Deep inelastic scattering and EIC
Part II

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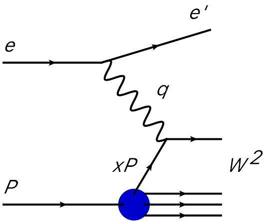
Plan

- DIS and QCD
- Evolution equations
- Unpolarized structure functions
- EIC studies



- Virtuality of the probe $(\gamma, Z^0, W^\pm)$

$$Q^2 = -q^2 = -(k_e - k_e')^2 > 0$$

- Bjorken variable

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2+W^2}$$

- Inclusive cross section for $ep \to e'X$ in which $(x, Q^2) \leftrightarrow (E_e', \theta_e')$

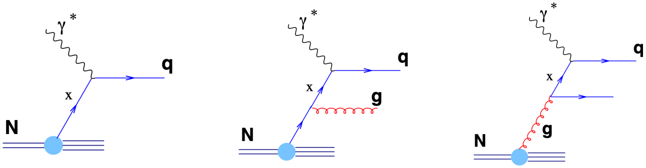
$$\frac{d\sigma}{dxdQ^2} = \frac{2\pi\alpha_{\rm em}^2}{xQ^4}\, Y_+ \left(F_2(x,Q^2) - \frac{y^2}{Y_+} F_L(x,Q^2)\right)$$

- In Feynman's parton model - Bjorken scaling

$$F_2(x) = \sum_f e_f^2\, x\,[q_f(x) + \bar{q}_f(x)]\,, \qquad F_L = 0$$

Parton model and QCD

- Partons in QCD: quarks, antiquarks and gluons



- Scale dependent parton distributions

$$q_f(x, Q^2), \qquad \bar{q}_f(x, Q^2), \qquad G(x, Q^2)$$

- Collinear factorization: coefficients functions $\otimes$ universal PDFs

$$F_2(x, Q^2) = \int_0^1 d\xi \sum_{f,\bar{f}} \left[e_f^2\, \xi\, \delta(\xi - x) + \alpha_s(Q^2) C_q(\xi, x) + \ldots \right] q_f(\xi, Q^2)$$

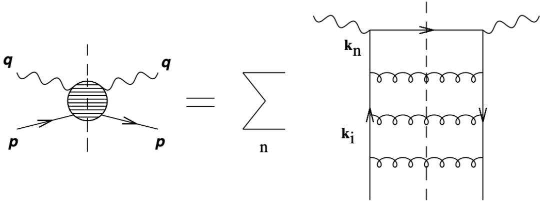
$$+ \int_0^1 d\xi \left[\alpha_s(Q^2) C_G(\xi, x) + \ldots \right] G_f(\xi, Q^2) + \underbrace{\mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)}_{higher\ \ twists}$$

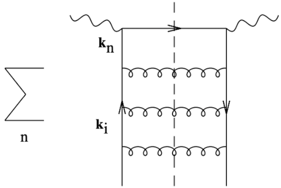
- In the lowest order in α_s in coefficient functions

$$F_2(x, Q^2) = \sum_f e_f^2\, x \left[q_f(x, Q^2) + \bar{q}_f(x, Q^2) \right], \qquad F_L = 0$$

- Q^2 dependence from evolution equations for PDFs
- From optical theorem:

$$F_2 = Q^2\, \sigma_{\gamma^* p} = \mathrm{Im}\, A(\gamma^* p \to \gamma^* p)$$





- Resummation of gluon emissions with large logarithms (LLA):

$$F_2(x, Q^2) = \sum_{n=0}^{\infty} a_n(x) \, \frac{1}{n!} \left[\alpha_s \ln \frac{Q^2}{m^2} \right]^n$$

- Ordered configuration of exchanged transverse momenta:

$$m^2 < k_{\perp 1}^2 < k_{\perp 2}^2 < \ldots < k_{\perp n}^2 < Q^2$$

gives large logarithms

$$\alpha_s^n \int_{m^2}^{Q^2} \frac{dk_{\perp n}^2}{k_{\perp n}^2} \ldots \int_{m^2}^{k_{\perp 3}^2} \frac{dk_{\perp 2}^2}{k_{\perp 2}^2} \int_{m^2}^{k_{\perp 2}^2} \frac{dk_{\perp 1}^2}{k_{\perp 1}^2} = \frac{\alpha_s^n}{n!} \left(\ln \frac{Q^2}{m^2} \right)^n$$

Mellin moments

- Mellin transformation

$$\bar{F}(N) = \int_0^1 dx\, x^{N-1} F(x)$$

- Integral convolution factorization

$$A(x) = \int_0^1 dz \int_0^1 dy\, \delta(x - yz)\, B(y) C(z) \quad \leftrightarrow \quad \tilde{A}(N) = \tilde{B}(N)\, \tilde{C}(N)$$

- Remark

$$A(x) = \int_x^1 \frac{dz}{z}\, B\left(\frac{x}{z}\right)\, C(z)$$

- In the Mellin moment N-space

$$\bar{F}_2(N,Q^2)=\sum_{n=0}^{\infty}\frac{\alpha_s^n\ln^n(Q^2/m^2)}{n!}\left(\frac{\gamma_N}{2\pi}\right)^n=\exp\left\{\frac{\alpha_s\gamma_N}{2\pi}\ln\frac{Q^2}{m^2}\right\}=\left(\frac{Q^2}{m^2}\right)^{\alpha_s\gamma_N/(2\pi)}$$

- Factoriztion with the pQCD scale $\mu_f^2\gg\Lambda_{QCD}^2$

$$\bar{F}_2(N,Q^2)=\underbrace{\left(\frac{Q^2}{\mu_f^2}\right)^{\alpha_s\gamma_N/(2\pi)}}_{coefficient\;\;function}\times\underbrace{\left(\frac{\mu_f^2}{m^2}\right)^{\alpha_s\gamma_N/(2\pi)}\times\bar{q}_0(N,m)}_{\bar{q}(N,\mu_f^2)}$$

- But $\partial\bar{F}_2/\partial\mu_f^2=0$ and from this evolution equation

$$\mu_f^2\,\frac{\partial\bar{q}(N,\mu_f^2)}{\partial\mu_f^2}=\frac{\alpha_s}{2\pi}\,\gamma_N\;\bar{q}(N,\mu_f^2)$$

- Setting $\mu_f^2=Q^2$ we obtain

$$\bar{F}_2(N,Q^2)=\bar{q}(N,Q^2)$$

Evolution as Markov process

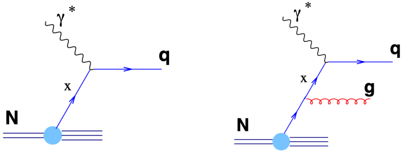
- Back to x-space (and setting $\mu_f^2 = Q^2$)

$$Q^2 \frac{\partial q(x,Q^2)}{\partial Q^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{z}\right) q(z,Q^2)$$

- Knowing PDF at Q^2 and want to know it at $Q^2 + \delta Q^2$

$$q(x,Q^2+\delta Q^2) = q(x,Q^2) + \frac{\delta Q^2}{Q^2} \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{z}\right) q(z,Q^2)$$

- Splitting function $P_{qq}(x/z)$ gives quark-to-quark transition probab.: $z \to x$

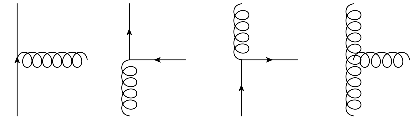


- Basis for parton showers in MC after including virtual corrections.

http://annapurna.ifj.edu.pl/∼golec/data/teaching/files/wyklady/procst.pdf

DGLAP evolution equations

- In general, more splittings are possible



- DGLAP evolution equations (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi, 1972-77)

$$\frac{\partial q_f(x,Q^2)}{\partial \ln Q^2} = P_{qq} \otimes q_f + P_{qG} \otimes G$$

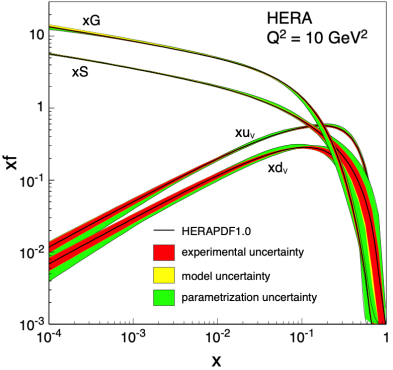
$$\frac{\partial \bar{q}_f(x,Q^2)}{\partial \ln Q^2} = P_{qq} \otimes \bar{q}_f + P_{qG} \otimes G$$

$$\frac{\partial G(x,Q^2)}{\partial \ln Q^2} = P_{Gq} \otimes \sum_f (q_f + \bar{q}_f) + P_{GG} \otimes G$$

- Initial conditions at $Q_0^2 \simeq 1 \ \mathrm{GeV}^2$:

$$q_f(x,Q_0^2), \qquad \bar{q}_f(x,Q_0^2), \qquad G(x,Q_0^2)$$

- Global fits to hard scattering data



- Valence quark distributions

$$u_v = u - \bar{u}\,, \qquad d_v = d - \bar{d}$$

- Sea quark distribution

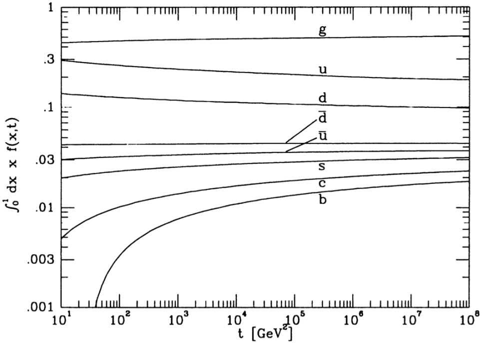
$$S = 2\,(\bar{u} + \bar{d} + \bar{s} + \ldots)$$

- Gluons and sea quarks dominate at small x
- Gluons carry half of proton's momentum - momentum sum rule

$$\underbrace{\int_0^1 dx\, x\, G(x,Q^2)}_{f_G\ \approx\ 0.5} + \underbrace{\int_0^1 dx\, x \sum\nolimits_f \left[q_f(x,Q^2) + \bar{q}_f(x,Q^2)\right]}_{f_q = 1 - f_G} = 1$$

Sum rules

- Momentum sum rule

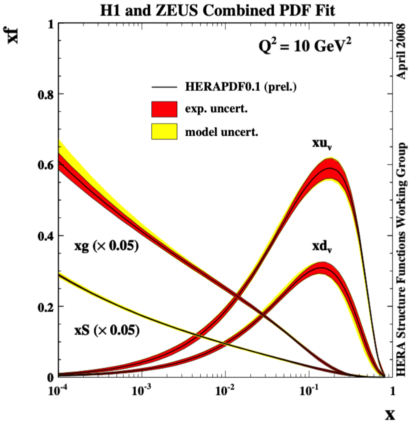


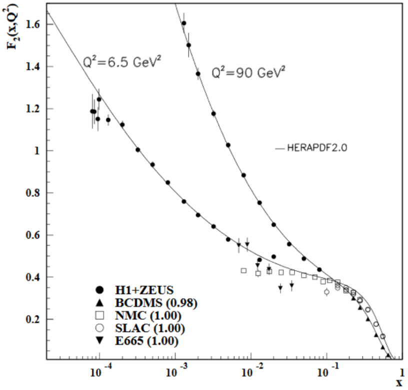
- Valence quark number sum rules, e.g. for the proton

$$\int_0^1 dx\, u_v(x, Q^2) = 2\,, \qquad \int_0^1 dx\, d_v(x, Q^2) = 1$$

- No total parton number sum rule - could be infinite.

Unpolarized structure functions





- Sea quark distribution seen for $x \to 0$: $F_2 = x\left(\frac{4}{9}u_v + \frac{1}{9}d_v + \frac{2}{3}S\right)$
- Strong dependence on Q^2 at small x - Bjorken scaling violation due to gluons

Bjorken scaling violation

- Logarithmic derivative

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} = \sum_f e_f^2\, x \left[\frac{\partial q_f(x,Q^2)}{\partial \ln Q^2} + \frac{\partial \bar{q}_f(x,Q^2)}{\partial \ln Q^2}\right]$$

- Using DGALP equations for $x \to 0$

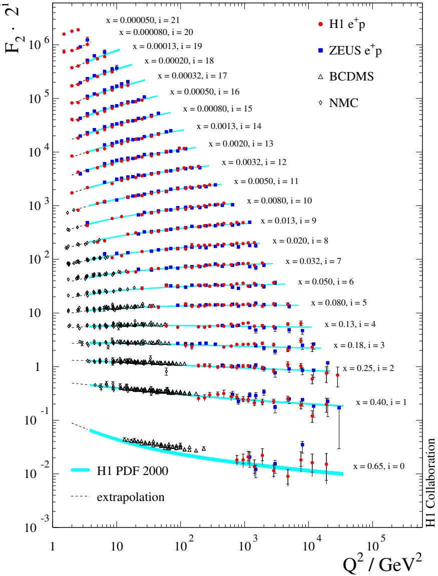
$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} \simeq 2 \sum_f e_f^2\, x \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{qG}\left(\frac{x}{z}\right) G(z,Q^2) \sim \bar{x} G(\bar{x},Q^2)\Big|_{\bar{x}\sim x}$$

where

$$P_{qG}(w) = \tfrac{1}{2}\left[w^2 + (1-w)^2\right]$$

- At small x Bjorken scaling violation is driven by gluons

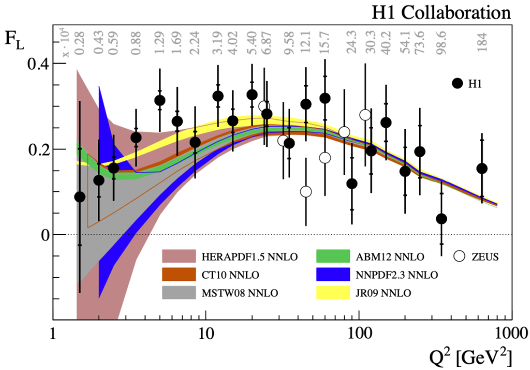
Bjorken scaling violation - the tale of gluons



Longitudinal structure function F_L

- NLO QCD collinear factorization formula

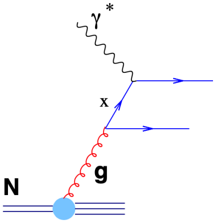
$$F_L(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dz}{z}\left[C_L^q\left(\frac{x}{z}\right)F_2^{LO}(z,Q^2) + C_L^g\left(\frac{x}{z}\right)zG(z,Q^2)\right] + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

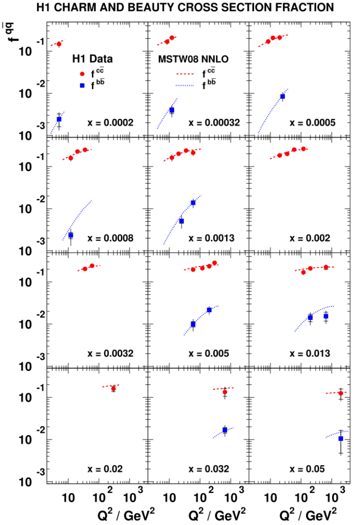


- Strong sensitivity to gluon distribution at small x. Higher twists at small Q^2?

- c, b quarks generated radiatively

$$\gamma^* g \to c\bar{c}\,,\ \ b\bar{b}$$





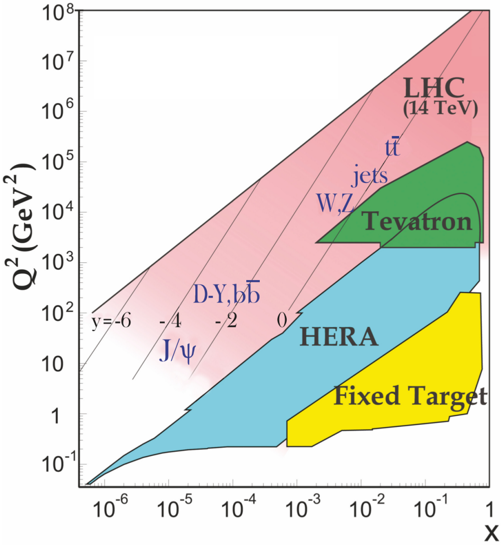
- Intrinsic charm?

- Charm contribution up to $25 - 30\%$ for small x and large Q^2.

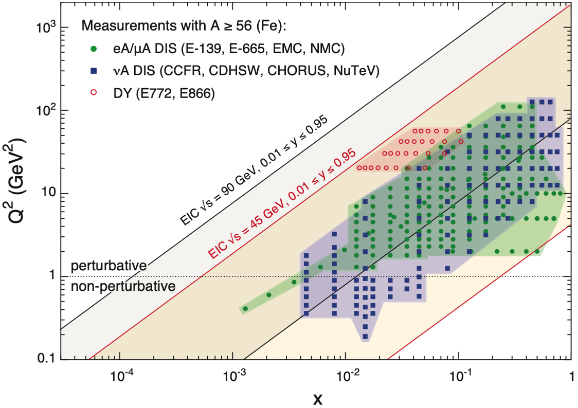
EIC studies

Kinematic plane

(PDG Book)

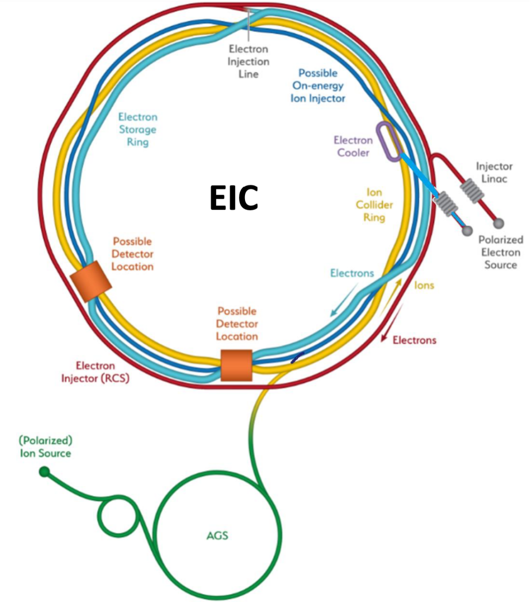


$$xyS = Q^2$$



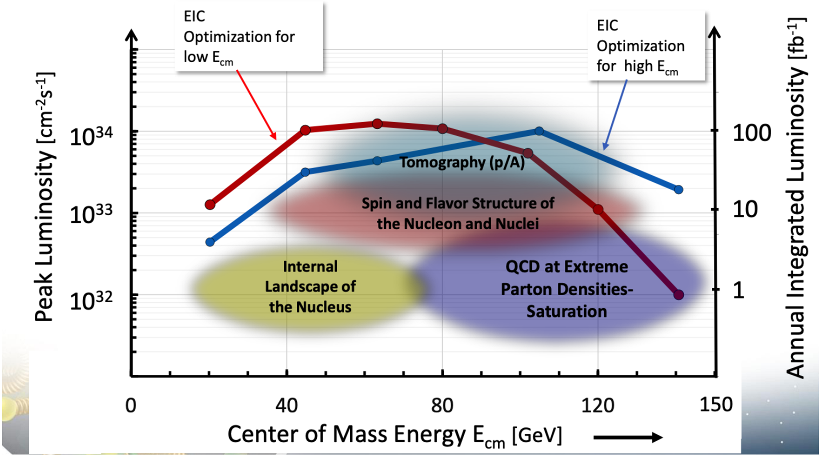
- EIC $\sqrt{S} = 20 - 140\,\mathrm{GeV}$ is smaller than HERA $\sqrt{S} = 318\,\mathrm{GeV}$
- Nuclear beams from p to Uranium - QCD structure of nuclei
- Polarized electron and hadron beams $> 70\%$ - spin physics program
- Maximum Luminosity $10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$

EIC Design

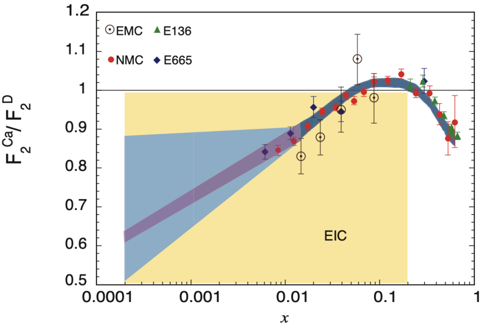


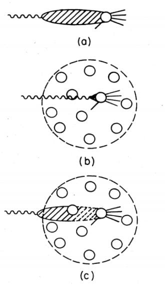
Electron
Injection
Line
Possible
On-energy
Ion Injector
Electron
Storage
Ring
Electron
Cooler
Injector
Linac
EIC
Ion
Collider
Ring
Polarized
Electron
Source
Possible
Detector
Location
Electrons
Ions
Possible
Detector
Location
Electrons
Electron
Injector (RCS)
(Polarized)
Ion Source
AGS

Luminosity versus E_{CM} center of mass energy



EMC effect

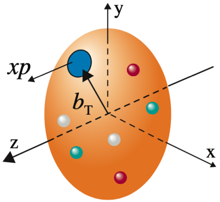




- Quark and gluon distributions in bound nucleon - nuclear PDFs
- QCD structure of nuclei is directly probed.
- In heavy ion collisions, it is probed through initial state formed in the collision.

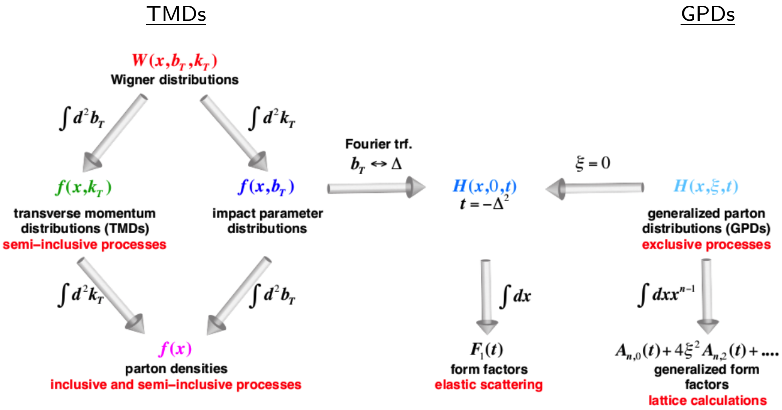
Nucleon/nucleus tomography

- PDFs: 1-dimensional parton structure in longitudinal momenta - $q(x), \bar{q}(x), G(x)$
- Multidimensional structure - Wigner function $W(x, k_T, b_T)$



- Information on transverse momentum k_T and transverse spatial b_T distributions
- Information about spin structure

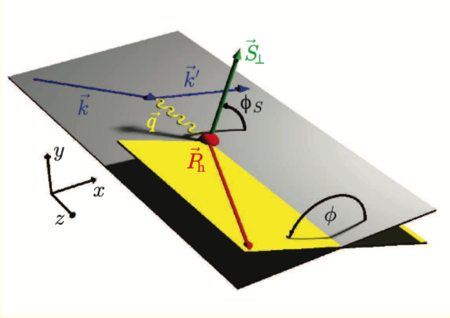
EIC perspective



- k_T-dependence through TMDs: $f(x, k_T) = \langle\, p \,|\, Partonic\ \ Operators \,|\, p \,\rangle$
- b_T-dependence through GDPs: $H(x, \xi, t = -\Delta^2) = \langle\, p \,|\, Partonic\ \ Operators \,|\, p' \,\rangle$
- Spins of partons and target come into play

SIDIS

- Semi-inclusive DIS (SIDIS): $e + N(\vec{S}) \to e' + h(\vec{P}) + X$

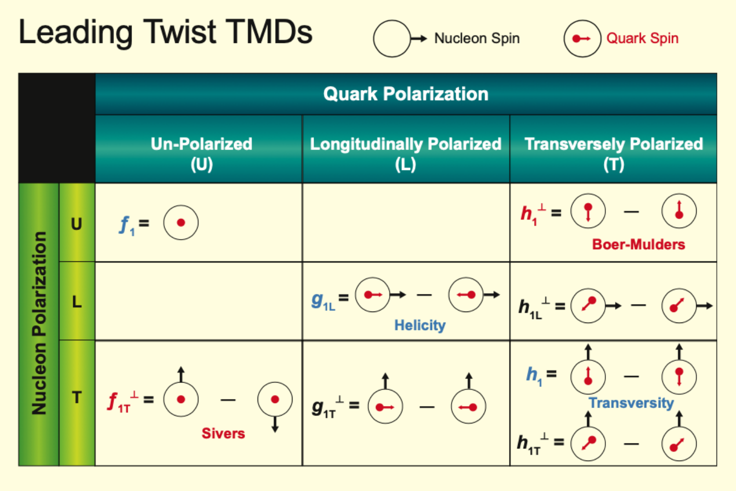


(Diehl, Sapeta, hep-ph/0503023)

- Probes 8 polarized and upolarized quark TMDs
- Access to gluon TMDs when $h = D\bar{D}$

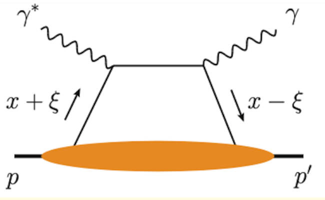
Quark TMDs

- Correlate intrinsic k_T of partons with their spin $\vec{s}$ and target spin $\vec{S}$



DVCS

- Deeply Virtual Compton Scattering (DVCS): $e + N \to e' + \gamma + N'$

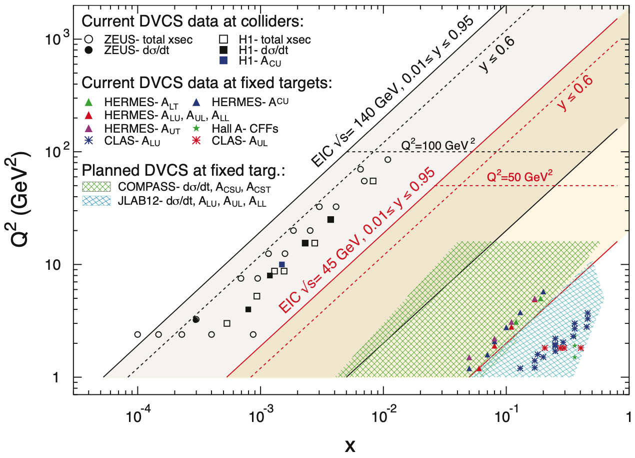


- Probes GPDs: $H^q(x, \xi, t)$ and $E^q(x, \xi, t)$
- Total angular momentum of nucleon carried by quarks

$$J^q = \frac{1}{2} \int dx\, x\, [H^q(x, \xi, 0) + E^q(x, \xi, 0)]$$

- Spin-orbit correlations of quarks and gluons in nucleon

DVCS studies at EIC



- Bridging the gap. More high precision data.

Nucleon spin studies

- Longitudinal spin of the nucleon - Jaffe-Manohar sum rule:

$$\frac{1}{2} = S_q + L_q + S_G + L_G$$

- Polarized parton distributions: $\Delta f(x, Q^2) = f^+(x, Q^2) - f^-(x, Q^2)$

$$S_q = \frac{1}{2}\int_0^1 dx\left[\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \ldots\right], \qquad S_G = \int_0^1 dx\,\Delta G$$

- g_1 structure function:

$$\frac{1}{2}\left[\frac{d\sigma^{\leftrightarrows}}{dxdQ^2} - \frac{d\sigma^{\rightrightarrows}}{dxdQ^2}\right] \simeq \frac{4\pi\alpha_{\rm em}^2}{Q^4}\,y(2-y)\,g_1(x, Q^2)$$

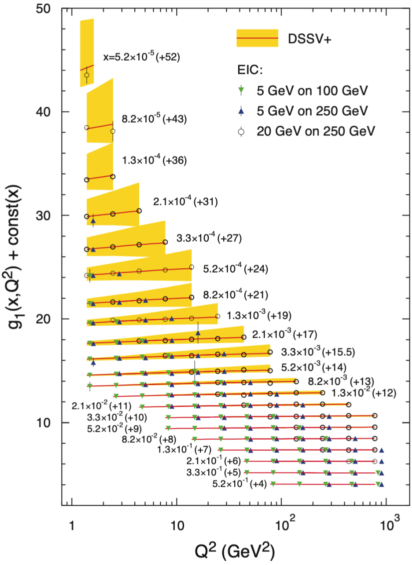
- Parton model relation:

$$g_1(x, Q^2) = \frac{1}{2}\sum_f e_f^2\left[\Delta q_f(x, Q^2) + \Delta \bar{q}_f(x, Q^2) - \frac{\alpha_s}{2\pi}\Delta G(x, Q^2)\right]$$

- Sum rule:

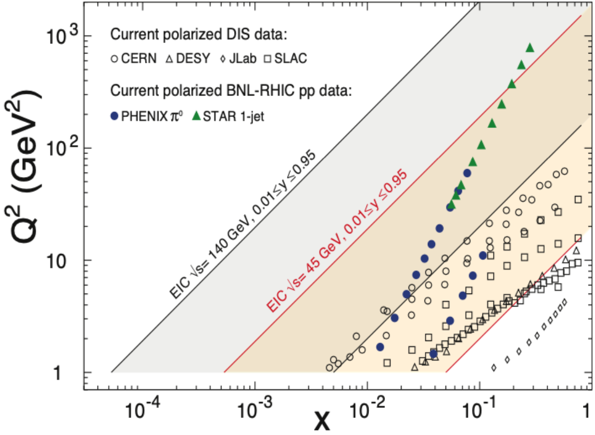
$$\int_0^1 dx\, g_1(x, Q^2) = S_q - \frac{\alpha_s}{4\pi} S_G$$

Simulated g_1 at EIC energies



- Scaling violation due to polarized gluon distribution $\Delta G(x, Q^2)$

Spin physics at EIC



- EIC will open new opportunities for spin physics studies

Summary

- EIC opens new opportunities to study partonic structure of nucleons and nuclei:
- Nuclear PDFS
- Nucleon/nucleus tomography
- Spin physics
- Parton saturation studies - pending

Plan for Part III

- Small x limit of DIS
- Parton saturation
- Diffractive processes