Deep inelastic scattering and EIC
Part III

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Plan

- Small x limit of DIS
- Parton saturation
- Diffractive processes in DIS

Basic books

- S. Donnachie, G. Dosch, P. Landshoff, O. Nachtmann, *Pomeron Physics and QCD*, Cambridge University Press 2002
- Y. V. Kovchegov, E. Levin, *Quantum Chromodynamics at High Energy*, Cambridge University Press 2012
- V. Barone, E. Predazzi, *High-Energy Particle Diffraction*, Springer 2002
- K. Golec-Biernat, *Habilitation thesis, 2001*
  http://annapurna.ifj.edu.pl/∼golec/data/teaching/files/wyklady/hab.pdf

Small-x limit of DIS

DIS for $x \sim 1$

- Collinear factorization formula:

$$F_2(x,Q^2) = \sum_f \int_0^1 d\xi \left[\underbrace{e_f^2\,\xi\,\delta(\xi - x) + \alpha_s(Q^2) f(\xi,x)\ldots}_{coefficient\ \ function\ \ from\ \ pQCD} \right] q_f(\xi,Q^2) + \ldots + \underbrace{\mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)}_{higher\ \ twist}$$

- DGLAP evolution equations:

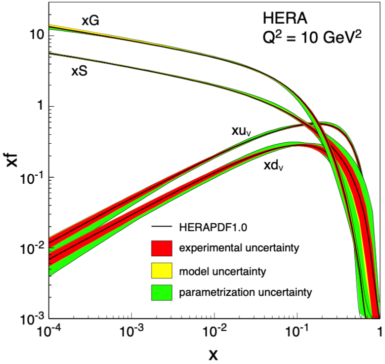
$$\frac{\partial q_f(x,Q^2)}{\partial \log Q^2} = P_{qq} \otimes q_f + P_{qG} \otimes G$$
$$\frac{\partial \bar{q}_f(x,Q^2)}{\partial \log Q^2} = P_{qq} \otimes \bar{q}_f + P_{qG} \otimes G$$
$$\frac{\partial G(x,Q^2)}{\partial \log Q^2} = P_{GG} \otimes G + P_{Gq} \otimes \sum_f (q_f + \bar{q}_f)$$

- Splitting functions:

$$P_{ij}(z,Q^2) = \underbrace{\alpha_s(Q^2)\,P_{ij}^{(0)}(z)}_{LL} + \underbrace{\alpha_s^2(Q^2)\,P_{ij}^{(1)}(z)}_{NLL} + \underbrace{\alpha_s^3(Q^2)\,P_{ij}^{(2)}(z)}_{NNLL} + \ldots$$

- Initial conditions from global fits.

Gluon dominance for $x \to 0$



- DGLAP eqs. in the limit $x \to 0$ and $Q^2 \to \infty$ (DLLA):

$$xG(x,Q^2) = \sum_n g_n \left[\alpha_s \ln(1/x) \ln(Q^2/\mu^2)\right]^n$$

- Solution :

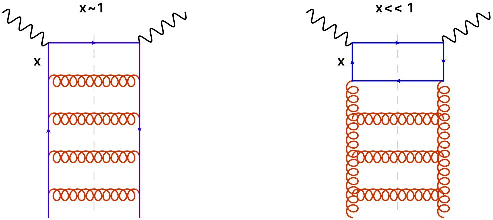
$$\frac{\partial^2 xG(x,Q^2)}{\partial \ln(1/x) \ln Q^2} = \bar{\alpha}_s \, xG(x,Q^2) \quad \to \quad xG(x,Q^2) \sim \exp\left\{2\sqrt{\bar{\alpha}_s \ln(1/x) \ln Q^2}\right\}$$

Small x limit of DIS

- Small-x or high energy limit: $Q^2 = \textit{fixed}$ but $W^2 \to \infty$

$$x \;\approx\; \frac{Q^2}{W^2} \;\to\; 0$$

- Valence quark versus sea quark DIS

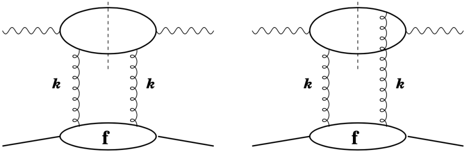


$$F_2 = \sum_n a_n \left[\alpha_s \ln(Q^2/\mu^2)\right]^n \qquad\qquad \sum_n b_n \left[\alpha_s \ln(1/x)\right]^n$$

- Large $\ln(1/x)$ from strong ordering of longitudinal momenta of gluons

$$x_1 \gg x_2 \gg \ldots \gg x_n \gg x$$

without ordering in transverse momenta



- k_t-factorization formula:

$$\frac{F_2(x,Q^2)}{Q^2} = \sigma^{\gamma^* p} = \int \frac{d^2k_t}{k_t^2}\, \Phi(k_t, Q^2)\, f(x,k_t)$$

- Photon impact factor Φ with off-shell gluon: $k^2 = -k_t^2$

- Unintegrated gluon distribution $f(x,k_t)$:

$$xG(x,Q^2) = \int \frac{d^2k_t}{k_t^2}\, \theta(Q - |k_t|)\, f(x,k_t)$$

- Dependence on x from the BFKL equation (Balitsky, Fadin, Kuraev, Lipatov, 1976-78)

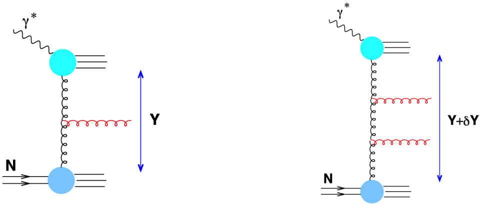
BFKL equation

- Strong ordering in $x \Leftrightarrow$ strong ordering in rapidity $Y = \ln(1/x)$

$$Y_1 \ll Y_2 \ll \ldots \ll Y_n$$

and no restrictions on transverse momenta - Regge kinematics

- Evolution equation in rapidity Y



$$\frac{\partial f(Y,k_t)}{\partial Y} = \frac{\alpha_s \pi}{N_c} \int_0^\infty \frac{dk_t'^2}{k_t'^2} k_t^2 \left\{ \frac{f(Y,k_t') - f(Y,k_t)}{|k_t'^2 - k_t^2|} + \frac{f(Y,k_t)}{\sqrt{4k_t'^4 + k_t^4}} \right\}$$

- Conformally invariant, infrared and ultraviolet safe

Solution to BFKL equation

- Color singlet solution - BFKL pomeron. For $Y = \ln W^2 \to \infty$

$$f(Y, k_t) \sim e^{\alpha_{I\!P}(0)Y} \times \frac{1}{\sqrt{2\pi DY}} \exp\left\{\frac{-\log^2(k_t^2)}{\sqrt{2D\,Y}}\right\}$$

- Power like growth with energy, given by pomeron intercept

$$\alpha_P(0) = 4\overline{\alpha}_s \ln 2 \approx 0.5 \qquad \to \qquad f(Y, k_t) \sim (W^2)^{0.5}\,,$$

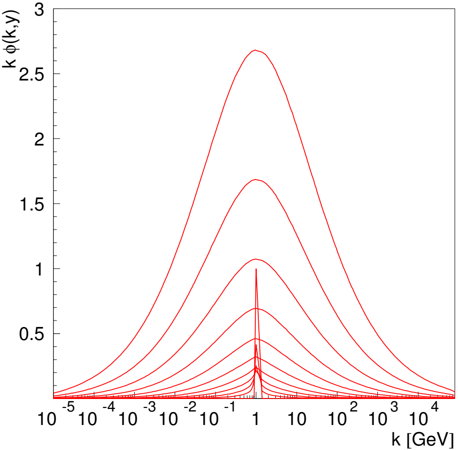
- In contradiction with the Froissart unitarity bound

$$\sigma \leqslant c \log^2(W^2)$$

- Diffusion of transverse momentum into infrared: $k_t < \Lambda_{QCD}$

Solution to BFKL equation

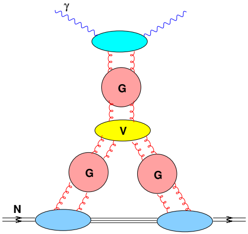
- Numerical solution for $y = 1, 2, \ldots, 10$



- Only gluon splitting: $g \to gg$, no gluon recombination: $gg \to g$

Parton saturation

- Gluon recombination before hard interation (Gribov, Levin, Ryskin, 1983, Mueller, Qiu, 1986)



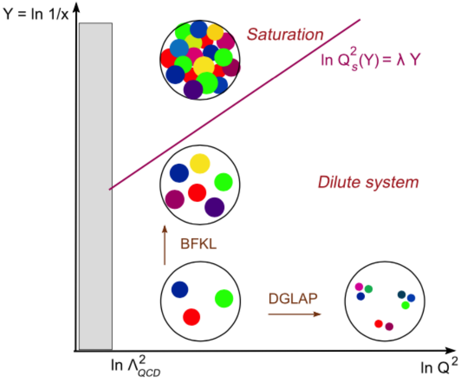
- Non-linear modification of the DGLAP equations in DLLA ($x \to 0$ and $Q^2 \to \infty$)

$$\frac{\partial^2 xG(x,Q^2)}{\partial \ln(1/x)\,\partial \ln Q^2} = \frac{3\alpha_s}{\pi} xG \left(1 - \frac{\alpha_s}{Q^2}\frac{xG}{\pi R^2}\right)$$

- Saturation of gluon density when non-linear term ≈ linear term

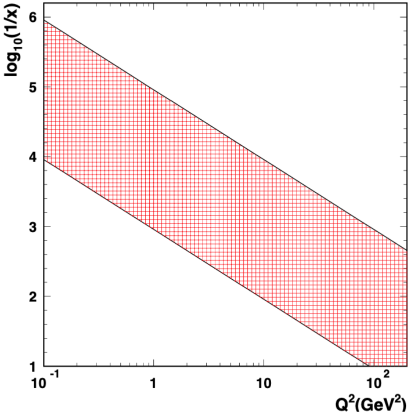
$$\frac{\alpha_s}{Q^2}\, xG(x) \approx \pi R^2 \quad => \quad \underbrace{Q_s^2(x) = \frac{\alpha_s}{\pi R^2}\, xG(x) = Q_0^2\, x^{-\lambda}}_{saturation\ \ scale}$$

Saturation scale



- Saturation scale $Q_s^2(Y)$ growth when $Y = \ln(1/x) \to 0$.
- What is the position of the saturation line?

Saturation studies at HERA



- Acceptance region at HERA excluded saturation studies in DLLA approach
- Studies for moderate values of Q^2 and $x \to 0$ are necessary

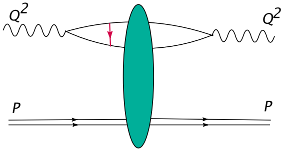
Dipole picture of DIS at small x

- k_t-factorization:

$$\sigma^{\gamma^* p} = \int \frac{d^2 k_t}{k_t^2}\, \Phi(k_t, Q^2)\, f(x, k_t)$$

- Transverse momentu is Fourier transformed: $k_t \to r_t$

$$\sigma_\lambda^{\gamma^* p}(x, Q^2) = \int d^2 r_t \int_0^1 dz \sum_f |\underbrace{\Psi_\lambda(r_t, z, Q^2, m_f)}_{photon\ \ wave\ \ function}|^2\, \hat{\sigma}(r_t, x)$$



- $q\bar{q}$ dipole of transverse size $r = |r_t|$ interacts with the dipole cross section

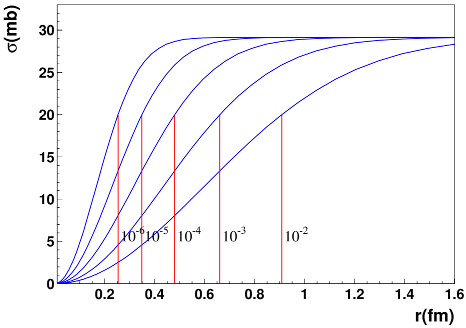
$$\hat{\sigma}(r, x) = \int \frac{d^2 k_t}{k_t^4}\, \alpha_s\, f(x, k_t)\, (1 - e^{-ik_t \cdot r_t})(1 - e^{ik_t \cdot r_t}) = \begin{cases} c\, r^2 & \text{for } r \to 0 \\ \sigma_0 & \text{for } r \to \infty \end{cases}$$

Saturation model of dipole cross section

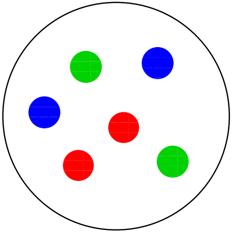
- Model with saturation scale (GB, Wüsthof 1998-99, GB, Sapeta 2018)

$$\hat{\sigma}(r,x) = \sigma_0 \left\{ 1 - \exp\left(-\frac{r^2}{4R_s^2(x)} \right) \right\}, \qquad R_s^2(x) = R_s^2\, x^{\lambda}$$

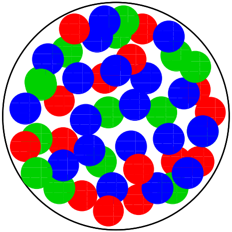
- Red parameters fitted to F_2 data with $x < 10^{-2}$.
- Saturation for $x \to 0$



Parton saturation cartoon



DIPOL



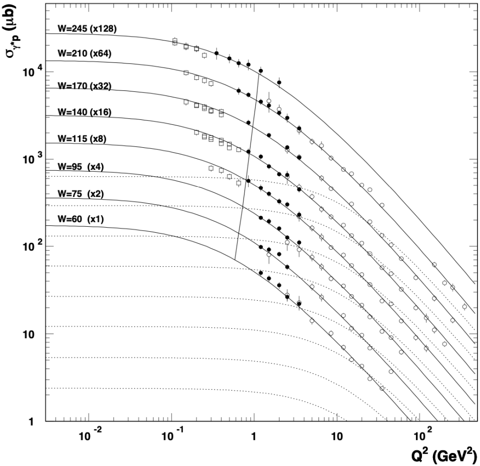
$x=10^{-2}$

$x=10^{-5}$

$r < R_s(x)$

$r > R_s(x)$

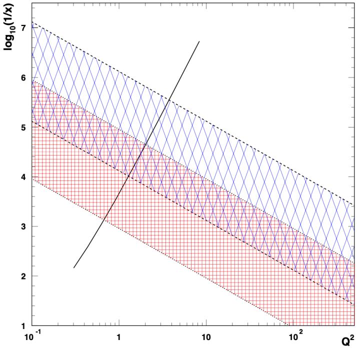
Transition to low Q^2



- Saturation line:

$$\sigma^{\gamma^* p} \sim \frac{x^{-\lambda}}{Q^2} = \frac{1}{Q^2}\left(\frac{W^2}{Q^2}\right)^{\lambda} \qquad \to \qquad \sigma^{\gamma^* p} \sim \ln\left(\frac{W^2}{Q^2}\right)$$

Saturation scale at HERA



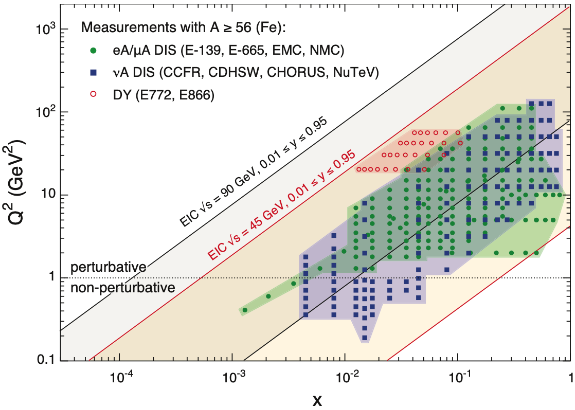
- Typical dipole size = distance between partons

$$\frac{1}{Q} = R_s(x)$$

- Saturation scale in pQCD domain at HERA with $\sqrt{s} = 318\,\mathrm{GeV}$

$$Q_s^2(x) = \frac{1}{R_s^2(x)} = 1\ \mathrm{GeV}^2 \left(\frac{3\cdot 10^{-4}}{x}\right)^{0.29}$$

Saturation scale at EIC

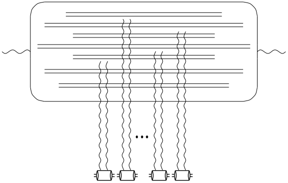


- Smaller $\sqrt{s} = 20 - 90\,\mathrm{GeV}$ is compensated by nucleus mass number $A \geqslant 56$

$$Q_s^2(x, A) = A^{1/3}\, Q_s^2(x) \sim \left(\frac{A}{x}\right)^{1/3}$$

Balitsky - Kovchegov equation

- Multiple scattering of each dipole on nucleons in a large nucleus
  (Balitsky 1995, Kovchegov 1999-2000)



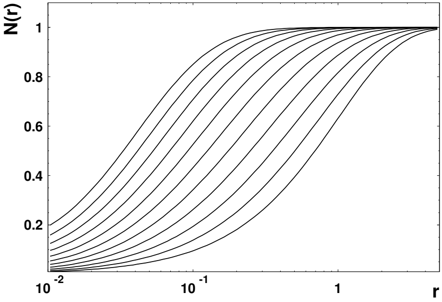
- Evolution equation for dipole scattering amplitude: $N(\vec{x},\vec{y},Y)$

$$\frac{\partial N(\vec{x},\vec{y})}{\partial Y} = \int d^2\vec{z}\, \frac{\alpha_s}{2\pi^2} \frac{(\vec{x}-\vec{y})^2}{(\vec{x}-\vec{z})^2(\vec{z}-\vec{y})^2} \left\{ \underbrace{N(\vec{x},\vec{z}) + N(\vec{z},\vec{y}) - N(\vec{x},\vec{y})}_{gives\;\; BFKL\;\; eq.} - \underbrace{N(\vec{x},\vec{z})\, N(\vec{z},\vec{y})}_{nonlinearity} \right\}$$

- Introducing $\vec{r}=(\vec{x}-\vec{y})$ and $\vec{b}=\frac{1}{2}(\vec{x}+\vec{y})$, we obtain the dipole cross section

$$\hat{\sigma}(\vec{r},Y) = \int d^2\vec{b}\, N(\vec{r},\vec{b},Y)$$

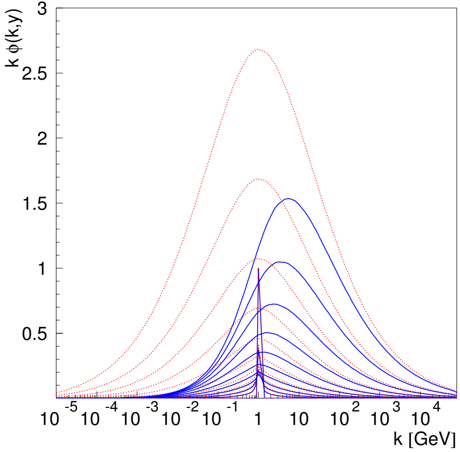
Solution to the BK equation



- Local unitarity: $0 \leqslant N(r, x) \leqslant 1$
- Saturation scale emerges from the BK equation: $Q_s^2(x) \sim x^{-4\ln 2\,\bar{\alpha}_s}$
- Problems with b-dependence - no exponential suppression
- Color Glass Condensate - effective QCD theory with JIMWLK equation
  (Mc Lerran, Venugopalan 1993-94 , Jalilian-Marian, Iancu, Mc Lerran, Weigert, Leonidov, Kovner 1997-98)

Solutions to the BK equation for gluon distribution

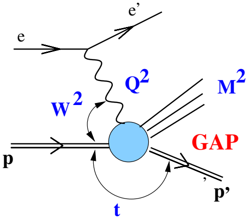
(Golec-Biernat, Motyka, Staśto, 2002)



- Power-like growth and infrared diffusion in BFKL solution cured by BK solution

Diffractive processes in DIS

- DIS with proton p' separated by large rapidity gap from diffractive system M^2



- Pomeron exchange with momentum $p_P = \xi P$ gives rapidity gap $(\xi \ll 1)$

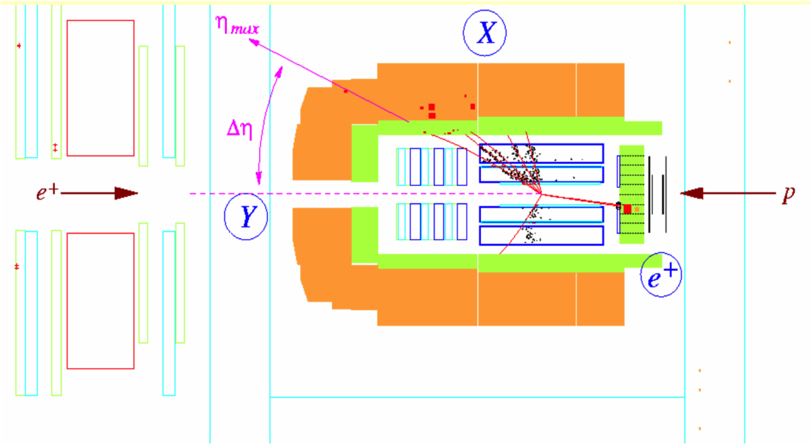
$$(q+\xi P)^2 = M^2 \quad \Rightarrow \quad \xi = \frac{Q^2+M^2}{Q^2+W^2}\,, \qquad \beta = \frac{x}{\xi} = \frac{Q^2}{Q^2+M^2}$$

- Diffractive structure functions with Pomeron PDFs and Pomeron flux

$$F_{2,L}^D(x,Q^2;\xi,t) = f_P(\xi,t)\sum_f e_f^2\,\beta\left\{q_f^P(\beta,Q^2)+\bar{q}_f^P(\beta,Q^2)\right\}$$

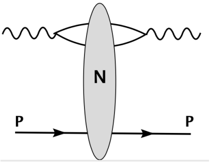
- Global fit analysis with DGLAP equations.

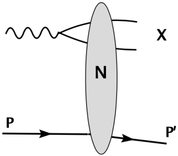
Diffractive event from ZEUS



Dipole approach to diffraction

- The same dipole scattering amplitude in inclusive and diffractive DIS





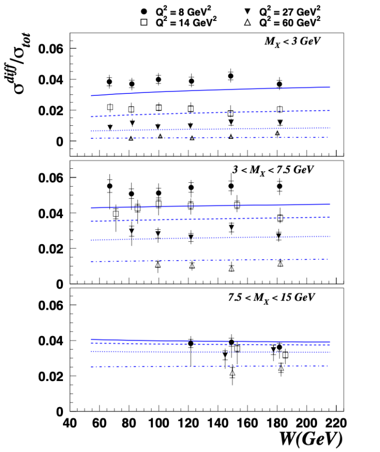
- Cross sections

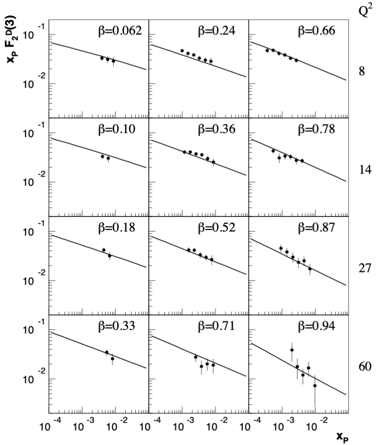
$$\sigma^{\gamma^* p} = \int dz\, d^2 r |\Psi_\gamma(r,z,Q^2)|^2\, \hat{\sigma}(r,x) \qquad \frac{d\sigma_D^{\gamma^* p}}{dt}\bigg|_{t=0} = \int dz\, d^2 r |\Psi_\gamma(r,z,Q^2)|^2\, \hat{\sigma}^2(r,x)$$

- Constant ratio with energy in the GBW dipole model

$$\frac{\sigma_D^{\gamma^* p}}{\sigma^{\gamma^* p}} \sim \frac{1}{\ln(Q^2/Q_s^2(x))}$$

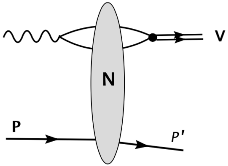
- With $(q\bar{q})_{T,L}$ and $(q\bar{q}G)_T$ diffractive components and GBW dipole cross section





(G-B, Wüsthoff 1999-2001, G-B, A. Luszczak 2008-10)

Exclusive diffractive processes



- Scattering amplitude for diffractive vector meson production $\gamma p \to V\, p'$

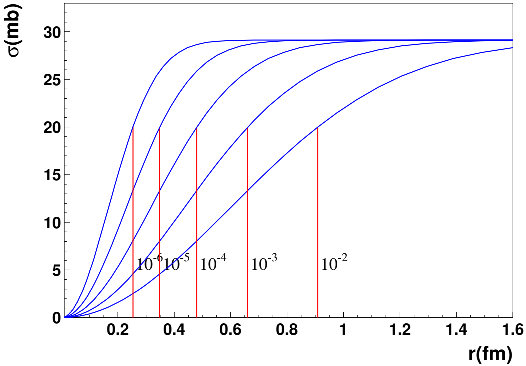
$$A_{T,L}(x, Q^2, \Delta) = i \int dz\, d^2r\, d^2b \left(\Psi_V^* \Psi_\gamma\right)_{T,L} e^{-i(b-(1-z)r)\cdot\Delta}\, N(r, b, x)$$

- Cross section

$$\left.\frac{d\sigma_{T,L}}{dt}\right|_{t=-\Delta^2} = \frac{1}{16\pi} \left|A_{T,L}\right|^2$$

- Overlap function $(\Psi_V^* \Psi_\gamma)(r)$ probes the dipole scattering amplitude $N(r, b, x)$ (Kowalski, Motyka, Watt, 2006)

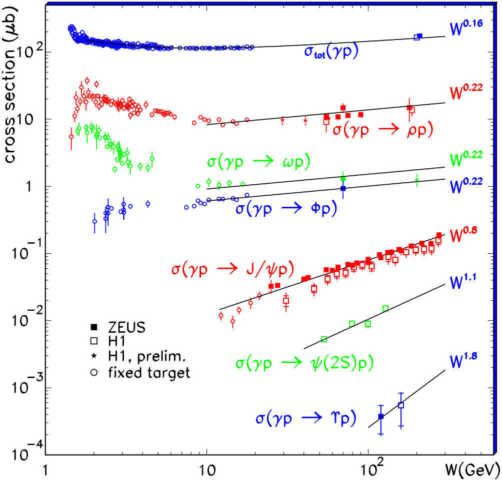
Probing different regions of r



- Small size dipoles (J/ψ) have stronger energy dependence than large ones (ρ)

$$\langle r \rangle_V = \frac{1}{\sqrt{Q^2 + M_V^2}}$$

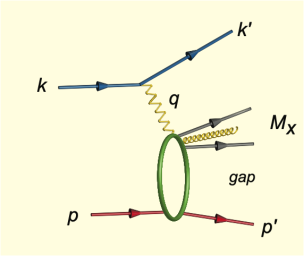
- Diffractive VM production: $\gamma p \to Vp$ where $V = \rho, \omega, \phi, J/\psi, \psi, \Upsilon$

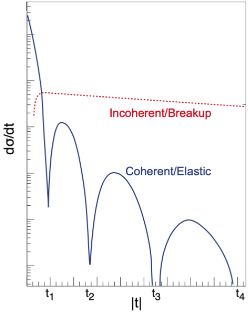


- Energy dependence due to transverse size of meson

Diffractive DIS at EIC

- Diffractive processes: 25 – 30% events in *eA*





- Saturation effects in pomeron exchange responsible for rapidity gap
- Coherent and incoherent VM production - nucleus stays intact or breaks up
- Onset of the black disc limit?

$$\frac{(\sigma_{\rm diff} + \sigma_{\rm el})}{\sigma_{\rm tot}} = \frac{1}{2}$$

Summary

- Exciting studies of QCD structure of nucleons and nuclei at the EIC
- Nuclear PDFs
- Nucleon/nucleus tomography
- Spin structure of nucleons
- Parton saturation studies