Introduction to physics of EIC
High energy QCD

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Basic books

- Y. V. Kovchegov, E. Levin,
  Quantum Chromodynamics at High Energy, Cambridge Univ. Press 2012
- S. Donnachie, G. Dosch, P. Landshoff, O. Nachtmann,
  Pomeron Physics and QCD, Cambridge Univ. Press 2002
- V. Barone, E. Predazzi,
  High-Energy Particle Diffraction, Springer 2002
- K. Golec-Biernat, *Habilitation thesis, 2001*
  https://annapurna.ifj.edu.pl/~golec/data/teaching/files/wyklady/hab.pdf

Small x limit of DIS

Two limits of DIS

- Bjorken limit - Bjorken scaling and its violation

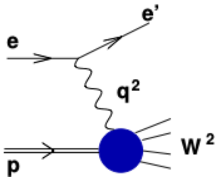
$$Q^2 \to \infty\,, \qquad\qquad x = \frac{Q^2}{2P\cdot q} = const$$

- Small x limit - strong rise of gluon distribution and structure functions

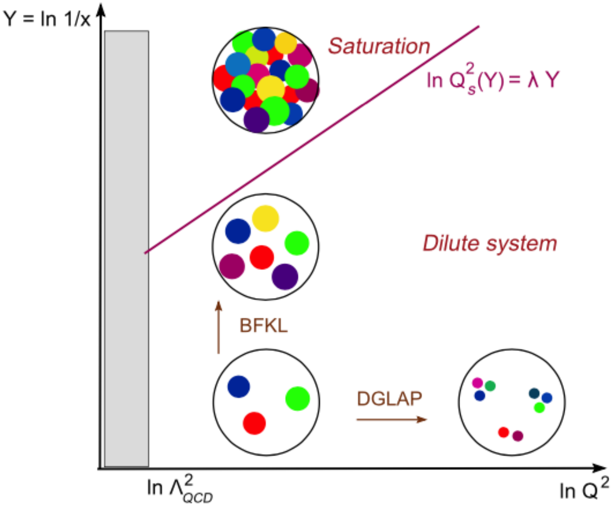
$$Q^2 = const\,, \qquad\qquad x = \frac{Q^2}{Q^2 + W^2} \approx \frac{Q^2}{W^2} \to 0$$

- Small x limit = High energy limit:

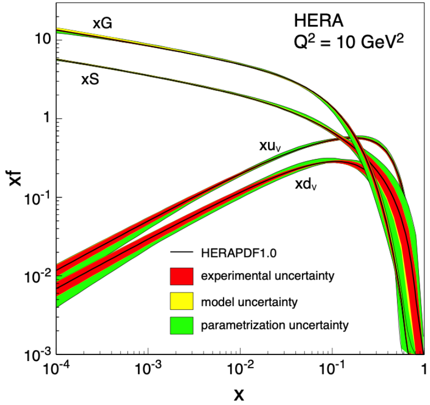
$$S_{\gamma^* p} = (q+P)^2 = W^2 \to \infty$$

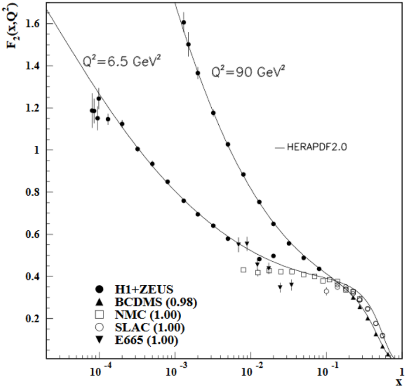


The two limits in kinematic plane



Gluon and sea quarks dominance for $x \to 0$





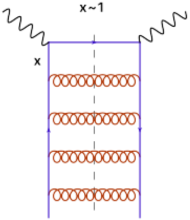
- Valence quarks seen for $x \sim 1$:

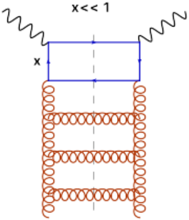
$$F_2 = x\left(\frac{4}{9}u_v + \frac{1}{9}d_v + \frac{4}{3}S\right)$$

- Sea quarks distribution seen for $x \to 0$.
- Strong Bjorken scaling violation at small x due to gluons

Small x limit of DIS

- Valence quark versus sea quark DIS





$$F_2 = \sum_n a_n \left[\alpha_s \ln(Q^2/m^2)\right]^n$$

$$F_2 = \sum_n b_n \left[\alpha_s \ln(1/x)\right]^n$$

DGALP evolution equations

BFKL evolution equation

collinear factorization

k_T−factorization

Collinear factorization

- Collinear factorization formula:

$$F_2(x,Q^2) = \int_0^1 d\xi \left[\sum_f C_f(\alpha_s, x, \xi)\, q_f(\xi, Q^2) + C_g(\alpha_s, x, \xi)\, G(\xi, Q^2) \right]$$

- Coefficient functions from pQCD

$$C_f = e_f^2\, \xi\, \delta(\xi - x) + \alpha_s(Q^2)\, C_f^{(1)}(x,\xi) + \ldots, \qquad C_g = \alpha_s(Q^2)\, C_g^{(1)}(x,\xi) + \ldots$$

- DGLAP evolution equations:

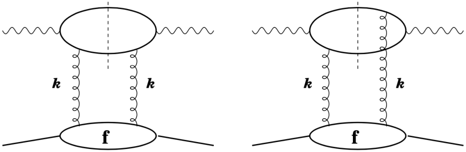
$$\frac{\partial q_f(x,Q^2)}{\partial \log Q^2} = P_{qq} \otimes q_f + P_{qG} \otimes G$$

$$\frac{\partial \bar{q}_f(x,Q^2)}{\partial \log Q^2} = P_{qq} \otimes \bar{q}_f + P_{qG} \otimes G$$

$$\frac{\partial G(x,Q^2)}{\partial \log Q^2} = P_{Gq} \otimes \sum_f (q_f + \bar{q}_f) + P_{GG} \otimes G$$

- Splitting functions from pQCD:

$$P_{ij}(z,Q^2) = \underbrace{\alpha_s(Q^2)\, P_{ij}^{(0)}(z)}_{LL} + \underbrace{\alpha_s^2(Q^2)\, P_{ij}^{(1)}(z)}_{NLL} + \underbrace{\alpha_s^3(Q^2)\, P_{ij}^{(2)}(z)}_{NNLL} + \ldots$$



- k_T−factorization formula for $x \ll 1$:

$$F_2(x, Q^2) = \int \frac{d^2 k_T}{k_T^2} \, \Phi_\gamma(k_T, Q^2) \, f(x, k_T)$$

- Photon impact factor $\Phi_\gamma(k_T, Q^2)$ with off-shell gluon: $k^2 = -k_T^2$
- Unintegrated gluon distribution $f(x, k_T)$:

$$xG(x, Q^2) = \int \frac{d^2 k_T}{k_T^2} \, \theta(Q - |k_T|) \, f(x, k_T)$$

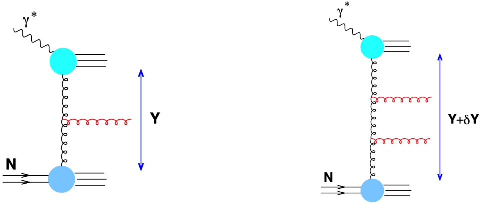
- $f(x, k_T)$ from the BFKL equation (Balitsky, Fadin, Kuraev, Lipatov, 1976-78)

- Large $Y = \log(1/x)$ from strong ordering in longitudinal momenta of gluons:

$$1 \gg x_1 \gg x_2 \gg \ldots \gg x$$

and no restrictions on their transverse momenta - multi-Regge kinematics.

- Summation of powers $(\alpha_s Y)^n$ gives BFKL evolution equation in rapidity Y:



$$\frac{\partial f(x,k_T^2)}{\partial Y} = \frac{\alpha_s \pi}{N_c} \int_0^\infty \frac{dk_T'^2}{k_T'^2} k_T^2 \left\{ \frac{f(x,k_T'^2) - f(x,k_T^2)}{|k_t'^2 - k_T^2|} + \frac{f(x,k_T^2)}{\sqrt{4k_T'^4 + k_T^4}} \right\}$$

- Conformally invariant, infrared and ultraviolet safe

BFKL equation solution

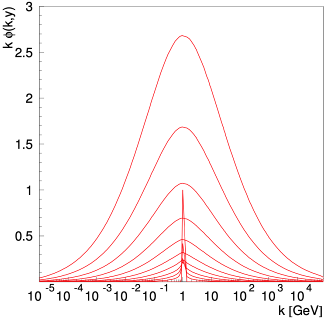
- Asymptotic solution for $Y = \log(1/x) \to \infty$

$$f(x, k_T^2) = \frac{x^{-(\alpha_P(0)-1)}}{\sqrt{2\pi D Y}} \exp\left\{ \frac{-\log^2(k_T^2)}{\sqrt{2D\,Y}} \right\}$$

- Power like growth with $x \to 0$ given by the BFKL pomeron intercept

$$\alpha_P(0) - 1 = \frac{\alpha_s N_c}{\pi} 4 \ln 2 \approx 0.5$$

- Diffusion of transverse momenta into infrared: $k_T < \Lambda_{QCD}$. Solutions for $Y = 0, 1, \ldots, 10$



Problems with BFKL result

- From k_T-factorization formula power-like behaviour of $F_2(x)$:

$$F_2(x) = \Phi_\gamma \otimes f \sim x^{-(\alpha_{\mathbb{P}}(0)-1)} \qquad \rightarrow \qquad \sigma_{\gamma^* p} \sim (W^2)^{(\alpha_{\mathbb{P}}(0)-1)}$$

- Froissart unitarity bound in hadronic reactions

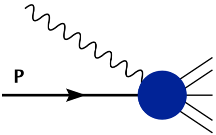
$$\sigma_{tot} < c \log^2(s)$$

- BFKL solution breaks unitarity bound. Unitarization of BFKL approach necessary.
- Sensitivity to infrared region $k_T < \Lambda_{QCD}$ questions validity of pQCD approach.

How to correct the BFKL aproach?

Dipole approach to high energy QCD

Change of frame in small x DIS



$$\sigma_{\gamma^* p} = \frac{4\pi^2\alpha_{em}}{Q^2} F_2$$

- Infinite momentum frame ($P^+ \to \infty$) - virtual probe γ^* resolves partonic structure of the proton.
- Proton rest frame ($P^+ \approx m_p$) - virtual probe γ^* developes partonic configurations long before the interaction with the proton.

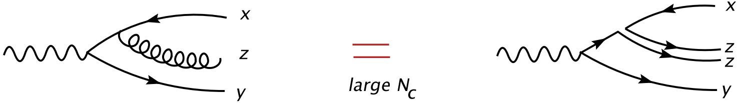
- Coherence length for such fluctuations:

$$l_c = \frac{1}{x m_p} \gg \frac{1}{m_p} = 0.2\,\mathrm{fm}$$

- Virtual photon light-cone wave function:

$$|\gamma^*\rangle = \Psi_{q\bar{q}}(z_q)\,|q\bar{q}\rangle + \Psi_{q\bar{q}g}(z_q, z_g)\,|q\bar{q}g\rangle + \ldots$$

- Soft gluon emissions with $z_g \ll z_q$ in onium (A. H. Mueller, Nucl.Phys. B415 (1994) 373)



- Dipole splitting $(xy) \to (xz) + (zy)$ in transverse space with probability distr.

$$\frac{dP}{dYd^2z} = \frac{N_c\alpha_s}{2\pi^2}\,\frac{(\vec{x}-\vec{y})^2}{(\vec{x}-\vec{z})^2(\vec{z}-\vec{y})^2} \equiv K(x,y,z)$$

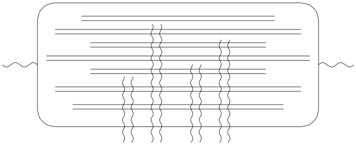
- Branching process with generating functional $Z(x,y,Y;u)$ and $Z(x,y,Y;0)=1$

$$\frac{dZ(x,y,Y;u)}{dY} = \int d^2z\, K(x,y,z)\Big\{Z(x,z,Y;u)\,Z(z,y,Y;u) - Z(x,y,Y;u)\Big\}$$

- Multi-dipole distributions in onium $(r_i = x_i - y_i\,,\quad b_i = (x_i+y_i)/2)$

$$n_k(r_1,b_1,\dots,r_k,b_k;Y) = \frac{1}{k!}\,\frac{\delta^k Z(x,y,Y,u)}{\delta u(r_1,b_1)\dots\delta u(r_k,b_k)}\bigg|_{u_i=0}$$

BFKL equation in dipole picture



- One-dipole distribution

$$n_1(r, b; Y) = \left. \frac{\delta Z(x, y, Y, u)}{\delta u(r, b)} \right|_{u=0}$$

- BFKL evolution equation for n_1

$$\frac{dn_1(x, y, Y)}{dY} = \int d^2z\, K(x, y, z) \{n_1(x, z, Y) + n_1(z, y, Y) - n_1(x, y, Y)\}$$

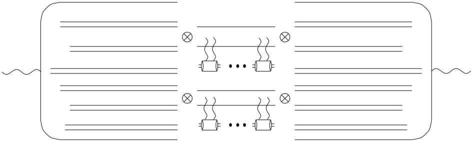
- BFKL growth in the number of color dipoles with $Y = \log(1/x) \to \infty$

$$n_1(Y) \sim \mathrm{e}^{(\alpha(0)-1)Y} = x^{-(\alpha(0)-1)}$$

- Using color dipoles instead of transverse coordinates of gluons allowed to formulate the BFKL equation in an elegant form.

The Balitsky-Kovchegov equation

- The BK equation is justified for onium scattering on nucleons in a large nucleus (Kovchegov, 1998-99)



- Each dipole interacts via multiple scattering on nucleons with scattering matrix in Glauber-Gribov-Mueller model, 1999

$$s_0(r,b,Y=0) = \exp\left\{-\frac{\alpha_s \pi^2}{2N_c} r^2 T(b) x G(x, 1/r^2)\right\}$$

- Scattering matrix of the parent dipole for $Y \to \infty$

$$S(x,y,Y) = \sum_{k=1}^{\infty} \frac{1}{k!} \int d^2 r_1 d^2 b_1 \ldots d^2 r_k d^2 b_k$$

$$\times \left.\frac{\delta^k Z(x,y,Y,u)}{\delta u(r_1,b_1)\ldots\delta u(r_k,b_k)}\right|_{u_i=0} s_0(r_1,n_1)\ldots s_0(r_k,b_k) = Z(x,y,Y,s_0)$$

The Balitsky-Kovchegov equation

- Hence the BK equation for the dipole scattering matrix

$$\frac{dS(x,y,Y)}{dY} = \int d^2z\, K(x,y,z) \left\{ S(x,z,Y)\, S(z,y,Y) - S(x,y,Y) \right\}$$

- Introducing the dipole forward scattering amplitude: $N(x,y,Y) = 1 - S(x,y,Y)$

$$\frac{dN(x,y)}{dY} = \int d^2z\, K(x,y,z) \left\{ \underbrace{N(x,z) + N(z,y) - N(x,y)}_{BFKL} - N(x,z)N(z,y) \right\}$$

- Nonlinear equation with two fixed points: $N = 0, 1$ which implies

$$0 \leqslant N(r,b,Y) \leqslant 1$$

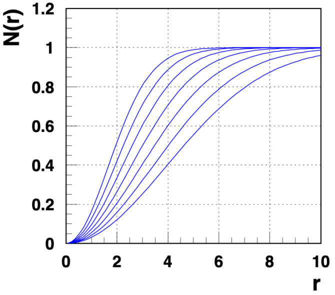
- Unitarity problem solved for small x DIS

$$\sigma_{\gamma^* p}(x, Q^2) = \int d^2r\, d^2b \int_0^1 dz\, |\Psi_{q\bar{q}}(r,z,Q^2|^2\, N(r,b,Y)$$

- Exponential growth of $\sigma_{\gamma^* p}$ with $Y = \log(1/x) \to \infty$ is tamed!

The BK equation solutions

- Neglecting $b-$dependence, $N(r,b) = N(r)$, solutions for $x = 10^{-2}, \ldots, 10^{-8}$

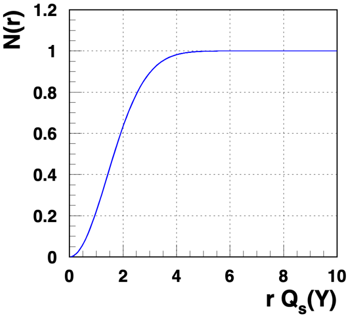


- Physical picture for a dipole probe of a given size: $r = 2/Q$

- For $x \to 0$ proton becomes "black" - physics of parton saturation.

Saturation scale

- BK developes saturation scale $Q_s^2(Y) = Q_0^2\, \mathrm{e}^{\lambda Y}$ such that $N(r, Y) = N(rQ_s(Y))$



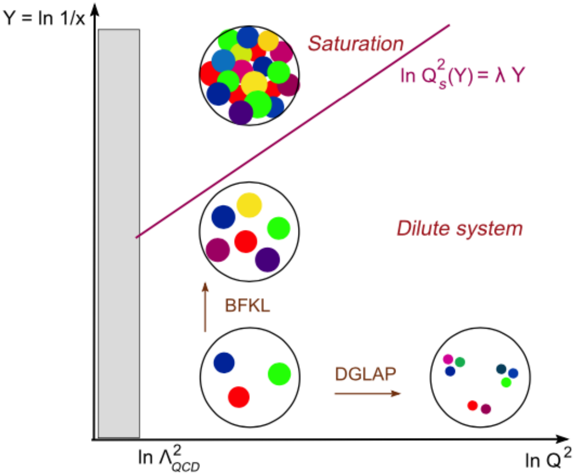
- For $Y \to \infty$ the dipole amplitude N saturates for smaller dipole sizes r
- Saturation scale defines transition to saturation region. For $r = 2/Q$ we consider

$$N(rQ_s(Y)) = 0.6 \quad \to \quad \frac{Q_s(Y)}{Q} = 1 \quad \to \quad \lambda Y = \ln(Q^2/Q_0^2)$$

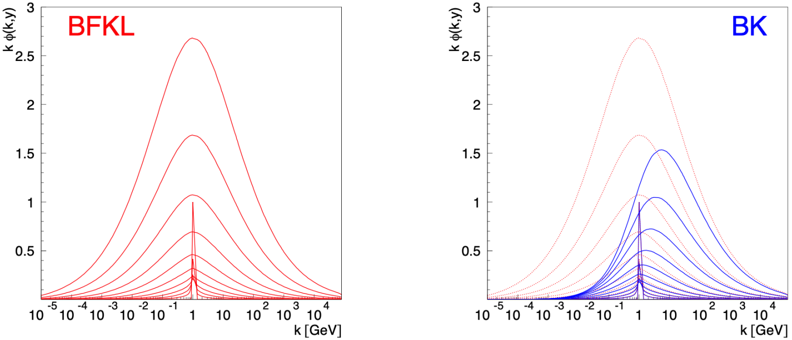
- This is seen in the kinematic plane.

Parton saturation in kinematic plane

(Gelis, Iancu, Jalilian-Marian, Venugopalan, 2010)



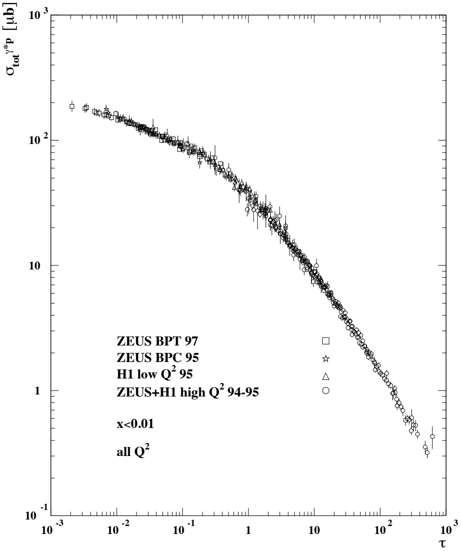
- Fourier transform: $N(r, Y) \to \phi(k_T, Y)$ (GB, Motyka Staśto, 2003)



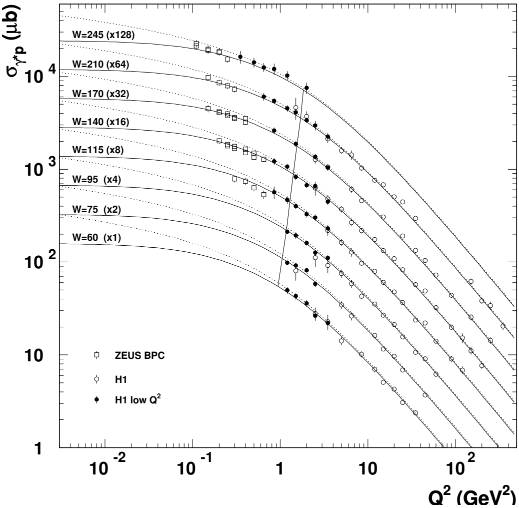
- Suppression of power-like growth in x and diffusion to infrared in the BK solution.
- Problems with the BK equation - nonperturbative tails in $b-$dependence

Experimental evidence od saturation - geometric scaling

(Staśto, GB, Kwieciński, 2001) $N(rQ_s) \quad\rightarrow\quad \sigma_{\gamma^* p}(\tau = Q^2/Q_s^2(x))$

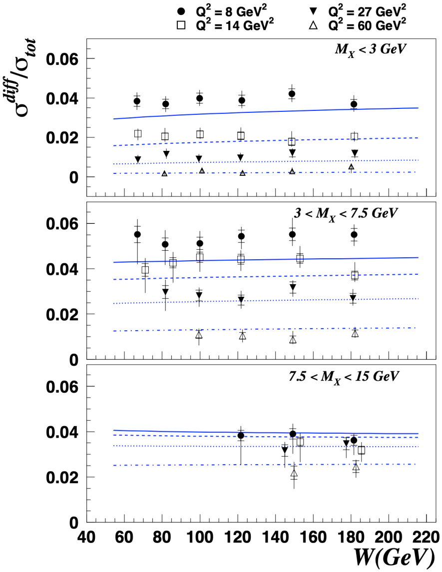


(GB, Wuesthoff, 1998)



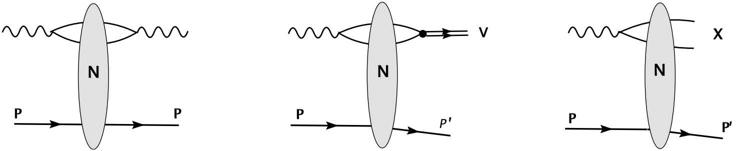
Experimental evidence of saturation - inclusive diffraction in DIS

(GB, Wuesthoff, 1999)

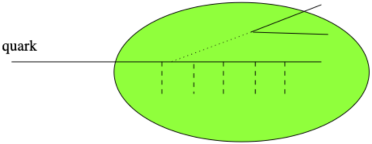


Experimental tests of saturation

- Exclusive diffraction in DIS (L. Motyka with collaborators)



- The same dipole scattering amplitude N in these reactions.
- Forward scatterring processes in ep/N and pp, e.g. forward jet/γ/DY production (K. Kutak with collaborators)



CGC in nucleus

- CGC (Color Glass Condensate) - effective QCD theory of gluon saturation (McLerran, Venugopalan, 1994)

Summary

- More detailed presentations in future lectures.
- EIC will open new opportunities with nucleons for QCD studies in DIS.
- Keep fingers crossed for the future of our world.