Double parton distributions in QCD

Krzysztof Golec-Biernat

Institute of Nuclear Physics in Kraków
and
Rzeszow University

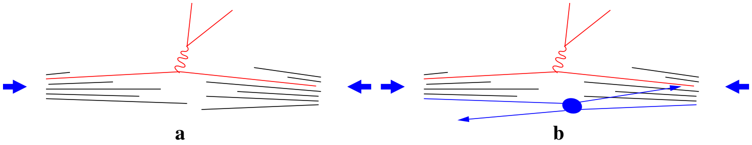
WE-Heraeus Physics School, Bad Honnef, September 24-30, 2017

Plan

- Double parton scattering
- Double parton distributions
- Evolution equations
- DPS cross section computation
- Transverse momentum dependent double distributions

- Hard processes with the scale $Q \gg \Lambda \sim 1\ \mathrm{GeV}$ due to collisions of quarks and gluons.

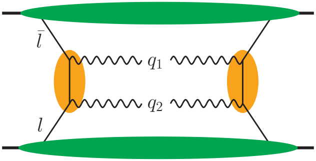
$$pp \rightarrow X_{hard} + Y$$



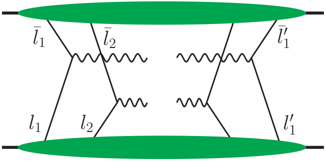
- At the LHC multiparton interactions (MPI) become increasingly important.
- If no hard scale is involved, MPI are crucial for modeling of underlying event.
- For rising center-of-mass energy MPI lead to more hard scatterings.

Double parton scattering

- For example, two vector bosons with transverse momenta q_1 and q_2



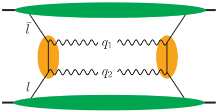
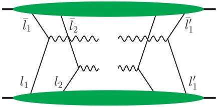
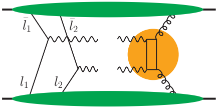
Single parton scattering (SPS)



Double parton scattering (DPS)

- Introduction to DPS: M. Diehl, D. Ostermeier, A. Schafer, JHEP 1203 (2012) 089

DPS versus SPS cross sections

	$\frac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i}$	$\frac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i}$
	$\frac{1}{\Lambda^2 Q^2}$	1
	$\frac{1}{\Lambda^2 Q^2}$	$\frac{\Lambda^2}{Q^2}$
	$\frac{1}{\Lambda^2 Q^2}$	$\frac{\Lambda^2}{Q^2}$

- Inclusive DPS is enhanced due to rising parton densities for $x \to 0$

$$d\sigma^{SPS} \sim x^{-2\lambda} \qquad\qquad d\sigma^{\mathrm{DPS}} \sim x^{-4\lambda}$$

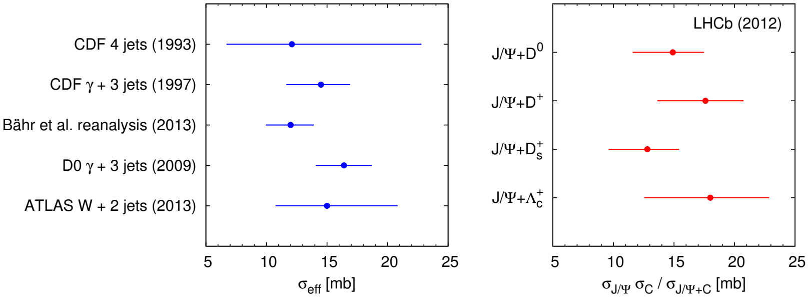
- SPS suppressed in same sign vector boson production $W^{\pm}W^{\pm}$.

Experimental evidence of DPS

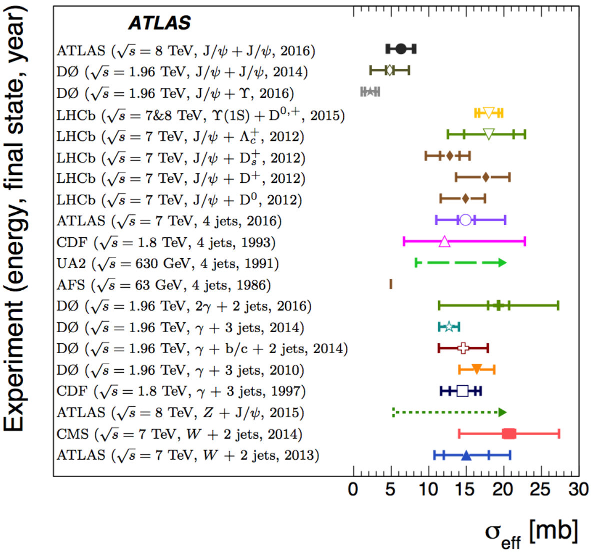
- Pocket formula

$$\sigma_{AB}^{\mathrm{DPS}} = \frac{1}{1+\delta_{AB}} \frac{\sigma_A^{\mathrm{SPS}} \sigma_B^{\mathrm{SPS}}}{\sigma_{\mathrm{eff}}}$$

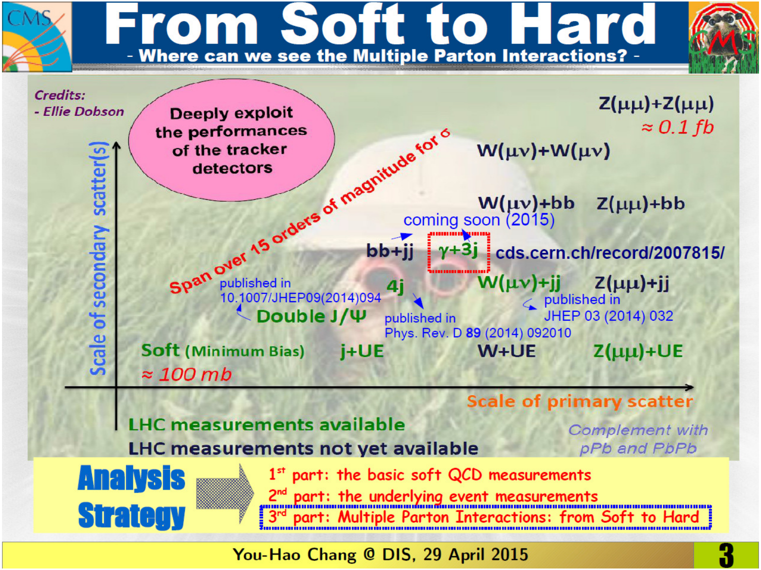
- Effective cross section: $\sigma_{\mathrm{eff}} \approx 15\,\mathrm{mb}$.



Recent update

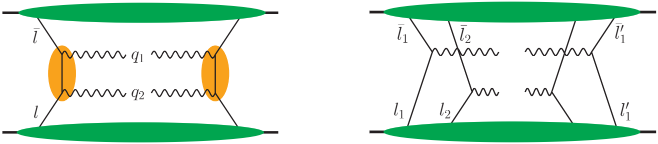


LHC strategy for DPS



SPS and DPS cross sections

- Partons with transverse momenta $k_\perp \sim \Lambda \ll Q$ (naive parton model)



- For SPS single parton distributions (PDF) with $0 < x < 1$

$$\frac{d\sigma^{SPS}_{AB}}{dx\, d\overline{x}} = \sum_{i\bar{i}} D_i(x)\, \sigma^{\rm AB}_{i\bar{i}}\, D_{\bar{i}}(\overline{x})$$

- For DPS double parton distributions (DPDF) with $0 < x_1 + x_2 < 1$

$$\frac{d\sigma^{DPS}_{AB}}{dx_1\, dx_2\, d\overline{x}_1\, d\overline{x}_2} = \sum_{i\bar{i}j\bar{j}} \int \frac{d^2\mathbf{q}}{(2\pi)^2}\, D_{ij}(x_1, x_2, \mathbf{q})\; \sigma^A_{i\bar{i}}\, \sigma^B_{j\bar{j}}\; D_{\bar{i}\bar{j}}(\overline{x}_1, \overline{x}_2; -\mathbf{q})$$

- Single PDF

$$D_i(x) = \int dz^- \, \mathrm{e}^{i(xP^+)z^-} \langle P| \underbrace{\bar{\Psi}_i(0,0,\mathbf{0})\gamma^+\Psi_i(0,z^-,\mathbf{0})}_{\mathcal{O}_i(0,z)} |P\rangle$$

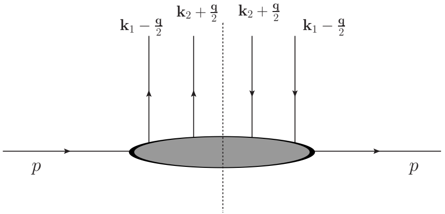
- $\mathcal{O}_i(0,z)$ connects two points on the light cone
- Double PDF

$$D_{ij}(x_1,x_2,\mathbf{q}) = \int dz_1^- \, dz_2^- \, \mathrm{e}^{i(x_1 z_1^- + x_2 z_2^-)P^+} \int dy^- d^2\mathbf{y} \, \mathrm{e}^{-i\mathbf{q}\cdot\mathbf{y}} \\ \times \langle P| \, \mathcal{O}_j(0,z_2) \, \mathcal{O}_i(y,z_1) \, |P\rangle$$

- $\mathcal{O}_j(y,z_2)$ does not connect light cone points since $y=(0,y^-,\mathbf{y})$ has transverse component $\mathbf{y}$.
- Transverse momentum $\mathbf{q}$ is Fourier conjugate to transverse separation $\mathbf{y}$.

Why does q exist ?

- $\langle P|\, \mathcal{O}_j(0,z_2)\, \mathcal{O}_i(y,z_1)\, |P\rangle$ has 4 fermionic operators. In momentum space

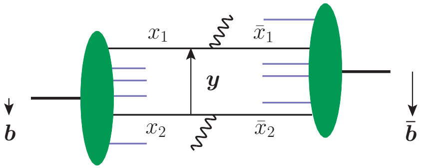


- Momentum conservation leads to 3 independent vectors: $\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}$.
- Integrated transverse momentum dependent distributions

$$D^{(ab)}_{(\alpha\beta)(ij)}(x_1,x_2,\mathbf{q}) = \int d^2\mathbf{k}_1\, d^2\mathbf{k}_2\, F^{(ab)}_{(\alpha\beta)(ij)}(x_1,x_2,\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})$$

- D_{ij} is color singlet, spin averaged transverse momentum integrated distribution.

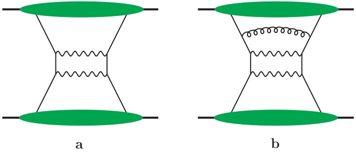
- $\mathbf{y}$ is the transverse separation between the two partons in a hadron



- Partons with the same transverse position interact with each other

$$\frac{d\sigma_{AB}^{DPS}}{dx_1\,dx_2\,d\overline{x}_1\,d\overline{x}_2} = \sum_{i\bar{i}j\bar{j}} \int d^2\mathbf{y}\, d^2\mathbf{b}\, d^2\mathbf{b}'\, \tilde{D}_{ij}(x_1, x_2, \mathbf{y}, \mathbf{b})\, \sigma_{i\bar{i}}^{A}\, \sigma_{j\bar{j}}^{B}\, \tilde{D}_{\bar{i}\bar{j}}(\overline{x}_1, \overline{x}_2, \mathbf{y}, \mathbf{b}')$$

- Partons with transverse momenta $\Lambda \ll k_\perp \ll Q$ (improved parton model)

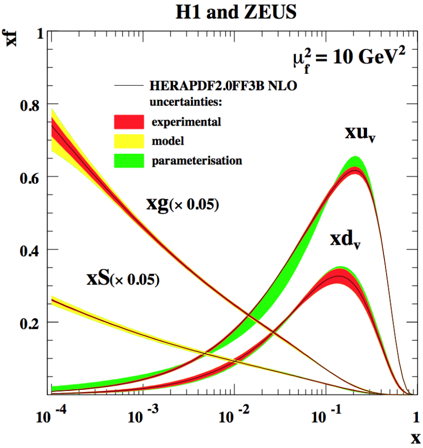


- Collinear divergences must be subtracted.
- PDF become factorization scale dependent, $D_f(x, Q)$.
- DGLAP evolution equations

$$\frac{\partial}{\partial \ln Q^2} D_f(x, Q) = \frac{\alpha_s}{2\pi} \sum_{f'} \int_x^1 du\, \mathcal{P}_{ff'}\left(\frac{x}{u}\right) D_{f'}(u, Q)$$

- $+$ initial conditions $D_f(x, Q_0)$, determined from global fits to data.

PDF from global fits



- Can we obtain the same precision for DPDF ?

Evolution equations for DPDF

- For equal scales, $Q_1 = Q_2 \equiv Q$, and $\mathbf{q} = \mathbf{0}$

$$D_{f_1 f_2}(x_1, x_2, Q, \mathbf{q} = 0) = \int d^2\mathbf{y}\, D_{f_1 f_2}(x_1, x_2, Q, \mathbf{y})$$

- Evolution equations (Snigirev hep-ph/0304172)

$$\frac{\partial}{\partial \ln Q^2} D_{f_1 f_2}(x_1, x_2, Q) = \frac{\alpha_s}{2\pi} \sum_{f'} \left\{ \int_{x_1}^{1-x_2} du\, \mathcal{P}_{f_1 f'}\left(\frac{x_1}{u}\right) D_{f' f_2}(u, x_2, Q) \right.$$
$$+ \int_{x_2}^{1-x_1} du\, \mathcal{P}_{f_2 f'}\left(\frac{x_2}{u}\right) D_{f_1 f'}(x_1, u, Q)$$
$$\left. + \mathcal{P}^R_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right) D_{f'}(x_1 + x_2, Q) \right\}$$

- Coupling with DGLAP equations for PDF in splitting term.

Sum rules obeyed by evolution equations

- How to determine initial conditions $D_{f_1 f_2}(x_1, x_2, Q_0)$?
- Momentum and quark number sum rules obeyed by single PDF

$$\sum_f \int_0^1 dx\, x\, D_f(x) = 1$$

$$\int_0^1 dx\, (D_q(x) - D_{\overline{q}}(x)) = N_q \qquad (= 2, 1, 0)$$

- Momentum and quark number sum rules obeyed DPDF

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, D_{f_1 f_2}(x_1, x_2) = (1 - x_2)\, D_{f_2}(x_2)$$

$$\int_0^{1-x_2} dx_1 \left\{ D_{q f_2}(x_1, x_2) - D_{\bar{q} f_2}(x_1, x_2) \right\} = (N_q - \delta_{q f_2} + \delta_{\bar{q} f_2})\, D_{f_2}(x_2)$$

- Sum rules for DPDF constrain $D_{f_1 f_2}(x_1, x_2, Q_0)$. Try to impose them.

Initial conditions for DPDF

- Most popular initial condition at Q_0 are built from the known single PDF (J.R. Gaunt, W.J. Stirling, arXiv:0910.4347)

$$D_{f_1 f_2}(x_1, x_2) = D_{f_1}(x_1)\, D_{f_2}(x_2)\, \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+n_1}(1 - x_2)^{2+n_2}}$$

- They obey sum rules for DPDF only approximately.
- Factorizable form for $x_1, x_2 \ll 1$

$$\text{ratio} = \frac{D_{f_1 f_2}(x_1, x_2)}{D_{f_1}(x_1)\, D_{f_2}(x_2)} = 1$$

- Study this ratio as a function of Q^2 during evolution $\rightarrow$ factorization test.
- Can we go beyond the factorized ansatz?

Pure gluonic case

(GB, Lewandowska, Serino, Snyder, Stasto, arXiv:1606.01679)

- MSTW08 gluon distr. at $Q_0 = 1\ \mathrm{GeV}$ with known parameters A_k, α_k, η

$$D_g(x) = \sum_{k=1}^{3} A_k\, x^{\alpha_k}\, (1-x)^{\eta}$$

- Solve momentum sum rule

$$\int_0^{1-x_2} dx_1\, x_1\, D_{gg}(x_1, x_2) = (1-x_2)\, D_g(x_2)$$

for the ansatz

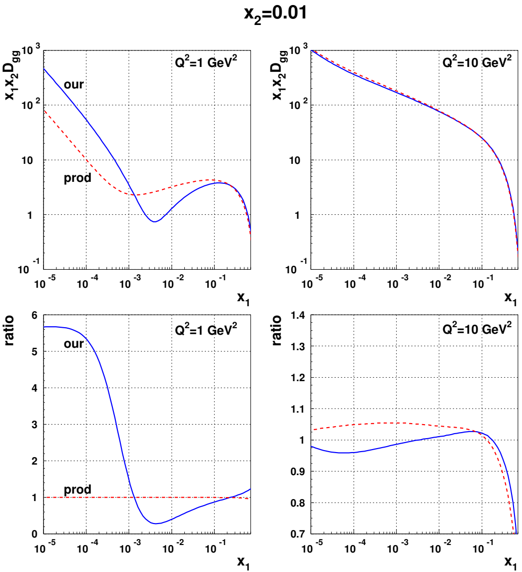
$$\int_0^{1-x_2} dx_1\, x_1 \underbrace{\sum_{k=1}^{3} \bar{N}_k\, (x_1\, x_2)^{a_k}\, (1-x_1-x_2)^{b_k}}_{D_{gg}(x_1,x_2)} = (1-x_2) \underbrace{\sum_{k=1}^{3} \bar{A}_k\, x_2{}^{a_k}\, (1-x_2)^{a_k+b_k+1}}_{D_g(x_2)}$$

- From the comparison with MSTW08 gluon

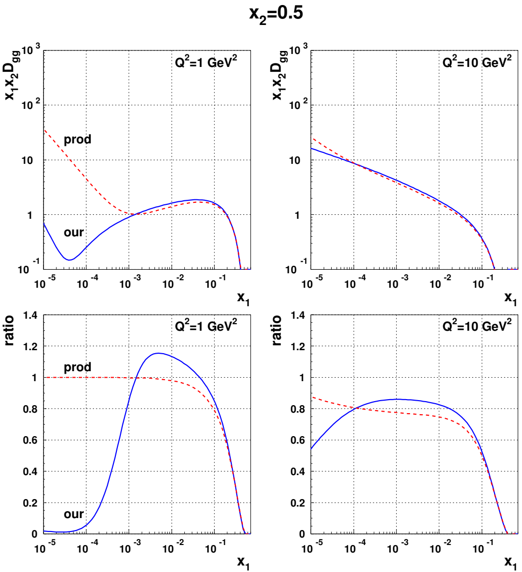
$$\bar{A}_k = A_k\,, \qquad a_k = \alpha_k\,, \qquad a_k + b_k + 1 = \eta$$

- Non-factorizable initial double gluon distribution.

Numerical results for $x_2 = 10^{-2}$



Numerical results for $x_2 = 0.5$



Adding quarks

- Light-cone Fock expansion of the proton state in terms of partonic states is the origin of the sum rules

$$|P\rangle = \sum_N \sum_{f_1 \ldots f_N} \int dx_1 \ldots dx_N \, \delta\left(1 - \sum_{k=1}^{N} x_k\right) \Psi_N(x_1 \ldots x_n; f_1 \ldots f_N) \, |x_1 \ldots x_N; f_1 \ldots f_N\rangle$$

- Try to model wave functions Ψ_N

$$D_f(x) = \sum_N \sum_{f_1 \ldots f_N} \int d\Pi_N \, |\Psi_N|^2 \left\{ \sum_{i=1}^{N} \delta(x - x_i) \, \delta_{ff_i} \right\}$$

$$D_{fh}(x, y) = \sum_N \sum_{f_1 \ldots f_N} \int d\Pi_N \, |\Psi_N|^2 \left\{ \sum_{i=1}^{N} \sum_{j \neq i}^{N} \delta(x - x_i)\delta(y - x_j) \, \delta_{ff_i} \delta_{hf_j} \right\}$$

- What Ψ_N leads to the MSTW08 parameterization?

A model

- Assume (Broniowski, Ruiz Ariola, GB, arXiv:1602.00254)

$$|\Psi_N(x_1 \dots x_N; f_1 \dots f_N)|^2 = A^N_{f_1 \dots f_N}\, x_1^{\alpha^N_{f_1}-1} x_2^{\alpha^N_{f_2}-1} \dots x_N^{\alpha^N_{f_N}-1}$$

- Single PDF

$$D_f(x) = \sum_N \sum_{(f_1 \dots f_N)'} \bar{A}^N_{f_1 \dots f_N}\, x^{\alpha^N_f - 1} (1-x)^{\alpha^N_{(f_1} + \dots + \alpha^N_{f_N)'} - 1}$$

- Small x powers $\alpha^N_{f_i}$ determine the large x powers

$$\eta^N_f = \alpha^N_{(f_1} + \dots + \alpha^N_{f_N)'}$$

- Such a relation is not valid for MSTW08 parameterization of PDF.

Summary on initial conditions

- Non-factorizable initial condition which fulfill momentum sum rule in the pure gluon case was constructed.

- Factorization of double gluon distribution at small x sets rather quickly

$$D_{gg}(x_1, x_2, Q) \approx D_g(x_1, Q) \cdot D_g(x_2, Q)$$

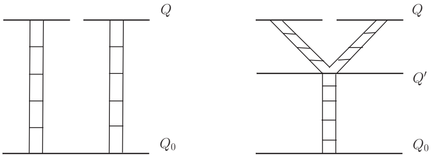
- Extension to the case with quarks has not been successful.

- Factorizable Gaunt-Stirling prescription is still mostly used.

DPS cross section computation

- Solution to the DPDF evolution equations

$$D_{f_1 f_2}(x_1, x_2, Q, \mathbf{q}=0) = D^{(1)}_{f_1 f_2}(x_1, x_2, Q) + D^{(2)}_{f_1 f_2}(x_1, x_2, Q)$$



- Sum of hadronic and point-like contributions

$$D(x_1, x_2, Q, \mathbf{q}) = U_1(Q, Q_0) \otimes \underbrace{D(x_1, x_2, Q_0)}_{initial\ \ DPDF} \otimes U_2^T(Q, Q_0) \times F^2(\mathbf{q})$$

$$+ \int_{max\{|\mathbf{q}|, Q_0\}}^{Q} dQ' \; U_1(Q, Q') \otimes \underbrace{D(x_1 + x_2, Q')}_{single\ \ PDF} \otimes P(Q') \otimes U_2^T(Q, Q')$$

- **q**-prescription from M.G. Ryskin, A.M. Snigirev, arXiv:1103.3495

Decomposed DPS cross section

- Partonic form factor (Frankfurt, Strikman)

$$F^2(q^2) = \frac{1}{(1+q^2/m^2)^4} \qquad\qquad m_g \approx 1.5\ \mathrm{GeV}$$

- DPS cross section written in terms of the two components

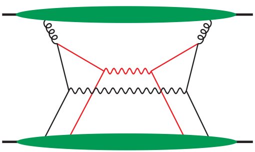
$$\sigma_{AB} = \int d^2\mathbf{q}\ (D^{(1)} + D^{(2)})\, \sigma_A\, \sigma_B\, (D^{(1)} + D^{(2)})$$
$$= \sigma_{AB}^{(11)} + \sigma_{AB}^{(12+21)} + \sigma_{AB}^{(22)}$$

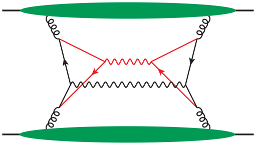
- Pocket formula from $\sigma_{AB}^{(11)}$ with factorized $D^{(1)}$ and

$$\frac{1}{\sigma_{\mathrm{eff}}} = \int d^2\mathbf{q}\, F^4(\mathbf{q}) \qquad \approx \qquad (15\,\mathrm{mb})^{-1}$$

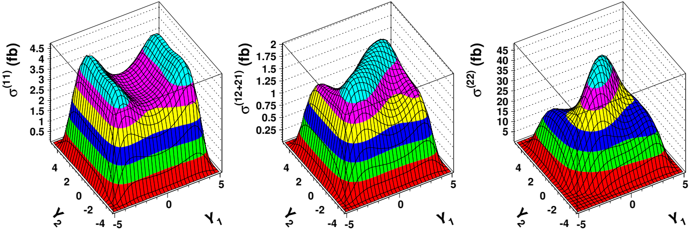
- How important are splitting contributions $\sigma_{AB}^{(12+21)}$ and $\sigma_{AB}^{(22)}$?

Splitting contributions





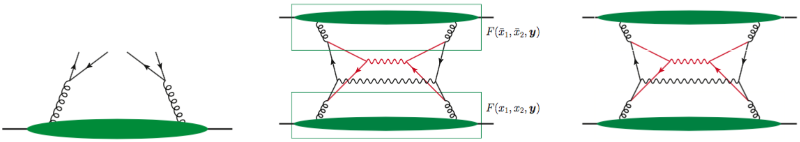
- W^+W^- production in DPS from GB, Lewandowska, arXiv:1407.4038



- $\sigma_{AB}^{(12+21)}$ and $\sigma_{AB}^{(22)}$ are important. (also Gaunt, Maciuła, Szczurek, arXiv:1407.5821)

How to avoid double counting ?

- Are the splitting graphs part of SPS or DPDF? (Diehl, Gaunt, Schönwald, arXiv:1702.06486)



- Procedure based on removing $1/\mathbf{y}^2$ divergence in the point-like DPDF

$$D^{(2)}_{f_1 f_2}(x_1, x_2, \mathbf{y}) = \frac{1}{\mathbf{y}^2} \frac{\alpha_s}{2\pi} \sum_{f'} D_{f'}(x_1 + x_2)\, P^R_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right)$$

by introducing monotonic cut-off function $\Phi(u) \in [0, 1]$ to

$$\sigma_{\rm DPS} \sim \int d^2\mathbf{y}\, |\Phi(Q\mathbf{y})|^2\, D_{f_1 f_2}(x_1, x_2, \mathbf{y})\, D_{f_3 f_4}(x_1, x_2, \mathbf{y})$$

and subtraction term to the cross section

$$\sigma = \sigma_{\rm DPS} - \sigma_{\rm sub} + \sigma_{\rm SPS}$$

- For $y \ll 1/Q$ $\sigma_{\rm DPS} \approx \sigma_{\rm sub}$ while for $y \gg 1/Q$ $\sigma_{\rm SPS} \approx \sigma_{\rm sub}$

.

Transverse momentum dependent double distributions (TMDD)

- Need of TMDD

$$D_{f_1 f_2}(x_1, x_2, \mathbf{y}) = \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \underbrace{F_{f_1 f_2}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{y})}_{\text{TMDD}}$$

be able to compute more exclusive cross sections

$$\frac{d\sigma^{DPS}_{AB}}{dx_i d\bar{x}_i d^2\mathbf{q}_i} = \sigma_A \sigma_B \int d^2\mathbf{k}_i \, d^2\bar{\mathbf{k}}_i \, d^2\mathbf{y} \, \delta^2(\mathbf{q}_i - \mathbf{k}_i - \bar{\mathbf{k}}_i) \, F(x_i, \mathbf{k}_i, \mathbf{y}) \, F(\bar{x}_i, \bar{\mathbf{k}}_i, \mathbf{y})$$

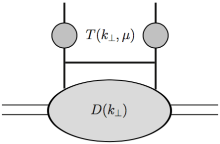
- Unfolding evolution equations for DPDF (for TMDD integrated over $\mathbf{y}$ or at $\mathbf{q} = 0$) (GB, Stasto, arXiv:1611.02033).

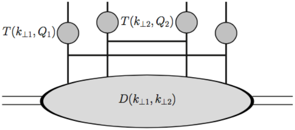
- Analysis with $\mathbf{y}$-dependent TMDD, following Collins-Soper method. TMDD factorization proved for $q_1 \ll Q_i$ by extending proof for TMD. (Buffing, Diehl, Kasemets, arXiv:1708.03528)

Unfolded TMDD

- Unfolding DGLAP evolution in the last step (Kimber,Martin,Ryskin). For TMD

$$F_a(x,\mathbf{k},Q) \equiv T_a(Q,\mathbf{k}) \sum_{a'} \int_x^{1-\Delta(\mathbf{k})} \frac{dz}{z} P_{aa'}(z,\mathbf{k})\, D_{a'}\Big(\frac{x}{z},\mathbf{k}\Big)\,,$$



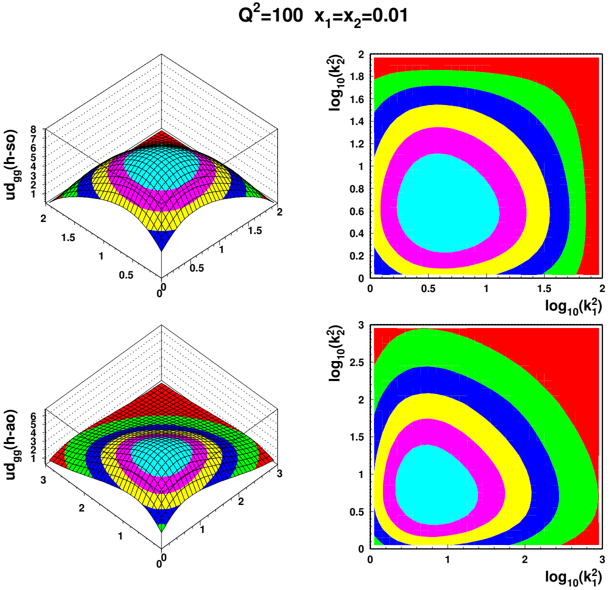


- $T_a(Q,\mathbf{k})$ is Sudakov form factor. For TMDD (GB, Stasto, arXiv:1611.02033)

$$F_{a_1a_2}(x_i,\mathbf{k}_i,Q_i) = T_{a_1}(Q_1,\mathbf{k}_1)\, T_{a_2}(Q_2,\mathbf{k}_2) \times$$
$$\times \sum_{b,c} \int_{\frac{x_1}{1-x_2}}^{1-\Delta(\mathbf{k}_1)} \frac{dz_1}{z_1} \int_{\frac{x_2}{1-x_1/z_1}}^{1-\Delta(\mathbf{k}_2)} \frac{dz_2}{z_2} P_{a_1b}(z_1,\mathbf{k}_1)\, P_{a_2c}(z_2,\mathbf{k}_2)\, D_{bc}\Big(\frac{x_1}{z_1},\frac{x_2}{z_2},\mathbf{k}_1,\mathbf{k}_2\Big)$$

- TMD and TMDD built out of known PDF and DPDF.

$F_{gg}(\mathbf{k}_1, \mathbf{k}_2)$ in pure gluon case



Summary

- Double parton scattering processes are being measured.
- Double parton distributions are well defined object with known properties.
- Significant progress in theoretical understanding of DPS and DPDF:
  - evolution equations
  - spin, color and momentum correlations
  - hadron and point-like contribution to DPDF
  - tranverse momentum double distributions
  - factorization theorems to compute DPS cross sections
- Still much to be done.

Thank you!