Introduction to physics of EIC
QCD parton distributions

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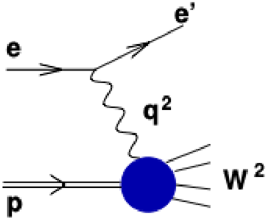
ep DIS



- Virtuality of the probe $(\gamma, Z^0, W^{\pm})$

$$Q^2 = -q^2 > 0, \qquad q = k_e - k_{e'}$$

- Bjorken variable

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2 + W^2 - m_p^2}$$

- Inclusive cross section for $ep \to e'X$ in which $(E'_e, \theta'_e) \leftrightarrow (x, Q^2)$

$$\frac{d\sigma}{dxdQ^2} = \frac{2\pi\alpha_{\rm em}^2}{xQ^4}\, Y_+ \left(F_2 - \frac{y^2}{Y_+} F_L\right) \;\sim\; L^{\mu\nu} W_{\mu\nu}$$

- Strong interactions in two structure function if γ exchange only

$$F_2(x, Q^2), \qquad\qquad F_L(x, Q^2) = F_2 - 2xF_1$$

Hadronic tensor

- Hadronic tensor in DIS cross section

$$W_{\mu\nu} = \sum_X \int d\Gamma_X (2\pi)^4 \delta^4(q + P - P_X) \left\langle P | J^\dagger_\mu(0) | X \right\rangle \langle X | J_\nu(0) | P \rangle$$

- Dirac Delta representation

$$\delta^4(q + P - P_X) = \int \frac{d^4x}{(2\pi)^4} \, \mathrm{e}^{i(q+P-P_X)\cdot x}$$

- After substitution

$$W_{\mu\nu} = \int d^4x \, e^{iq\cdot x} \sum_X \int d\Gamma_X \langle P | e^{i\hat{P}\cdot x} J^\dagger_\mu(0) \, e^{-i\hat{P}\cdot x} | X \rangle \, \langle X | J_\nu(0) | P \rangle$$

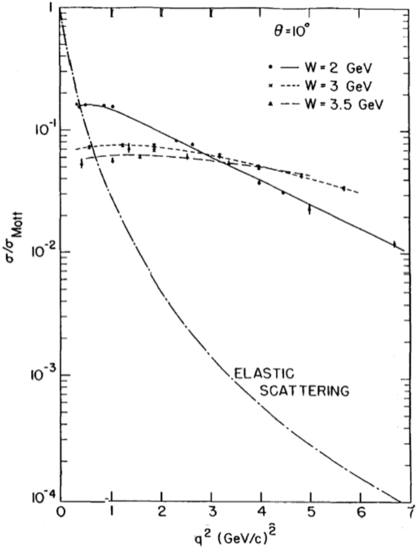
- From the current translational invariance

$$W_{\mu\nu} \quad = \int d^4x \, e^{iq\cdot x} \sum_X \int d\Gamma_X \left\langle P | J^\dagger_\mu(x) | X \right\rangle \langle X | J_\nu(0) | P \rangle$$

- From completness relation of hadronic states X

$$W_{\mu\nu}(q, P) \quad = \int d^4x \, e^{iq\cdot x} \left\langle P | J^\dagger_\mu(x) J_\nu(0) | P \right\rangle \quad \rightarrow \quad \mathrm{OPE}$$

DIS versus elastic scattering



- Elastic scattering: $e + p \to e + p$:

$$\frac{\sigma}{\sigma_{Mott}} = \frac{1}{\left(1 + \frac{Q^2}{0.71 GeV^2}\right)^4}$$

- Mean electric charge radius:

$$\sqrt{\langle r^2 \rangle} \approx 0.8 \cdot 10^{-15} m$$

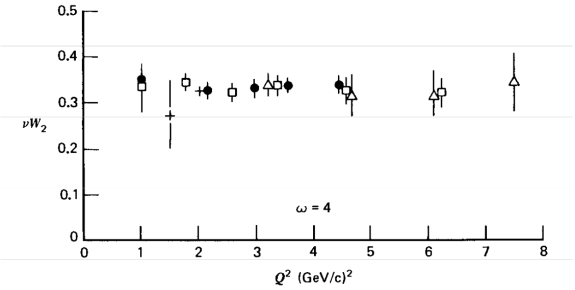
- No such a scale in DIS:

$$e + p \to e + X$$

- Scattering on a point-like object?

Bjorken scaling at SLAC data (1969)

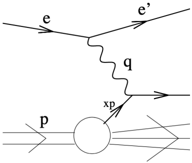
- Structure function $F_2 = \nu W_2$ does not depend on Q^2 for $x = 1/\omega = 0.25$



- Bjorken scaling (no dependence on Q^2) is a smoking gun.
- Explained by parton model

Parton model

- Feynman, 1969 - elastic scattering on a point-like parton



- Proton momentum fraction carried by parton $\xi = x$

$$(\xi P + q)^2 = 0 \quad \rightarrow \quad \xi = \frac{-q^2}{2P \cdot q} = x$$

- In the infinite proton momentum frame - proton is a collection of free partons with probability density $q_f(\xi)$. Incoherent scattering:

$$F_2(x) = \sum_f \int_0^1 d\xi \left[e_f^2 \, \xi \, \delta(\xi - x) \right] q_f(\xi) = \sum_f e_f^2 x \left[q_f(x) + \bar{q}_f(x) \right]$$

- Bjorken scaling is explained. Callan-Gross relation is fulfilled for spin 1/2 partons

$$F_L(x, Q^2) = F_2 - 2xF_1 = 0$$

QCD improved parton model I

- Asymptotic freedom of QCD explains weakness of interactions between quarks.
- Partons are valence u, d quarks and pairs of $q\bar{q}$ quarks - sea quarks

$$u(x) = u_v(x) + u_s(x) \qquad d(x) = d_v(x) + d_s(x) \qquad s(x) = s_s(x)$$
$$\bar{u}(x) = u_s(x) \qquad \bar{d}(x) = d_s(x) \qquad \bar{s}(x) = s_s(x)$$

- Quark number sum rule for the proton

$$\int_0^1 dx\, u_v(x) = 2, \qquad \int_0^1 dx\, d_v(x) = 1$$

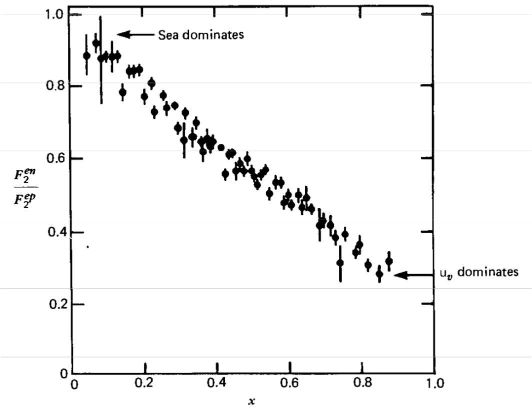
- Proton and neutron structure function - isospin symmetry: $u \leftrightarrow d, \quad \bar{u} \leftrightarrow \bar{d}$

$$F_2^p = \tfrac{4}{9}x(u+\bar{u}) + \tfrac{1}{9}x(d+\bar{d}) + \tfrac{1}{9}x(s+\bar{s})$$
$$F_2^n = \tfrac{4}{9}x(d+\bar{d}) + \tfrac{1}{9}x(u+\bar{u}) + \tfrac{1}{9}x(s+\bar{s})$$

- Their ratio for $u_s = d_s = s_s \equiv S$:

$$\frac{F_2^n}{F_2^p} = \frac{\frac{1}{9}u_v + \frac{4}{9}d_v + \frac{4}{3}S}{\frac{4}{9}u_v + \frac{1}{9}d_v + \frac{4}{3}S}$$

Feynman's partons = Gell-Mann's quarks



- Dominance of $S(x)$ for $x \to 0$ and $u_v(x)$ for $x \to 1$ and implies

$$\frac{1}{4} < \frac{F_2^n}{F_2^p} < 1$$

QCD improved parton model II

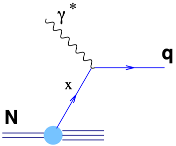
- Momentum sum rule:

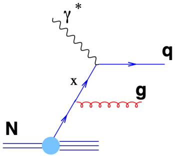
$$\sum_f \int_0^1 dx\, x\, (q_f(x) + \bar{q}_f(x)) \approx 0.5$$

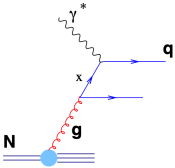
- Logaritmic Bjorken scaling violation in the Bjorken limit: $Q^2 \to \infty$ and $x = const$

$$F_2 = F_2(x, \log(Q^2))$$

- Gluons have to be taken into account:





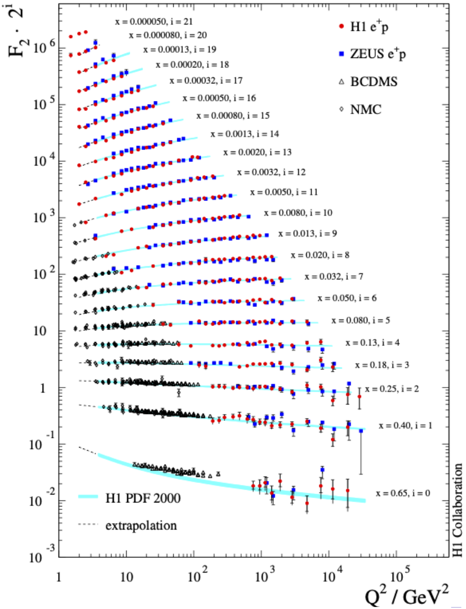


- Scale dependent parton distribution functions (PDFs)

$$q_f(x, Q^2), \qquad \bar{q}_f(x, Q^2), \qquad G(x, Q^2)$$

- Bjorken scaling violations explained by PDFs $\log Q^2$ dependence.

Bjorken scaling violation - the tale of gluons

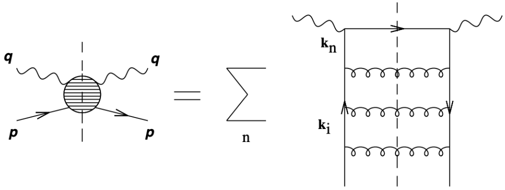


DGLAP evolution equations - non-singlet case

- From optical theorem

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\,\sigma_{\gamma^* p}(x, Q^2) = \mathrm{Im}\,A(\gamma^* p \to \gamma^* p)$$

- For valence quark only - summation over gluon emissions must be done



- Leading logarithmic approximation for ordered exchanged transverse momenta

$$m^2 < \mathbf{k}_1^2 < \mathbf{k}_2^2 < \ldots < \mathbf{k}_n^2 < Q^2$$

- Q^2 is the upper limit for transverse part of the phase space. As a result

$$\alpha_s^n \int_{m^2}^{Q^2} \frac{d\mathbf{k}_n^2}{\mathbf{k}_n^2} \cdots \int_{m^2}^{\mathbf{k}_3^2} \frac{d\mathbf{k}_2^2}{\mathbf{k}_2^2} \int_{m^2}^{\mathbf{k}_2^2} \frac{d\mathbf{k}_1^2}{\mathbf{k}_1^2} = \frac{\alpha_s^n}{n!} \left[\ln \frac{Q^2}{m^2}\right]^n$$

Evolution equations - non-singlet case

- F_2 in the LLA:

$$F_2(x, Q^2) = \sum_{n=0}^{\infty} a_n(x) \, \frac{1}{n!} \left[\alpha_s \ln \frac{Q^2}{m^2} \right]^n$$

- Longitudinal part in the Mellin moment space

$$\tilde{a}_n(N) = \int_0^1 dx \, x^{N-1} a_n(x) = \left[\frac{\gamma_N}{2\pi} \right]^n$$

- Final result with infrared regulator $m^2 \to 0$:

$$\tilde{F}_2(N, Q^2) = \exp \left\{ \frac{\alpha_s \gamma_N}{2\pi} \ln \frac{Q^2}{m^2} \right\} = \left(\frac{Q^2}{m^2} \right)^{\alpha_s \gamma_N / (2\pi)}$$

- Final result has to be multiplied by bare quark distribution $\bar{q}_0(N, m^2)$

$$\tilde{F}_2(N, Q^2) = \left(\frac{Q^2}{m^2} \right)^{\alpha_s \gamma_N / (2\pi)} \times \bar{q}_0(N, m^2)$$

- Two scales mixed up - short distance given by Q^2 and long distance given by m^2

Collinear factorization

- Separation of short and long distance physics with factorization scale $\mu^2 \gg \Lambda^2$

$$\tilde{F}_2(N, Q^2) = \underbrace{\left(\frac{Q^2}{\mu^2}\right)^{\alpha_s \gamma_N/(2\pi)}}_{coefficient\ \ function} \times \underbrace{\left(\frac{\mu^2}{m^2}\right)^{\alpha_s \gamma_N/(2\pi)} \times \bar{q}_0(N, m^2)}_{\bar{q}(N+1,\mu^2)}$$

- Renormalized quark distribution $\bar{q}(N, \mu^2)$ obeys evolution equation

$$\mu^2 \frac{\partial \bar{q}(N, \mu^2)}{\partial \mu^2} = \frac{\alpha_s}{2\pi} \gamma_N \; \bar{q}(N, \mu^2)$$

- Evolution equation in the x-space

$$\mu^2 \frac{\partial q(x, \mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{z}\right) q(z, \mu^2)$$

- F_2 does not depend on the factorization scale. Choosing $\mu^2 = Q^2$

$$\tilde{F}_2(N, Q^2) = \bar{q}(N+1, Q^2) \quad \leftrightarrow \quad F_2(x, Q^2) = xq(x, Q^2)$$

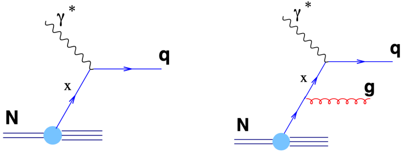
Evolution as Markov process

- QCD improved formula after transformation to x-space

$$F_2(x, Q^2) = \sum_f e_f^2 \, x \left[q_f(x, Q^2) + \bar{q}_f(x, Q^2) \right] + \mathcal{O}\left(\frac{\Lambda^2}{Q^2} \right)$$

- Knowing PDF at Q^2 and want to know it at $Q^2 + \delta Q^2$

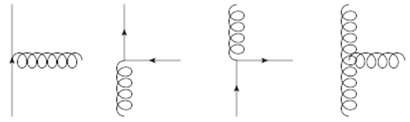
$$q(x, Q^2 + \delta Q^2) = q(x, Q^2) + \frac{\delta Q^2}{Q^2} \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{z}\right) q(z, Q^2)$$



- Splitting function $P_{qq}(x/z)$ gives quark-to-quark transition probab. $z \to x$
- Basis of MC parton showers after including virtual corrections.

DGLAP evolution equations

- In general, more splittings



- DGLAP evolution equations with evolution "time" $t = \log(Q^2/Q_0^2)$
  (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi, 1972-77)

$$\frac{\partial q_f(x,t)}{\partial t} = P_{qq} \otimes q_f + P_{qG} \otimes G$$

$$\frac{\partial \bar{q}_f(x,t)}{\partial t} = P_{qq} \otimes \bar{q}_f + P_{qG} \otimes G$$

$$\frac{\partial G(x,t)}{\partial t} = P_{Gq} \otimes \sum_f (q_f + \bar{q}_f) + P_{GG} \otimes G$$

- Non-sinlet evolution equation for valence quark distributions: $q_{vf} = q_f - \bar{q}_f$

$$\frac{\partial q_{vf}(x,t)}{\partial t} = P_{qq} \otimes q_{vf}\,, \qquad P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$

Solving DGLAP equations

- Initial conditions at $Q_0^2 \simeq 1\ \mathrm{GeV}^2 \quad (t = 0)$ with several parameters

$$q_f(x,0), \qquad \bar{q}_f(x,0), \qquad G(x,0)$$

- Momentum and quark number sum rules conserved by evolution:

$$\int_0^1 dx\, x \left[\sum_f \Big(q_f(x,Q^2) + \bar{q}_f(x,Q^2) \Big) + G(x,Q^2) \right] = 1$$

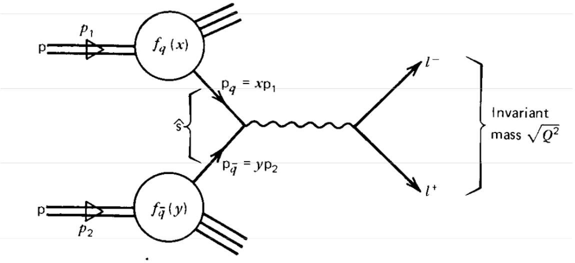
$$\int_0^1 dx \left[q_f(x,Q^2) - \bar{q}_f(x,Q^2) \right] = N_f$$

- If imposed for initial conditions - reduce the number of parameters.

- Global fits of the input parameters to hard scattering data.

PDFs universality

- PDFs can be used in hadronic reactions, e.g. in the Drell-Yan production

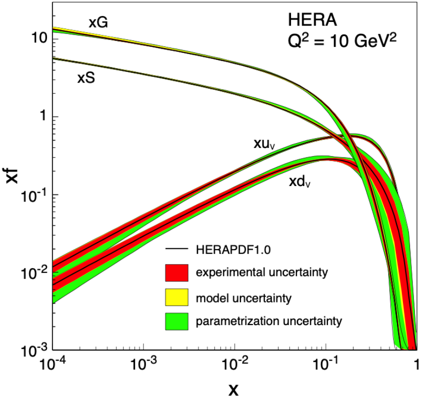


$$\sigma_{DY} = \int_0^1 dx \int_0^1 dy \left[q(x, Q^2)\, \bar{q}(y, Q^2) + (x \leftrightarrow y) \right] \hat{\sigma}(q\bar{q} \to l^+ l^-)$$

Data used in global fits

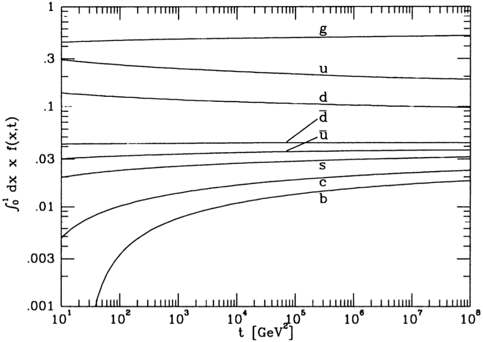
H1, ZEUS	$F_2^{e^+p}(x,Q^2)$, $F_2^{e^-p}(x,Q^2)$ NC + CC
BCDMS	$F_2^{\mu p}(x,Q^2)$, $F_2^{\mu d}(x,Q^2)$
NMC	$F_2^{\mu p}(x,Q^2)$, $F_2^{\mu d}(x,Q^2)$, $F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2)$
SLAC	$F_2^{e^-p}(x,Q^2)$, $F_2^{e^-d}(x,Q^2)$
E665	$F_2^{\mu p}(x,Q^2)$, $F_2^{\mu d}(x,Q^2)$
CCFR, NuTeV, CHORUS	$F_2^{\nu(\bar{\nu})N}(x,Q^2)$, $F_3^{\nu(\bar{\nu})N}(x,Q^2)$
	$\to$ q, $\bar{q}$ at all x and g at medium, small x
H1, ZEUS	$F_{2,c}^{e^\pm p}(x,Q^2)$, $F_{2,b}^{e^\pm p}(x,Q^2)$ $\to$ c, b
E605, E772, E866	Drell-Yan $pN \to \mu\bar{\mu} + X \to$ $\bar{q}$ (g)
E866	Drell-Yan p, n asymmetry $\to$ $\bar{u}, \bar{d}$
CDF, D0	$W^\pm$ rapidity asymmetry $\to$ u/d ratio at high x
CDF, D0	Z^0 rapidity distribution $\to$ u, d
CDF, D0	inclusive jet data $\to$ g at high x
H1, ZEUS	DIS + jet data $\to$ g at medium x
CCFR, NuTeV	dimuon data $\to$ strange sea s, $\bar{s}$

Parton distributions from global fits



- Gluons and sea quarks dominate at small x.
- Gluons carry the missing half of proton's momentum.

Momentum sum rule

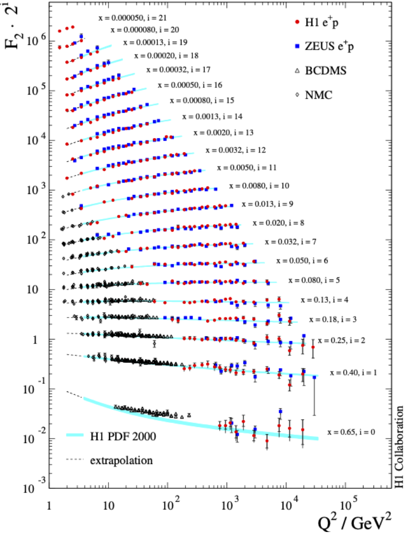


- For $Q^2 \to \infty$ the quark and gluon momentum fractions equal

$$f_q = \frac{3n_f}{16 + 3n_f}\,, \qquad\qquad f_G = \frac{16}{16+3n_f} \approx 0.5$$

- No total parton number sum rule - could be infinite.

Bjorken scaling violation - the tale of gluons



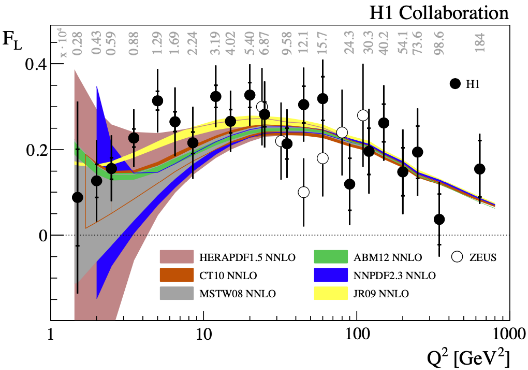
- For $x \ll 1$

$$\frac{\partial F_2(x)}{\partial logQ^2} \sim P_{qG} \otimes G \sim \bar{x}G(\bar{x}, Q^2)\Big|_{\bar{x}\sim x}$$

Longitudinal structure function F_L

- NLO QCD collinear factorization formula

$$F_L(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} \left[C_L^q\left(\frac{x}{z}\right) F_2^{LO}(z,Q^2) + C_L^g\left(\frac{x}{z}\right) zG(z,Q^2) \right] + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

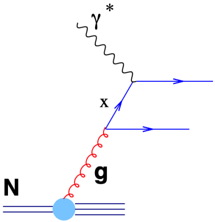


- Strong sensitivity to gluon distribution at small x.
- Higher twists at small Q^2?

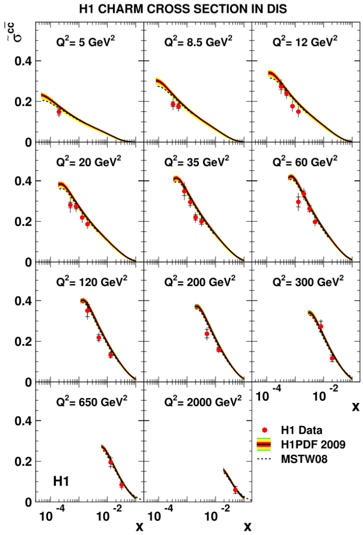
Charm contribution to F_2

- c, b quarks generated radiatively

$$\gamma^* g \to c\bar{c},\ b\bar{b}$$

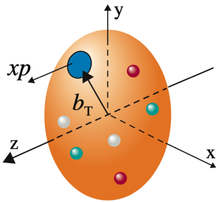


- Intrinsic charm?
- Charm contribution up to 25% for $x \ll 1$ and large Q^2.



Summary and broader perspective

- PDFs provide 1-dim. parton structure in longitudinal momenta - $q(x), \bar{q}(x), G(x)$
- Multidimensional structure - Wigner function $W(x, k_T, b_T)$



- Information on transverse momentum k_T and transverse spatial b_T distributions
- More exclusive PDFs probed through exclusive processes like DVCS, SIDIS, VMP
- Information about spin structure
- EIC program: nucleon/nucleus tomography