Double parton distributions

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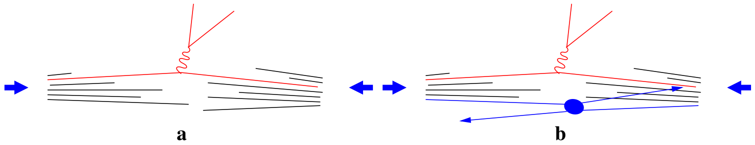
Outline

- Introduction
- Single and double parton scattering
- Double parton distribution functions (DPDFs)
- Evolution equations
- **q**-dependence of DPDFs
- Electroweak boson production in DPS
- Summary

Motivation

- QCD is the basic theory of strong interactions.
- Hard processes (with the scale $Q \gg \Lambda \sim 1\ \mathrm{GeV}$) are interpreted as collisions of quark and gluons (partons) from colliding hadrons.
- In single parton scattering (SPS) partons are described by parton distribution functions (PDFs)

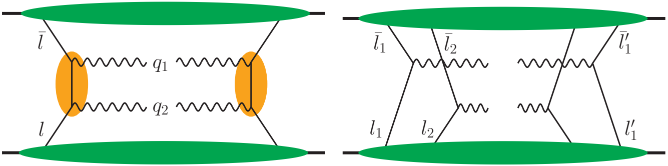


$$D_f(x, Q)\,, \qquad\qquad x = \text{longitudinal momentum fraction}$$

(plots thanks to Markus Diehl).

Multiparton interactions (MPI)

- At the LHC multiparton interactions become increasingly important
- For scattering with low scale, MPI are part of underlying event.
- For high center-of-mass energy MPI may become hard.
- Double parton scattering (DPS) as the first step.



- For not totally inclusive cross sections DPS as important as SPS

$$\frac{d\sigma}{dY_1 dY_2 d^2q_1 d^2q_2} \sim \frac{1}{Q^4\Lambda^2}$$

Double parton scattering

- However, after integrating over q_1, q_2 DPS is power suppressed

$$\frac{d\sigma^{\rm DPS}}{dY_1 dY_2} \sim \frac{\Lambda^2\Lambda^2}{Q^4\Lambda^2} = \frac{\Lambda^2}{Q^4}, \qquad \frac{d\sigma^{\rm SPS}}{dY_1 dY_2} \sim \frac{Q^2\Lambda^2}{Q^4\Lambda^2} = \frac{1}{Q^2}$$

- Nevertheless, for small momentum fractions $x \to 0$ (large $\sqrt{s}$)

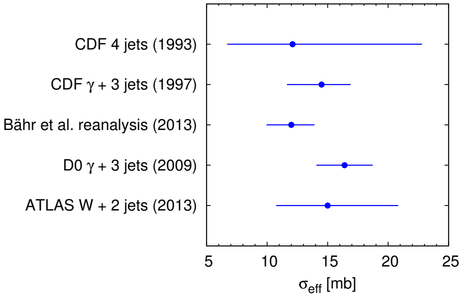
$$xD_f(x) \sim x^{-\lambda}$$

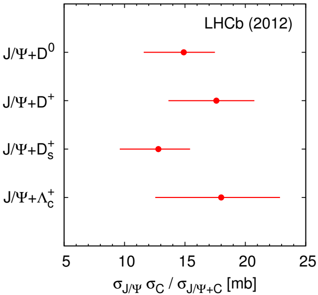
and

$$d\sigma^{DPS} \sim x^{-4\lambda}, \qquad d\sigma^{\rm SPS} \sim x^{-2\lambda}$$

- DPS increased due to energy enhancement.

Experimental evidence of DPS



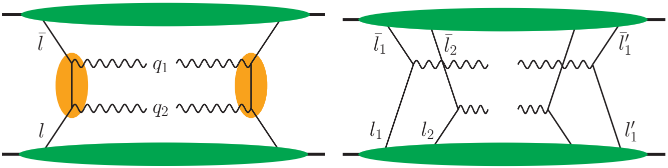


- Pocket formula

$$\sigma_{AB}^{\rm DPS} = \frac{\sigma_A^{\rm SPS}\,\sigma_B^{\rm SPS}}{\sigma_{\rm eff}}, \qquad\qquad \sigma_{\rm eff} \approx 15\ {\rm mb}$$

SPS and DPS cross sections

- Incoming partons with transverse momenta $\Lambda \ll k_\perp \ll Q$.



- Single and double PDFs in collinear factorization cross sections:

$$\frac{d\sigma_{AB}^{SPS}}{dx\, d\overline{x}} = D_i(x,Q)\,\sigma_{i\bar{i}}^{AB}(x\overline{x}s,Q)\,D_{\bar{i}}(\overline{x},Q)$$

$$\frac{d\sigma_{AB}^{DPS}}{dx_1\,dx_2\,d\overline{x}_1\,d\overline{x}_2} = \int \frac{d^2q}{(2\pi)^2}\, D_{ij}(x_1,x_2,Q_1,Q_2,q)\;\sigma_{i\bar{i}}^{A}\,\sigma_{j\bar{j}}^{B}\;D_{\bar{i}\bar{j}}(\overline{x}_1,\overline{x}_2,Q_1,Q_2;-q)$$

Double parton distribution functions

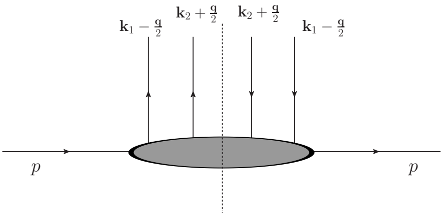
- Phenomenolgically

$$D_{f_1 f_2}(x_1, x_2, Q_1, Q_2, \mathbf{q}) = D_{f_1}(x_1, Q_1)\, D_{f_2}(x_2, Q_2)\, F^2(\mathbf{q}) \times \theta(1 - x_1 - x_2)$$

Now

$$\sigma_{AB}^{DPS} = \sigma_A^{\rm SPS} \sigma_B^{\rm SPS} \int \frac{d^2\mathbf{q}}{(2\pi)^2} F^4(\mathbf{q}) \;\longleftarrow\; \frac{1}{\sigma_{\rm eff}}$$

- In QCD - lower (upper) part of the forward scattering amplitude



DPDFs - definition

- Start from the correlator

$$\langle\langle z_1, z_2, y\rangle\rangle = \left\langle p|\overline{q}(-\tfrac{1}{2}z_2)\,\Gamma^a_\alpha\, q(\tfrac{1}{2}z_2)\,\overline{q}(y-\tfrac{1}{2}z_1)\,\Gamma^b_\beta\, q(y+\tfrac{1}{2}z_1)|p\right\rangle$$

- Fourier transform to momentum space: $z_i^- \leftrightarrow k_i^+ = x_i\, p^+$

$$F^{ab}_{\alpha\beta}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{q}) = \int dz_1^- dz_2^-\, \mathrm{e}^{i(x_1 z_1^- + x_2 z_2^-)p^+} \int d^2\mathbf{z}_1 d^2\mathbf{z}_2\, \mathrm{e}^{-i(\mathbf{z}_1\mathbf{k}_1 + \mathbf{z}_2\mathbf{k}_2)}$$

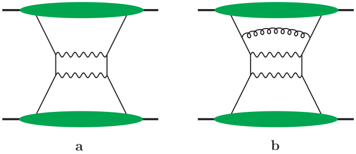
$$\times \int dy^- d^2\mathbf{y}\, \mathrm{e}^{i\mathbf{y}\mathbf{q}}\, \langle\langle z_1, z_2, y\rangle\rangle \Big|_{z_1^+ = z_2^+ = y^+ = 0}$$

- Transverse momentum integrated (color singlet) DPDFs:

$$D(x_1, x_2, \mathbf{q}) = \int d^2\mathbf{k}_1 d^2\mathbf{k}_2\, F^1(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{q})$$

Evolution equations

- QCD emissions - collinear and ultraviolet divergences subtracted



- DGLAP evolution equations for PDFs

$$\frac{\partial}{\partial \ln Q^2} D_f(x,Q) = \frac{\alpha_s(Q)}{2\pi} \sum_{f'} \int_x^1 du\, \mathcal{P}_{ff'}(x,u)\, D_{f'}(u,Q)$$

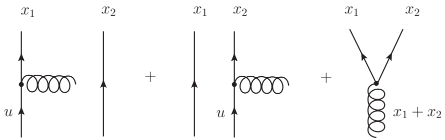
- Similar evolution equations for the DPDFs.

Evolution equation for DPDFs

- For equal scales: $D_{f_1 f_2}(x_1, x_2, t, \mathbf{q}) = D_{f_1 f_2}(x_1, x_2, Q, Q, \mathbf{q})$

$$\begin{aligned}
\frac{\partial}{\partial t} D_{f_1 f_2}(x_1, x_2, t, \mathbf{q}) &= \sum_{f'} \int_{x_1}^{1-x_2} du\, \mathcal{P}_{f_1 f'}(x_1, u)\, D_{f' f_2}(u, x_2, t, \mathbf{q}) \\
&+ \sum_{f'} \int_{x_2}^{1-x_1} du\, \mathcal{P}_{f_2 f'}(x_2, u)\, D_{f_1 f'}(x_1, u, t, \mathbf{q}) \\
&+ \sum_{f'} \mathcal{P}^{R}_{f' \to f_1 f_2}(x_1/(x_1 + x_2))\, D_{f'}(x_1 + x_2, t)
\end{aligned}$$

- DGLAP type evolution equation with additional splitting term



Solution

- Solution in the Mellin moment space

$$\tilde{D}_{f_1 f_2}(n_1, n_2) = \int_0^1 dx_1\, x_1^{n_1} \int_0^1 dx_2\, x_2^{n_2}\, \theta(1 - x_1 - x_2)\; D_{f_1 f_2}(x_1, x_2)$$

- Evolution equation in the Mellin space

$$\partial_t\, \tilde{D}(n_1, n_2, \mathbf{q}) \;=\; \gamma(n_1)\, \tilde{D}(n_1, n_2, \mathbf{q}) + \tilde{D}(n_1, n_2, \mathbf{q})\, \gamma^T(n_2)$$

$$+\;\; \tilde{\gamma}^R(n_1, n_2)\tilde{D}(n_1 + n_2)$$

where the splitting functions

$$\gamma(n) = \int_0^1 dz\, z^n P(z) \qquad\qquad \tilde{\gamma}^R(n_1, n_2) = \int_0^1 dz\, z^{n_1}(1 - z)^{n_2} P^R(z)$$

Solution

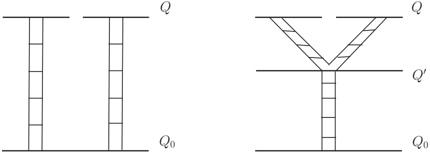
- Linear non-homogeneous equation - general solution:

$$\tilde{D}(n_1, n_2, t, \mathbf{q}) = \mathrm{e}^{\gamma(n_1)t}\, \tilde{D}_0(n_1, n_2, \mathbf{q})\, \mathrm{e}^{\gamma^T(n_2)t} \longleftarrow \textit{homogeneous}$$

$$+ \int_0^t dt'\, \mathrm{e}^{\gamma(n_1)(t-t')}\, \tilde{\gamma}(n_1, n_2)\, \tilde{D}(n_1 + n_2, t')\, \mathrm{e}^{\gamma^T(n_2)(t-t')}$$

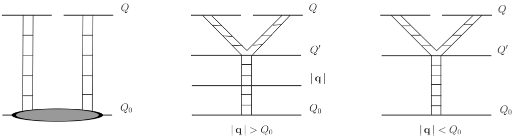
$\uparrow$ non-homogeneous

- Graphical illustration



q-dependence of the solution

- Ryskin-Snigirev proposal (2011)



$$\tilde{D}(n_1, n_2, t, \mathbf{q}) = e^{\gamma(n_1)t}\, \tilde{D}_0(n_1, n_2)\, e^{\gamma^T(n_2)t}\, F_{2g}^2(\mathbf{q})$$

$$+ \int_{t_0(\mathbf{q})}^{t} dt'\, e^{\gamma(n_1)(t-t')}\, \tilde{\gamma}(n_1, n_2)\, \tilde{D}(n_1 + n_2, t')\, e^{\gamma^T(n_2)(t-t')}$$

- Homogeneous solution suppressed by the partonic form factor

$$F_{2g}(q^2) = \frac{1}{\left(1 + q^2/m_g^2\right)^2}$$

with $m_g \approx 1.5\ \mathrm{GeV}$ to obtain $\sigma_{\text{eff}} \approx 15\ \mathrm{mb}$.

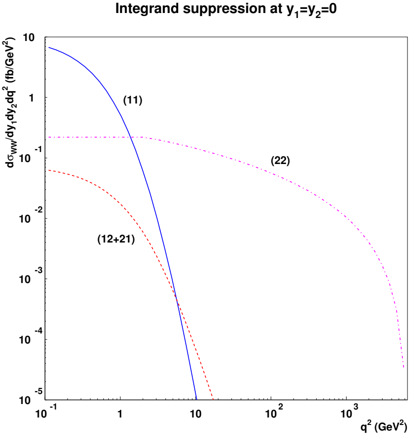
DPS cross sections

- DPS cross section written in terms of $D = D^{(1)} + D^{(2)}$

$$\sigma_{AB} = \int d^2\mathbf{q}\,(D^{(1)} + D^{(2)})\,\sigma_A\,\sigma_B\,(D^{(1)} + D^{(2)})$$
$$= \sigma_{AB}^{(11)} + \sigma_{AB}^{(12+21)} + \sigma_{AB}^{(22)}$$

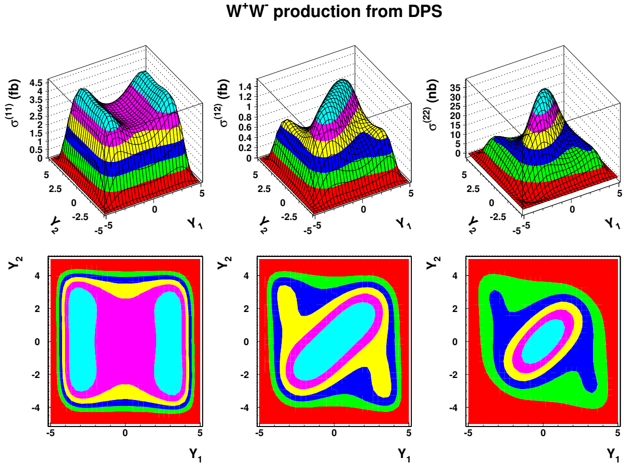
- Pocket formula corresponds to $\sigma_{AB}^{(11)}$.
- How important are the splitting contributions, $\sigma_{AB}^{(12+21)}$ and $\sigma_{AB}^{(22)}$?

W^+W^- production from DPS



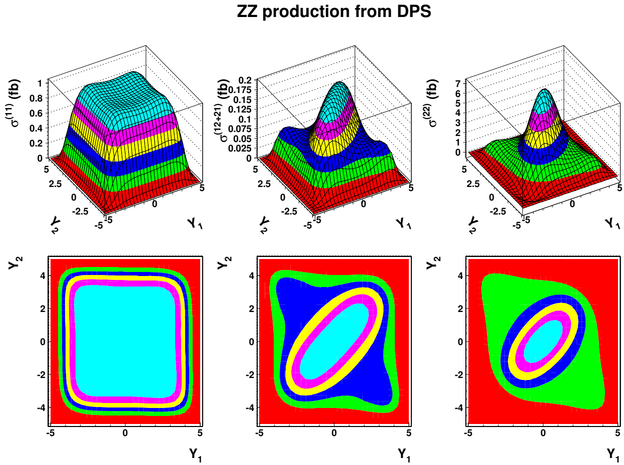
- No q^2- suppression for pure splitting contribution $\sigma_{AB}^{(22)}$.

W^+W^- production from DPS



- $\sigma^{(22)}_{WW} \gg \sigma^{(11)}_{WW}$. Pocket formula no longer valid.

ZZ production from DPS



- $\sigma^{(22)}_{ZZ} \gg \sigma^{(11)}_{ZZ}$. Pocket formula no longer valid.

Summary and outlook

- We analyze DPS using within collinear factorization with DPDFs and evolution equations in the LLA.
- We implemented the proposal of Ryskin and Snigirev for the **q**-dependence of DPDFs.
- We show that for electroweak boson production the pure splitting contribution dominates the cross section.
- For future: SPS/DPS interference, transverse momentum dependent DPDFs, spin and color correlations, Sudakov resummations, more processes to analyse.