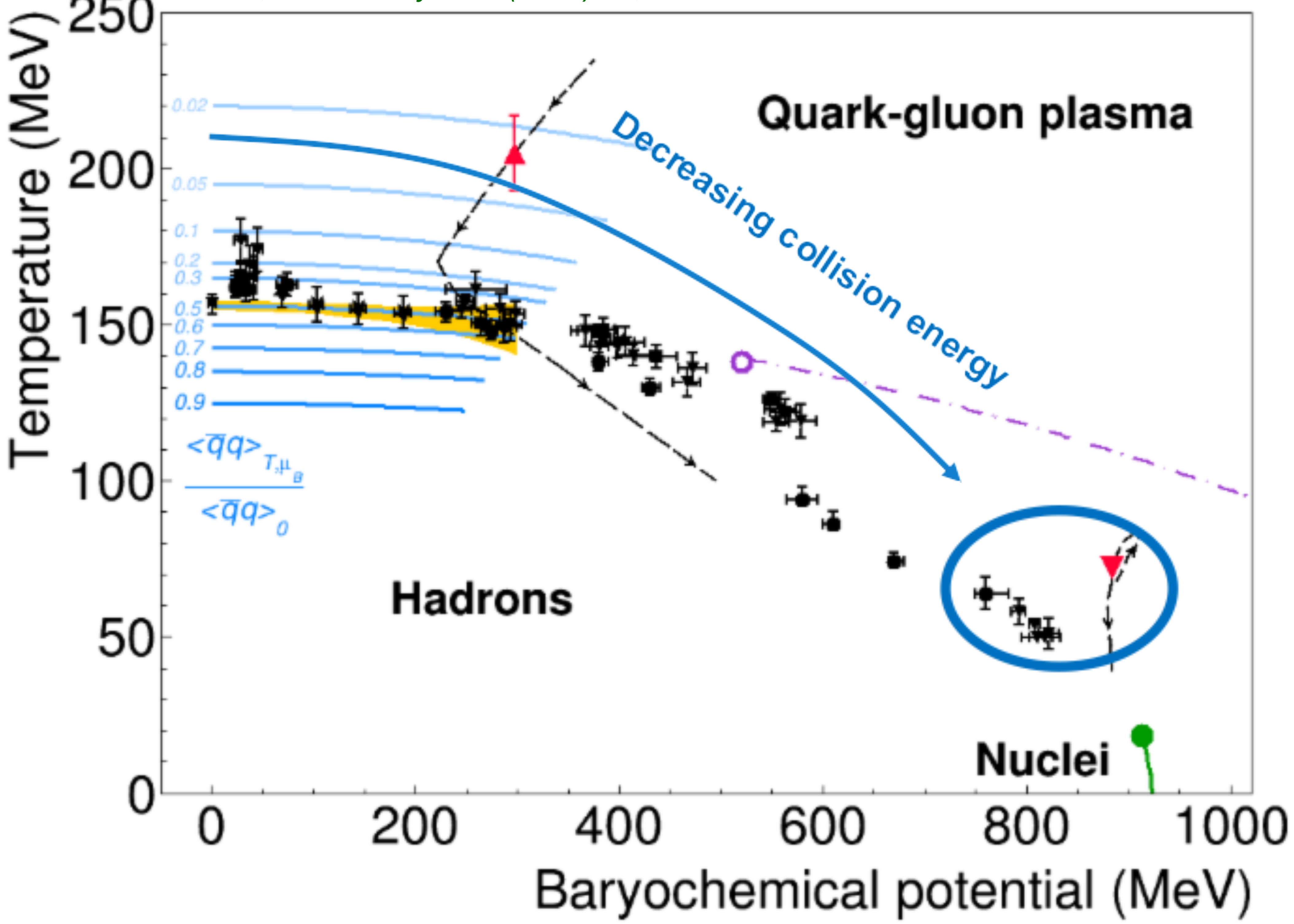
One can **fit the ratios of measured particle yields** and **extract free parameters** giving location in the phase diagram.



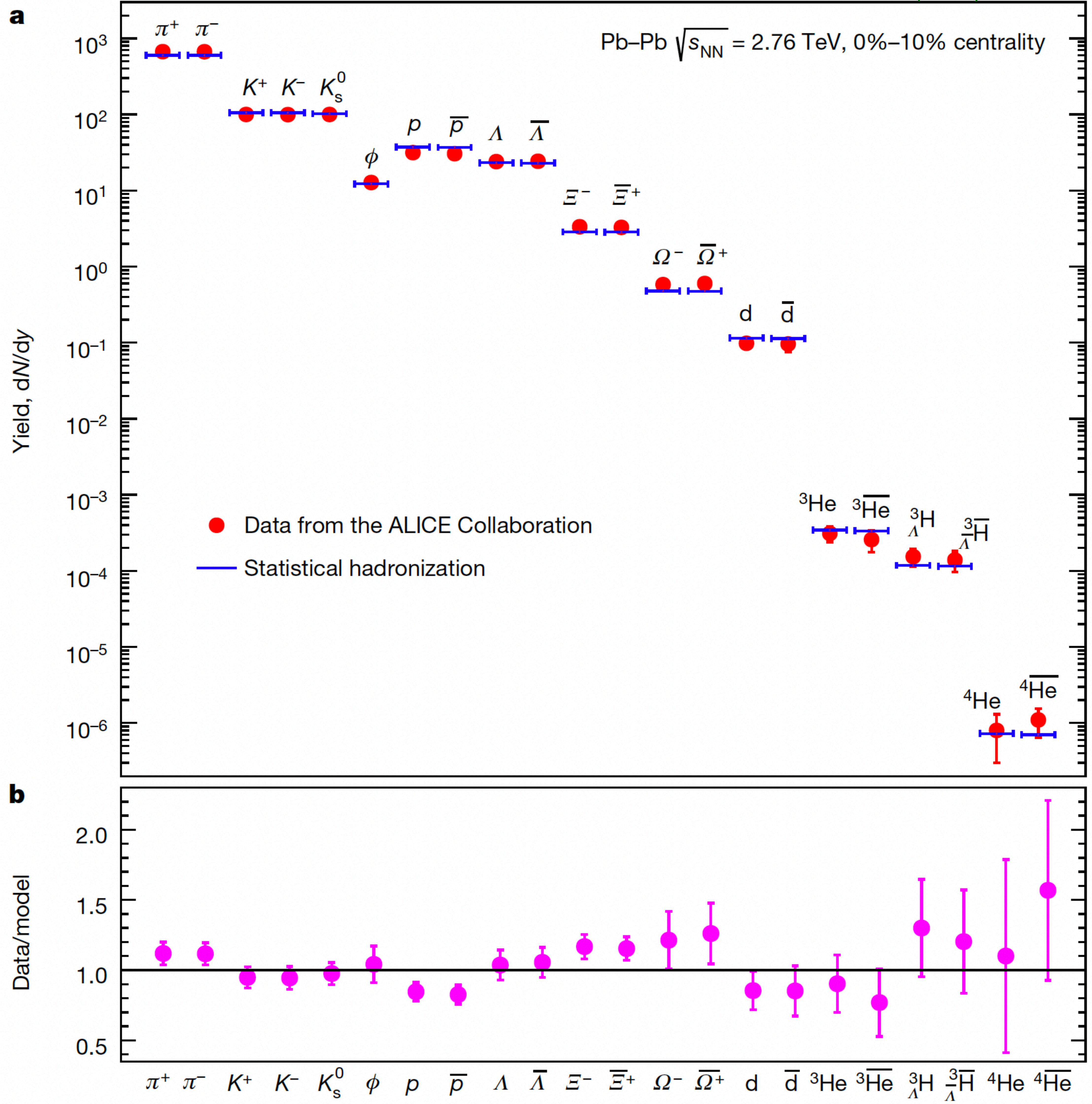
M. Michalec PhD thesis

MAPPING PHASE DIAGRAM WITH STATISTICAL HADRONIZATION MODEL

HADES, Nature Phys. 15 (2019) 10, 1040-1045



A. Andronic et al., Nature 561 (2018) no.7723

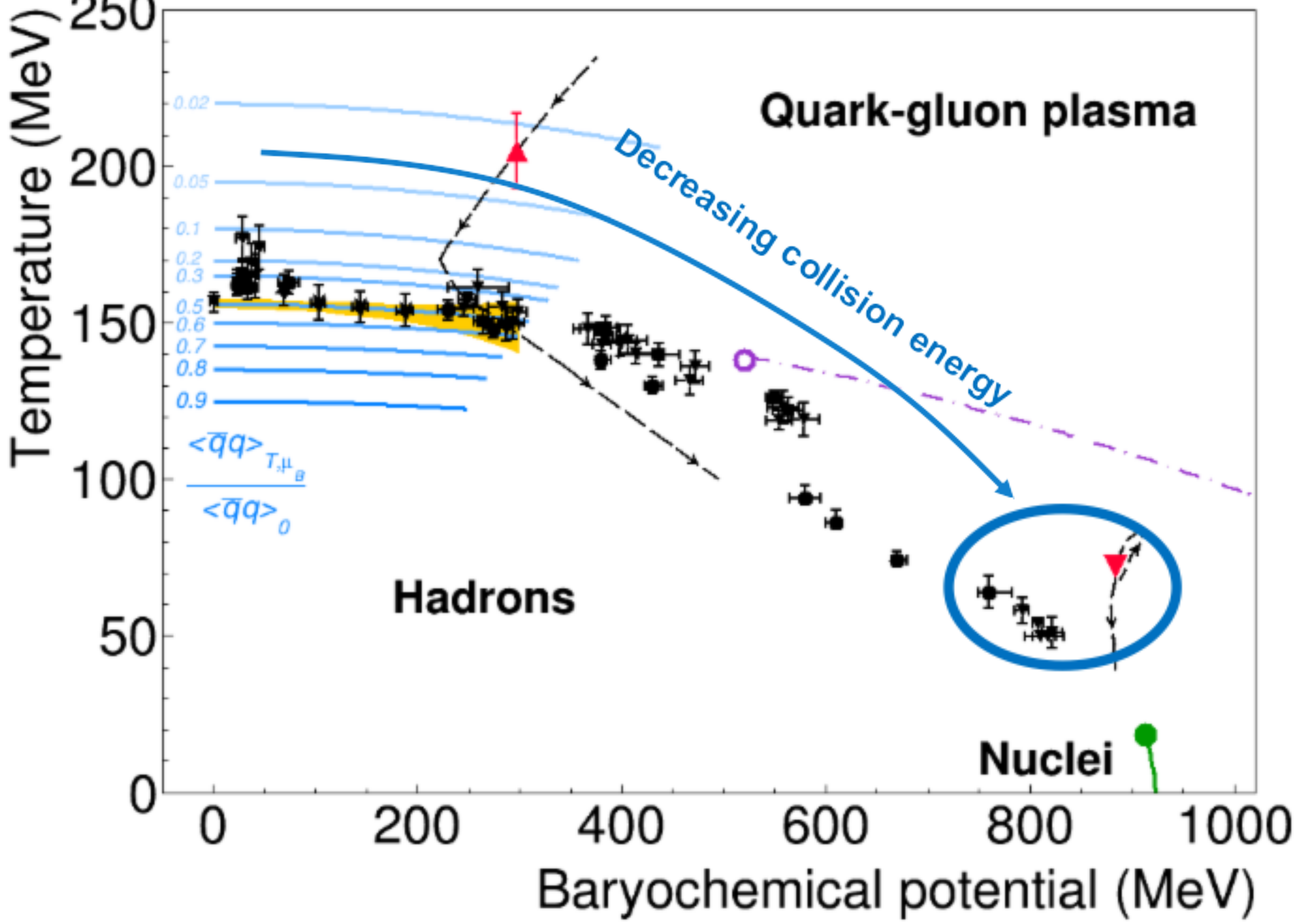


The **hadron yields** can be explained over several orders of multiplicity by fixing just **a few thermodynamic parameters.**

The role of hadronic resonanses is crucial **at high energies** (~400 states are included).

At lower energies their role is diminished.

MAPPING PHASE DIAGRAM WITH STATISTICAL HADRONIZATION MODEL



HADES, Nature Phys. 15 (2019) 10, 1040-1045

Is it valid to assume equilibrium at low energies?

- Low number of newly produced particles in the interaction zone (~40 in central events, mainly pions)

On the other hand:

- Original nucleons stopped in the interaction zone (~300 particles in central events)
- Longer life-time of the system – enough to thermalize

The problem of whether the fireball in few-GeV energy regime is thermalized remains a matter of debate.

The study of **hadron yields** and **spectra** is crucial to answer this question.

DYNAMICAL MODELING OF HEAVY-ION COLLISIONS

Standard prescription at high beam energies (RHIC/LHC):

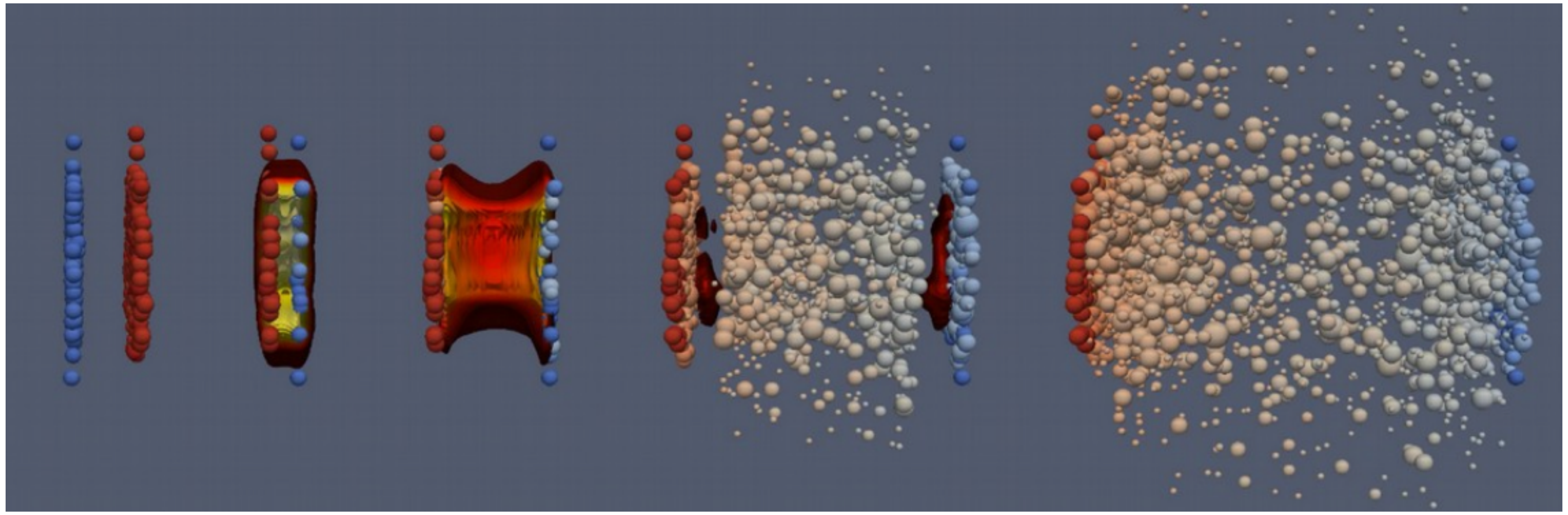
- Non-equilibrium initial conditions
- Viscous hydrodynamic evolution
- Equilibrium
- Hadronic final-state rescattering

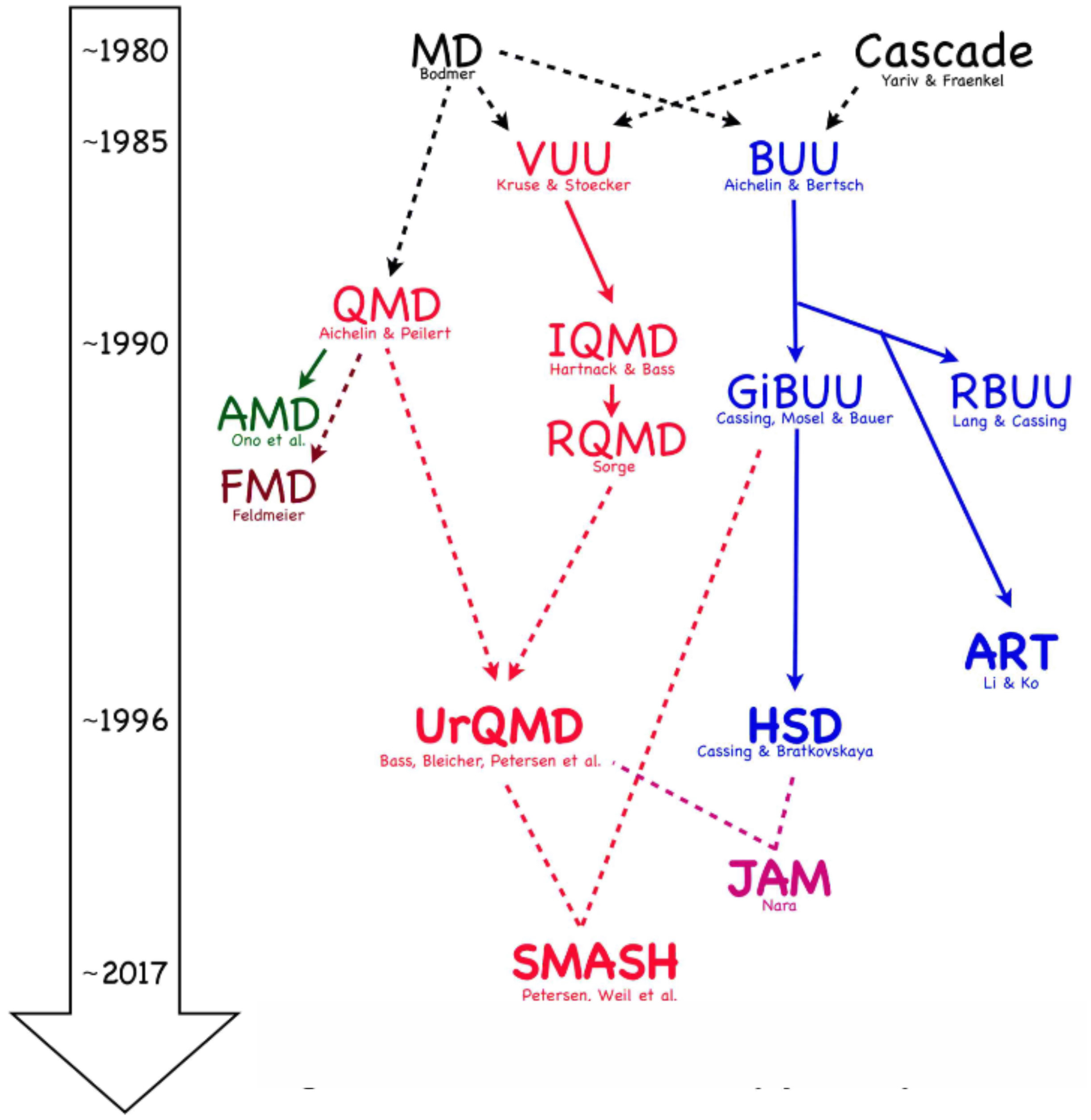
Standard prescription at low beam energies (GSI/FAIR):

- Hadronic transport
- Importance of:
  - Resonance dynamics
  - Nuclear potentials

  …

MADAI collaboration, Hannah Petersen and Jonah Bernhard



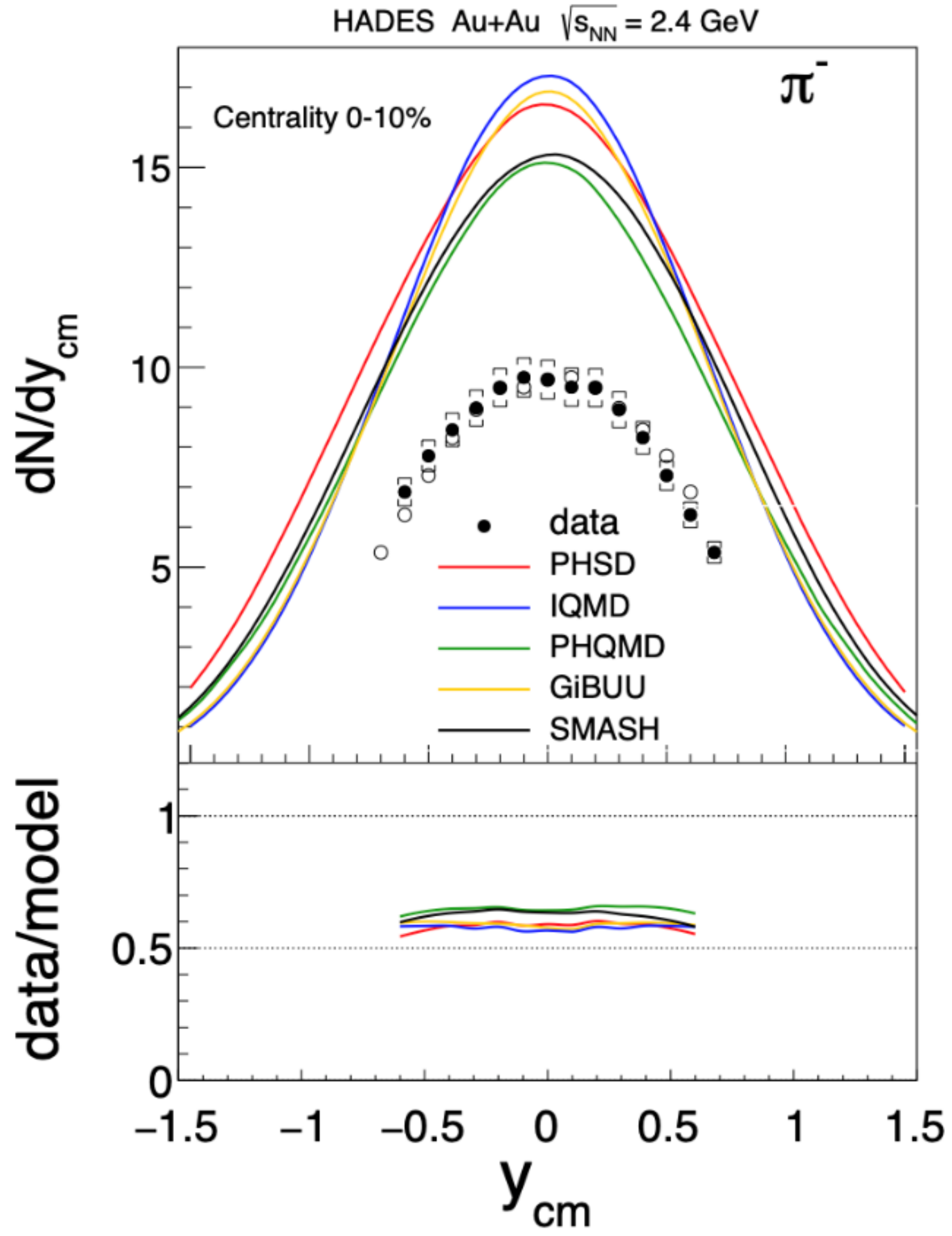


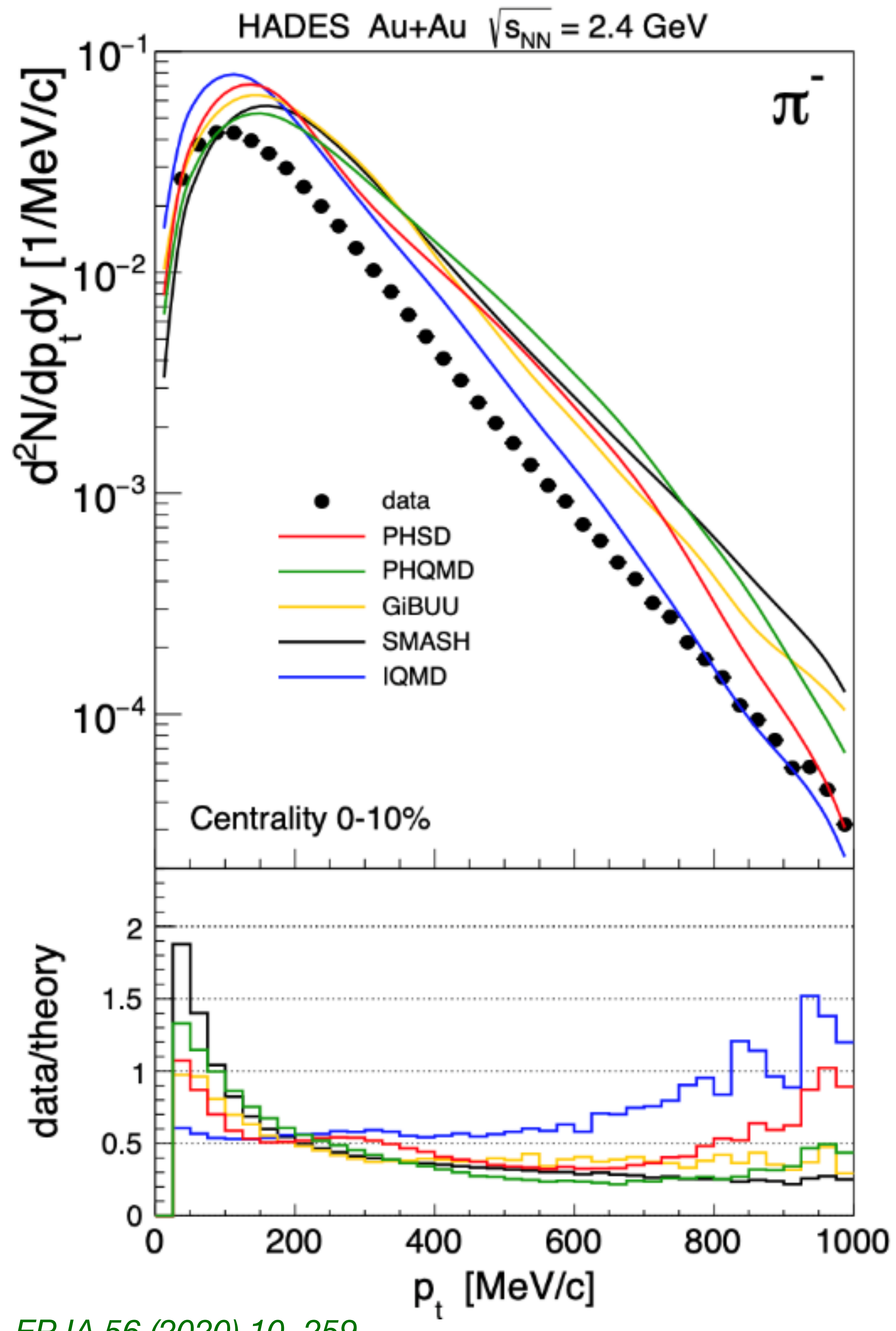
Steffen A. Bass

A FEW-GEV REGIME IS STILL PUZZLING FOR TRANSPORT MODELS

Basic description is obtained with transport models (UrQMD) and the emphasis is usually put on non-equilibrium features.

S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255 (1998),
O. Buss, et al, Phys. Rept. 512, 1 (2012),
H. Petersen, D. Oliinychenko, M. Mayer, J. Staudenmaier, and S. Ryu, Nucl. Phys. A 982, 399 (2019),
C. Hartnack, R. K. Puri, J. Aichelin, J. Konopka, S. A. Bass, H. Stoecker, and W. Greiner, Eur. Phys. J. A 1, 151 (1998)
W. Cassing and E. L. Bratkovskaya, Phys. Rept. 308, 65 (1999).





HADES Collaboration, EPJA 56 (2020) 10, 259
https://www.hepdata.net/record/ins1796710

HYDRO-INSPIRED MODELS

Instead of determining freeze-out conditions from hydrodynamic simulations or transport one can ***model* the freeze-out conditions (hypersurface and flow).**

In the original formulation of the **blast-wave model by Siemens and Rasmussen (SR)** the **freeze-out was spherical and the flow was radial.**

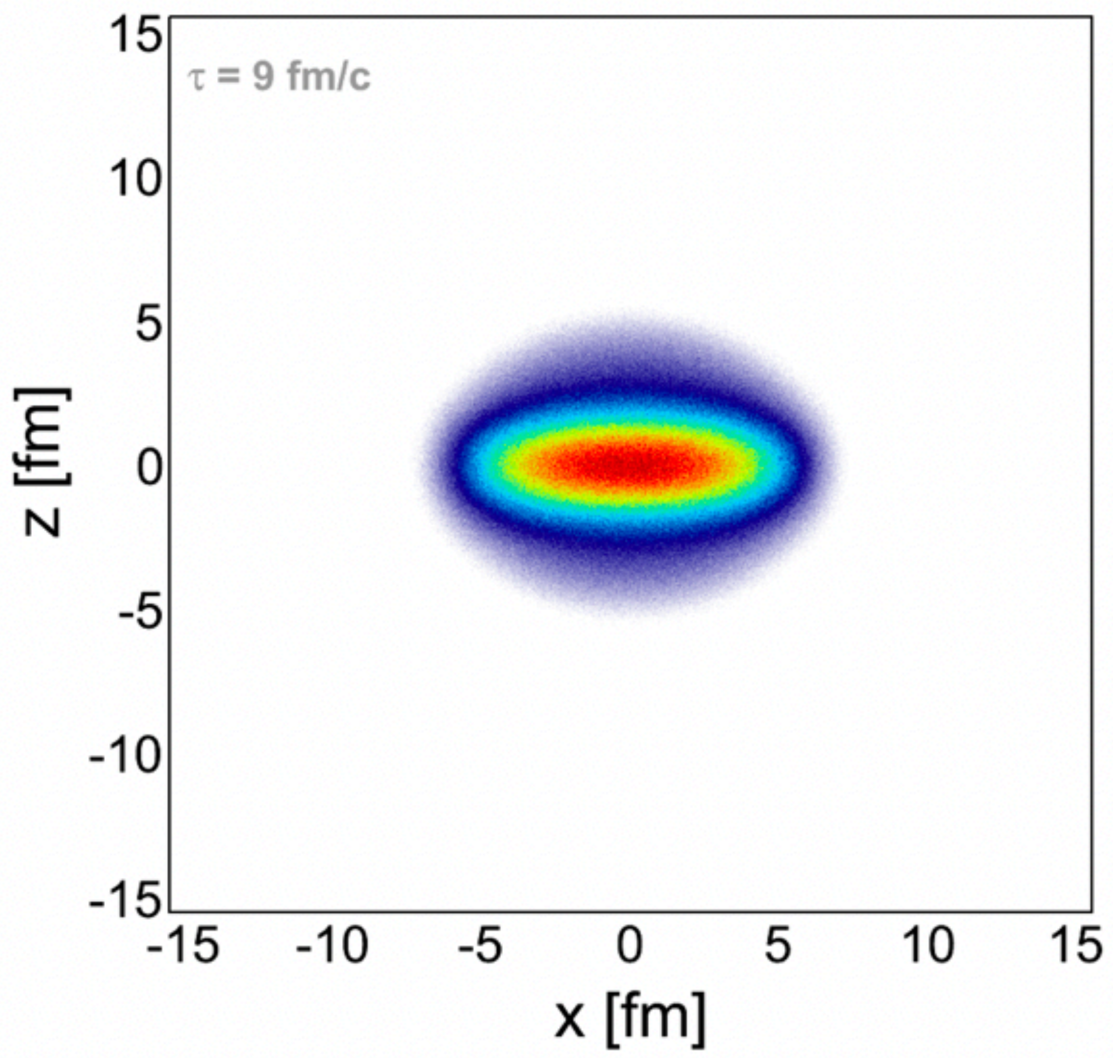
P. Siemens and O. Rasmussen, PRL 42, 880 (1979)

This approach was **modified for higher energies** (RHIC and LHC) **assuming boost-invariance and cyllindrical symmetry** and used extensively by experimentalists.

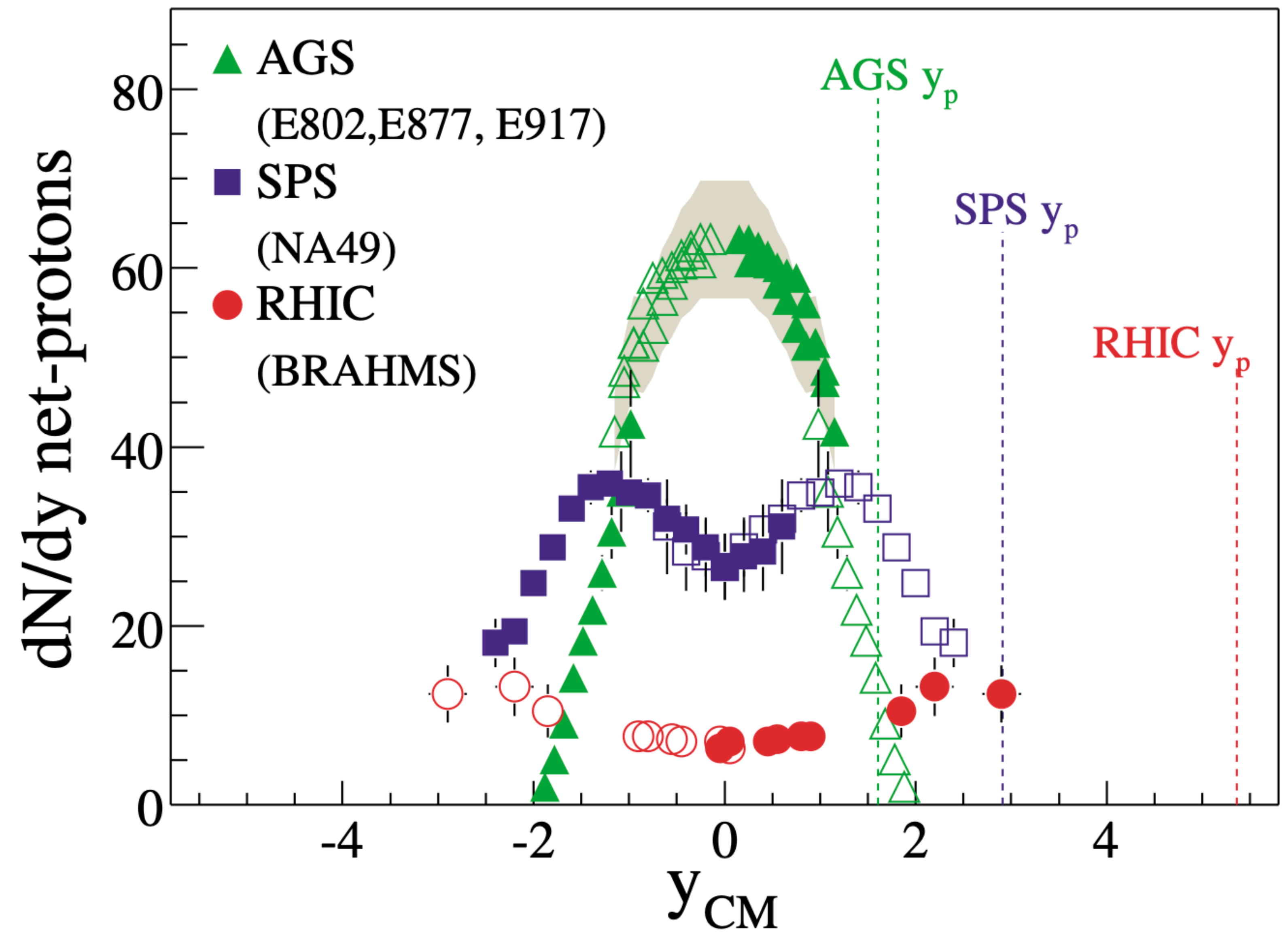
E. Schnedermann, J. Sollfrank, U. Heinz, PRC 48, 2462 (1993)

We aim to re-examine RS model in the context of low-energy collision measurements performed by HADES where boost-invariance is not observed.

T. Galatyuk et al., EPJA 52 (2016) 5, 131



I. G. Bearden et al. (BRAHMS), PRL 93, 102301 (2004)



COOPER-FRYE FORMULA

Invariant momentum spectrum of particles emitted from an expanding source through the hypersurface Σ_μ

F. Cooper and G. Frye, PRD 10, 186 (1974).

$$\boxed{E_p \frac{dN}{d^3p} = \int d^3\Sigma_\mu(x)\, p^\mu f(x,p)} \qquad E_p = \sqrt{m^2 + \boldsymbol{p}^2}.$$

Assuming a **spherically symmetric source** the freeze-out points are defined by the space-time coordinates

$$\boxed{x^\mu = (t, \boldsymbol{x}) = (t(\zeta), r(\zeta)\boldsymbol{e}_r)} \qquad \boldsymbol{e}_r = (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta) \qquad \zeta \longrightarrow (t(\zeta), r(\zeta))$$

$$d^3\Sigma_\mu = -\epsilon_{\mu\alpha\beta\gamma} \frac{\partial x^\alpha}{\partial a} \frac{\partial x^\beta}{\partial b} \frac{\partial x^\gamma}{\partial c}\, da\, db\, dc \;\Longrightarrow\; \boxed{d^3\Sigma_\mu = (r'(\zeta), t'(\zeta)\boldsymbol{e}_r)\, r^2(\zeta) \sin\theta\, d\theta\, d\phi\, d\zeta}$$

We assume **sudden freezeout** $\boxed{t(r) = \text{const}}$

With the hadron four-momentum parametrized as $p^\mu = (E_p, p\,\boldsymbol{e}_p)$ $\quad \boldsymbol{e}_p = (\cos\varphi \sin\vartheta, \sin\varphi \sin\vartheta, \cos\vartheta)$

We get $\boxed{d^3\Sigma(x) \cdot p = E_p \sin\theta\, d\theta\, d\phi\, r^2 dr}$

LOCAL THERMAL EQUILIBRIUM

Assume that the hadron system formed is very close to **local thermodynamic equilibrium**

$$\boxed{f(x,p) = \frac{g_s}{(2\pi)^3}\left[\Upsilon^{-1}\exp\left(\frac{p\cdot u}{T}\right) - \chi\right]^{-1}}$$

The fugacity is defined as

G. Torrieri, S. Steinke, W. Broniowski, W. Florkowski, J. Letessier, and J. Rafelski, CPC 167, 229 (2005).

$$\boxed{\Upsilon = \gamma_q^{N_q+N_{\bar{q}}}\gamma_s^{N_s+N_{\bar{s}}}\exp\left(\frac{\mu}{T}\right)} \qquad \mu = \sum\nolimits_Q Q\mu_Q \qquad Q \in \{B, I_3, S\}$$

We allow for **strangeness undersaturation** (characteristic feature at low beam energies).

J. Rafelski, J. Letessier, and A. Tounsi, Acta Phys. Pol. B28 (1997) 2841

HUBBLE-LIKE RADIAL FLOW

We introduce a **spherically symmetric flow**

$$\boxed{u^\mu = \gamma(r)(1, v(r)\boldsymbol{e}_r)}$$

In the **original SR blast-wave model**, it was assumed that the thermodynamic parameters as well as the radial flow velocity are constant

$$(T = \text{const}, \mu = \text{const}, v = v_0 = \text{const})$$

We take **Hubble-like flow**

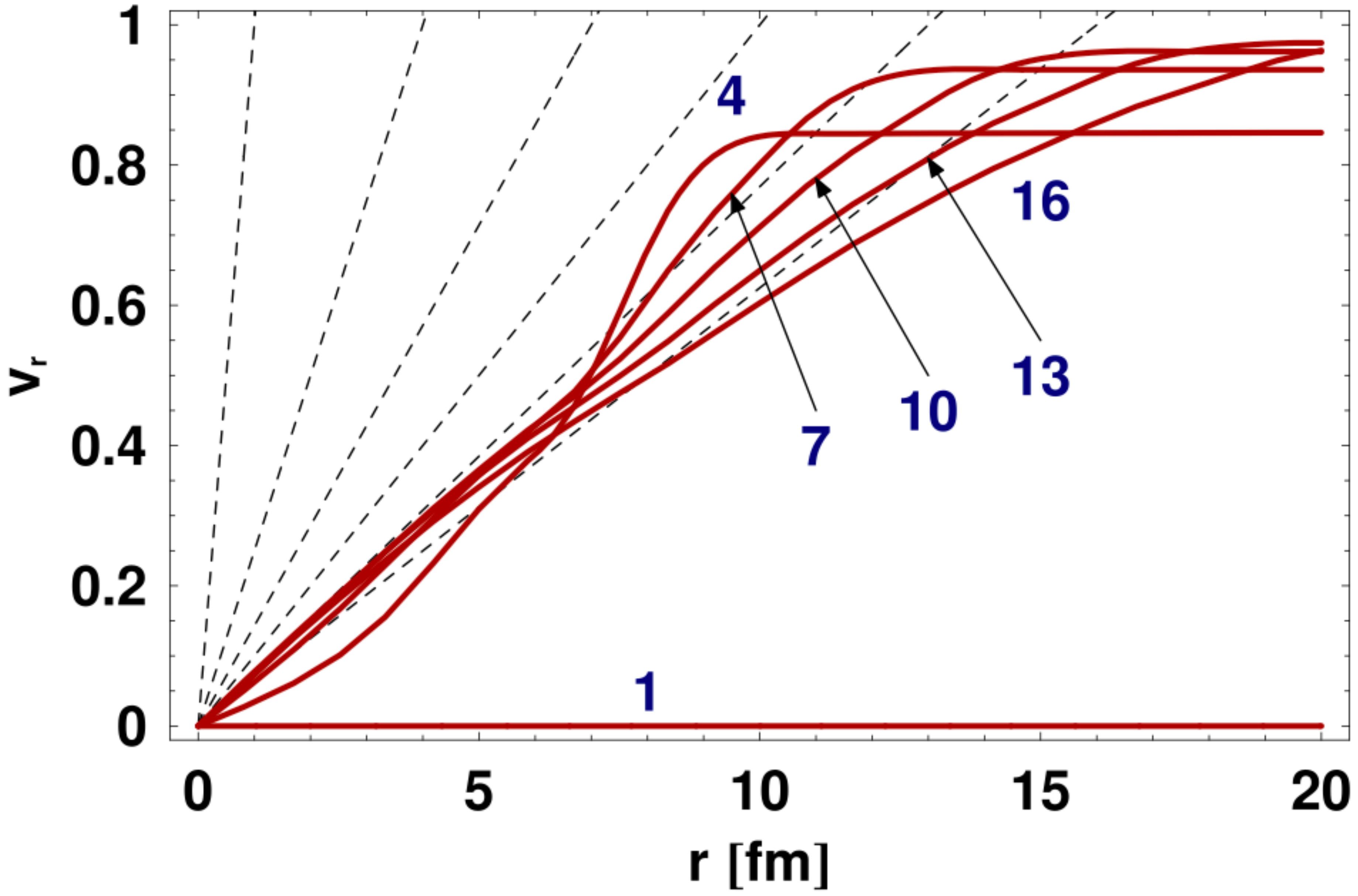
$$\boxed{v(r) = \tanh(Hr)}$$

The parameter **H plays a role of the Hubble constant** in the theory of expanding Universe.

As a result we get

$$\boxed{p \cdot u = \gamma(E_p - pv\kappa)} \qquad \kappa \equiv \boldsymbol{e}_p \cdot \boldsymbol{e}_r$$

M. Chojnacki, W. Florkowski, and T. Csorgo, PRC 71, 044902 (2005).



Condition of constant radial flow breaks requirement that the flow at the center of the system should vanish.

Results of hydrodynamic calculations indicate that the **radial flow linearly grows with radius** for small values of r.

THERMINATOR

Our freeze-out prescription is implemented in the **THERMINATOR** Monte Carlo hadron generator which allows for studies of hadron production taking place on **arbitrary freeze-out hypersurfaces** defined in the four-dimensional space-time.

A. Kisiel, T. Taluc, W. Broniowski, and W. Florkowski, Comput. Phys. Commun. 174, 669 (2006).
M. Chojnacki, A. Kisiel, W. Florkowski, and W. Broniowski, Comput. Phys. Commun. 183, 746 (2012).

THERMINATOR generates primordial particles at the freeze-out hypersurface (~400 states are included).

Unstable particles are free-streaming and are allowed to decay contributing to feed-down.

The code includes contributions from decays of all heavier resonances — most of them are very small or negligible.

The largest contribution comes from decays of the lowest-lying baryonic resonance, i.e., Delta (1232).

THERMODYNAMIC PARAMETERS

We obtain thermodynamic model parameters from the ratios of experimental yields measured by HADES in Au+Au collisions at 2.4 GeV.

We assume that the **protons finally bound in the emitted deuterons, tritons, and ^{3}Helium nuclei were initially frozen out as unbound nucleons**; hence, they are included in the proton yield

In the studies of the ratios of hadronic yields the invariant volume cancels (if the thermodynamic parameters are constant on the freeze-out hypersurface!).

$$N = \int d^3\Sigma_\mu(x) \int \frac{d^3p}{E_p} p^\mu f(x,p).$$

$$N = n(T,\Upsilon) \int d^3\Sigma_\mu(x) u^\mu(x) \equiv n(T,\Upsilon)\mathcal{V},$$

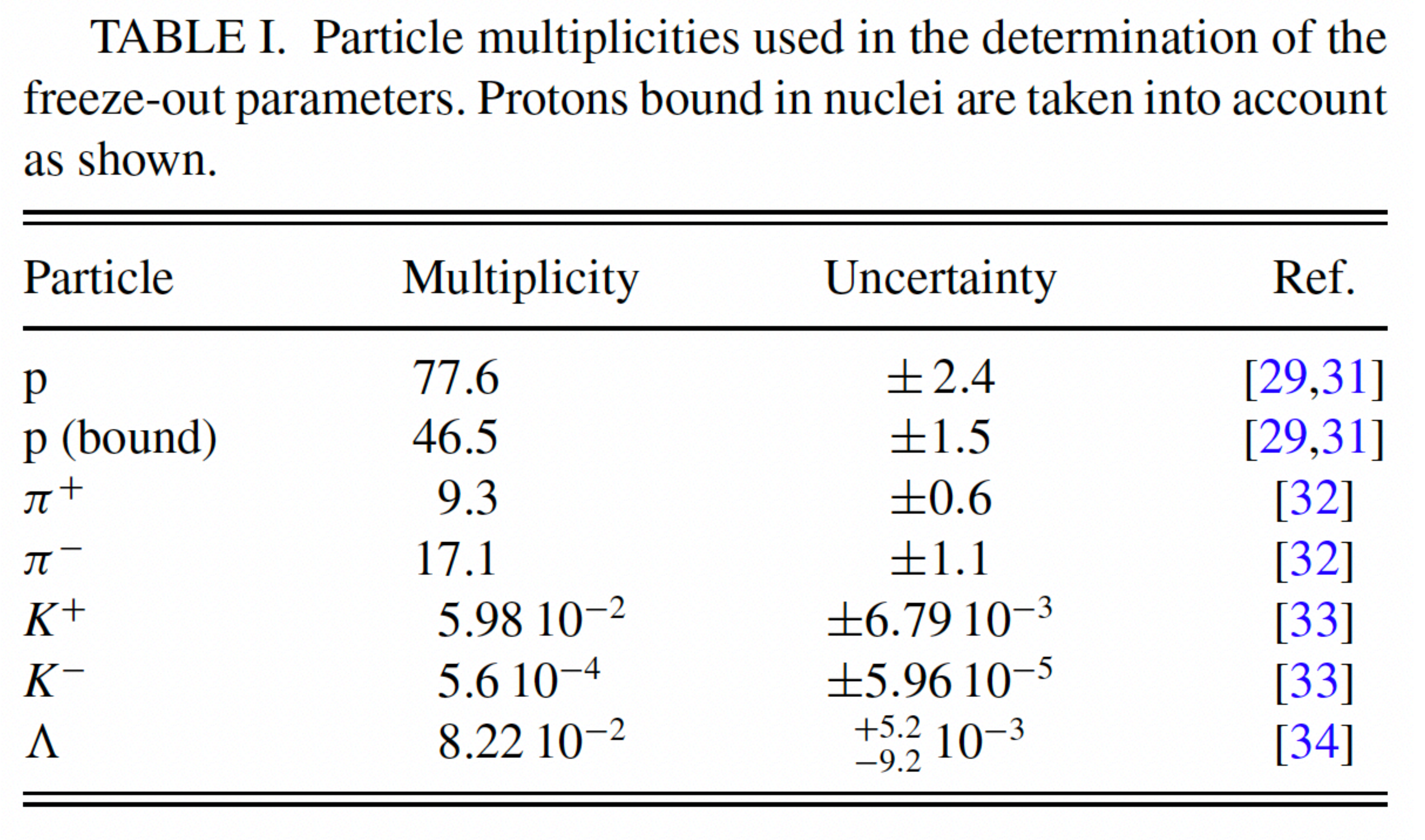


TABLE I. Particle multiplicities used in the determination of the freeze-out parameters. Protons bound in nuclei are taken into account as shown.

Particle	Multiplicity	Uncertainty	Ref.
p	77.6	±2.4	[29,31]
p (bound)	46.5	±1.5	[29,31]
π^+	9.3	±0.6	[32]
π^-	17.1	±1.1	[32]
K^+	$5.98\ 10^{-2}$	$\pm 6.79\ 10^{-3}$	[33]
K^-	$5.6\ 10^{-4}$	$\pm 5.96\ 10^{-5}$	[33]
Λ	$8.22\ 10^{-2}$	$^{+5.2}_{-9.2}\ 10^{-3}$	[34]

$\sqrt{s_{NN}} = 2.4$ GeV full phase space for the 10% Au-Au collisions

[29] M. Szala (HADES), *Light nuclei formation in heavy ion collisions measured with HADES*
[31] M. Szala (HADES), *Springer Proc.Phys.* 250 (2020) 297-301
[32] J. Adamczewski-Musch et al., (HADES) *EPJA 56 (2020) 10, 259*
[33] J. Adamczewski-Musch et al. (HADES), *PLB 778, 403 (2018).*
[34] J. Adamczewski-Musch et al. (HADES), *PLB 793, 457 (2019).*

THERMODYNAMIC PARAMETERS

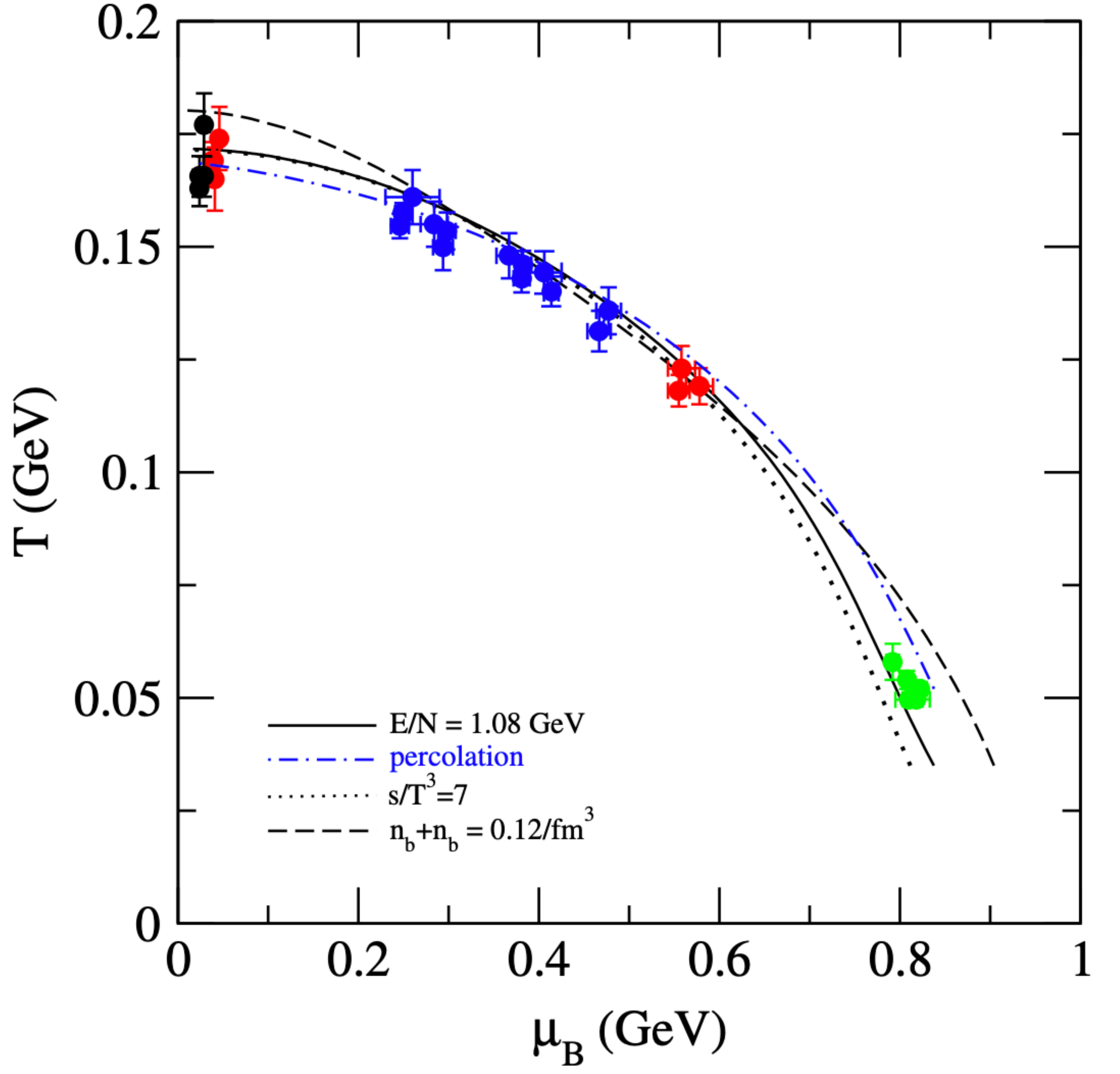
$$T = 49.6 \pm 1 \text{ MeV}, \mu_B = 776 \pm 3 \text{ MeV},$$

$$\mu_{I_3} = -14.1 \pm 0.2 \text{ MeV}, \mu_S = 123.4 \pm 2 \text{ MeV},$$

$$\gamma_s = 0.16 \pm 0.02$$

J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, PRC 73, 034905 (2006).
P. Castorina, A. Iorio, D. Lanteri, H. Satz, and M. Spousta, PRC 101, 054902 (2020).

J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, PRC 73, 034905 (2006).



TRANSVERSE-MOMENTUM AND RAPIDITY SPECTRA

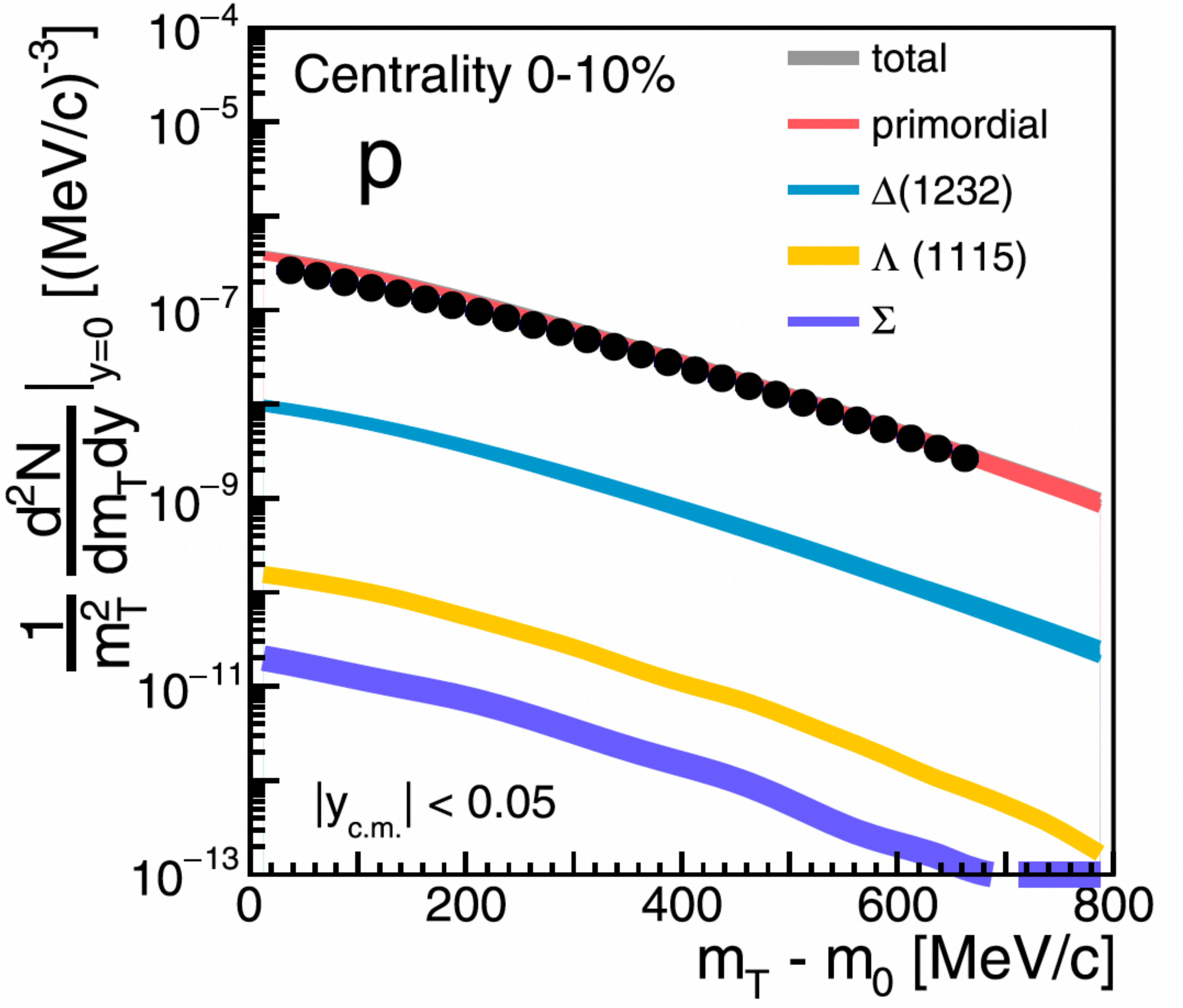
For a fixed value of **H**, the absolute normalization of the yields determines the value of **R**.
Hence we may treat **R = R(H)** and we are left with only one independent parameter **H.**

Value of **H** is obtained from the fit of the proton transverse-mass spectrum by minimization of the quadratic deviation

$$Q^2(H) = \sum_i \frac{(Q_{i,\mathrm{model}}(H) - Q_{i,\mathrm{exp}})^2}{Q^2_{i,\mathrm{exp}}}$$

R = 16.02 fm

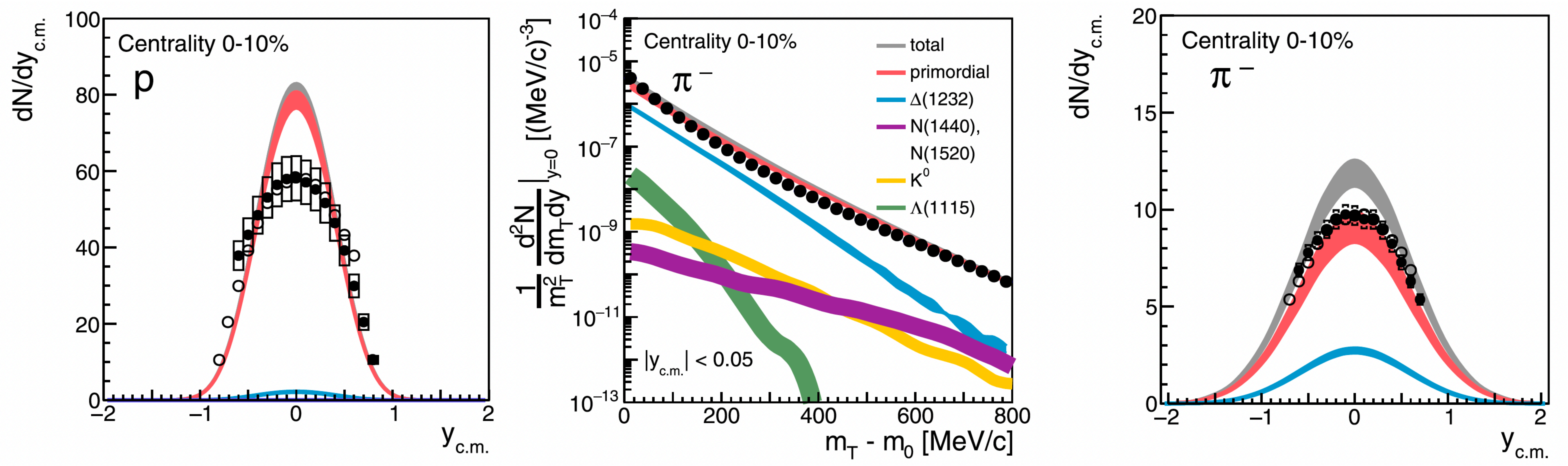
H = 0.04 1/fm



$(Q = 0.20)$

TRANSVERSE-MOMENTUM AND RAPIDITY SPECTRA

Having determined the value of H, we can predict other model spectra.



$Q = 0.28$

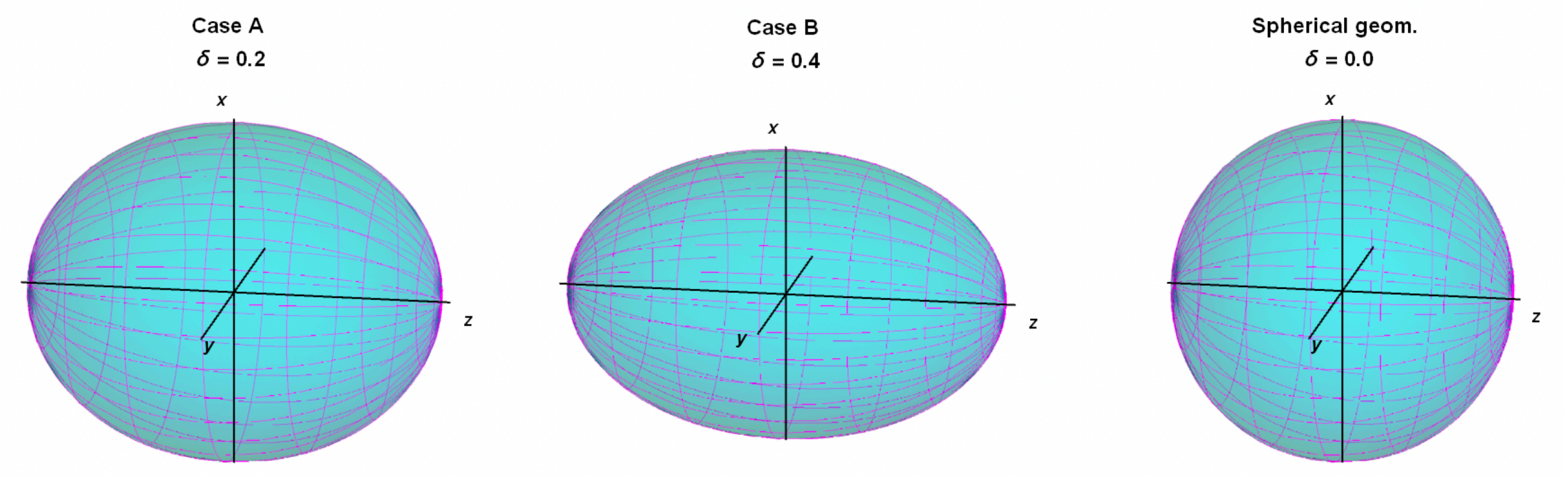
The fact that the rapidity distribution is equally well described (compared to the transverse-mass distribution) points out the **approximate (!) spherical symmetry** of the produced system.

IMPROVEMENT: SPHEROIDAL EXPANSION

$$x^\mu = \left(t, r\boxed{\sqrt{1-\epsilon}}\sin\theta\,\hat{\mathbf{e}}_\rho, r\boxed{\sqrt{1+\epsilon}}\cos\theta\right)$$

$$u^\mu = \gamma(\zeta,\theta)\left(1, v(\zeta)\boxed{\sqrt{1-\delta}}\sin\theta\,\hat{\mathbf{e}}_\rho, v(\zeta)\boxed{\sqrt{1+\delta}}\cos\theta\right)$$

$$\hat{\mathbf{e}}_\rho = (\cos\phi, \sin\phi)$$



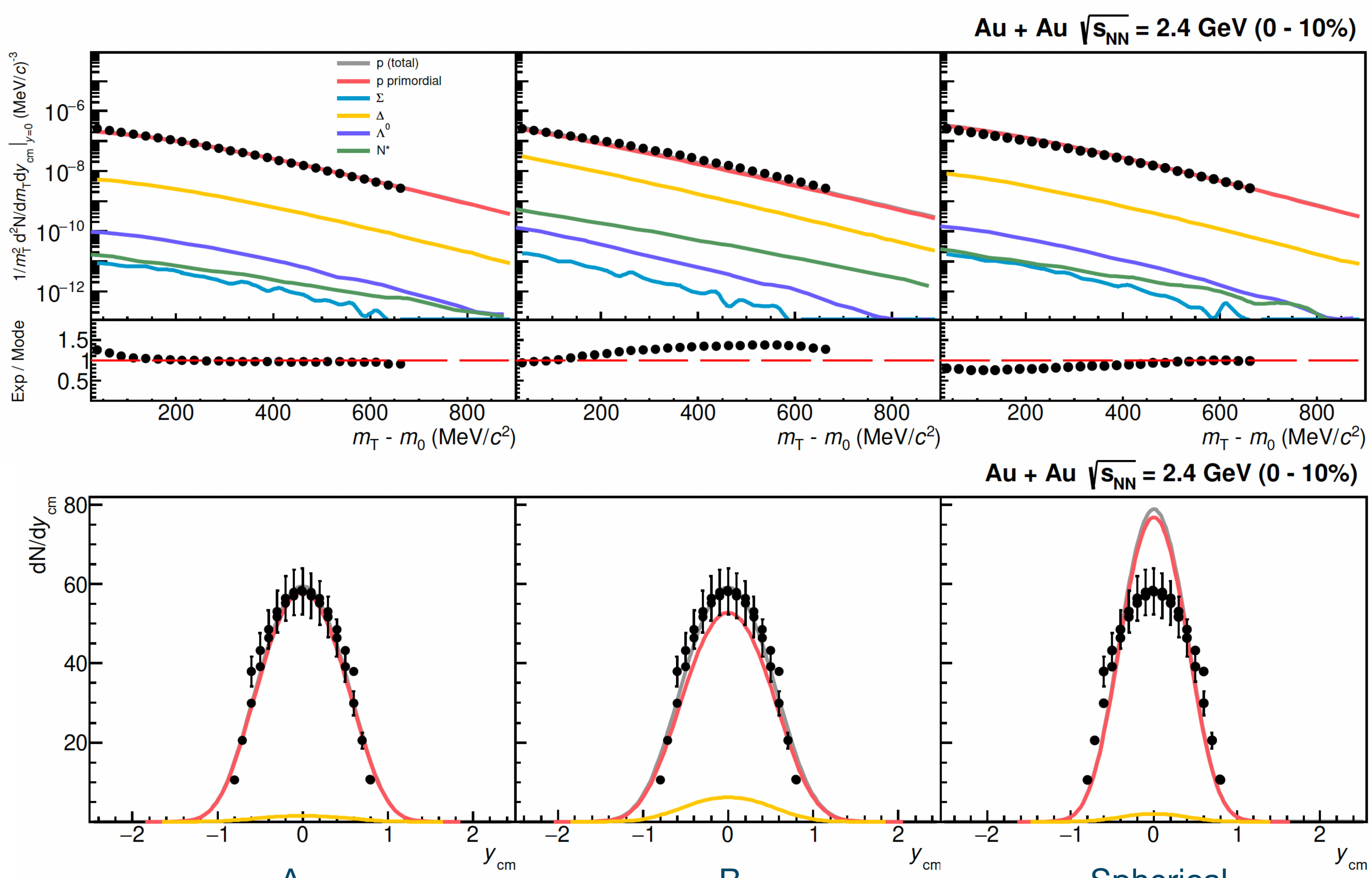
We take for comparison transverse mass distributions of protons, + and - pions in five center-of-mass rapidity intervals:
[0:45;0:35],
[0:25;0:15],
[0:05; 0:05],
[0:15; 0; 25],
[0:35; 0:45]

Parameter	Spherical geometry, Ref. [29]	Case A	Case B
T (MeV)	49.6	49.6	70.3
R (fm)	16.0	15.7	6.06
μ_B (MeV)	776	776	876
μ_S (MeV)	123.4	123.4	198.3
μ_{I_3} (MeV)	-14.1	-14.1	-21.5
γ_S	0.16	0.16	0.05
$\chi^2/N_{\rm df}$	$N_{\rm df}=0$	$N_{\rm df}=0$	1.13/2
H (GeV)	0.008	0.01	0.0225
δ	0	0.2	0.4
$\sqrt{Q^2}$	0.285	0.238	0.256

$S = 0$ and $Q/B = 0.4$

A. Motornenko, J. Steinheimer, V. Vovchenko, R. Stock, and H. Stoecker PLB 822, 136703 (221),

RESULTS

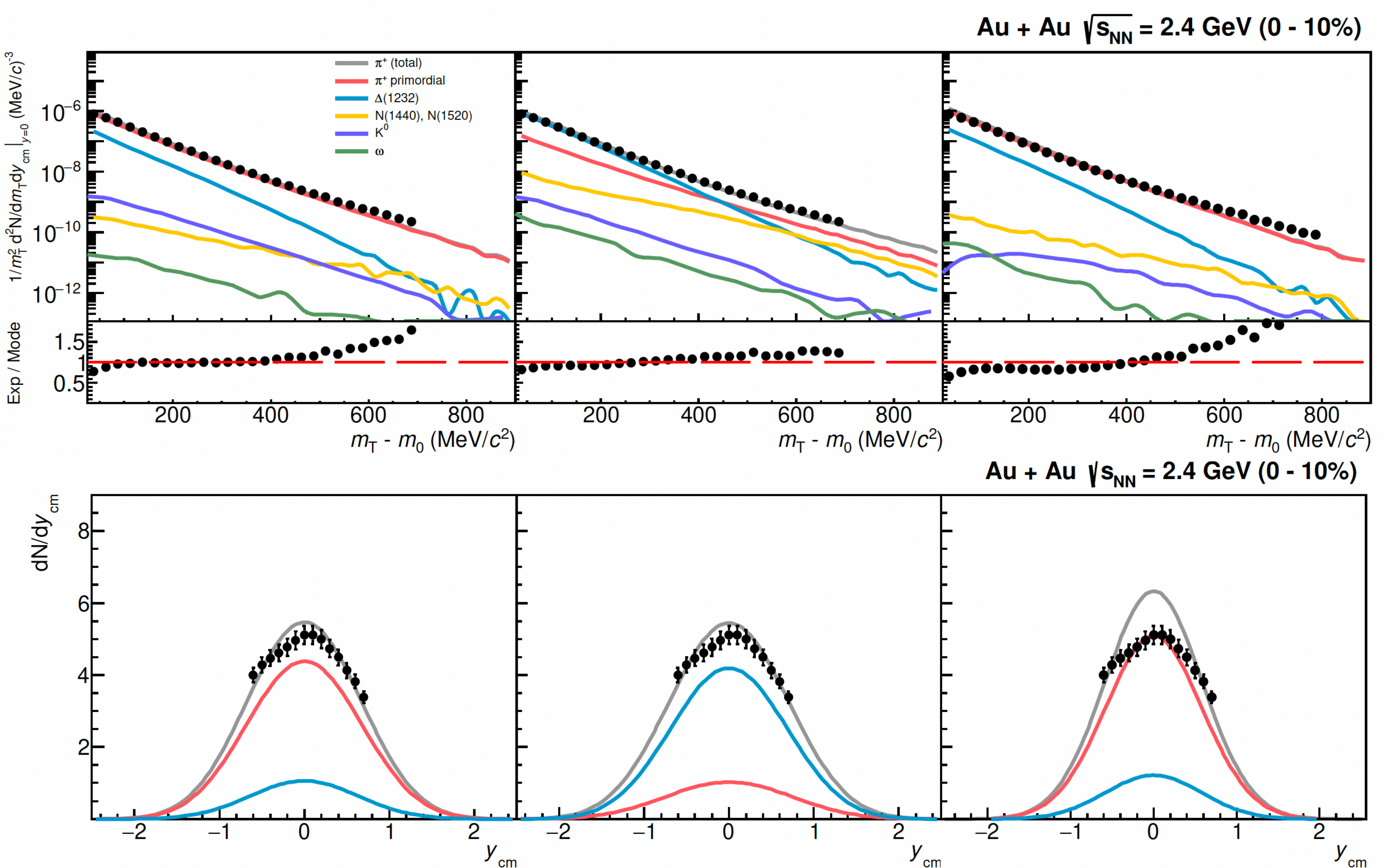


A

B

Spherical

RESULTS

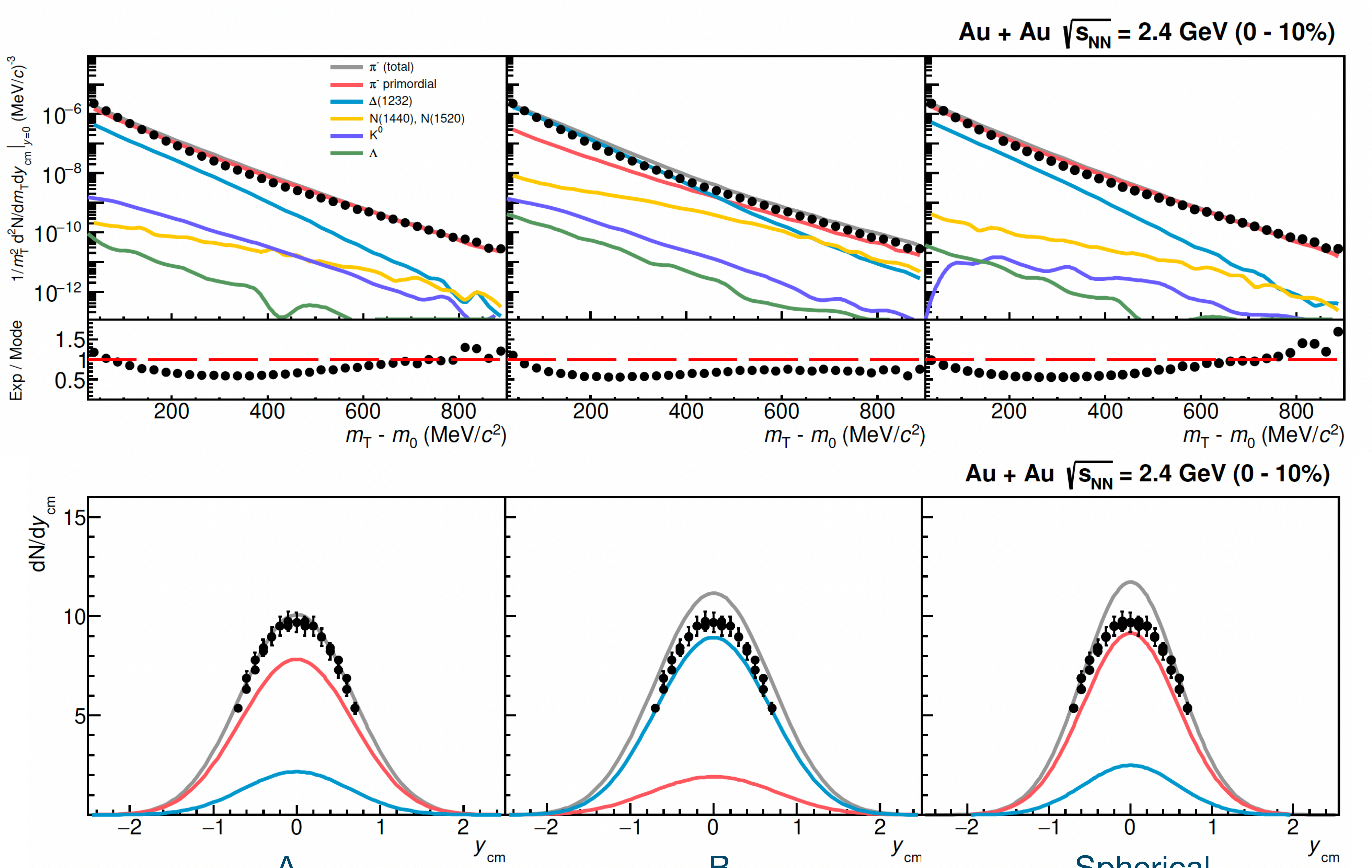


A

B

Spherical

RESULTS



A

B

Spherical

CONCLUSIONS

- We have **studied the rapidity and transverse mass spectra of protons and pions** produced in Au-Au collisions at 2.4 GeV and measured by HADES.

- We have found that they can be **well reproduced in a extended SR model** that assumes **single freeze-out of hadrons from a hypersurface spheroidal along beam direction**.

- Our framework modifies and extends RS approach by incorporation of the Hubble-like expansion of matter, inclusion of the resonance decays, and spheroidal deformation of the source.

- We have found that the presence of the **Delta resonance affects the spectra of pions**, while the contributions from other resonances can be neglected.

- The **obtained thermodynamic parameters agree well with the universal freeze-out curve established by other groups**.

- Our results bring **evidence for substantial thermalization of the matter produced in the few-GeV energy range and its nearly spherical expansion.**

THANK YOU FOR YOUR ATTENTION.

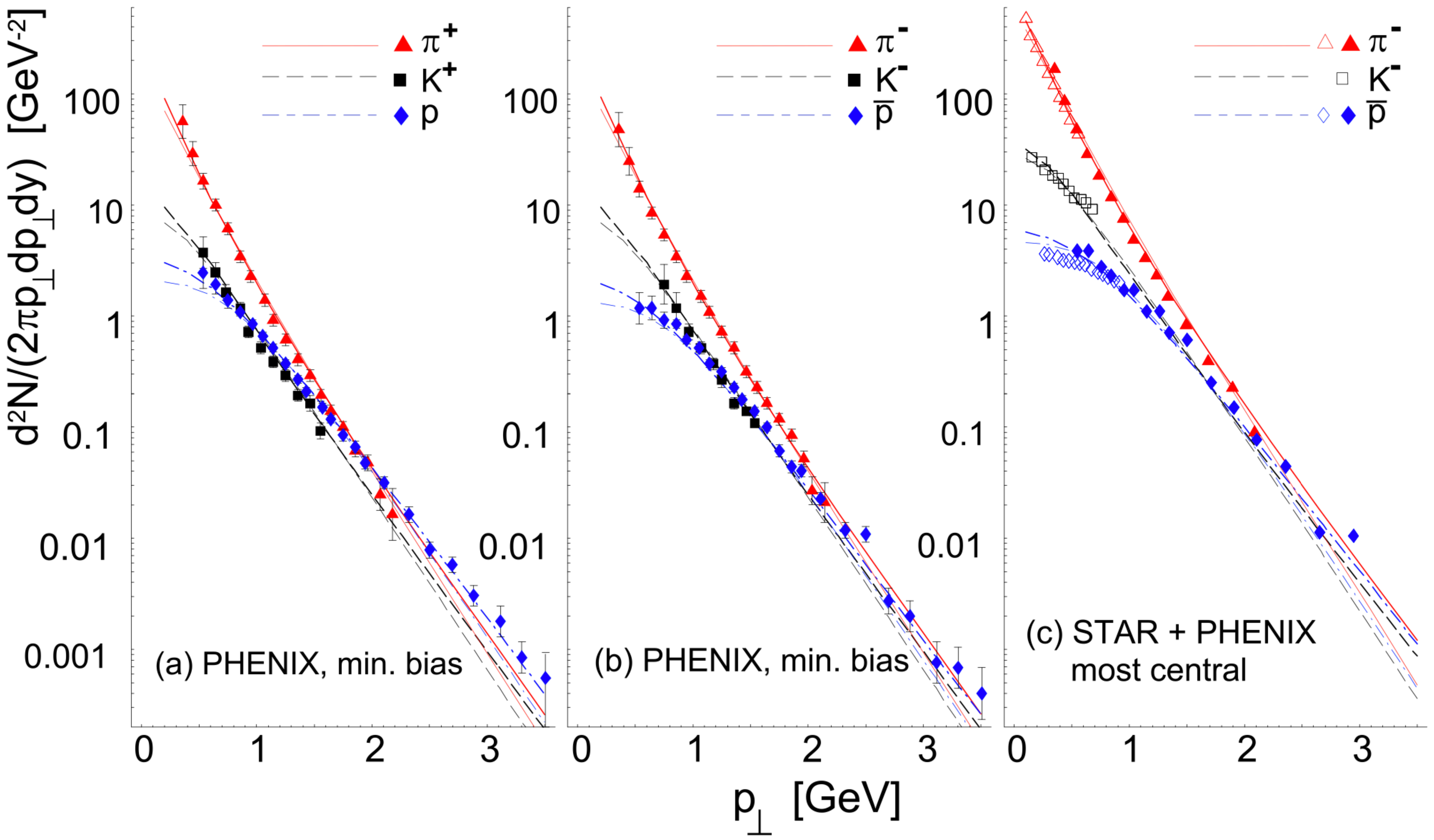
SINGLE-FREEZE-OUT SCENARIO

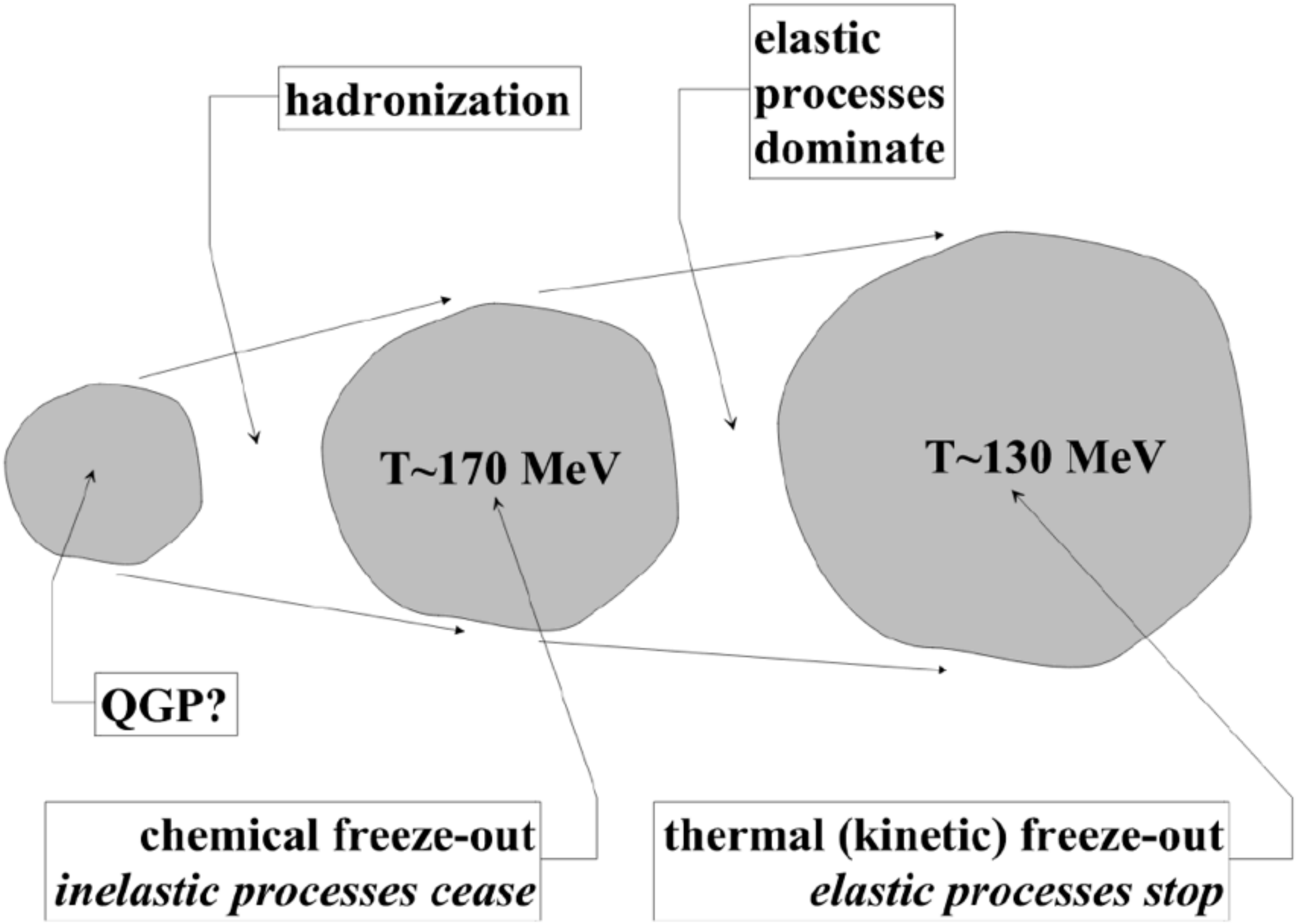
1) Studies of ratios of hadron yields define **chemical freeze-out.**

2) Studies of spectra of hadrons define **kinetic freeze-out.**

We adopt the **single-freeze-out scenario** where chemical and kinetic freeze-outs coincide - successful at high energies.

W. Broniowski, W. Florkowski, PRL 87, 272302 (2001)

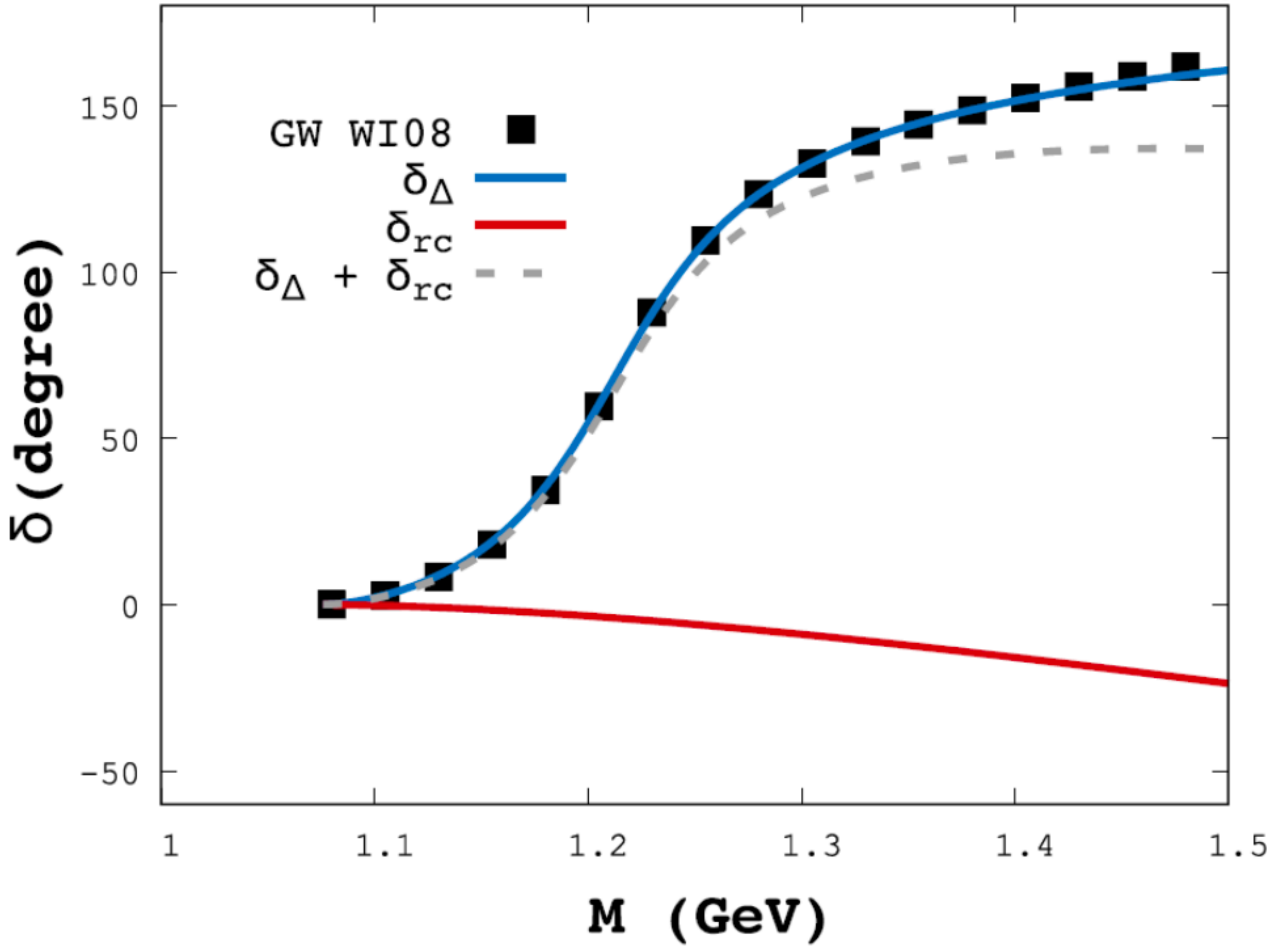




DELTA RESONANCE TREATMENT

πN phase shift in the P_{33} channel

[Pok Man Lo *et al.*, PRC 96, 015207,
GW WI08: R.L. Workman et al. PRC 86, 035202]



Spectral function: $B_l(M) = 2\frac{d}{dM}\delta_l$