Deep inelastic scattering and EIC
Part I

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Plan

Part I

- Basics of DIS
- Quantum chromodynamics

Part II

- DIS and QCD
- Evolution equations
- Unpolarized structure functions
- EIC studies

Part III

- Small x limit of DIS
- Parton saturation
- Diffractive processes

Basic books and prerequisites

- F. Halzen, A. D. Martin, *Quarks and Leptons*, JOHN WILEY and SONS 1984
- R. G. Roberts, *The Structure of the Proton*, Cambridge University Press 1990
- R. K. Ellis, W.J. Stirling, B. R. Webber, *QCD and Collider Physics*, Cambridge University Press 1996
- John Collins, *Foundation of Perturbative QCD*, Cambridge University Press 2011
- K. Golec-Biernat, *Habilitation thesis, 2001*
  http://annapurna.ifj.edu.pl/~golec/data/teaching/files/wyklady/hab.pdf

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- Four-momentum: $p^\mu = (E, p_x, p_y, p_z)$

$$p^2 = E^2 - p_x^2 - p_y^2 - p_z^2 = m^2 \qquad (\geqslant 0 \ \text{ or } \ < 0)$$

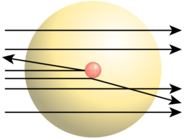
- Four-momenta: $p^\mu = (E_p, p_x, p_y, p_z)$ and $q^\mu = (E_q, q_x, q_y, q_z)$

$$p \cdot q = g_{\mu\nu} p^\mu q^\nu = E_p E_q - p_x q_x - p_y q_y - p_z q_z$$

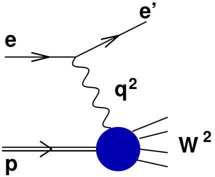
Basics of DIS

Basic idea of DIS

- Rutherford 1912 - scattering of ^{4}He nuclei on atoms of ^{79}Au



- SLAC 1967-69 - scattering of leptons on nucleons



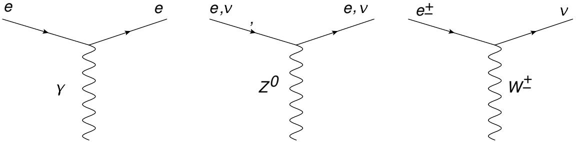
- Photon virtuality: $Q^2 \equiv -q^2 = -(k_e - k_{e'})^2 \;>\; 0$
- Invariant mass squared: $W^2 \equiv (P+q)^2 = M_X^2 \;\geqslant\; m_p^2 \simeq 1\ \mathrm{GeV}^2$
- Deep Inelastic Scattering (DIS): $Q^2, W^2 \;>\; m_p^2$

Electroweak and strong interactions

- Scattering amplitude:

$$A(l\,N \to l'X) = \underbrace{\bar{u}_{l'}(k',\lambda)\,\Gamma^{\mu}\,u_l(k,\lambda)}_{leptonic\ \ current}\,\frac{ig_{\mu\nu}}{Q^2+m_B^2}\,\underbrace{\langle\, X\,|\,J^{\nu}(0)|\,P,s\,\rangle}_{hadronic\ \ ME}$$

- Leptonic current: neutral currents (NC) + charged currents (CC)



$$\Gamma^{\mu} = \qquad e\,\gamma^{\mu} \qquad\qquad e\,\gamma^{\mu}(v_l - a_l\,\gamma_5) \qquad\qquad g\,\gamma^{\mu}(1-\gamma_5)$$

- Electromagnetic interactions: P-parity conserving, $m_{\gamma} = 0$
- Weak interactions: P-parity violation, $m_Z = 91.19\,\mathrm{GeV}$, $m_{W^{\pm}} = 80.38\,\mathrm{GeV}$
- Strong interaction in hadronic ME
- λ, s are particle polarizations

Leptonic and hadronic tensor

- Cross section for $lN \to l'X$ unpolarized DIS with γ exchange only

$$\frac{d\sigma}{dE_e'\, d\Omega_e'} \sim \frac{1}{(Q^2)^2}\, L^{\mu\nu}(k,k')\, W_{\mu\nu}(q,P)$$

- Leptonic tensor for $m_l = 0$

$$L^{\mu\nu}(k,k') = \left[k^\mu k'^{\,\nu} + k^\mu k'^{\,\nu} - g^{\mu\nu}(k\cdot k')\right] \qquad (+\, i\lambda\, \epsilon_{\mu\nu\alpha\beta}\, k^\alpha k'^\beta)$$

- Hadronic tensor

$$W_{\mu\nu}(q,P) = \sum_s \int d^4x\, e^{iq\cdot x} \langle\, P, s|\, J_\mu(x)\, J_\nu(0)\, |P, s\,\rangle$$

- Current conservation and P-parity conservation

$$q^\mu W_{\mu\nu} = 0 \qquad\qquad W_{\mu\nu} = W_{\nu\mu}$$

Structure functions

- Having q_μ, P_ν and $g_{\mu\nu}$ at the disposal:

$$W_{\mu\nu} = -\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right) F_1 + \frac{1}{P\cdot q}\left(p_\mu + q_\mu \frac{P\cdot q}{Q^2}\right)\left(p_\nu + q_\nu \frac{P\cdot q}{Q^2}\right) F_2$$

- Two unknown (scalar) dimesionless structure functions

$$F_{1,2} = F_{1,2}(Q^2, W^2, m_p^2)$$

- Weak boson exchanges: P-parity violating tensor $\epsilon_{\mu\nu\alpha\beta}$ at the disposal

$$W_{\mu\nu} \;\to\; W_{\mu\nu} + i\epsilon_{\mu\nu\alpha\beta}\frac{P^\alpha q^\beta}{P\cdot q} F_3$$

- Reduced cross section with three structure functions, e.g. for NC

$$\frac{1}{\sigma_0}\frac{d\sigma^{NC}}{dE_e'\,d\Omega_e'}(e^\pm p) \;=\; \tilde{F}_2 - \frac{y^2}{Y_+}\tilde{F}_L \mp \frac{xY_-}{Y_+}\tilde{F}_3 \qquad (1)$$

where

$$0 \leqslant x, y \leqslant 1\,, \qquad Y_\pm = 1 \pm (1-y)^2\,, \qquad \tilde{F}_L = \tilde{F}_2 - 2x\tilde{F}_1$$

Kinematic variables in DIS

- Bjorken variable x and inelacticity y are Lorentz invariants:

$$x = \frac{Q^2}{2P\cdot q} \overset{\text{Rest}}{=} \frac{Q^2}{2m_p E_\gamma} \geqslant 0\,, \qquad\qquad y = \frac{P\cdot q}{P\cdot k_e} \overset{\text{Rest}}{=} \frac{E_\gamma}{E_e} = \left(1 - \frac{E_e'}{E_e}\right) \in [0,1]$$

- Bound $0 \leqslant x, y \leqslant 1$ justified:

$$(P+q)^2 \geqslant m_p^2 \quad => \quad (q^2 + 2P\cdot q) \geqslant 0 \quad => \quad \frac{-q^2}{2P\cdot q} \leqslant 1$$

- For $S = (P+k_e)^2 \gg m_p^2$

$$x\,y\,S = Q^2 \tag{2}$$

- By measuring scattered electron energy E_e' and scattering angle θ_e':

$$Q^2 = 2E_e E_e'(1-\cos\theta_e')\,, \qquad\qquad x = \frac{E_e'}{E_p}\left[\frac{1-\cos\theta_e'}{2-(E_e'/E_e)(1+\cos\theta_e')}\right]$$

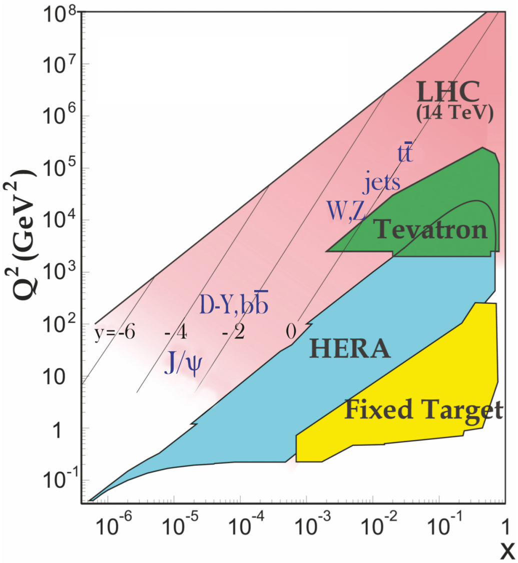
- Equivalent descriptions of DIS

$$(E_e', \theta_e') \;\leftrightarrow\; (x, Q^2) \;\leftrightarrow\; (y, Q^2) \;\leftrightarrow\; (x, y)$$

- For $\theta_e' \to 0$ we have $Q^2 \to 0$ and/or $x \to 0$ (photoproduction and small-x limit).

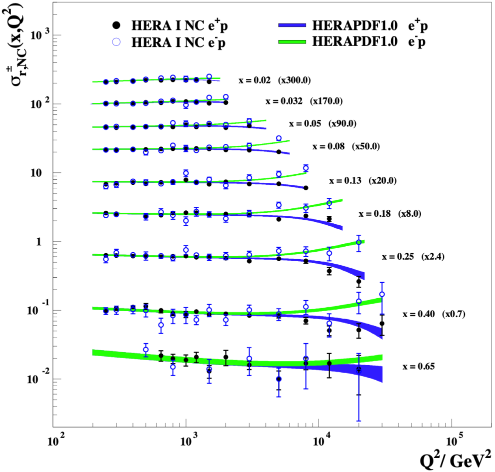
Kinematic plane

(PDG Book)



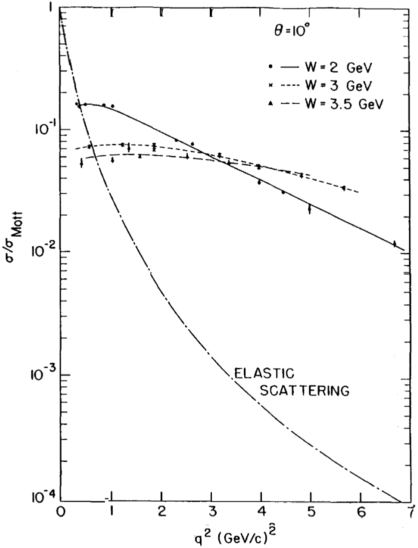
Neutral current data with (1)

H1 and ZEUS Collaborations, JHEP 01 (2010) 109



- For $Q^2 \ll m_Z^2$ only γ exchange: $F_2(x = const, Q^2)$ is almost flat
- Bjorken scaling in Bjorken limit: $x = const$ and $Q^2 \to \infty$

DIS versus elastic scattering



- Elastic scattering $W = M_X = m_p$:

$$\frac{\sigma}{\sigma_{Mott}} = \frac{1}{\left(1 + \frac{Q^2}{0.71\,GeV^2}\right)^4}$$

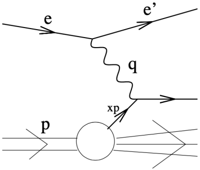
- Mean charge radius

$$\sqrt{\langle r^2 \rangle} \approx 0.8 \cdot 10^{-15} m$$

- No scale in DIS.
- Scattering on point-like object?

Parton model - "Bohr model for QCD"

- In infinite momentum frame, proton is a collection of free partons.
- Scattering on a point-like parton (Feynman, 1968-69)



- Parton carries proton momentum fraction ξ = Bjorken variable x

$$(\xi P + q)^2 = 0 \quad \rightarrow \quad \xi = \frac{Q^2}{2P\cdot q} = x = \frac{Q^2}{Q^2 + W^2 - m_p^2}$$

- Bjorken scaling in structure functions:

$$F_2(x) = \int_0^1 d\xi \sum_f \left[e_f^2\, \xi\, \delta(\xi - x) \right] q_f(\xi) = \sum_f e_f^2\, x\, \{q_f(x) + \bar{q}_f(x)\}$$

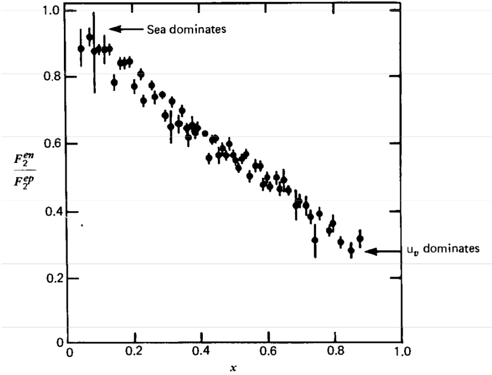
where $q_f(x)$ are parton distributions. **Callan-Gross relation**:

$$\text{partons have spin } 1/2 \quad \rightarrow \quad F_2 - 2xF_1 = F_L = 0$$

Feynman's partons = Gell-Mann's quarks

- DIS on protons and neutrons. From izospin symmetry:

$$\frac{1}{x}F_2^{ep} = \left[\frac{4}{9}u_v + \frac{1}{9}d_v\right] + \frac{2}{3}S \qquad \frac{1}{x}F_2^{en} = \left[\frac{1}{9}u_v + \frac{4}{9}d_v\right] + \frac{2}{3}S$$



- Ratio: $$1/4 < F_2^{en}/F_2^{ep} < 1$$

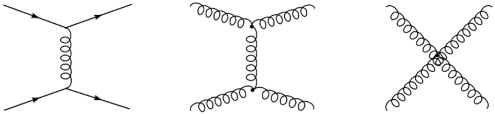
Nobel Prize 1990

Jerome I. Friedman, Henry W. Kendall, Richard E. Taylor

"For pioneering investigations concerning deep inelastic scattering of electrons on protons which have been essential for the development of the quark model."

QCD

- Parton model - proton is an incoherent assembly of free partons
- QCD - $SU(3)_c$ gauge field theory of colored spin 1/2 quarks and spin 1 gluons



- Asymptotic freedom for $Q^2 \to \infty$ (Gross, Politzer, Wilczek, 1973)

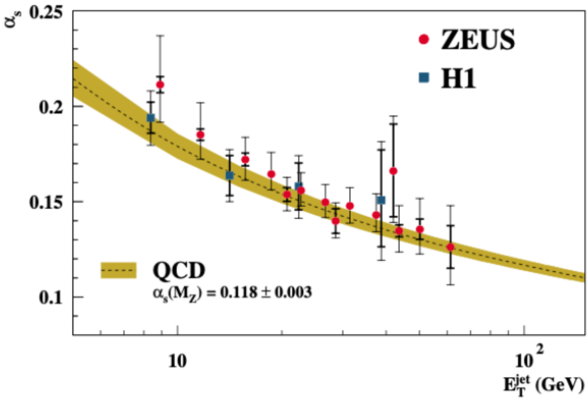
$$\alpha_s(Q^2) = \frac{g_s^2}{4\pi} = \frac{12\pi}{(33 - 2n_f)\log(Q^2/\Lambda^2_{QCD})} \;\to\; 0$$

where $\Lambda_{QCD} \simeq 200 - 400 \text{ MeV}$

- Confinement for $Q^2 \to 0$ (no quarks and gluons in final states - hadronization).

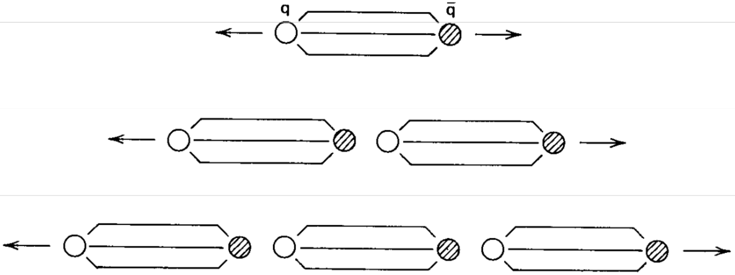
Asymptotic freedom

- $\alpha_s(Q)$ from jet production cross section: $Q = E_T^{\rm jet}$



- $\alpha_s(M_Z) = 0.118 \pm 0.003$ versus $\alpha_{\rm em}(M_Z) \approx 1/128 \approx 0.008$

String model of hadronization



- Confinement - Millenium problem (1 mln USD)
- Hadronization in Monte Carlo programs

Plan for Part II

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- Evolution equations
- Unpolarized structure functions
- EIC studies