Surfing the traveling wave with Robi

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High density QCD

High density QCD is based on the idea of parton saturation:

- gluons form high density system in which easily recombine
- nonlinear evolution equations appear: (Color Glass Condensate)
- unitarity restored (Froissart bound)

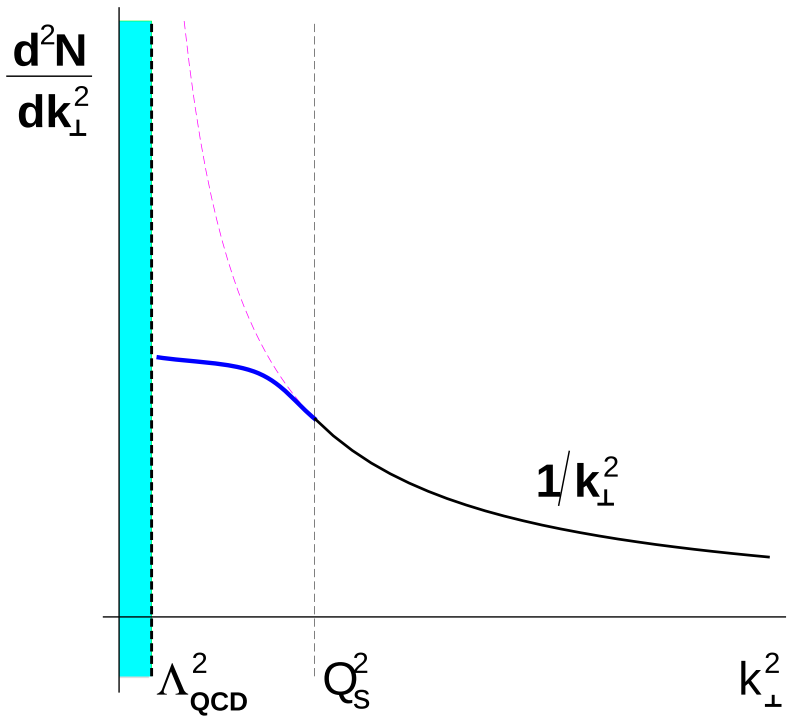
Two basic features of high density QCD:

- saturation scale - intrinsic scale for gluon system: $Q_s^2(x)$
- geometric scaling - DIS $\gamma^* p$ cross section scales

$$\sigma^{\gamma p}(x, Q^2) = \sigma^{\gamma p}\left(\frac{Q^2}{Q_s^2(x)}\right)$$

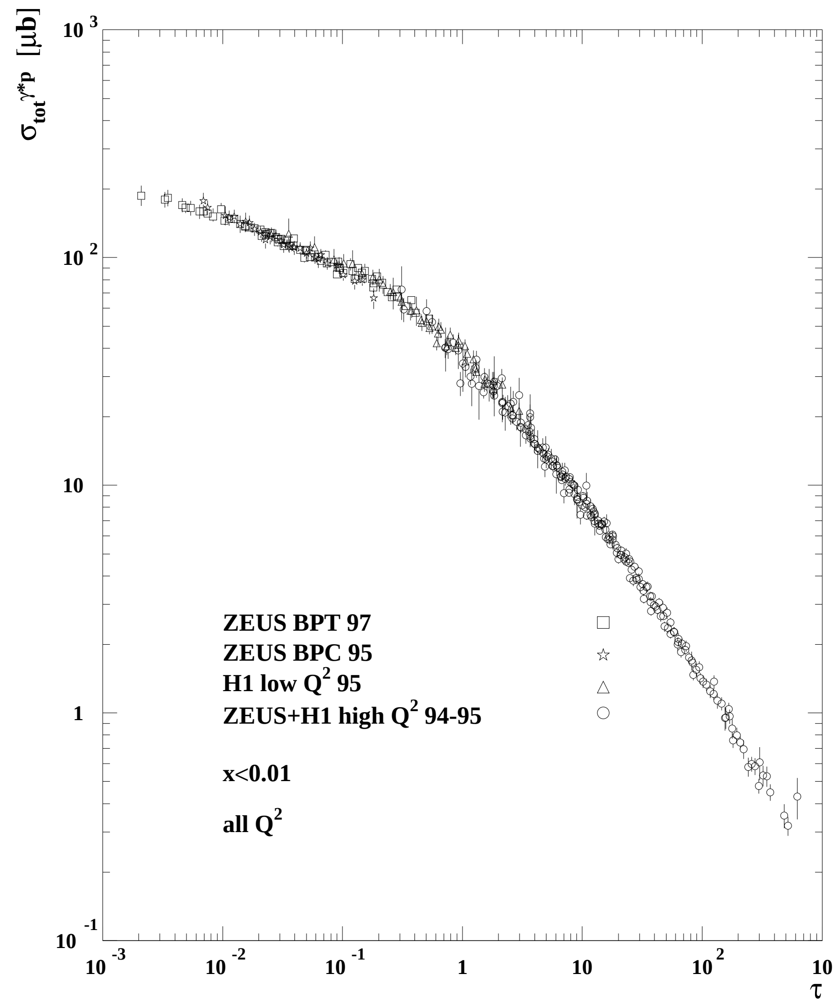
Saturation scale and geometric scaling

Gluon density



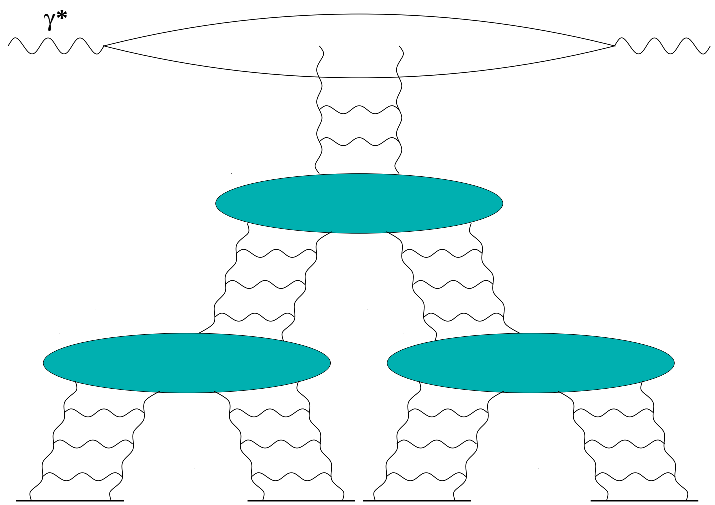
Saturation for $k_\perp < Q_s(x)$

$\gamma^* p$ cross section



Scaling in $\tau = Q^2/Q^2(x)$

Nonlinear evolution equations



Balitsky-Kovchegov equation for the dipole scattering amplitude $N_{xy}(Y)$

$$\frac{\partial N_{xy}}{\partial Y} = \overline{\alpha}_s \int d^2 z \, \frac{(x-y)^2}{(x-z)^2 (y-z)^2} \left\{ \underbrace{N_{xz} + N_{yz} - N_{xy}}_{BFKL} - N_{xz} \, N_{yz} \right\}$$

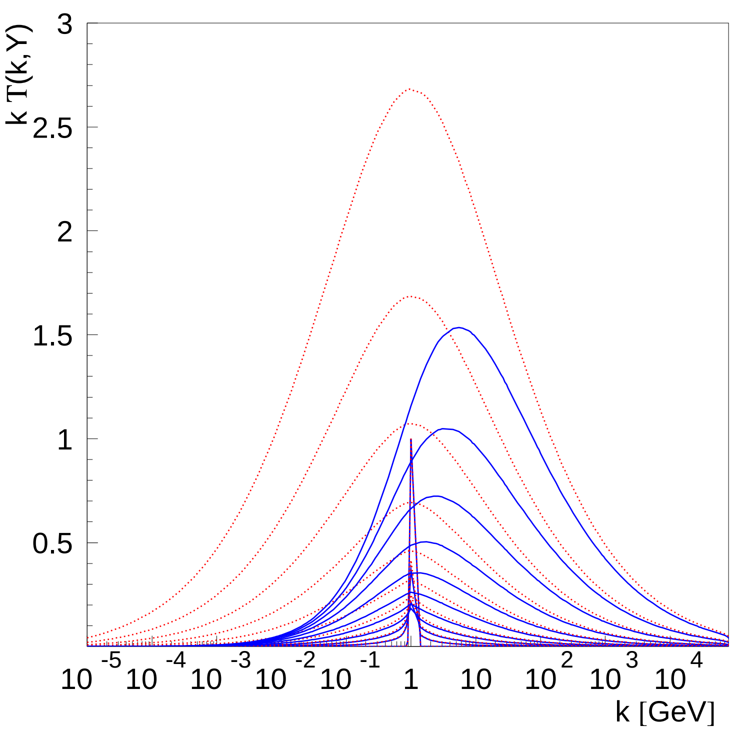
Cross section:

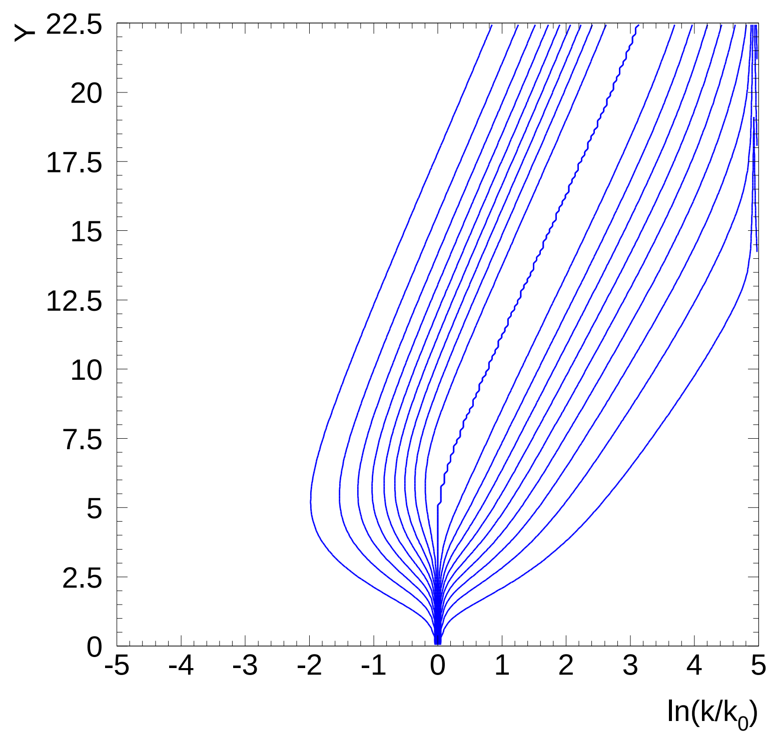
$$\sigma^{\gamma p} = \frac{1}{Q^2} \int d^2 x \, d^2 y \, |\Psi_\gamma(x-y, Q^2)|^2 \, N_{xy}(Y)$$

Saturation scale from BK equation

- Fourier transform: $T(k_\perp, Y) = \int d^2r \, \mathrm{e}^{ik_\perp \cdot r} \, N(r,Y)/r^2 \,, \quad r = |x-y|$
- BK equation in spherical and uniform case *(GB, Motyka, Staśto (2003))*

$$\partial_Y T = \underbrace{\chi\left(-\partial_{\ln k_\perp}\right) T}_{BFKL} - T^2$$





Infrared suppression

Scaling: $T = T(k_\perp / Q_s(Y))$

Traveling wave

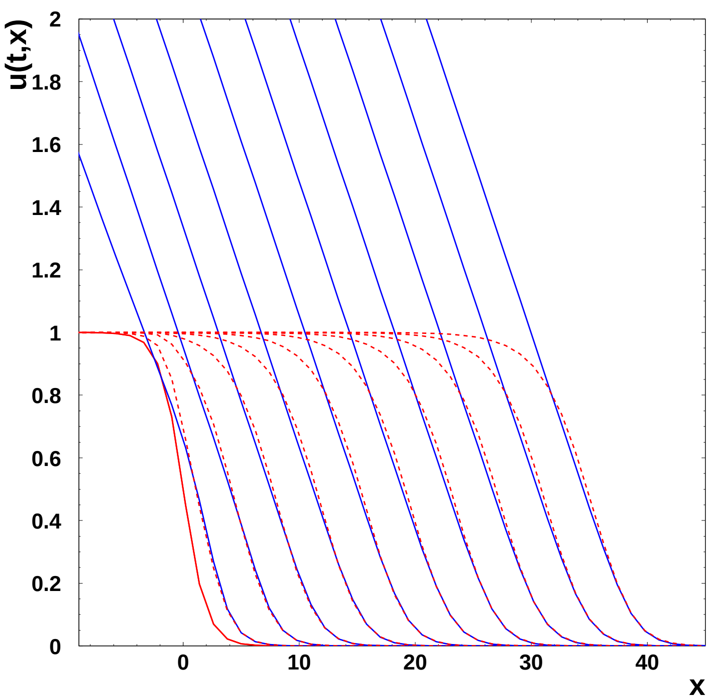
- Expand BFKL kernel: *(Munier, Peschanski (2003-04))*

$$\chi(\gamma) = \chi_c + \chi'_c(\gamma - \gamma_c) + \frac{1}{2}\chi''_c(\gamma - \gamma_c)^2$$

  and change variable: $u(x,t) = T(k_\perp, Y)$.

- BK equation belongs to the universality class of FKPP equation:

$$\partial_t u(x,t) \;=\; \partial_{xx} u \;+\; u\,(1-u)$$



- Traveling wave: $u = u(x - v_\infty t)$
- Geometric scaling: $T = T\left(\frac{k_\perp}{Q_s(Y)}\right)$
- Saturation scale from the tail $u \ll 1$

$$\ln Q_s^2(Y) = \frac{\bar{\alpha}_s\,\chi_c}{\gamma_c} Y - \frac{3}{2\gamma_c}\ln Y - \frac{A(\gamma_c)}{\sqrt{Y}}$$

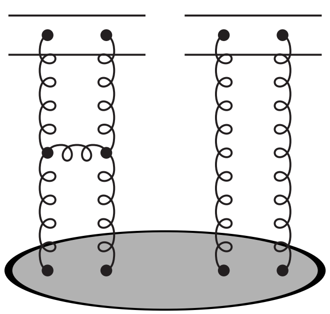
Saturation and fluctuation

Scattering amplitude T is averaged over partonic configurations:

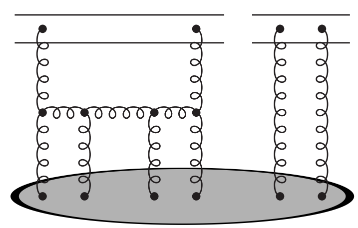
$$\langle T^2 \rangle \simeq \langle T \rangle \langle T \rangle$$

BK equation is an approximation $\rightarrow$ Balitsky – JIMWALK hierarchy

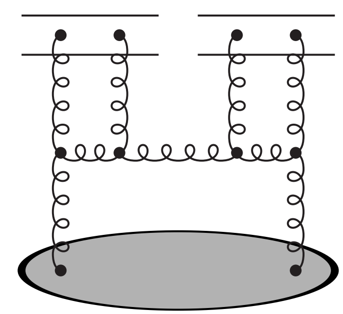
$$\langle T \rangle \rightarrow \langle T^2 \rangle \rightarrow \langle T^3 \rangle \rightarrow \langle T^4 \rangle \rightarrow \cdots$$



evolution T^2



splitting $T^2 \rightarrow T^3$



merging $T^2 \rightarrow T$

Merging is not present in the Balitsky's hierarchy.

Stochastic FKPP equation

- HE scattering is a stochastic process with death/birth processes.

$$\partial_Y T(k,Y) = \chi\,(-\partial_{\ln k_\perp})\,T \;-\; T^2 \;+\; \alpha_s\sqrt{2T}\,\eta$$

where η is white noise

$$\langle \eta(k,Y) \rangle = 0 \qquad \langle \eta(k,Y)\,\eta(k',Y') \rangle = \delta(\ln k - \ln k')\,\delta(Y - Y')$$

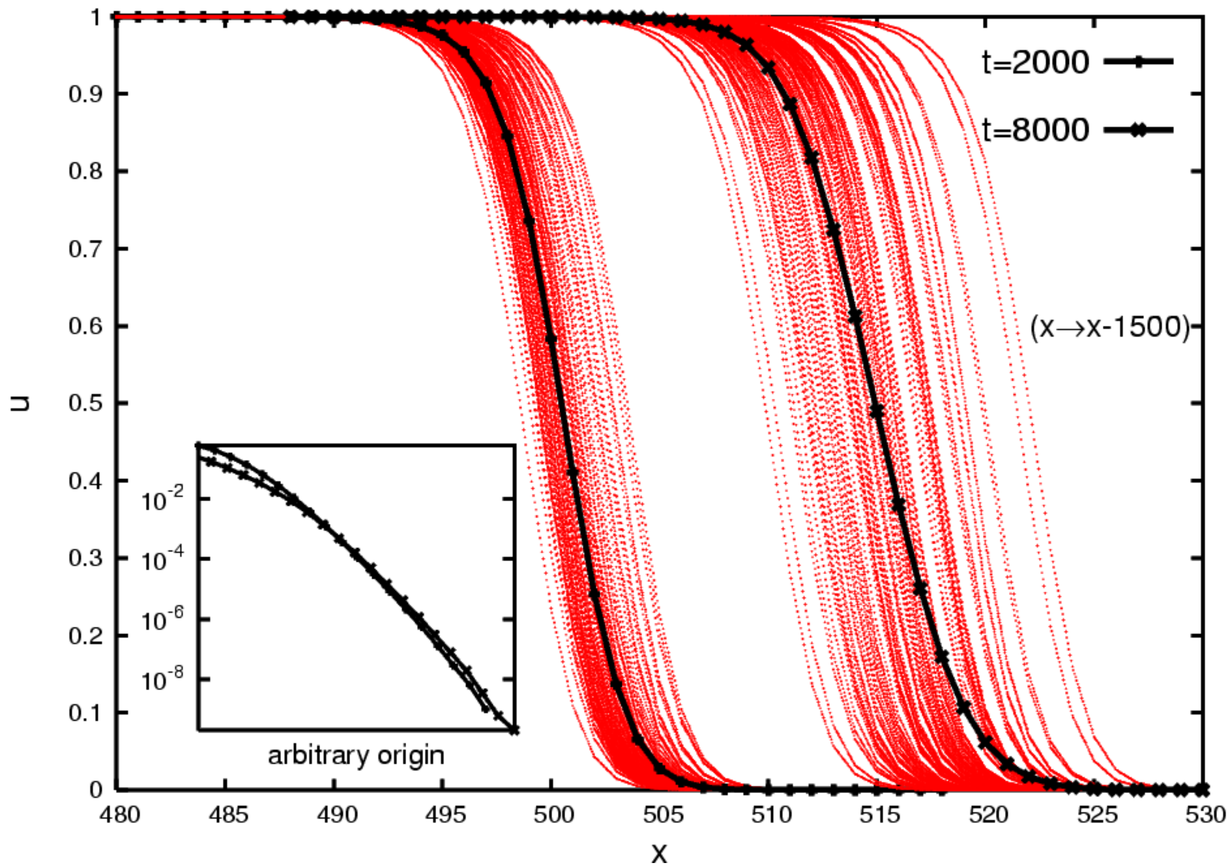
- The equation for the scattering amplitude T is in the universality class of the stochastic FKPP equation *(Iancu, Mueller, Munier (2004))*:

$$\partial_t u \;=\; \partial_{xx} u \;+\; u\,(1-u) \;+\; \sqrt{\frac{2}{N} u(1-u)}\,\eta$$

- The front of the traveling wave $T \sim u$ is very sensitive to fluctuations when the number of partons n is low: $\quad T \;\simeq\; \alpha_s^2\, n \;\ll\; 1$

Consequences of stochasticity

(Munier (2005))



- saturation scale $Q_s(Y)$ is a random variable
- scattering amplitude is given by an average over noise
- geometric scaling is violated due to fluctuations in the dilute domain

Geometric scaling violation

(Brunet, Derida, Enberg, GB, Iancu, Marquet, Mueller, Munier, Peschanski, Soyez, Shoshi, Triantafyllopoulos, Xiao (2004-06))

- Average amplitude – diffusive scaling instead of geometric scaling:

$$\langle T(k_\perp, Y)\rangle = T\left(\frac{\ln k_\perp^2 - \langle \ln Q_s^2(Y)\rangle}{\sqrt{\bar{\alpha}_s Y/\ln^3(1/\alpha_s^2)}}\right)$$

- Average saturation scale with dispersion $\sigma^2 \sim Y$

$$\langle \ln Q_s^2(Y)\rangle = \left(\frac{\bar{\alpha}_s \chi_c}{\gamma_c} - \frac{\bar{\alpha}_s \pi^2 \gamma_c \chi_c''}{2\ln^2(1/\alpha_s^2)}\right) Y$$

- Fluctuations important when $\bar{\alpha}_s Y \gg \ln^2(1/\alpha_s^2) \gg 1$

Conclusions

- The origin of saturation scale and geometric scaling can be understood through the relation to statistical physics.
- High energy QCD scattering can be viewed as stochastic process in which rare partonic configurations play important role.
- Geometric scaling breaking can be given precise meaning (diffusive scaling).

A very happy birthday Robi!