Sum rules and factorization of double parton distributions

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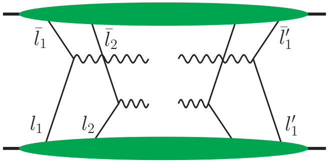
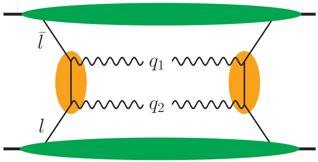
Plan

1. Double parton scattering and double parton distriburtions
2. Evolution equations and sum rules
3. Small x factorization and momentum sum rule

Double parton scattering - DPS

- With increasing $\sqrt{s}$ multiparton interactions become increasing important.
- The first step - DPS in addition to SPS





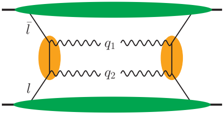
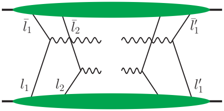
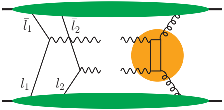
- Collinear factorization with double parton distribution functions (DPDFs):

$$\frac{d\sigma^{AB}_{DPS}}{dx_1\, dx_2\, d\bar{x}_1\, d\bar{x}_2} = \sum_{f_i \bar{f}_i} \int d^2\Delta\, D_{f_1 f_2}(x_1, x_2, \Delta)\, \sigma^{A}_{f_1 \bar{f}_1}(q_1)\, \sigma^{B}_{f_2 \bar{f}_2}(q_2)\, D_{\bar{f}_1 \bar{f}_2}(\bar{x}_1, \bar{x}_2, -\Delta)$$

- In addition, two hard scale dependence in DPDFs: q_1, q_2

(M. Diehl, D. Ostermeier, A. Schäfer, 1111.0910/JHEP)

DPS versus SPS

	$\frac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i}$	$\frac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i}$
	$\frac{1}{\Lambda^2 Q^2}$	1
	$\frac{1}{\Lambda^2 Q^2}$	$\frac{\Lambda^2}{Q^2}$
	$\frac{1}{\Lambda^2 Q^2}$	$\frac{\Lambda^2}{Q^2}$

- Inclusive DPS is enhanced due to rising guon density for $x \to 0$:

$$d\sigma^{AB}_{DPS} \sim g^4(x) \qquad\qquad d\sigma^{AB}_{\mathrm{SPS}} \sim g^2(x)$$

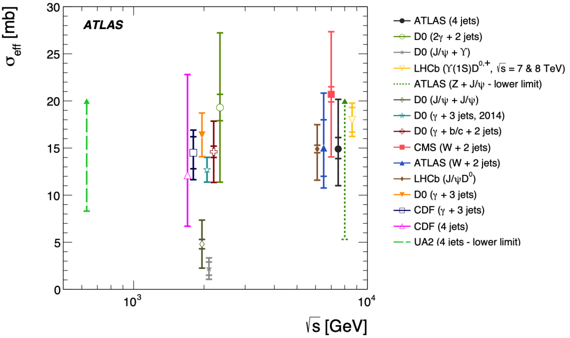
Standard approach to DPS

- In the first approximation:

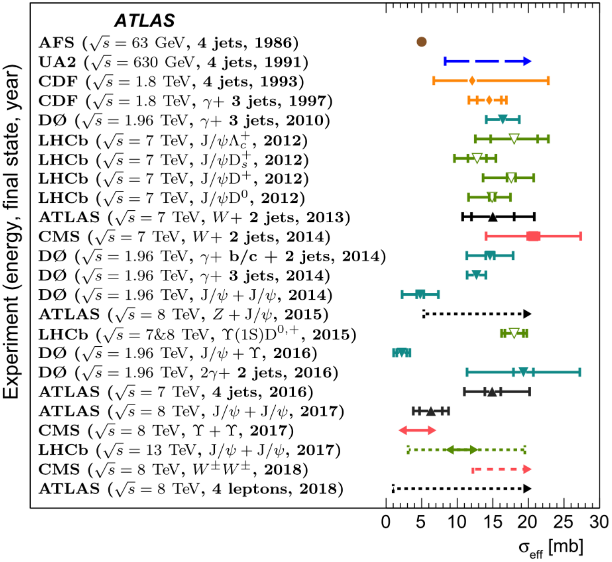
$$D_{f_1 f_2}(x_1, x_2, \Delta) = D_{f_1}(x_1)\, D_{f_2}(x_2)\, F(\Delta)$$

- Pocket formula:

$$\sigma^{AB}_{DPS} = \sigma^{A}_{SPS}\, \sigma^{B}_{SPS} \int d^2\Delta\, F(\Delta) F(-\Delta) = \frac{\sigma^{A}_{SPS}\, \sigma^{B}_{SPS}}{\sigma_{eff}}$$

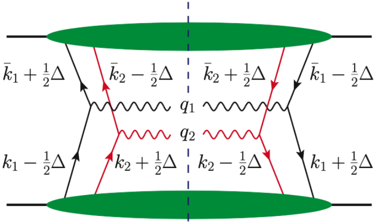


- Goal: to go beyond this approximation.


ATLAS
AFS ($\sqrt{s} = 63$ GeV, 4 jets, 1986)
UA2 ($\sqrt{s} = 630$ GeV, 4 jets, 1991)
CDF ($\sqrt{s} = 1.8$ TeV, 4 jets, 1993)
CDF ($\sqrt{s} = 1.8$ TeV, $\gamma +$ 3 jets, 1997)
DØ ($\sqrt{s} = 1.96$ TeV, $\gamma +$ 3 jets, 2010)
LHCb ($\sqrt{s} = 7$ TeV, $J/\psi \Lambda_c^+$, 2012)
LHCb ($\sqrt{s} = 7$ TeV, $J/\psi D_s^+$, 2012)
LHCb ($\sqrt{s} = 7$ TeV, $J/\psi D^+$, 2012)
LHCb ($\sqrt{s} = 7$ TeV, $J/\psi D^0$, 2012)
ATLAS ($\sqrt{s} = 7$ TeV, $W +$ 2 jets, 2013)
CMS ($\sqrt{s} = 7$ TeV, $W +$ 2 jets, 2014)
DØ ($\sqrt{s} = 1.96$ TeV, $\gamma +$ b/c + 2 jets, 2014)
DØ ($\sqrt{s} = 1.96$ TeV, $\gamma +$ 3 jets, 2014)
DØ ($\sqrt{s} = 1.96$ TeV, $J/\psi + J/\psi$, 2014)
ATLAS ($\sqrt{s} = 8$ TeV, $Z + J/\psi$, 2015)
LHCb ($\sqrt{s} = 7\&8$ TeV, $\Upsilon(1S)D^{0,+}$, 2015)
DØ ($\sqrt{s} = 1.96$ TeV, $J/\psi + \Upsilon$, 2016)
DØ ($\sqrt{s} = 1.96$ TeV, $2\gamma +$ 2 jets, 2016)
ATLAS ($\sqrt{s} = 7$ TeV, 4 jets, 2016)
ATLAS ($\sqrt{s} = 8$ TeV, $J/\psi + J/\psi$, 2017)
CMS ($\sqrt{s} = 8$ TeV, $\Upsilon + \Upsilon$, 2017)
LHCb ($\sqrt{s} = 13$ TeV, $J/\psi + J/\psi$, 2017)
CMS ($\sqrt{s} = 8$ TeV, $W^\pm W^\pm$, 2018)
ATLAS ($\sqrt{s} = 8$ TeV, 4 leptons, 2018)
Experiment (energy, final state, year)
0
5
10
15
20
25
30
σ_{eff} [mb]

DPDFs

- Double parton distributions are basic objects in DPS:



- DPDFs integrated over transverse momenta:

$$D_{f_1 f_2}(x_1, x_2, \Delta) = \int d^2\mathbf{k}_1\, d^2\mathbf{k}_2\, F_{f_1 f_2}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \Delta)$$

- Spectral condition:

$$0 < x_1 + x_2 \leqslant 1$$

DPDFs in $\mathbf{y}$ space

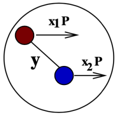
- Fourier transformed DPDFs (color singlet, spin averaged):

$$\tilde{D}_{f_1 f_2}(x_1, x_2, \mathbf{y}) = \int d^2\Delta\, \mathrm{e}^{-i\Delta\cdot\mathbf{y}}\, D_{f_1 f_2}(x_1, x_2, \Delta)$$

- Definition through twist-2 partonic operators:

$$\tilde{D}_{f_1 f_2}(x_1, x_2, \mathbf{y}) = \int dy^- \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi}\, \mathrm{e}^{i[z_1^-(x_1 P^+)+z_2^-(x_2 P^+)]} \langle P|\, \mathcal{O}_{f_1}(0, z_1)\, \mathcal{O}_{f_2}(y, z_2)\, |P\rangle$$

and $\quad z_1 = (0, z_1^-, \mathbf{0}), \;\; z_2 = (0, z_2^-, \mathbf{0})$ and $y = (0, y^-, \mathbf{y})$



- Probabilistic interpretation only for $\mathbf{y}$ averaged DDPFs:

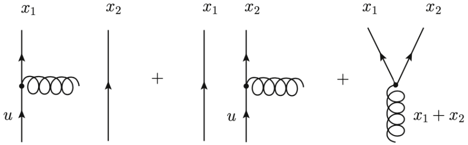
$$\int d^2\mathbf{y}\, \mathrm{e}^{i\Delta\cdot\mathbf{y}}\Big|_{\Delta=0} \tilde{D}_{f_1 f_2}(x_1, x_2, \mathbf{y}) = D_{f_1 f_2}(x_1, x_2, \Delta = 0)$$

Evolution equations for DPDFs

- DGLAP-like equations for $D_{f_1 f_2}(x_1, x_2, Q) \equiv D_{f_1 f_2}(x_1, x_2, \Delta = 0; Q, Q)$

$$\frac{\partial D_{f_1 f_2}(x_1,x_2,Q)}{\partial \ln Q^2} = \frac{\alpha_s(Q)}{2\pi} \sum_f \left\{ \int_{x_1}^{1-x_2} \frac{du}{u} P_{f_1 f}\left(\frac{x_1}{u}\right) D_{f f_2}(u, x_2, Q) + \int_{x_2}^{1-x_1} \frac{du}{u} P_{f_2 f}\left(\frac{x_2}{u}\right) D_{f_1 f}(, x_1, u, Q) + \frac{1}{x_1+x_2} P^R_{f\to f_1 f_2}\left(\frac{x_1}{x_1+x_2}\right) D_f(x_1+x_2, Q) \right\}$$

- In non-homogenous term single PDFs, evolved with DGLAP equations:



(A. Snigirev 2003, J. Gaunt and W.J. Stirling 2009, F.A. Ceccopieri 2011, . . .)

Sum rules

- Momentum and valence quark number sum rules for PDFs:

$$\sum_f \int_0^1 dx\, x\, D_f(x, Q) = 1$$

$$\int_0^1 dx_1 \left[D_q(x, Q) - D_{\bar{q}}(x, Q) \right] = N_q$$

- Evolution equations for DPDFs obey similar sum rules:

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, D_{f_1 f_2}(x_1, x_2, Q) = (1 - x_2)\, D_{f_2}(x_2, Q)$$

$$\int_0^{1-x_2} dx_1 \left[D_{q f_2}(x_1, x_2, Q) - D_{\bar{q} f_2}(x_1, x_2, Q) \right] = (N_q - \delta_{q f_2} + \delta_{\bar{q} f_2})\, D_{f_2}(x_2, Q)$$

- Analogous sum rules with respect to x_2.
- Initial conditions for evolution equations should obey these sum rules:

$$D_{f_1 f_2}(x_1, x_2, Q_0), \qquad\qquad D_f(x, Q_0)$$

Initial conditions - some prescriptions

- Naive factorized ansatz:

$$D_{f_1 f_2}(x_1, x_2, Q_0) = D_{f_1}(x_1, Q_0)\, D_{f_2}(x_2, Q_0)\, \theta(1 - x_1 - x_2)$$

  Sum rules badly violated.

- Gaunt-Stirling prescription (0910.4347/JHEP):

$$D_{f_1 f_2}(x_1, x_2, Q_0) = D_{f_1}(x_1, Q_0)\, D_{f_2}(x_2, Q_0)\, \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+\alpha(f_1)}\,(1 - x_2)^{2+\alpha(f_2)}}$$

  Better agreement with sum rules, although still approximate.

- Analysis in pure gluonic case - momentum sum rule fulfilled exactly:

  (KGB, E.Lewandowska, M.Serino, Z.Snyder, A Staśto, 1507.08583/PLB)

Pure gluonic case

- MSTW08 fit of single PDFs at initial $Q_0 = 1\ \mathrm{GeV}$:

$$D_g(x, Q_0) = \sum_{k=1}^{3} A_k\, x^{\alpha_k}\, (1-x)^{\beta_k}$$

- Double gluon distribution (Dirichlet distribution form):

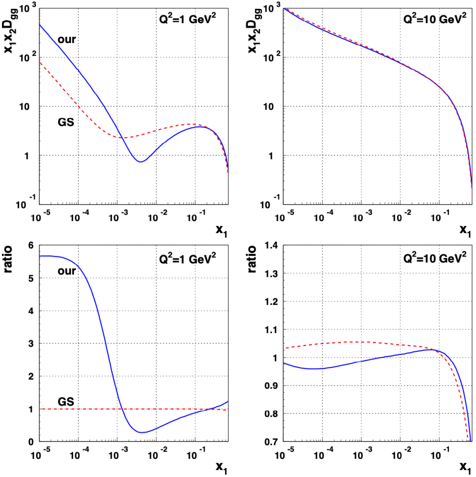
$$D_{gg}(x_1, x_2, Q_0) = \sum_{k=1}^{3} A_k \frac{\Gamma(\beta_k + 2)}{\Gamma(\alpha_k + 2)\Gamma(\beta_k - \alpha_k)}\, (x_1 x_2)^{\alpha_k} (1 - x_1 - x_2)^{\beta_k - \alpha_k - 1}$$

  obeys the momentum sum rule exactly:

$$\int_0^{1-x_2} dx_1\, x_1 D_{gg}(x_1, x_2, Q_0) = (1 - x_2)\, D_g(x_2, Q_0)$$

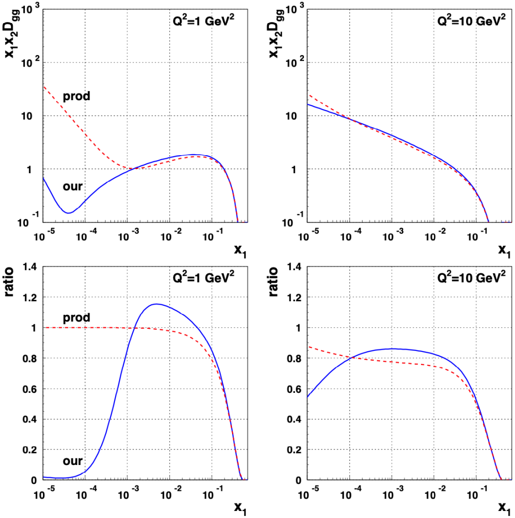
- Parameters of the single gluon distribution are only used.

Numerical results for $x_2 = 10^{-2}$



- No small x factorization in our ansatz at the initial scale.
- Small x factorization at final scale: $D_{gg}(x_1, x_2, Q) \approx D_g(x_1, Q)\, D_g(x_2, Q)$

Numerical results for $x_2 = 0.5$



- Factorization restoration in DPDFs is a small x phenomenon.

Momentum sum rule and small x factorization

- Thesis:

  Momentum sum rules are necessary conditions for restoration of small x factorization of DPDFs.

- Case 1: Initial conditions with one term which obey momentum sum rules:

$$D_g(x, Q_0) = A_g \, x^{\alpha_g} (1-x)^{\beta_g}$$

$$D_{gg}(x_1, x_2, Q_0) = A_g \, \frac{\Gamma(\beta_g + 2)}{\Gamma(\alpha_g + 2)\Gamma(\beta_g - \alpha_g)} \, (x_1 x_2)^{\alpha_g} (1 - x_1 - x_2)^{\beta_g - \alpha_g - 1}$$

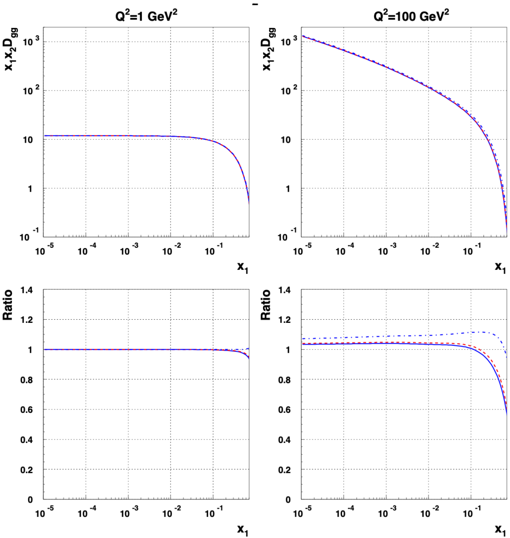
- From momentum sum rule for single gluon distribution (with 1 on the rhs):

$$A_g = \frac{\Gamma(\alpha_g + \beta_g + 3)}{\Gamma(\alpha_g + 2)\Gamma(\alpha_g + 1)}$$

- For $\alpha_g = -1$ small x factorization at initial scale:

$$D_{gg}(x_1, x_2, Q_0) \approx A_g^2 \, (x_1 x_2)^{\alpha_g} \approx D_g(x_1, Q_0) D_g(x_2, Q_0)$$

Numerical results for $x_2 = 10^{-2}$ with momentum sum rule



- Small x factorization at final scale: $D_{gg}(x_1, x_2, Q) \approx D_g(x_1, Q) D_g(x_2, Q)$.

Violation of momentum sum rule

- Case 1: $\alpha_g = -1$ in the initial conditions:

$$D_g(x, Q_0) = A_g\, x^{\alpha_g}(1-x)^{\beta_g}$$
$$D_{gg}(x_1, x_2, Q_0) = A_g^2\,(x_1 x_2)^{\alpha_g}(1-x_1-x_2)^{\beta_g-\alpha_g-1}$$

- Case 2: modification of large x behaviour of double gluon distribution:

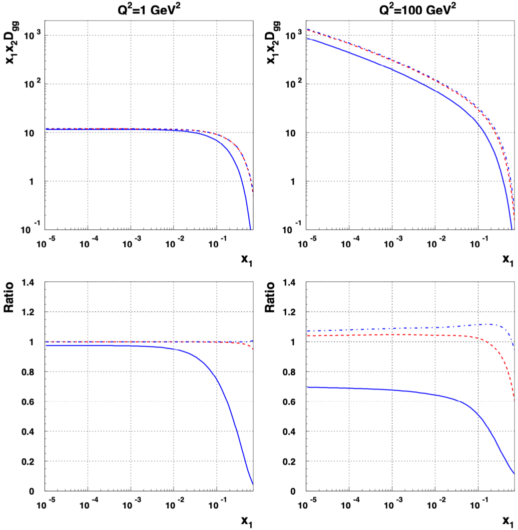
$$\beta_g - \alpha_g - 1 = 2.5 \quad \to \quad 5$$

- Momentum sum rule is violated at initial scale:

$$\int_0^{1-x_2} dx_1\, x_1 D_{gg}(x_1, x_2, Q_0) \;\neq\; (1-x_2)\, D_g(x_2, Q_0)$$

- Small x factorization at initial scale is kept by construction.

Numerical results for $x_2 = 10^{-2}$ without momentum sum rule



- Small x fact. violated at final scale: $D_{gg}(x_1, x_2, Q) \neq D_g(x_1, Q) D_g(x_2, Q)$.

Analytical insight

- Mellin moments:

$$\tilde{D}_g(n, Q) = \int_0^1 dx\, x^{n-1} D_g(x, Q)$$

$$\tilde{D}_{gg}(n_1, n_2, Q) = \int_0^1 dx_1 \int_0^1 dx_2\, x_1^{n_1-1} x_2^{n_2-1}\, \theta(1 - x_1 - x_2)\, D_{gg}(x_1, x_2, Q)$$

- Momentum sum rules in Mellin moment space:

$$\tilde{D}_g(2, Q) = 1 \tag{1}$$

$$\tilde{D}_{gg}(n_1, 2, Q) = \tilde{D}_g(n_1, Q) - \tilde{D}_g(n_1 + 1, Q) \tag{2}$$

- Solution of evolution equations for $Q \to \infty$:

$$\tilde{D}_{gg}(n_1, n_2, Q) \;\approx\; \mathrm{e}^{[\gamma(n_1)+\gamma(n_2)]t} \left\{ \tilde{D}_{gg}(n_1, n_2, Q_0) + \tilde{D}_g(n_1 + n_2 - 1, Q_0) \right\}$$

- Small x factorization at Q:

$$\tilde{D}_{gg}(n_1, n_2, Q) \;\approx\; \mathrm{e}^{[\gamma(n_1)+\gamma(n_2)]t} \left\{ \tilde{D}_g(n_1, Q_0)\, \tilde{D}_g(n_2, Q_0) \right\}$$

Analytical insight

- Small x factorization when initial conditions obey:

$$\tilde{D}_{gg}(n_1, n_2, Q_0) + \tilde{D}_g(n_1 + n_2 - 1, Q_0) \approx \tilde{D}_g(n_1, Q_0)\, \tilde{D}_g(n_2, Q_0)$$

- Setting $n_2 = 2$ and using sum rule (1), $\tilde{D}_g(2, Q_0) = 1$, we obtain

$$\tilde{D}_{gg}(n_1, 2, Q_0) \approx \tilde{D}_g(n_1, Q_0) - \tilde{D}_g(n_1 + 1, Q_0)$$

  which is sum rule (2).

- Momentum sum rules are necessary cond. for small x fact. at $Q \gg Q_0$:

$$D_{gg}(x_1, x_2, Q) \approx D_g(x_1, Q)\, D_g(x_2, Q)$$

Including quarks

- In order to include quarks in our analysis, the parameters of single PDFs must obey unrealistic condition:

$$\beta_k^{f_2} - \beta_k^{f_1} = \alpha_k^{f_2} - \alpha_k^{f_1}$$

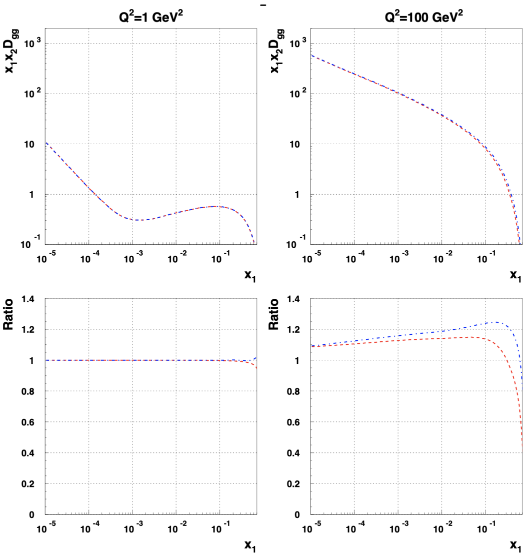
- The program of the construcion of the initial DPDFs out of the well known single PDFs with the sum rules exactly fulfilled is unrealistic.

- In practice, only GS ansatz for initial distribution, which approximately fulfils sum rules:

$$D_{f_1 f_2}(x_1, x_2, Q_0) = D_{f_1}(x_1, Q_0)\, D_{f_2}(x_2, Q_0)\, \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+\alpha(f_1)}\,(1 - x_2)^{2+\alpha(f_2)}}$$

- Small x factorization after evolution up to $10 - 20\%$ for GS ansatz.

Numerical results for $x_2 = 10^{-2}$ for evolution with quarks



- For GS (red) and fully factorized (blue) initial conditions.

Δ dependence of DPDFs

- For $\Delta \neq 0$ the small x factorization might be broken.
- In M. Diehl, T. Kasemets, S. Keane, 1401.1233/JHEP, the following ansatz was proposed for $x_{1,2} < 0.1$, due to parton correlations in y-space:

$$D_{f_1 f_2}(x_1, x_2, \Delta, Q) = D_{f_1}(x_1, Q)\, D_{f_2}(x_2, Q) \exp\left\{-h_{f_1 f_2}(x_1, x_2)\, \Delta^2\right\}$$

- This has an implications for the pocket formula for DPS cross sections:

$$\sigma_{eff} = \sigma_{eff}(x_1, x_2)$$

- More studies are necessary:

  M. Diehl, J.R. Gaunt, D.M. Lang, T. Plößl, A. Schäfer, 2001.10428/EPJC

Summary

- Factorization of DPDFs is small x phenomenon.
- We showed that the momentum sum rules are necessary conditions for the small x factorization of evolved DPDFs.
- This conclusion motivates the importance of the construction of initial DPDFs which fullfil the momentum sum rules.
- The pocket formula for DPS cross sections makes sense as a first approximation for $x_1, x_2 \ll 1$.
- More detailed studies are necessary.

Thank you for your attention